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Observational signatures of disc and jet misalignment in images of accreting black holes


1 Anton Pannekoek Institute for Astronomy, University of Amsterdam, Science Park 904, NL-1098 XH Amsterdam, the Netherlands
2 Mullard Space Science Laboratory, University College London, Holmbury St. Mary, Dorking, Surrey RH5 6NT, UK
3 Institut für Theoretische Physik, Goethe-Universität Frankfurt, Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany
4 Institute for Theory and Computation, Harvard University, 60 Garden Street, Cambridge, MA 02138, USA
5 Center for Interdisciplinary Exploration & Research in Astrophysics (CIERA), Physics & Astronomy, Northwestern University, Evanston, IL 60202, USA
6 Gravitation Astroparticle Physics Amsterdam (GRAPPA) Institute, University of Amsterdam, Science Park 904, NL-1098 XH Amsterdam, the Netherlands
7 Astrophysics, Department of Physics, University of Oxford, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH, UK

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ABSTRACT
Black hole (BH) accretion is one of nature’s most efficient energy extraction processes. When gas falls in, a significant fraction of its gravitational binding energy is either converted into radiation or flows outwards in the form of BH-driven jets and disc-driven winds. Recently, the Event Horizon Telescope (EHT), an Earth-sized submillimetre radio interferometer, captured the first images of M87’s BH. These images were analysed and interpreted using general relativistic magnetohydrodynamics (GRMHD) models of accretion discs with rotation axes aligned with the BH spin axis. However, since infalling gas is often insensitive to the BH spin direction, misalignment between accretion disc and BH spin may be a common occurrence in nature. In this work, we use the general relativistic radiative transfer code BHOSS to calculate the first synthetic radio images of (highly) tilted disc/jet models generated by our GPU-accelerated GRMHD code H-AMR. While the tilt does not have a noticeable effect on the system dynamics beyond a few tens of gravitational radii from the BH, the warping of the disc and jet can imprint observable signatures in EHT images on smaller scales. Comparing the images from our GRMHD models to the 43 and 230 GHz EHT images of M87, we find that M87 may feature a tilted disc/jet system. Further, tilted discs and jets display significant time variability in the 230 GHz flux that can be further tested by longer-duration EHT observations of M87.

Key words: accretion, accretion discs – black hole physics – MHD – methods: numerical – galaxies: active – galaxies: jets.

1 INTRODUCTION
There is observational evidence for misalignment between the accretion disc and black hole (BH) spin axis in both active galactic nuclei (AGNs) and BH X-ray binaries (XRBs; e.g. Hjellming & Rupen 1995; Greene, Bailyn & Orosz 2001; Maccarone 2002; Caproni et al. 2006; van den Eijnden et al. 2017; Russell et al. 2019), and tidal disruption events (e.g. Pasham et al. 2019; Shea & Akiyama 2020), imaging the near-horizon region (i.e. r ≲ 20 r g , where r g ≡ GM/c 2 is the gravitational radius of the BH, M is its mass, G is the gravitational constant, and c is the speed of light) for SMBHs has become a reality, making it possible to directly test for misalignment in Sagittarius A* (Sgr A*) and M87. In fact, Park et al. (2019) briefly discussed possible misalignment in M87.

Misalignment brings about important changes in the dynamics of the system via general relativistic (GR) frame dragging, which induces nodal Lense–Thirring precession (LT; Lense & Thirring 1918) of test particles on tilted orbits around the central object, with a radially dependent angular frequency ΩLT ∝ 1/r 3. Growing interest in the physics of accretion under the effects of LT precession led to modelling tilted accretion discs via general relativistic magnetohydrodynamics (GRMHD) simulations of both thick (e.g. Fragile & Anninos 2005; Fragile et al. 2007; McKinney, Tchekhovskoy & Blandford 2013; Polko & McKinney 2017; Liska et al. 2018; White,

* E-mail: k.chatterjee@uva.nl
† John Harvard Distinguished Science and ITC Fellow.

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Quataert & Blaes 2019) and thin (e.g. Liska et al. 2019b) tilted discs, some of which carry promising indications for the origin of specific kinds of QPOs (e.g. Liska et al. 2020). Of course, the absence of QPOs due to disc precession does not rule out misalignment, since tilted geometrically thick discs tend to have extremely long precession periods, resulting in quasi-stationary disc warps. Due to GR warping of the disc via pressure waves (Papaloizou & Lin 1995; Ivanov & Illarionov 1997; Lubow & Ogilvie 2000), Dexter & Fragile (2011) showed, via general relativistic radiative transfer (GRRT) of GRMHD simulations, that tilting the disc brings about significant changes in the appearance of the inner 20 $r_g$ around a BH, e.g. when scaled to the mass and distance of Sgr A* (Dexter & Fragile 2013). GRMHD simulations found that tilted discs accrete on to the BH via two high-density plunging streams of infalling material (e.g. Fragile & Blaes 2008; Liska et al. 2018). Further, it has been shown that the narrow and highly warped morphology of these plunging streams can lead to the accumulation of gas near the point of highest disc tilt, developing a pair of standing shocks (Fragile & Blaes 2008; Generozov et al. 2013; White et al. 2019). Consequently, higher inflow temperatures resulting from shock heating dominate the emission (e.g. Dexter & Fragile 2013; White et al. 2020).

The magnetic field strength plays a vital role in a tilted disc system because magnetic fields provide an extra torque to the GR warping, and hence help in aligning the disc. In particular, Liska et al. (2018) found that for tilted precessing thick discs, the higher the disc magnetic field strength, the stronger the jets are and the more they tend to push the inner parts of the disc to align more closely with the BH spin axis. This can be understood because when the magnetic flux on to the BH is large enough to hinder accretion from the disc (Narayan, Igumenshchev & Abramowicz 2003), known as the magnetically arrested disc (MAD) condition, the associated jet efficiently extracts the BH’s rotational energy (Tchekhovskoy & McKinney 2012) and becomes powerful enough to force the inner part of the disc to align with the BH spin axis (McKinney et al. 2013), while the large-scale jet remains aligned with the disc. Further, it was shown that in the absence of magnetic fields, the disc alignment is much weaker as compared to the MHD case (Sorathia, Krolik & Hawley 2013). Strongly magnetized jets can dictate dynamics along with jet-disc interactions, potentially developing shocks similar in nature to the pair of standing shocks present in the plunging streams. Misalignment can therefore introduce potential degeneracies in observed images and spectra, warranting further exploration so that we are able to more clearly interpret current and future EHT horizon-scale images. However, to date, images illustrating the warping of misaligned disc/jet systems (see also White et al. 2020 for misaligned disc images) and corresponding observational indications of tilt in spectra have not been studied and compared to observed images (e.g. EHTC 2019b).

In this work, we explore, for the first time, a variety of initial misalignment angles for a BH disc/jet system in high resolution using our GPU-accelerated GRMHD code h-AMR (Liska et al. 2019a). We further calculate observable images using the GRRT code BHOSS (Younsi et al. 2016, 2020). In Section 2, we give an overview of our methodology and simulation set-up. We present our results in Section 3. In Section 4, we compare our images to the recently published EHT image of M87 (EHTC 2019a). We conclude in Section 5.

### 2 METHODOLOGY AND NUMERICAL SET-UP

We use our state-of-the-art massively parallel, GPU-accelerated 3D GRMHD code h-AMR (Liska et al. 2018, 2019a; Chatterjee et al. 2019) to solve the GRMHD equations in a fixed Kerr space–time.

#### Table 1

<table>
<thead>
<tr>
<th>Model</th>
<th>$a$</th>
<th>Resolution</th>
<th>$r_{in}$ (r$_g$)</th>
<th>r$_{max}$ (r$_g$)</th>
<th>r$_{out}$ (r$_g$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T0</td>
<td>0</td>
<td>448 x 144 x 240</td>
<td>12.5</td>
<td>25</td>
<td>10$^5$</td>
</tr>
<tr>
<td>T30</td>
<td>30</td>
<td>18, 26, 78</td>
<td>45</td>
<td>22.7, 28.4</td>
<td>1.19</td>
</tr>
<tr>
<td>T60</td>
<td>60</td>
<td>13, 28, 45</td>
<td>27.8</td>
<td>43.4, 42.2</td>
<td>1.48</td>
</tr>
</tbody>
</table>

The H-AMR section in Porth et al. (2019) presents a description of the code attributes, such as adaptive mesh refinement (AMR), local adaptive time-stepping, and a staggered mesh for the magnetic field evolution, as well as comparisons to benchmark results for a standard accretion disc problem. We adopt the geometrical unit convention, $G = c = 1$, and further normalize the BH mass to $M = 1$, thereby normalizing the length-scale to the gravitational radius $r_g$. We carry out the simulations in logarithmic Kerr–Schild coordinates with a numerical resolution $N_r x N_{\phi} x N_{\theta}$ of 448 x 144 x 240 (Table 1), sufficient to resolve the magnetorotational instability (MRI; Balbus & Hawley 1991) in the disc. Our grid is axisymmetric and uniform in log ($r_{in}$), extending from 0.75 $r_{in}$ to $10^5 r_g$, where the event horizon radius $r_{out} = r_g (1 + \sqrt{1 - a^2})$, with $a = 0.9375$. A further description of the grid is given in Liska et al. (2018). We use outflowing boundary conditions (BCs) at the inner and outer boundaries; transmissive polar BCs in the $\theta$-direction and periodic BCs in the $r$-direction. To quantify the MRI resolution, we calculate the quality factors $Q_{r, \theta, \phi}$, where $Q_r = (2\pi r_{in}/u_0) / (\Delta r) u_0$ measures the number of cells per MRI wavelength in direction $i = [r, \theta, \phi]$, volume-averaged over the disc ($r < 150 r_g$) with weight $w = r$, the gas density (see equations 18–20 in Liska et al. 2019b). Here, $u_0$ is the Alfvén velocity, $\Delta r$ the cell size, and $\Omega$ the angular velocity of the fluid. We achieve $Q_r > 10$ (Table 1) during our chosen time period, fulfilling the numerical convergence criteria (see e.g. Hawley, Guan & Krolik 2011).

In all models, we start with a standard Fishbone & Moncrief (1976) torus in hydrostatic equilibrium around the central (spinning) Kerr BH. The torus inner edge is located at $r_{in} = 12.5 r_g$ and the gas pressure ($p_g$) maximum is at $r_{max} = 25 r_g$, with the ideal gas law adiabatic index set to $\Gamma = 5/3$ (i.e. non-relativistic). The non-zero magnetic field vector potential is given by $A_\phi \propto r^2 (p_g)^{3/2}$ and normalized to max ($p_g$) max ($p$) = 100, where $p_g = (\Gamma - 1)/u_0$ is the gas pressure, $p_g \equiv b^2/2$ is the magnetic pressure, and $u_0$ is the fluid internal energy. Furthermore, the magnetic field 4-vector, $b^4$, is defined in Lorentz–Heaviside units where a factor of $1/\sqrt{4\pi}$ is absorbed into its definition. Note that our model parameters are slightly different from the tilted disc models used in Fragile et al. (2007), where the authors use $r_{in} = 15 r_g$ and $r_{max} = 25 r_g$, an adiabatic index of $\Gamma = 5/3$, and vector potential $A_\phi \propto r$, resulting in a smaller magnetic flux content in the torus. We also note that White et al. (2019) use similar parameters to Fragile et al. (2007), but with $\Gamma = 4/3$. We tilt our discs at three different angles: $\theta_{tilt} = 0^\circ$ (model T0), 30°
illustrating the misalignment of both the high-density disc and the low-density jet with respect to the BH spin direction, which points upwards in this figure. Each BH system is shown in the plane of the BH spin and the jet angular momentum vector, or, in other words, the $\varphi = 0^\circ$ plane rotated by the corresponding jet precession angle $\omega_j$ (see Table 1). Fig. 1 also delineates the jet funnel (dark blue in the figure), given by $b^2/\rho c^2 = 1$ (red line) and the unbound material given by the Bernoulli parameter $Be := -\dot{m}_b = 1.02$ (black line), with the latter being taken as a proxy for the disc–wind region (light blue–green region). In the definition of $Be$, $h$ and $u_\ell$ are the enthalpy and the temporal component of the covariant velocity (interpreted as the conserved particle energy), respectively. Over time, in all three models the accretion disc develops turbulence via the MRI, leading to gas accretion on to the BH (Fig. 2a) in the form of plunging streams (Fragile et al. 2007; Liska et al. 2018). Figs 2(b) and (c) show that our chosen initial field configuration evolves to create a near-MAD disc in each case, with the dimensionless magnetic flux through the horizon $\phi_{BH} = \Phi_{BH}/(M c^3)^{1/2} \approx \phi_{max} \approx 50$ (Tchekhovskoy, Narayan & McKinney 2011), producing highly efficient jets with $P_{outflow}/(\langle (\perp c)^2 \rangle \sim 1$, powered by BH rotational energy extracted via the Blandford–Znajek mechanism (Blandford & Znajek 1977).

Here, we make use of the following definitions. The BH accretion rate (positive for inflow of gas towards the BH):

$$M := -\int \rho u^r \mathrm{d}A_{\phi \psi},$$

the magnetic flux at the event horizon:

$$\Phi_{BH} := \frac{1}{2} \int |B_r| \mathrm{d}A_{\phi \psi},$$

and the outflow power:

$$P_{outflow} := Mc^2 - \dot{E},$$

where the energy accretion rate is defined as $\dot{E} = \int \int T_r^r \mathrm{d}A_{\phi \psi}$ (taken to be positive for inflow of energy towards the BH), $u^r$ and $B^r$ are the radial velocity and magnetic field, respectively, $T_r^r$ is the total radial energy flux, $\mathrm{d}A_{\phi \psi} = \sqrt{-g} \, \mathrm{d}\theta \, \mathrm{d}\varphi$ is the surface area element, and $g \equiv |g_{\mu\nu}|$ is the metric determinant.

Figs 2(d) and (e) show the disc/jet tilt and precession angles, spatially averaged over [50, 150] $r_g$, and demonstrate that the large-scale jet is perpendicular to the large-scale disc on average (in agreement with Liska et al. 2018), with some oscillatory behaviour of the jet tilt and precession angles illustrating the jet’s dynamic nature. Over time, the inner part of the misaligned discs tends to align with the plane perpendicular to the BH spin axis, with the tilt angle decreasing by roughly 25 per cent compared to the initial tilt. Further, even though we started with a compact disc, subsequent disc evolution causes the disc to puff up due to viscous spreading, as illustrated by the disc barycentric radius (Fig. 2f; see also Porth et al. 2019), given by

$$r_{\text{disc}} = \int \int r \, \rho \, \mathrm{d}A_{\phi \psi}.$$  

(6)

As the disc becomes substantially larger, it stops precessing and the disc jet tilt angle become roughly constant at $t \gtrsim 9 \times 10^7 t_g$. Note that even though the disc and the jet no longer precess, they remain misaligned with respect to the BH spin vector, clearly illustrating that the absence of QPOs or any indication of precession does not rule out the presence of misalignment. Effectively, we end up with three disc/jet models with time- and spatially averaged jet tilt and precession angles ($T_i, P_j$) as follows: $T_0$: $\{0.9^\circ, 0^\circ\}$, $T_{\pm 0}$: $\{22.7^\circ, 28.4^\circ\}$, and $T_{\pm 60}$: $\{43.4^\circ, 42.2^\circ\}$ (see Table 1).

3 RESULTS

Fig. 1 shows the gas density $\rho$, the plasma-$\beta$ ($\beta_\rho$), and the ion temperature $T_i$ (in Kelvin) of the simulations at $t = 10^5 t_g$, clearly

Figure 1. We show the 2D cross-sectional plots of the gas density and plasma-β in code units, and the ion temperature $T_i$ (in Kelvin) for each tilt model: $T_0$, $T_{30}$, and $T_{60}$, spanning $100 r_g \times 100 r_g$. The 2D plane created by the BH spin vector and the large-scale jet angular momentum vector is displayed, rotating the $x$-$z$ plane by the corresponding jet precession angle $\psi_0 = [0\,^\circ, 28.4\,^\circ, 42.2\,^\circ]$ for $T_0$, $T_{30}$, and $T_{60}$, respectively (given in Table 1). The red and black lines in all plots denote the magnetization, $b^2/\rho c^2 = 1$, and the Bernoulli parameter, $Be = 1.02$, contours, respectively.

Fig. 3 shows the radial profiles of the tilt $\mathcal{T}$ (top row) and precession $\mathcal{P}$ (bottom row) for the models $T_{30}$ (left column) and $T_{60}$ (right column), time-averaged over $[99,960, 100,960] t_g$, along with their $1\sigma$ standard deviation. This interval at such late times is chosen to allow the simulations to reach a quasi-steady state at least within the inner $100 r_g$. We average the radial profiles over $1000 t_g$ in order to average over the short time-scale variations in the flow that otherwise cause the angles to fluctuate at small radii. LT torques affect the near-BH region, with radial tilt oscillations in both the disc and the jet peaking at $r \sim 10-20 r_g$ (Figs 3a and c). This behaviour was shown previously for model $T_{30}$ by Liska et al. (2018) and is consistent with simulations of small misalignment (Fragile et al. 2007; White et al. 2019). The peak $\mathcal{T}$ values are close to $30^\circ$, instead of $\sim 40^\circ$ at earlier times ($5 \times 10^4 t_g$; Liska et al. 2018), suggesting gradual alignment of the disc over time, until the viscous spreading of the disc saturates. Figs 3(a) and (c) clearly show that the jet follows...
of the disc, along with the jet for models T30 and T60, as $T$ for $T_{\text{init}} = 0^\circ$ is not well-defined. Both the angles are radially averaged over $[50, 150] \, r_g$.

It is noteworthy that the jet orientation is more variable than that of the disc. The barycentric radius of the disc $r_{\text{disc}}$ (equation 6) (f) shows that all three models display similar viscous spreading, and therefore indicate similar rates of outward angular momentum transfer.

the disc orientation at larger radii, and therefore the large-scale jet is always misaligned with respect to the BH spin direction, while the inner jet undergoes a small degree of alignment. This result is similar to the conclusions of McKinney et al. (2013), who considered misalignment under MAD conditions. However, this contradicts the assumption by White et al. (2020) that the jets follow the BH spin direction at large radii. This assumption can significantly affect the orientation of the BH images as the inferred observer inclination depends on the jet orientation in the sky (see Section 4.2). In this work, we have considered relatively powerful jets with $\phi_{\text{BH}} \gtrsim 30^\circ$ (Table 1) as compared to White et al. (2019), where $\phi_{\text{BH}} \lesssim 8^\circ$, assuming that all the magnetic flux goes into powering the jets ($P_{\text{outflow}} \propto \phi_{\text{BH}}^2$; Blandford & Znajek 1977; Tchekhovskoy, Narayan & McKinney 2010).

Henceforth, we will refer to the plane of the BH spin and the jet angular momentum vector, i.e. the 2D cross-section shown in Fig. 1, as the reference $\varphi = 0^\circ$ plane, which would be useful as a notation for the BH images in the next section.

3.1 GRRT imaging: synchrotron maps and SEDs

In jet observations, the inclination ($\theta_0$) is usually defined as the angle between the observer's line of sight and the large-scale jet. We therefore align our camera position according to the jet tilt $\tau$ and precession $\varphi$ angles, spatially and temporally averaged between $[50, 150] \, r_g$ and $[99\,960, 100\,960] \, r_g$. For our observer camera, we redefine the spherical grid: the $\varphi = 0^\circ$ plane is equivalent to the GRMHD $y = 0$ plane rotated by the corresponding precession $\varphi$ angle (Table 1), and the polar axis, i.e. the $x = y = 0$ line, is rotated by the corresponding tilt angle $\tau$. Therefore, we now have a camera grid which is aligned with the large-scale jet. Note that the precession angle is undefined for non-tilted jets (e.g. White et al. 2019), hence we took $\tau = 0^\circ$ for T0. The camera field of view (FOV) is set to be $75 \, r_g \times 75 \, r_g$, which is sufficient since warping occurs within $r \lesssim 20 \, r_g$, while the image resolution is $1024 \times 1024$ pixels. For a BH with the same mass and at the same distance as M87, $100 \, r_g$ corresponds to 382 micro-arcseconds ($\mu$as), which corresponds to an FOV of 286.5 $\mu$as. The simplest example to showcase the alignment of the camera grid with the large-scale jet is when we observe our
disc–jet system at an inclination of $\theta_0 = 90^\circ$ (i.e. edge-on to the outer disc).

Fig. 4 shows synthetic mm-radio images for $R_h = 10$ applied to an M87-like disc/jet system at an accretion rate of $6 \times 10^{-4} M_\odot$ yr$^{-1}$, with the observer positioned perpendicular to the jet axis (or edge-on to the large-scale disc) at four frequencies: 43, 86, 230, and 345 GHz. We have chosen $R_h = 10$, which yields contributions from both the disc and the jet to the total emission. We also show the 3D visualizations of the disc/jet system in the rightmost column of Fig. 4, as observed from the same direction as the GRRT camera. The rotating accretion flow surrounding the BH is clearly visible in the image centre is due to gravitational lensing by the BH and is a characteristic of images of accreting BHs (Narayan, Johnson & Gammie 2019). Hereafter, we refer to this flux depressed region as the BH ‘shadow’ (see Fig. 4). If we focus at the centre of the Atacama Large Millimetre/submillimetre Array (ALMA) results in constraining the size and structure of the central emitting region in Sgr A$^*$ to an impressive limit. Further constraints from the anticipated EHT Sgr A$^*$ 230 GHz images will be vital in capturing possible disc misalignment. We also show GRRT images of our models at different inclination angles later on in Section 4.3 and Appendix C.

In Appendix A1, we show sets of images for disc-dominated emission ($R_h = 1$; Fig. A1) and jet-dominated emission ($R_h = 100$; Fig. A2). With higher $R_h$ values, the disc electron temperature drops and the jet becomes more visible with respect to the disc. Due to the decrease...
in disc electron temperature, the streak feature also diminishes in brightness at 230 and 345 GHz.

Fig. 5 shows the corresponding spectral energy distribution (SED) generated for each model for the three different values of $R_0$. Since the mass and distance scales, as well as the accretion rate, are fixed for this set of GRRT images, higher $R_0$ values result in lower peak synchrotron fluxes. Viewing from edge-on with respect to the disc, for higher tilts the aforementioned warping causes one of the inner jets ($r \lesssim 20 \, r_g$) to point towards the observer and the other jet to point away, as seen from Figs 4 and A2. Hence, the net radio emission remains roughly similar for the three tilted models at the same $R_0$.

For larger misalignment angles, the models have lower synchrotron peaks with a shift of the peak towards the radio as well as lower near-infrared (NIR) and optical emission. Hence for an edge-on BH disc/jet system, the disc/jet warp creates a substantial change in the image as well as the SED. Note that, instead of using a constant accretion rate, if we were to fit to the same flux at the EHT frequency image as well as the SED. Note that, instead of using a constant disc/jet system, the disc/jet warp creates a substantial change in the peaks with a shift of the peak towards the radio as well as lower.

3.2 The case of M87

In this section, we apply our tilted models to M87 (EHTC 2019a). To match the crescent position in the M87 image as well as the direction of the jet, EHTC (2019b) found that for a positive BH spin ($\alpha > 0$), the observer inclination angle $\theta_0 > 90^\circ$. From the jet/cOUNTER-jet flux intensity ratio at 43 GHz, Mertens et al. (2016) estimated that the jet is $\sim 17^\circ$ offset from the line of sight (i.e. the viewing angle). Using these two results, we chose an inclination angle of $\theta_0 = 163^\circ$, i.e. $17^\circ$ offset from the jet moving in the opposite direction to the BH spin vector, such that we view the disc material to rotate clockwise. Hence, for M87, we shift our camera position to $\theta_0 = 163^\circ$ for each of the different tilt cases, with the jet direction given by the time-averaged tilt $T_j$ and precession $P_j$ angles from Table 1. As a first exploration of the simulations, in this section we restrict ourselves to the observer’s line of sight being in the plane shared by the large-scale jets and BH spin axes (later we will relax this condition). This still leaves two possible configurations: one where the BH spin vector and the bottom jet reside in the same half of the image plane ($\phi_0 = 0^\circ$) and the other where the BH spin vector and the bottom jet are in opposite halves ($\phi_0 = 180^\circ$). We find that $\phi_0 = 180^\circ$ is more favourable since the forward jet must appear on the right-hand side of the synthetic image plane while keeping the crescent shape in the bottom half of the image. This choice does not make a difference for the $T_0$ case, since the jet is roughly axisymmetric. Furthermore, we rotate the image to match the M87 outer-jet position angle of PA = 288° (Walker et al. 2018).

Fig. 6 shows the GRRT M87 images for $R_0 = 10$, time-averaged over [99 960, 100 960] $t_g$, with the accretion rate for each model set to match the 221 GHz flux shown in the M87 SED (Fig. 7). The synchrotron maps and SEDs have the same FOV and resolution as quoted in the previous section. The shadow size is about 40 $\mu$as as expected. However, due to the disc warp, some flux is present in front of the shadow for both misaligned models, which may itself be a crucial diagnostic since the brightness ratio from the ring to the depression (i.e. the shadow region) is expected to be increasingly better measured with future EHT observations. The extent of the southern bright crescent in the blurred 230 GHz image becomes smaller with increasing tilt angle. It is particularly noteworthy that the asymmetry in the upper and lower halves of the photon ring increases with increase in tilt. However, both of these features depend on the underlying electron temperature distribution, and therefore investigation of more sophisticated treatments, e.g. two-temperature plasma physics (e.g. Chael et al. 2018), is required.

The $R_0 = 1$ and $R_0 = 100$ cases are shown in Appendix A2, namely, Figs A3 and A4. The plunging region becomes more visible with increasing tilt angle as the streak in the face-on view grows more distinct, similar to tilted models seen in Section 3.1 with larger value of $R_0$. Since the disc and jet at low radii display a smaller tilt and larger precession angle as compared to the large-scale jet (see Fig. 3), a part of the disc/jet is slightly warped towards the observer and lensing effects no longer completely dominate the 230 GHz synthetic image. A notable difference from previous work is the absence of the bright ‘double’ crescent feature, one on each side of the shadow, seen in face-on images of tilted discs (Dexter & Fragile 2013, Fig. 5). This double crescent structure was attributed to standing shocks in the plunging region for tilted disc models in Fragile et al. (2007). Considering that White et al. (2020), who used similar parameters to Fragile et al. (2007), also found the double crescent feature to be absent, this leads us to speculate that such strong standing shock features as seen by Dexter & Fragile (2013) might stem from a difference in grid resolution and/or the numerical methods, e.g. the shock-capturing scheme, employed in the different GRMHD codes, or could be a result of our assumed electron temperature model (used in the EHT M87 papers; EHTC 2019b), as suggested by White et al. (2020). In our models, the flow in the plunging streams is not subject to strong shock-heating, and therefore the ion temperature does not steeply rise in this region and there is no enhanced synchrotron emission. Hence, the plunging streams do not form the double crescent feature, but instead we see a single crescent feature consistent with Doppler boosting, without significant additional heating. Similar to the results of White et al. (2020), the change in the shape of the crescent feature with tilt translates to a change in the BH shadow shape as is evident from the blurred 230 GHz images in Fig. 6. Further, we note that the key
Imaging tilted discs and jets

Figure 6. Left figure: Time-averaged, synthetic radio images of misaligned BH disc/jet systems using $R_h = 10$. The observer position is chosen such that the viewing angle is $17^\circ$ offset from the direction of the large-scale bottom jet ($r \gtrsim 50 r_g$), i.e. $(\theta_0, \phi_0) = (163^\circ, 180^\circ)$, calculated over [99 960, 100 960] $t_g$. Images are rotated to fit the position angle PA = $288^\circ$ of the large-scale jet in M87. From left to right: images at frequencies of 43, 86, 230, and 345 GHz, along with the blurred 230 GHz image (see Fig. 4). From top to bottom: underlying GRMHD simulations with initial tilt angles of 0, 30, and 60. Rightmost figure: density contour plot visualizations for (top to bottom) $T_0$, $T_{30}$, and $T_{60}$ viewed at the same observer position as for the synthetic GRRT images and rotated to fit the large-scale jet position angle PA = $288^\circ$ (red dashed line).

Figure 7. SEDs for misaligned BH disc/jet systems applied to M87 (Fig. 4) show little change with increasing $R_h$. The GRRT images are scaled to M87 with an accretion rate chosen to fit for the 221 GHz data point in the SED. From left to right: SEDs for increasing $R_h$ values for each of models $T_0$, $T_{30}$, and $T_{60}$. The radio, sub-mm, infrared, and optical data points are taken from Prieto et al. (2016).

4 DISCUSSION

4.1 Time series analysis of the M87 GRRT images

Fig. 8 shows the 230 GHz light curve derived from the core-unresolved spectrum (Fig. 7) for each tilt model at $R_h = 1, 10$, and 100, applied to the case of M87. The $T_0$ model shows the least amount of variation at 230 GHz due to strong gravitational lensing effects, irrespective of the temperature prescription, and is consistent with the results of EHTC (2019b). For the $T_{30}$ case, the variability is also small, except for a roughly monotonic flux increment for $t > 245$ d due to a flaring event. It is interesting to see that this flux increase in $T_{30}$ is more prominent for $R_h = 10$ and 100, since the jet plays a greater role in the net flux at higher $R_h$ values. Overall, $T_{60}$ consistently displays the largest 1σ deviation (>10 per cent) over the chosen time period for a given $R_h$ model, with a maximum factor of two change in the 230 GHz flux (see Table 2). This is a result of the turbulent inflow (outflow) in the disc (jet) being partially directed towards the observer. Hence the variability amplitude of the 230 GHz light curve may provide a diagnostic of disc misalignment in M87 and, perhaps, other AGNs. Future timing analysis of M87 EHT data may shed further light on whether there is indeed a tilted disc present.
Figure 8. Higher tilts show more variability in the near-horizon flux when applied to the M87 BH. The 230 GHz light curve is shown over 1000 $t_g$, equivalent to 353.55 d when scaled to M87, or roughly one year, for models: T0, T30, and T60 using (from top to bottom) $R_h$ values of 1, 10, and 100, respectively. The start time $t_0 = 99970.58 t_g \approx 35354.92$ d. T0 shows the least amount of variation as most of the 230 GHz flux is dominated by gravitational lensing effects rather than disc turbulence, while T60 shows a flux change by a factor of more than 2 over the entire time period.

Table 2. Higher disc–BH misalignment results in a more variable 230 GHz light curve. We show the time-averaged fluxes from the 230 GHz light curve for each tilt and $R_h$ model (see Fig. 8) as well as the $1\sigma$ standard deviation normalized by the averaged flux. T0 shows the lowest deviation, while T60, the highest, indicating that deviations $\geq 10$ per cent might be a hint of possible misalignment.

<table>
<thead>
<tr>
<th>Model</th>
<th>$R_h$ = 1</th>
<th>$R_h$ = 10</th>
<th>$R_h$ = 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>T0</td>
<td>1.649$^{+0.232}_{-0.203}$</td>
<td>1.664$^{+0.302}_{-0.260}$</td>
<td>1.765$^{+0.257}_{-0.214}$</td>
</tr>
<tr>
<td>T30</td>
<td>1.624$^{+0.26}_{-0.274}$</td>
<td>1.633$^{+0.433}_{-0.32}$</td>
<td>1.622$^{+0.477}_{-0.349}$</td>
</tr>
<tr>
<td>T60</td>
<td>1.656$^{+0.402}_{-0.329}$</td>
<td>1.723$^{+0.532}_{-0.644}$</td>
<td>1.727$^{+0.721}_{-0.744}$</td>
</tr>
</tbody>
</table>

In Appendix B, we perform further analysis on the flux variability time-scales for each light curve and construct the power spectrum as well as the structure function. All of the models show substantial variability over time-scales ranging from days to months, with the fractional root mean square (rms) amplitude $\approx 7$–16 per cent. We then model the power spectra and the structure function to find that the characteristic time-scale, i.e. the time-scale at which the variability transitions from red noise on short time-scales to white noise on long time-scales. The best-fitting characteristic time-scales from the power spectra and the structure functions are roughly of the order of a hundred days and a few tens of days respectively, as compared to the time-scale of $\sim 45$ d obtained by Bower et al. (2015) where the 230 GHz light curve of M87 spanned over 10 yr. Due to the short duration of our light curves, we only capture the short time-scale variability, with the power spectra well modelled by the characteristic slope $= -2$ of red noise (see Appendix B for more details).

Figure 9. The location of the brightest pixel in the synthetic 230 GHz M87 images varies the most for T0. We show the brightest pixel locations (blue circles) over time against a background of the average blurred 230 GHz image for each of the nine models considered in this work. Note that each blue dot represents a different time snapshot. Top to bottom: tilt models T0, T30, and T60. Left to right: $R_h$ = 1, 10, and 100 models for the electron temperature prescription. For the T0 case, gravitational lensing dominates the emission and hence turbulence in the flow changes the brightest spot location considerably. Doppler boosting of the plunging stream dominates the tilted images, which creates a preferential zone that points towards the observer where the boost is the maximum, and hence restricts the brightest spot location.

4.2 Best-fitting images for M87

In Section 3.2, we considered GRRT maps of misaligned jets observed at an inclination of $163^\circ$ offset from the large-scale jet direction, fixing the observer position with respect to the BH spin and the jet. In this subsection, we relax this assumption and move the observer’s position azimuthally around the bottom jet at an inclination of $163^\circ$, looking for the best fit-by-eye convoluted images for each model relative to the 230 GHz M87 observations (EHTC...
Imaging tilted discs and jets

Figure 10. Comparing our models with the EHT M87 images indicates that M87 favourably hosts a misaligned disc and jet. We show the best fit-by-eye GRRT snapshot images from all three models when compared to the 230 GHz M87 BH image, with the 43 GHz jet pointed at PA ~288° with \( R_0 = 10 \). The BH spin direction and projection on to the image plane is also shown, with X meaning the BH spin is pointing into the plane. Best-fitting 230 GHz GRRT images of aligned BH models are consistent with the EHT M87 image to within 1.5σ. If this deviation persists with future EHT observations of M87, a misaligned disc/jet could become a more viable model for M87.

2019a), while ensuring that the 43 GHz jet image has a PA ~288° (Walker et al. 2018). The accretion rate is taken to be the same as in Fig. 6. Fig. 10 shows the best fits for a single representative time snapshot (at \( t = 100\,460 \, t_g \)) in the case of each tilt model. First, the tilted model images show significant dependence on the camera longitude \( \phi_0 \) (see Section 4.3), since the underlying warped disc/jet is not axisymmetric. On the other hand, the aligned disc/jet model is roughly axisymmetric and so, the images are independent of the camera-\( \phi_0 \), as expected.

Secondly, the best-fitting T0 230 GHz image (Fig. 10, first row, left column) has a PA offset with respect to the best-fitting EHT GRRT image (EHTC 2019b, Fig. 1, right-hand panel). EHTC (2019b) found that models with the spin vector of the BH oriented away from the observer (\( \theta_0 \gtrsim 90° \)) are favoured by comparing to both 230 and 43 GHz images of M87, which we also find. The statistically best-fitting 230 GHz images were found to have a mean PA in the range of 203°–209° (see Fig. 9 in EHTC 2019b) with a standard deviation of 54°, which means that aligned GRMHD simulations are consistent with the PA ~288° found for the M87 jet at 43 GHz (Walker et al. 2018) within ~1.5σ. If we rotate our T0 230 GHz image so that we match the orientation of the 43 GHz M87 jet to the 43 GHz GRRT image, we find that the southern crescent is shifted to the bottom right quarter of the image, and hence does not match well with the EHT observed M87 image. Interestingly, disc misalignment provides more flexibility to fit the jet PA better as the crescent position is dependent on the choice of the camera longitude \( \phi_0 \), as we mentioned above. Fig. 10 (second row) shows that for the T30 model, which possesses an average tilt angle of 22.7°, the image has a bright crescent shape located in the bottom left of the image at \( \phi_0 = 280° \) and fits remarkably well with the M87 jet PA, possibly hinting at the presence of a tilted disc in M87. Indeed, if the 230 GHz best-fitting aligned disc/jet model always resides at a 1.5σ deviation in PA away from the 43 GHz M87 jet PA over future EHT M87 observations, disc/jet misalignment in M87 could become a significant possibility.

Thirdly, the brightness asymmetry in the upper and lower halves of the photon ring increases with increasing misalignment (as mentioned above in Section 3.2). Using the crescent position and the ring brightness asymmetry in the 230 GHz image along with the jet PA orientation might help us to pinpoint the tilt angle of the jet, which can lead to better BH spin estimates. As an example, the T60 case appears more asymmetric by eye than the reconstructed images of M87, which leads us to favour smaller tilts. In this work, we propose that M87 possesses a disc with a misalignment \( \lesssim 60° \), given the assumption that the BH spin is \( a = 0.9375 \) and the inclination angle is 17° offset to the large-scale bottom jet (i.e. the viewing angle). Images at frequencies lower than 230 GHz might provide more definitive proof of misalignment by directly capturing the disc warping region, located within roughly 20 \( r_g \) of the BH (see Fig. 3). However, the typical beam sizes for radio interferometric images at 43 and 86 GHz are presently too large to adequately resolve the inner 20 \( r_g \). Imaging the jet base region with sufficient resolution to capture the warping of the disc and the jet as well as checking for time variability in the EHT observations (see Section 4.1) will go a long way towards testing our prediction of a tilted disc in M87.

4.3 Varying the observer inclination and longitude

Fig. 11 shows the blurred 230 and 43 GHz images for model T30 observed over a range of inclination angles, \( \theta_0 = 180°–90° \), as well as also azimuthally rotating the observer around the large-scale bottom jet with \( \phi_0 = 0°–360° \), assuming \( R_0 = 10 \) and scaled to M87 at an accretion rate of 3.75 \( \times 10^{-4} \, M_\odot \, yr^{-1} \) (see middle row of Fig. 6). Using this catalogue of images, we can pick out the best-fitting images as compared to the EHT M87 image. First, we see that ring brightness asymmetry constraint favours smaller inclinations, along with a few acceptable images at \( \theta_0 \sim 45° \) and \( \phi_0 \lesssim 30° \) or \( \gtrsim 330° \). If matching the jet PA to 288° and requiring the bright crescent to be in the bottom left of the image, higher \( \phi_0 \) values are clearly favoured. This result shows that accounting for the large-scale jet orientation is crucial in constraining the acceptable values of \( \theta_0 \) and \( \phi_0 \). Secondly, at higher inclinations, the spectral shape for each tilt model is sufficiently distinguishable (see Fig. 5) and perhaps the M87 SED might be able to rule out some observer inclinations. Not only can we use these images to fit for M87, but given that Sgr A* extends over a similar angular size in the sky (due to comparatively close values for the ratio of the BH mass and the distance), we can also fit these images to upcoming EHT observations of Sgr A* in the same manner.
Figure 11. Images at 230 and 43 GHz for different values of the observer inclination $\theta_0$ and longitude $\phi_0$, for a single snapshot of model T30 with $R_\text{h} = 10$. The box sizes are $110 \times 110$ and $150 \times 150 \mu\text{as}^2$ for the 230 and the 43 GHz images, respectively. A strong dependence of the images on $\phi_0$ due to the non-axisymmetric nature of the disc/jet warping is clearly seen. The BH spin vector direction is also shown with a white arrow and the X (O) indicating whether the BH spin vector is pointing into (out of) the image plane. The arrow length is indicative of the BH spin projection on to the image plane. The images have been rotated to fit the 43 GHz jet PA from Walker et al. (2018).
5 CONCLUSIONS
In this work, using our GPU-accelerated GRMHD code h - amr, we have simulated three BH accretion discs, with the disc mid-plane initially misaligned by 0°, 30°, and 60° with respect to the z = 0 plane. Using these three models, we calculated the first synthetic radio images of misaligned accretion disc/jet models scaled to the M87 BH mass, performing radiation transfer using the GRRT code BHOSS. Our analysis of the 230 GHz synthetic images suggests that M87* could host an accretion disc with a reasonably large misalignment angle ( ≤ 60°). In light of these results, the misalignment angle, neglected in the current EHT theoretical models (EHTC 2019b), may therefore be an important additional parameter within the observational fitting procedure of GRMHD simulation models worthy of further exploration. Tilt brings about a change in the BH shadow appearance, which could be verified with future EHT observations of M87*. We note that, recently, White et al. (2020) considered the case of weakly magnetized misaligned discs and found similar results to our work.

This study has assumed that the emission originates solely from thermal synchrotron radiation, further assuming an electron temperature prescription which calculates the electron temperature from the GRMHD-based ion temperature. These assumptions play an important role in the appearance of the M87 230 GHz synthetic images and warrant further study in order to fully address the question of possible misalignment in M87. From semi-analytical modelling of the M87 spectra (e.g. Lucchini, Krauss & Markoff 2019), the 230 GHz emission is expected to have a significant non-thermal synchrotron contribution, describable by a hybrid thermal–non-thermal electron distribution function (e.g. Davelaar et al. 2019). As Lucchini et al. (2019) mention, the X-ray spectrum is more likely to be dominated by the non-thermal emission rather than by an SSC inner jet. Therefore, self-consistently accounting for both the EHT image characteristics and the X-ray spectra will be the most constraining avenue for modelling the mixed thermal/non-thermal synchrotron emission in M87. Our future work will include two-temperature physics, which would allow independent evolution of the electron and ion temperatures, and perhaps shall add an additional layer of complexity in the system (e.g. Ressler et al. 2015; Chael et al. 2018; Ryan et al. 2018). Finally, with promising new telescope additions to the EHT array in the near future and the possible advent of space-VLBI telescopes over the next few decades, resolving smaller scale structures close to the base of the jet will gradually become a reality, playing a key role in the search for a misaligned disc in M87 and other SMBH systems.

ACKNOWLEDGEMENTS
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DATA AVAILABILITY
Data used to plot the images in this work are available in Zenodo at http://doi.org/10.5281/zenodo.3757013.

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Kerr J., Thirring H., 1918, Phys. Z., 19, 156

APPENDIX A: CONSIDERING DISC- AND JET-DOMINATED EMISSION

Here, we discuss the GRRT images for $R_h$ values of 1 and 100, for the two observer positions mentioned in the text, namely, edge-on to the outer disc in Section 3.1 and 17$^\circ$ offset to the bottom large-scale jet in Section 3.2.

A1 Edge-on case

For $R_h = 1$, we have disc-dominated emission as the electron temperature is set to be the same as the ion temperature (see equation 1), which is higher in the disc as compared to the jet. The disc warp is visible as the outer accretion disc becomes increasingly optically thin, with the jet-dominated emission for $R_h = 100$. Higher $R_h$ images exhibit a curved streak similar to the one discussed above for $R_h = 1$, but now offset from the shadow as the feature originates in the plunging streams and the jet edge.

A2 M87 case

The most prominent difference between $R_h = 1$ and $R_h = 100$ images lies in their brightest features at 43 GHz: the photon ring in the case $R_h = 1$ calculated over [99 960, 100 960] t_g and zoomed-in to the inner 100 × 100 $\mu$as$^2$ region. The camera is positioned at (θ_0, φ_0) = (90$^\circ$, 0$^\circ$). From left to right: images at frequencies of 43, 86, 230, and 345 GHz, along with the convolved 230 GHz image. From top to bottom: underlying models $T_0$, $T_30$, and $T_60$. With $R_h = 1$, the electron temperature is taken to be the same as that of the ion temperature everywhere, hence the disc shines brighter than the jet, which usually has a lower ion temperature.
of $R_h = 1$ and the extended streak in the case of $R_h = 100$. The 43 GHz image features an optically thick accretion flow in front of the shadow region, which is absent in the corresponding $R_h = 1$ image, indicating that the streak originates in the sheath region.

**APPENDIX B: EXTENDED TIMING ANALYSIS: POWER SPECTRA AND STRUCTURE FUNCTIONS**

In this section, we perform a Fourier transform of the 230 GHz light curves from Section 4.1 for our M87 models. Fig. B1 shows the fractional rms normalized power spectra for each of our models, using $R_h = 1, 10,$ and $100$. The cadence ($\Delta t_{\text{rms}}$) of our images is $10 t_g \approx 3.54$ d, when scaled to M87, which enables us to capture the short time-scale variability in the emission. Each power spectrum can be well-fitted with a white noise component, where the power is independent of the variability frequency, and a red noise component, where the power scales as frequency $^{−2}$, smoothly joined together by a modified sigmoid function. The modified sigmoid function $S_{\text{var}}$ is defined as a function of the variability frequency ($\nu_{\text{var}} := 1/\Delta t$):

\[
S_{\text{var}}(\nu_{\text{var}}. n_{\text{var}}) := \left[1 + \exp \left[-(\tau_{\text{rms}} n_{\text{var}})^{\nu_{\text{var}}} \right] \right]^{-1},
\]

where $\tau_{\text{rms}}$ is defined as the characteristic time-scale where the variability transitions from red noise, which dominates at shorter time-scales, to white noise, which is more characteristic of long time-scales. We use $n_{\text{var}} = 6$ to reduce the transition zone. Table B1 shows the fractional rms variability amplitude calculated from the power spectra and the corresponding characteristic transition time-scale $\tau_{\text{rms}}$, determined using a least-squares fitting method. The overall trend is that with higher initial disc tilt, the rms-percentage (rms per cent) increases, while higher $R_h$ values result in larger rms per cent as well (except for T0, where the rms per cent is relatively unchanged). This suggests that introducing disc misalignment and/or a higher jet contribution to the total emission results in more variability on longer time-scales. Of course, the caveat is that the slope of $−2$ for the power law is the steepest slope one can measure reliably with a boxcar window Fourier method (which is what we used), where steeper slopes ($\leq 2$) get converted to slope $=−2$ due to low-frequency leakage.

Bower et al. (2015) analysed the variability of M87’s 1.3 mm ($\approx 230$ GHz) light curve over a period of 10 yr using the submil-limetre array (SMA) and found the characteristic time-scale to be...
et al. (2015) showed that the structure function for M87 increases
summation is taken over all light-curve data points. Fig. 2 in Bower
MNRAS 499, 362–378 (2020) of the form
\[ S(\Delta t) = \frac{1}{N} \sum [s(t) - s(t + \Delta t)^2], \]  
where \( N \) is the number of data points in the light curve and the
summation is taken over all light-curve data points. Fig. 2 in Bower
et al. (2015) showed that the structure function for M87 increases
steadily on short time-scales due to red noise, saturating at long
time-scales due to white noise. This structure function was modelled
as
\[ S^2(\Delta t) = S^2_\infty [1 - \exp(-\Delta t/\tau_{sf})], \]  
with \( S^2_\infty \) is the power in the light curve on long-time-scales and \( \tau_{sf} \)
is the characteristic time-scale for the transition from red to
white noise. Bower et al. (2015) used a Bayesian approach to
model the light curve due to the noisy nature of observed data and
irregular spacing of the data points in time (following Dexter
et al. 2014).

Here, we choose to adopt the simpler approach of Chael, Narayan
& Johnson (2019), since our sampling cadence is constant, and
directly fit the structure functions (shown in Fig. B2) for each tilt
and \( R_0 \) model with equation (B3) using a least-squares method.
Using this technique, we see that \( \tau_{sf} \) is of the order of a few tens
of days (Table B2), in stark contrast to the \( \tau_{rms} \) values (Table B1).
This result exposes the primary caveat of our modelling approach
for both the power spectra and the structure functions: the short
time duration of our analysed light curves. Fig. 9 of Chael et al. (2019)
shows that white noise is achieved beyond time-scales of 300 d,
while the light curves cover 4.8 yr, significantly longer than the
duration of our light curves.

Emmanoulopoulos, McHardy & Uttley (2010) showed that one
cannot reliably estimate time-scales with the structure function,
which is illustrated by the incompatibility between \( \tau_{sf} \) and \( \tau_{rms} \).
Our power spectra from the year-long light curve cannot explain \( \tau_{sf} \)
values as we clearly have a sufficiently long-enough duration
to see the noise transition from red to white on time-scales of 10–
20 d, suggesting that the time-scales are longer than that given by
the structure functions. However, we must note that we have very
few power estimates at the lowest frequencies, and their statistical
error (arising from the stochastic nature of the light curve) is equal
to the powers themselves, which makes it difficult to find a good fit
for the time-scale. Further, the use of least-squares minimization is
unsuitable for finding the turnover frequency as the error distribution
is exponential rather than Gaussian in behaviour. However, the least-
squares technique is no doubt the most straightforward fitting method
and provides good order-of-magnitude estimates.

**APPENDIX C: A GLOSSARY OF IMAGES FROM OUR MODELS**

Here, we provide a collection of images at 230 and 43 GHz for the
aligned model T0 (Fig. C1) and misaligned model T60 (Fig. C2).
Fig. C1 clearly shows that images for an aligned BH disc/jet system
is independent of the camera longitude \( \psi_0 \), while Fig. C2 further
illustrates the powerful effect of misalignment of BH images, with
more pronounced disc/jet warping as compared to Fig. 11.

We also note that the images for the camera inclination \( \theta_0 = 0^\circ \)
do not change with \( \psi_0 \) due to the direction of the camera ‘up’ vector. In
order to illustrate the orientation of the camera, we can consider an
analogue of the situation: imagine a person walking on the surface of
the Earth with their eyes always directed towards the North Pole. In
this analogy, their feet are pointing downwards, towards the centre
of the Earth: this vector can be thought of as the camera ‘viewing’
vector that is normal to the image plane, pointing into the plane.
Their eyes are always looking straight ahead, towards North: this
is the same as the camera ‘up’ vector which is the positive y-axis
in each of our images. In each M87 image, the camera ‘up’ vector
is rotated such that the large-scale GRMHD jet projected on to the
image plane fits the M87 jet PA. At \( \theta_0 = 0^\circ \) (i.e. at the North Pole),
the definition of a longitude fails, and hence, changing \( \psi_0 \) does not
change the orientation of the image.

**Table B2.** The best-fitting characteristic time-scale (\( \tau_{sf} \) in
days) from the structure functions (Fig. B2) constructed from
the 230 GHz M87 light curve for each disc/jet model and \( R_0 \)
model (Fig. 8), fitted with equation (B3) using a least-squares
method. The bounding box for the fitting is [3.54, 150] d, with
the lower end indicating the cadence of the data sampling.
The upper end is chosen such that we avoid the turnover of the
structure function at time-scales comparable to the time
duration of the light curve (~1 yr).

<table>
<thead>
<tr>
<th>Model</th>
<th>( R_0 = 1 )</th>
<th>( R_0 = 10 )</th>
<th>( R_0 = 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T0</td>
<td>26.3</td>
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<td>10.7</td>
</tr>
<tr>
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<td>14.0</td>
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<tr>
<td>T60</td>
<td>38.3</td>
<td>19.9</td>
<td>12.7</td>
</tr>
</tbody>
</table>

approximately 45\(^{16}\)\(^{24}\) d, modelling the light curve \( s(t) \) as a damped
random walk (DRW) process. They used a structure function \( S^2(\Delta t) \)
of the form
\[ S^2(\Delta t) = \frac{1}{N} \sum [s(t) - s(t + \Delta t)^2], \]  
where \( N \) is the number of data points in the light curve and the
summation is taken over all light-curve data points. Fig. 2 in Bower
et al. (2015) showed that the structure function for M87 increases
steadily on short time-scales due to red noise, saturating at long
time-scales due to white noise. This structure function was modelled
as
\[ S^2(\Delta t) = S^2_\infty [1 - \exp(-\Delta t/\tau_{sf})], \]  
with \( S^2_\infty \) is the power in the light curve on long-time-scales and \( \tau_{sf} \)
is the characteristic time-scale for the transition from red to
white noise. Bower et al. (2015) used a Bayesian approach to
model the light curve due to the noisy nature of observed data and
irregular spacing of the data points in time (following Dexter
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Here, we choose to adopt the simpler approach of Chael, Narayan
& Johnson (2019), since our sampling cadence is constant, and
directly fit the structure functions (shown in Fig. B2) for each tilt
and \( R_0 \) model with equation (B3) using a least-squares method.
Using this technique, we see that \( \tau_{sf} \) is of the order of a few tens
of days (Table B2), in stark contrast to the \( \tau_{rms} \) values (Table B1).
This result exposes the primary caveat of our modelling approach
for both the power spectra and the structure functions: the short
time duration of our analysed light curves. Fig. 9 of Chael et al. (2019)
Figure C1. Single snapshot images at 230 and 43 GHz scaled to M87 and rotated to fit the M87 jet PA for different values of the observer inclination $\theta_0$ and longitude $\phi_0$, for aligned model T0 with $R_0 = 10$. The box sizes are $110 \times 110$ and $150 \times 150 \mu$as$^2$ for the 230 and the 43 GHz images, respectively. The white arrow indicates the BH spin vector direction with the arrow length illustrating the spin projection onto the image plane. The X (O) indicates whether the BH spin vector is pointing into (out of) the image plane. These images are similar to the ones considered in EHTC (2019b), where GRMHD models were used to interpret the M87 image. Refer to Section 4.3 for further details.
Figure C2. Single snapshot images at 230 and 43 GHz scaled to M87 and rotated to fit the M87 jet PA for different values of the observer inclination $\theta_0$ and longitude $\phi_0$, for tilted model T60 with $R_h = 10$. The box sizes are $110 \times 110$ and $150 \times 150$ $\mu$as$^2$ for the 230 and the 43 GHz images, respectively. The arrow direction and length indicates the BH spin direction and projection on to the image plane, with X (O) meaning into (out of) the plane. The warping of the disc and jet significantly alter the 43 GHz images.

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