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Sustainability of pension systems with voluntary participation[☆]

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ABSTRACT

Motivated by declining support for mandatory participation in pension arrangements, we explore whether the intergenerational risk-sharing benefits that these arrangements offer suffice to ensure their survival when participation becomes voluntary. Funded systems with asset buffers are particularly interesting since these buffers make contributions more sensitive to financial returns. Equilibria are characterised by thresholds on the young's willingness to contribute. Standard values for our parameters yield two such equilibria; only the one with the higher threshold is consistent with the initial young being prepared to start the system. An advancement relative to the related literature is that the equilibria feature a non-zero probability of collapse. Finally, we explore the social welfare maximising values for the pension parameters for various levels of uncertainty and risk aversion.

1. Introduction

Participation in most pension arrangements is mandatory, but this obligation to participate is increasingly being questioned. Individuals demand more flexibility to make their own choices, and there is a growing fear among current working-age participants that future workers will refuse to make up for potential deficits when current workers retire themselves. In an opinion poll among the Dutch population (Motivaction, 2011), 80% of the young respondents expected that current pension arrangements will become unaffordable.[2] Similarly, a poll among Canadian respondents (Nanos, 2012) showed an increasing fraction of them becoming less confident that company pension plans will be able to deliver on their promised pension payments in the future.

This paper explores whether current mandatory pension systems would still survive when the requirement of individuals to participate is lifted. In particular, the young could prefer to leave the system, instead of honouring the entitlements of the elderly. The answer to this question is crucial, because it helps to determine whether or to what extent the obligation to participate can be safely relaxed without risking a collapse of the system and, hence, losing the benefit that it brings. In our analysis, this benefit derives from the possibility of retired participants to share financial risks with subsequent generations.[3] The expectation that the future young will honour the claims of the current young, is what persuades the latter to continue to participate and enjoy the benefit from intergenerational risk sharing. However, this benefit vanishes when the system collapses.

If our analysis points to a high probability of survival when participation is made voluntary, then current participation obligations can be safely relaxed in response to public pressure for more freedom of choice, without endangering the continuity of the current arrangements. However, if a system collapse becomes likely when participation is made voluntary, while the benefits associated with maintaining the current system are large, then there

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2 Interestingly, for a sample of Dutch wage earners de Bresser and van Soest (2013) estimate that the subjective uncertainty about their future pension replacement rate is strongly decreasing in their age.

3 Participation in a pension arrangement may bring other benefits as well, such as insurance against longevity risk and protection against myopic behaviour. We abstract from these elements, since it is also possible to solve these problems in pension systems without intergenerational risk sharing. These systems may also collapse due to an unwillingness to participate, but that unwillingness does not follow from an intergenerational conflict. However, analysing this is beyond the scope of this paper.

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is a case for designing more broadly acceptable participation obligations.[4]

Our paper analyses an intergenerational risk-sharing arrangement that on two accounts is more realistic than the arrangements studied in the related literature. First, this literature typically assumes the existence of a long-lived institution (e.g., a government) that can enforce participation and finds that the gains from intergenerational risk sharing can be substantial.[5] However, the transfers from the young to an unfortunate old generation can be so substantial that from an ex-interim point of view they make the young worse off than under autarky, i.e. when they do not participate. By allowing young individuals to opt out, we avoid such an outcome that would likely be unrealistic from a political feasibility perspective. Second, to the best of our knowledge, this paper is the first to model an intergenerational risk-sharing contract with voluntary participation, in which in equilibrium there is a non-zero probability that young individuals actually opt out. The key concept in our paper is the individual's "willingness to participate", which determines in which situations the next generation refuses to participate, so that the system collapses.

We set up an overlapping generations model in which individuals live for two periods. We allow for two sources of uncertainty: demographic risk, as captured by the birth rate, and financial market risks, as captured by the return on savings. In the first period of their life individuals decide whether to participate in a collective pension arrangement. They do so when utility under participation – given their beliefs about the willingness to participate of the future young – exceeds utility under autarky. Limiting ourselves to recursive settings in which each generation faces the same decision problem, this decision translates into a threshold on the contribution that a young individual applies, given its belief about the threshold of the future young. We confine ourselves to the study of equilibria in which, given the equilibrium belief about the future threshold, the system persists if the contribution is lower than the threshold, while it collapses permanently when the contribution exceeds the threshold. In addition, we are only interested in equilibria that are stable.

Our set-up can be applied to a wide range of pension arrangements. We consider three specific, but empirically important, arrangements. The first is a defined-benefit pay-as-you-go (PAYG) scheme, which is the dominant public arrangement in most advanced economies. The scheme has no assets, because each period aggregate contribution adjusts to exactly match aggregate benefits. The second application considers a funded pension arrangement with a minimum return guarantee (as long as it survives), while the third application generalises this arrangement by adding a required asset buffer. This buffer allows to honour entitlements under adverse shocks. This is a particularly relevant extension in our view, because an increasing number of countries is shifting towards fully-funded pension arrangements, while some countries, such as Canada, Norway, Sweden and the U.S., pre-fund their PAYG social security arrangements (Yermo, 2008). Total accumulated pension assets in the Organisation for Economic Cooperation and Development (OECD) are 25–30 trillion U.S. dollars (OECD, 2012, Table A14). However, financial market uncertainties and rising life expectancy put pension buffers under pressure. This is, for example, the case for the Netherlands, which has one of the largest funded pension pillars in the world, and the U.S., where the worries about the underfunding of the public-sector pension funds are growing (Novy-Marx and Rauh, 2011). Supervisors of funded pension arrangements are particularly interested in policies that restore pension buffers without undermining the willingness to participate by the young, whose contributions are needed to restore the buffers.

The consequences for the young's pension contributions of a buffer requirement are of particular interest. If the asset returns are high, the buffer creates some "free" money that can be used to lower their contributions. However, the incoming cohort not only has to guarantee the pension benefits of the retired, but it also has to replenish the buffer when the asset returns are low, implying a contribution that is actually higher than in the case without buffers. Overall, the sensitivity of the contributions to the asset returns is increasing in the size of the required buffer.[6] Moreover, in contrast to common perception, individual contributions can actually be *increasing* in the size of the young cohort. If the fund has a buffer larger than required, then this excess buffer is used to lower the contributions of the young. The excess buffer per young individual is smaller when the young generation is larger and, hence, the individual contribution is higher. Of course, when the buffer is smaller than required ("underfunding"), a larger young generation requires a smaller increase in the individual contribution to bring the buffer to its required level.

For all three pension systems, we explore how the welfare maximising values of the pension parameters vary with uncertainty and risk aversion. An increase in the demographic uncertainty has only minor effects on the welfare maximising values of the pension parameters, because the demography does not directly affect individual utility. Both an increase in financial market uncertainty and an increase in risk aversion raise the benefit from risk sharing. Hence, the optimal benefit level and, along with it, the maximum willingness of the young to contribute rise. For funded systems, higher risk aversion lowers the optimal minimum return on the pension contributions. Hence, the likelihood that the future young, who have to ensure this minimum return, also participate rises and, thus, the risk of low consumption in retirement falls. Owing to the rather large difference between the expected financial return and the expected population growth rate, the social welfare gains from participating in the optimal funded scheme are substantially larger than from participating in the optimal PAYG scheme. Imposing a buffer requirement on the funded scheme raises social welfare further, though only by a relatively small amount. The reason is that the benefit to the first young generation of participation is lower when there is a buffer requirement, as this generation has to build up the initial buffer. Our results suggest that the collapse of well-designed pension arrangements with intergenerational risk sharing produces a substantial welfare loss. However, in equilibrium, the chance of a system collapse tends to be low and, hence, from the perspective of its survival there is little need to make individual participation mandatory.

Discontinuity risk in pension systems has been investigated from different perspectives. Beetsma and Romp (2016) provide a survey. One strand of the literature studies social contracts that are sustained by the threat that a deviation from the contract causes a permanent collapse of the system. An early contribution is Veall (1986), in which equilibria with social security are sustained by the threat that reducing the pension benefit

[4] For example, the most recent Dutch government coalition agreement foresees the possibility for a pension fund participant to take up part of her entitlement as a lump-sum payment at retirement date.

[5] Intergenerational risk sharing in pay-as-you-go pension schemes is analysed by Enders and Lapan (1982), Thøgersen (1998), Bohn (2001, 2009), Sanchez-Marcos and Sanchez-Martin (2006), Gottardi and Kubler (2011), Krueger and Kubler (2006), Ball and Mankiw (2007), Olovsson (2010) and Teulings and De Vries (2006). Gollier (2008), Cui et al. (2011), Mehlkopf (2013) and Bonenkamp and Westerhout (2014) explore intergenerational risk sharing in funded pension arrangements. Hassler and Lindbeck (2005), Matsen and Thøgersen (2004), Wagener (2004) and Beetsma et al. (2012) investigate intergenerational risk sharing in the presence of multiple pension pillars.

[6] In fact, in the Netherlands there is a fierce public debate about the appropriate target level of the pension buffers.

to the current elderly causes a reduction in all future benefits. More closely related to the current paper are contributions that explore the loss of (intergenerational) risk-sharing benefits associated with potential discontinuity. Demange and Laroque (2001) analyse risk sharing in a PAYG system with voluntary contributions. Bovenberg et al. (2007) explore the problem of negative buffers that may deter new cohorts from entering the pension arrangement, while Westerhout (2011) quantifies the feasible amount of risk-sharing when the old are bound by their pension contract, but the young are free to choose whether they will participate. The latter refuse to join the pension fund when it is under financial distress. Unlike in this paper, Westerhout (2011) assumes that the return on the fund's assets exceeds that on private savings, which makes it relatively attractive to join the fund. Siegmann (2011) and Molenaar et al. (2011) explore the thresholds on the ratio of assets over liabilities for which it is optimal for a participant to quit a pension fund. Their models are very different from ours, while they analyse only the incentive to quit the fund, but not the existence and characterisation of equilibria in which individuals can freely optimise whether or not to quit the fund. Finally, Beetsma et al. (2012) explore the specific case of a funded pension system without a buffer and with financial market uncertainty as the only source of risk. Their system collapses either immediately or never.

Our approach should be sharply distinguished from the strand in the literature that explores discontinuity risk in the context of a political-economy framework. Galasso and Profeta (2002) survey this literature. Cooley and Soares (1999), Tabellini (2000), Bohn (1999, 2003) and Demange (2009) show how social security can be sustained through a popular vote. A standard approach is that individuals at different ages compare the net present value of their future social security benefits with the net present value of the remaining contributions they still have to make. The benchmark alternative is a permanent breakdown of social security. As working individuals become older, the net benefit from social security becomes increasingly favourable. Our approach does not rely on sustaining pensions through a popular vote, but through the net benefits from intergenerational risk sharing, an aspect that is absent from these papers.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 characterises the participation decision. In Section 4 we explore three common applications: a PAYG arrangement, a pension fund without asset buffers and a pension fund with buffers. Section 5 analyses the welfare maximising values of the pension parameters for these arrangements. Finally, Section 6 concludes the main body of the paper. The Appendix contains the proofs.

2. The model

We present a simple overlapping generations model in which individuals live for two periods. They receive an exogenous endowment income in the first period of their life and decide whether or not to participate in a pension arrangement. In addition, they decide about their level of "personal savings", i.e. the amount of savings outside the pension arrangement in which they potentially participate. In the second period of their life they consume the proceeds of their savings and their pension benefit in case they decided to participate.

2.1. Individuals

Cohort-specific variables have two subscripts. The first indicates the period of birth, the second the age ("y" for young, "o" for old). We refer to the cohort born in period υ as "cohort υ". Lifetime utility of an individual from this cohort is:

$$\mathcal{U}_\upsilon = U(c_{\upsilon,y}, c_{\upsilon,o}), \tag{1}$$

where $c_{\upsilon,y}$ and $c_{\upsilon,o}$ are consumption when young and old, respectively. From the perspective of an individual when she is young, consumption when old will be stochastic and depend on savings when young, the participation decision by the next cohort and the state of the world when old. From an ex-ante perspective, also consumption when young will generally be stochastic and depend on whether the pension arrangement is still in existence or not. To unlink the rate of risk aversion and the intertemporal elasticity of substitution, we do not impose the standard time-additive expected-utility specification; the utility function $U(\cdot,\cdot)$ aggregates over all possible states of the world and deals with uncertainty of $c_{\upsilon,o}$. We only impose that the utility function is continuous, strictly increasing and concave in its arguments and satisfies the Inada conditions. In our examples in Section 4 we will use Epstein–Zin type preferences, which include the standard time-additive constant relative risk aversion preferences.

In the first period of her life an individual decides whether or not to participate in a pension scheme. She can make this decision only once in her life. That is, if she opts in, she cannot opt out later, while if she does not opt in, she has to stay out for her entire life and spend her life in autarky. In particular, by choosing to participate the individual will be obliged to participate not only in the first period of her life, but also in the second period of her life. We can think of participation as signing a legally-binding contract that prohibits leaving the system when retired. Specifically, this implies that it is not possible for the elderly participants to dismantle the system and distribute its assets, if any, among themselves. The only way in which the system can be terminated is when the young refuse to participate at the start of their life.

At birth in period t, the individual receives an exogenous endowment E, which we assume to be the same irrespective of the period of birth. We can think of the endowment capturing an exogenous wage earned by inelastically supplying a given amount of labour. We abstract from modelling a labour–leisure trade off, as this would yield only limited additional insight for the problem at hand. If the representative individual decides to participate, then she has to pay a contribution τ_t to the pension system. This contribution is determined by the pension system and the current state of the economy; it is not a decision variable of the individual.

The individual has full discretion over the endowment remaining after paying the pension contribution, i.e., how much to consume now and how much to save for future consumption. Specifically, the individual has the opportunity to invest in a risky asset that generates an exogenous, but possibly stochastic, strictly positive gross return of $R_{t+1} > 0$ in the next period. Given the length of one period corresponding to a full generation, i.e. 25–30 years, throughout we assume that financial returns are independently and identically distributed over time.

2.2. The demography

Each generation consists of a mass of individuals. We denote the size of the current old generation by N_{t-1}. This generation gives birth to a new generation of size N_t at the start of the period t. The gross birth rate $b_t \equiv N_t/N_{t-1}$ is stochastic, and independently and identically distributed over time, an assumption that should again be reasonable given the length of a period.[7] Given our two-cohort model, it is also the inverse of the old-age dependency ratio $N_{t-1}/N_t = 1/b_t$. Population growth is $n_t = N_t/N_{t-1} - 1$, hence, $1 + n_t = b_t$.

[7] Relaxing this assumption would substantially complicate the analysis, as then the contribution threshold that we derive below for the young to participate in a pension scheme would depend on current fertility.

2.3. The pension system

The pension system collects contributions $\tau_t \geq 0$ from the current young and pays a pension benefit $\theta_t \geq 0$ to each current old. These depend on the specific pension arrangement and the state of the economy. Depending on its type, the system may or may not manage assets. If there are any assets, then these are managed by a pension fund. If the new cohort decides to participate, total assets ($A \geq 0$) of the fund evolve as

$$A_{t+1} = R_{t+1}[A_t + N_t\tau_t - N_{t-1}\theta_t], \tag{2}$$

where $A_t \geq 0$ is assets at the start of period t. Assets at the beginning of period $t+1$ equal the gross return R_{t+1} on assets A_t at the start of period t plus total contributions in period t minus total pension benefits paid in period t. In the special case of a PAYG system, the fund simply has zero assets to manage, i.e. $A_t = 0$ for all t. It is often easier to focus on assets per current young person who participates. Dividing both sides by N_{t+1} gives

$$a_{t+1} = \frac{R_{t+1}}{b_{t+1}}\left(a_t + \tau_t - \frac{\theta_t}{b_t}\right), \tag{3}$$

where $a_t \equiv A_t/N_t$. However, if the new cohort decides not to participate, the system collapses in period t (a situation that is, henceforth, denoted by a superscript " c"), and all the assets still in the fund, A_t, are liquidated and evenly paid out to the members of the current old generation. In that case, every member of the old generation receives A_t/N_{t-1}, which is equal to

$$\theta_t^c = a_t b_t. \tag{4}$$

Eq. (3) shows that in general the asset position of the system features an auto-regressive component. Our solution method below requires the contribution and payout to be such that the assets of the pension fund ($a_t + \tau_t - \theta_t/b_t$) always return to their initial level. The examples that we study later satisfy this requirement.

3. The participation decision

We focus on the participation decision of a representative young individual. Because all young individuals are identical, the optimal decisions of the other individuals of its generation coincide with the optimal decision of this particular individual. Hence, if this individual decides to participate, given the participation decision of the other members of its generation, then all the other generation members will also decide to participate, and, hence, the entire young generation participates. Vice versa, if this particular individual decides not to participate. The decision to participate or not depends on the utility the individual can get from participation versus the utility under autarky.

The relevant variables for the participation decision of an individual newborn are the asset return, the birth rate and the asset position of the pension arrangement, which together make up the state of the economy denoted by $\phi_t = \{R_t, b_t, a_t\}$. At the moment of the participation decision, the young generation knows the current old-age dependency ratio and the current assets of the pension system. For simplicity, we focus on a fully recursive economy. Contributions paid to the pension system are such that the assets managed by the fund always return to the same level. For the sake of readability we drop time subscripts and use an apostrophe (e.g., ϕ') to denote the next period's values. Moreover, throughout this paper we normalise the endowment E to 1. Given our recursive setup, the next period's state ϕ' is independent of the current state ϕ, hence $Pr(\phi'|\phi) = Pr(\phi')$. We focus on contribution and payout rules that only depend on ϕ, so the contribution and payout can be written as a function of the state: $\tau = \tau(\phi) \geq 0$, $\theta = \theta(\phi) \geq 0$ and $\theta^c = \theta^c(\phi) \geq 0$.

In this section we will show that for the participation decision all relevant information on the current state ϕ can be summarised by the required contribution $\tau(\phi)$ and that the participation decision reduces to a simple decision rule that compares this required contribution to a threshold contribution τ^*. We derive an efficient procedure to find all thresholds that satisfy a stability criterion. Our proposed method accommodates both discrete and continuous shocks to the old-age dependency ratio and financial returns.

3.1. Autarky

Consider an individual born in state ϕ. She has to divide her endowment of 1 over current consumption and savings s, on which she receives a stochastic gross return R'. Given our assumptions, and suppressing the cohort subscript, utility under autarky is independent of the current state and this utility is uniquely determined by the following maximisation problem

$$\mathcal{U}^a = \max_{c_y, c_o, s} U(c_y, c_o) \tag{5}$$

$$\text{s.t. } c_y + s = 1,$$
$$c_o = R's.$$

Neither a potential pre-existing pension arrangement, nor its state, matter for a young individual, since the assets, if any, of the fund are divided over the current old generation. This consumption-saving decision problem is standard and, hence, we do not explicitly state its solution.

3.2. Participation

If the individual born in state ϕ decides to participate in the already existing pension arrangement, she has to pay a contribution $\tau(\phi)$ to the pension arrangement. The level of this contribution is not up to the individual to decide, but is determined by the state of the economy and the pension arrangement according to a system-specific rule. In Section 4 we will provide three examples of such rules. The benefit of participation is twofold. First, a systematic redistribution to the old may make young and old better off if the rate of population growth is higher than the exogenous real interest rate. Second, the pension system may provide some insurance against particularly bad shocks during the retirement period. However, the payout of the pension system depends on whether the next generation also participates. If the next generation decides to participate, the current young receive a pension benefit $\theta(\phi')$ when they are old. This benefit depends on the particular type of pension arrangement and it may depend on the future state of the economy. Hence, assuming that the next generation participates, consumption of the current young once they are old is $c_o = R's + \theta(\phi')$. The pension benefit $\theta(\phi')$ is financed out of the next cohort's contribution $\tau(\phi')$ and the possible assets of the pension system. Henceforth, we refer to the pension system's assets as the " buffer". We consider pension arrangements of the defined-benefit (DB) type, such as the subnational civil servants' pension funds in the U.S. This implies that there is no direct link between the benefit received when old and the individual contribution when young. It is precisely the absence of this direct link, that allows for the intergenerational risk sharing that is the focus of this paper.[8] Obviously, for a given benefit, an increase in the individual contribution lowers the utility of the current young.

8 Intergenerational risk sharing would be absent, for example, in the case of an individual defined contribution scheme in which participants contribute to their own account and receive the accumulated account at retirement. In this case, contributing more would result in a higher benefit, ceteris paribus.

If the cohort born in the next period decides not to participate, then the pension fund's assets are distributed evenly over the existing participants.[9] Because in our simple set-up there is only one remaining cohort of participants, namely the current young, there is no intergenerational conflict over the assignment of property rights of the remaining pension assets. In this case, consumption of the current young when old is $c_o = R's + \theta^c(\phi')$. In the case of a funded arrangement the buffer could be so large that θ^c even exceeds the benefit θ under continuation. Hence, it is possible that the old generation would actually prefer a collapse of the system. However, the obligation not to leave the system prevents them from liquidating it when the young decide to participate.

The value of participation, $\mathcal{U}^p$, depends on the current state of the economy, which determines the current contribution, and on the belief of the current young about the willingness of the next young to participate, parameterised by a decision rule $\lambda(\phi') \in \{0, 1\}$. A value of 0 indicates that the next young do not participate, a value of 1 indicates their participation. Utility of a current young individual when she decides to participate follows from the maximisation problem

$$\mathcal{U}^p_{\lambda,\tau,\theta,\theta^c}(\phi) = \max_{c_y,c_o,s} U(c_y, c_o) \tag{6}$$
$$\text{s.t. } c_y + s = 1 - \tau(\phi),$$
$$c_o = R(\phi')s + \lambda(\phi')\theta(\phi') + [1 - \lambda(\phi')]\theta^c(\phi').$$

The notation $\mathcal{U}^p_{\lambda,\tau,\theta,\theta^c}(\phi)$ signals that the value of participation depends on the beliefs about the next generation's willingness to participate $\lambda(\cdot)$, the characteristics of the pension system $\tau(\cdot)$, $\theta(\cdot)$ and $\theta^c(\cdot)$, and the current state of the economy $\phi = \{R, b, a\}$. Note that we assume that the current young individual is certain about the participation decision rule of the future young generation; uncertainty only comes from the stochastic return on savings and population growth.

Given our assumptions, $\mathcal{U}^p_{\lambda,\tau,\theta,\theta^c}(\phi)$ only depends on the current state ϕ via the current contribution $\tau(\phi)$, so we can define a function $\mathcal{U}^p_{\lambda,\theta,\theta^c}$ (note the number of subscripts) of the required contribution:

$$\mathcal{U}^p_{\lambda,\theta,\theta^c}(\tau) = \max_{c_y,c_o,s} U(c_y, c_o) \tag{7}$$
$$\text{s.t. } c_y + s + \tau = 1,$$
$$c_o = R's + \lambda(\phi')\theta(\phi') + [1 - \lambda(\phi')]\theta^c(\phi'),$$

and we have $\mathcal{U}^p_{\lambda,\tau,\theta,\theta^c}(\phi) = \mathcal{U}^p_{\lambda,\theta,\theta^c}(\tau(\phi))$. The advantage of using expression (7) over (6) is that $\mathcal{U}^p_{\lambda,\theta,\theta^c}$ is continuous and differentiable in its input argument τ. Since every agent is atomistic, we can ignore the effect of the individual contribution τ on benefits θ and θ^c. Positive marginal utility ensures that $\mathcal{U}^p_{\lambda,\theta,\theta^c}$ is strictly decreasing in τ; a higher contribution has a purely negative wealth effect.

3.3. Equilibrium

The key element in our analysis is the decision rule which is based on the belief of the current young about the next cohort's willingness to participate. Let Λ denote the set of all functions mapping all possible realisations of the state ϕ to a participation decision $\{0, 1\}$. The decision rule of a young individual who believes that the next young generation uses the decision rule to participate λ is given by a function $T : \Lambda \to \Lambda$ with T defined as

$$T(\lambda)(\phi) = \begin{cases} 0, & \text{if } \mathcal{U}^p_{\lambda,\theta,\theta^c}(\tau(\phi)) < \mathcal{U}^a, \\ 1, & \text{if } \mathcal{U}^p_{\lambda,\theta,\theta^c}(\tau(\phi)) \geq \mathcal{U}^a. \end{cases} \tag{8}$$

This function generates the decision rule that is optimal for the current young individual given its belief about the next generation's decision rule. We assume that if an individual is indifferent between participating or not, she chooses to participate. Because $\mathcal{U}^p_{\lambda,\theta,\theta^c}$ is continuous and strictly decreasing in τ, θ and θ^c are bounded, and $R(\phi) > 0$, the Inada conditions ensure that there is always a unique contribution $\tau^* = \tau^*(\lambda)$ implicitly defined by

$$\mathcal{U}^p_{\lambda,\theta,\theta^c}(\tau^*) = \mathcal{U}^a. \tag{9}$$

Using this threshold τ^*, we can write $T(\lambda)$ as a simple indicator function, i.e. $T(\lambda)(\phi) = 1 \Leftrightarrow \tau(\phi) \leq \tau^*(\lambda)$.

Since each generation's decision problem is the same, we assume that each generation applies the same decision rule. This gives us the following definition:

Definition 1. A *valid decision rule* is a fixed point of T, i.e. a valid decision rule satisfies $\lambda = T(\lambda)$.

Since $T(\lambda)$ is a simple indicator function, and a valid decision rule is a fixed point of T, we can write every valid decision rule as a simple indicator function, which we summarise in the following proposition:

Proposition 1. *Every valid decision rule λ satisfies*

$$\lambda(\phi) = \begin{cases} 0, & \text{if } \tau(\phi) > \tau^*(\lambda), \\ 1, & \text{if } \tau(\phi) \leq \tau^*(\lambda). \end{cases} \tag{10}$$

Besides symmetry, we also need a selection mechanism to discard "unstable" or "silly" valid decision rules. We require that if someone starts with a belief about the next generation's willingness to participate λ_0, which is "close to" a valid decision rule λ, then the series defined by $\lambda_{i+1} = T(\lambda_i)$ converges to λ. This gives the following definition:

Definition 2. A *stable valid decision rule* λ is an attractive fixed point of T, i.e. there exists an $\varepsilon > 0$ such that for every $\lambda_0 \in \Lambda$ with $|\tau^*(\lambda_0) - \tau^*(\lambda)| < \varepsilon$, the series $\lambda_{i+1} = T(\lambda_i)$ converges to λ, as $i \to \infty$.

Intuitively, stability implies that a member of the young cohort who happens to have a belief about the next generation's threshold τ^0 close enough to the equilibrium threshold τ^*, and tries to determine the threshold contribution that makes her indifferent between participation and autarky given this belief τ^0, eventually ends up at the equilibrium value τ^* by using the latter as threshold in her belief.

Given a valid decision rule, utility of participation $\mathcal{U}^p$ is now fully determined by the required contribution of the current generation τ and the belief about the willingness to participate of the next generation measured by contribution threshold τ^*. To make this dependence explicit we may write the optimisation problem of a current young individual as

$$\mathcal{U}^p(\tau, \tau^*) = \max_{c_y,c_o,s} U(c_y, c_o) \tag{11}$$
$$\text{s.t. } c_y + s + \tau = 1,$$
$$c_o = R's + I(\tau(\phi'), \tau^*)\theta(\phi') + [1 - I(\tau(\phi'), \tau^*)]\theta^c(\phi'),$$

where $I(\cdot, \cdot)$ denotes the indicator function

$$I(\tau, \tau^*) = \begin{cases} 0, & \text{if } \tau > \tau^*, \\ 1, & \text{if } \tau \leq \tau^*. \end{cases}$$

For the sake of readability, we dropped the dependence on the pension system's payout rules θ and θ^c from $\mathcal{U}^p(\cdot, \cdot)$.

A higher contribution τ lowers lifetime income and always lowers lifetime utility, but the effect of a higher perceived threshold τ^* applied by the next-period's young is ambiguous in general. A higher τ^* always widens the region over which the system

[9] In the case of a PAYG system, there is no buffer and the pension benefit of the current young will be zero if the next generation opts out.

survives, but the payout after a collapse of the pension system to the then old may or may not exceed the payout under continuation. In Section 4.3 we present a pension system in which the pension fund has a buffer. As long as the fund has a positive buffer, the old actually prefer the young *not* to participate, so that the fund can be closed and they can claim the buffer for themselves. Hence, in general, the effect of a higher contribution threshold of the next young on the utility of the current young is ambiguous.

We can now state the key proposition of this paper:

Proposition 2. *Let $\mathcal{U}^a$ and $\mathcal{U}^p(\cdot,\cdot)$ be defined as in Eqs.* (5) and (11). *A threshold τ^* corresponds to a valid decision rule if and only if*

1. *for all $\tau \le \tau^*$, $\mathcal{U}^p(\tau,\tau^*) \ge \mathcal{U}^a$,*
2. *for all $\tau > \tau^*$, $\mathcal{U}^p(\tau,\tau^*) < \mathcal{U}^a$.*

This valid decision rule is stable in the neighbourhood of τ^* if $|\mathcal{U}_2^p| < |\mathcal{U}_1^p|$ in $\tau = \tau^*$ and not stable if $|\mathcal{U}_2^p| > |\mathcal{U}_1^p|$, where $\mathcal{U}_1^p$ and $\mathcal{U}_2^p$ are the first derivatives of $\mathcal{U}^p(\tau,\tau^*)$ with respect to its first and second arguments, respectively.

Proof. See Appendix A.1. □

For interior values of τ, the two conditions in Proposition 2 reduce to the condition $\mathcal{U}^p(\tau,\tau) = \mathcal{U}^a$ for $\tau = \tau^*$. However, to also accommodate corner solutions, we split this condition into two parts. Condition 1 handles situations where the pension system and the economy are such that there is some maximum contribution τ_{max} and $\mathcal{U}^p(\tau_{max},\tau_{max}) \ge \mathcal{U}^a$. If this holds, then Condition 1 in Proposition 2 holds, since $\mathcal{U}^p$ is downward sloping in its first argument, which ensures that, regardless of the required contribution, it is always optimal for the individual to participate. The second condition is relevant if there is some minimum required contribution and autarky is the better option even at this minimum contribution. That is, $\mathcal{U}^p(\tau_{min},\tau_{min}) < \mathcal{U}^a$. In this case no generation would ever be willing to participate in such a pension system. The last part of the proposition gives an easy condition to check for stability based on the derivatives of $\mathcal{U}^p$ in the candidate fixed point.

To calculate equilibrium thresholds we define

$$\Delta^p(\tau) \equiv \mathcal{U}^p(\tau,\tau) - \mathcal{U}^a. \tag{12}$$

Finding a threshold boils down to a simple one-dimensional root finding problem by solving $\Delta^p(\tau) = 0$, as well as checking the values of Δ^p at potential corner solutions. The derivative of Δ^p is

$$\frac{d\Delta^p}{d\tau} = \mathcal{U}_1^p(\tau,\tau) + \mathcal{U}_2^p(\tau,\tau). \tag{13}$$

Given that $\mathcal{U}_1^p < 0$, the last part of Proposition 2 implies that any root $\hat{\tau}$ of Δ^p on an upward sloping part of the function $\Delta^p(\cdot)$ cannot be a stable equilibrium, because it implies that $|\mathcal{U}_2^p| > |\mathcal{U}_1^p|$. Hence, it can be discarded. Thus, a negative first derivative of $\Delta^p(\cdot)$ at a root provides only a necessary, but not a sufficient, condition for the stability of the root.

Finally, we can define the probabilities that the system continues and that it collapses. To this end, we define $D(\tau^*) = \{\phi|\tau(\phi) \le \tau^*\}$ as all the states in which the system continues and $D^c(\tau^*) = \{\phi|\tau(\phi) > \tau^*\}$ as its complement, i.e. all the states in which the system collapses. The probability that the system collapses is $Pr(\phi \in D^c(\tau^*))$, while the probability that the system continues is equal to $Pr(\phi \in D(\tau^*))$.

This non-zero equilibrium probability of a collapse of the system is our main contribution to the literature. Other papers (e.g., see Demange and Laroque (2001) and Prescott and Ríos-Rull (2005)) restrict their intergenerational transfer schemes such that it is ex-interim always optimal to participate and, hence, the system never collapses. Our pension scheme resembles the set of transfers in the "organisation equilibrium" of Prescott and Ríos-Rull (2005). New participants observe the transfers in the organisation (here: our pension scheme) and decide whether to participate or not. In Prescott and Ríos-Rull (2005), the transfers are chosen such that it is never optimal to restart the system. We do not impose such a restriction. As such, our setup is closer to the game theoretical equilibrium in Boldrin and Rustichini (2000), where new generations decide whether to participate or not. An important difference with their paper is that they restrict their attention to pay-as-you-go systems where the population votes over the preferred contribution rate.

3.4. Existence

The three applications below all turn out to have a stable valid decision rule. Unfortunately, proving existence of a valid decision rule for the general case is impossible; even in this simple set-up it is possible that there is no valid (so surely not a stable valid) decision rule. Here, we construct a simple example to demonstrate this.

Assume an economy with no uncertainty, so population growth and financial returns are constant. Specifically, assume that the net financial returns are zero, while population growth is strictly positive. Lifetime income is simply the sum of income when young and old. In the absence of uncertainty, utility is fully determined by this lifetime income. That is, we can compare lifetime income under participation and autarky to determine the validity of a decision rule. Finally, focus on a pension system with a pension fund that must keep a positive buffer that is constant in per capita terms of the young. If the system does not collapse when they have become old, it pays out nothing, while if it collapses, they receive the buffer. This is a special case of the third example analysed below.

As there is only one state, the participation decision by the young is easy. If they do not participate, their lifetime income is 1. If they do decide to participate, they "inherit" the buffer in the pension fund, but this buffer requires them to make an additional contribution when young, since population growth is positive. Now assume that they believe that their offspring wants to participate, i.e. $\lambda = 1$. Their offspring will take control of the buffer, hence the pension fund does not pay out anything and lifetime income is equal to the endowment minus the additional contribution to maintain the pension fund's buffer. This is clearly less than 1, so it is optimal for the current young not to participate. If the next generation decides not to participate ($\lambda = 0$), then the current young, when old, receive the buffer which consists of the buffer they inherited plus their own contribution. This payout certainly exceeds their own contribution, so lifetime income exceeds 1. Hence, it is optimal for the current young to participate. This shows that there is no valid decision rule in this economy.[10]

4. Three applications

In this section we will use the above model to analyse three specific pension arrangements, focussing on the possibility that a new generation of young individuals decides not to participate.

[10] One could wonder why this economy operates a funded system at all and how the pension fund ever managed to accumulate its buffer. A possible reason is that somewhere in the past an external party (e.g., the government) created this buffer or that the birth rate unexpectedly changed. However, this example is solely intended to illustrate the potential non-existence of a valid decision rule.

The above analysis can be applied, because in all three applications the assets managed within the system return to the same constant value.

In all our applications we will assume the same structure of the underlying economy with the same underlying uncertainties. Given our two-cohort model, one period consists of 30 years. We model the gross return R as a shifted log-normal process, using Campbell et al. (2003) for the calibration of the portfolio return. The risk-free interest rate is set to 2.1% and the risk premium on equity is 6.8%, with a standard deviation of 18.2%. Given the long investment horizon of individuals and pension funds, we assume that both parties invest 75% of the assets under their control in equity and the remaining 25% in a risk-free asset. Hence, the equity share is exogenous. Ideally, we would make it endogenous. However, in models of this type it is difficult to produce an individually-optimal portfolio composition that is realistic. Inflation is set at 2.0%. Hence, the mean annual real return on the whole portfolio is 5.2%. To prevent very low consumption levels we assume that, over a 30 year horizon, the minimum gross return is 25%. For a 30-year period and a log-normal distribution for the gross real return, this translates into a gross return with mean 4.57 and standard deviation 3.69. The gross birth rate also follows a shifted log-normal distribution, calibrated to the average annual population growth rate of 1.06% and its standard deviation of 0.47% in the Netherlands from 1900 to 2012. Over a 30-year period, this implies a shifted log-normal distribution with a mean gross birth rate of 1.37 and a standard deviation of 0.035. We assume that there is a minimum birth rate over the 30-year horizon of 75%, which corresponds to a shrinkage of the population by 1% per year.[11]

For each system that we describe below, we explain the qualitative effect of the level of financial and demographic uncertainty. While demographic developments are highly predictable in the short run, over the typical time horizon relevant for pension arrangements, demographic uncertainty can still become quite large, which motivates us to also incorporate this source of uncertainty into the model. For simplicity, we assume that the processes for financial market returns and population growth are independent of each other, so their joint probability density function $p(R, b)$ can be written as $p(R, b) = p_R(R)p_b(b)$.[12]

Finally, for utility we assume Epstein–Zin preferences

$$\mathcal{U}_v = \left(c_{v,y}^{1-\rho} + \beta \mathrm{E}_v \left[c_{v+1,o}^{1-\eta} \right]^{\frac{1-\rho}{1-\eta}} \right)^{\frac{1}{1-\rho}}, \tag{14}$$

where β denotes the discount factor, ρ the intertemporal elasticity of substitution and η a measure of risk aversion. We set the discount factor over our 30-year period at 0.5, which translates into an annual discount factor of roughly 0.977, and use $\rho = 1.5$ and $\eta = 10$. This is in line with most of the related literature, which assumes a coefficient of risk aversion substantially above one (see e.g. Weil (1989) and de Menil et al. (2016)). Estimates of especially risk aversion show large variation (see, e.g. Beetsma and Schotman, 2001), so we vary it in our sensitivity analysis. Our focus will be on expected utility when old, $\mathrm{E}_v[c_{v+1,o}^{1-\eta}]$, since this term depends on the states in which the system continues or not.

4.1. Defined benefit pay-as-you-go

Pay-as-you-go systems are the dominant public pension arrangement in the OECD. In a PAYG system, the pension benefits of the old generation are fully covered by the contributions of the young, so the system is financially balanced in each period. The system starts without any assets, hence $a = 0$ in all periods. We assume that it is of the DB type with a fixed pension benefit, so $\theta(\phi) = \theta$ for all ϕ. An advantage of a DB scheme is that this provides the elderly with a constant income on top of the uncertain resources associated with their savings. Indeed, real-world PAYG systems tend to contain important DB elements, for example by setting a minimum benefit or linking the benefit to the minimum wage, as is the case in the Netherlands. We assume that in case the system collapses, since there are no assets and the young refuse to contribute, the elderly receive no pension benefit, so $\theta^c(\phi) = 0$. Their old-age consumption would then be financed entirely out of their private savings. Such an extreme outcome may be slightly unrealistic in view of the political power of the elderly. However, any alternative fall-back arrangement would be rather arbitrary, while it is a priori not clear that there exists an equilibrium contribution threshold corresponding to such an alternative arrangement.

In our DB-PAYG scheme the contribution paid by new entrants varies to absorb all the shocks:

$$\tau = \theta/b. \tag{15}$$

Since $b \sim 0.75 + \mathrm{LogN}(\mu_b, \sigma_b^2)$, the contribution $\tau \in \langle 0, \theta/0.75]$. The lifetime budget constraint imposes an extra upper limit on the contribution. Any contribution higher than the initial endowment of $y = 1$ can be excluded from the analysis, because it violates the lifetime budget constraint when the system collapses in the next period and the pension benefit is zero. Savings must be positive to ensure positive consumption when old if the system collapses, but they cannot exceed $1 - \tau$ to prevent negative consumption when young. The presence of the PAYG system allows a reduction in savings and, hence, a reduction in the exposure to financial market risk at a given expected consumption level when old. However, the old may now be exposed to the fertility shock.

Given this specification, we can write

$$\mathrm{E}\left[c_o^{1-\eta}\right] = \int_{D(\tau^*)} (R's + \theta)^{1-\eta} p(\phi') + \int_{D^c(\tau^*)} (R's)^{1-\eta} p(\phi'),$$

where s is given by the first-order condition associated with the individual's intertemporal trade-off.[13] The contribution depends only on the birth rate, so given a threshold τ^*, there is a threshold for the birth rate $b^* = \theta/\tau^*$ below which the system collapses,

11 In terms of the usual lognormal distribution we have $R = 0.25 + \tilde{R}$, with $\tilde{R} \sim \mathrm{LogN}(1.19, 0.55)$ and $b = 0.75 + \tilde{b}$, with $\tilde{b} \sim \mathrm{LogN}(-0.47, 0.003)$, which gives the minimum, mean and standard deviation mentioned above. Using a shifted lognormal distribution for the gross birth rate is standard in the literature (e.g., see Limpert et al., 2001). We explored the sensitivity of the contribution thresholds to the use of a uniform distribution for the gross birth rate. Thresholds hardly change as long as the mean and variance are kept the same and very low gross birth rates are unlikely.

12 Systematic empirical evidence on the relationship between population growth and the equity premium seems rather hard to obtain. Yu (2002) finds evidence of a positive relationship between population growth and bond and stock returns for both small and large companies in the U.S., but unfortunately does not present direct evidence on the relationship between the equity premium and population growth. More work has been done on the study of the relationship between the age composition of the population and the equity premium. Geanakoplos et al. (2004) report a higher equity premium for the U.S. when the ratio of the middle-aged to the young is relatively low, while evidence for other large developed countries suggests that an increasing ratio of middle-aged to young is accompanied by high stock market returns over the period of the nineteen eighties and nineties. Kuhle et al. (2007) present a quantitative theoretical analysis for the U.S. that suggests an increase in the equity premium when a relatively small young cohort enters the labour market. In view of the lack of a consensus figure for the correlation between population growth and the equity premium, we stick to our assumption that the two processes are independent.

13 Concretely, savings s is determined by substituting $c_y = 1 - \theta/b - s$ for $c_{v,y}$ and the right-hand side of the expression for $\mathrm{E}\left[c_o^{1-\eta}\right]$ for $\mathrm{E}_v\left[c_{v+1,o}^{1-\eta}\right]$ into expression (14), and maximising the result with respect to s.

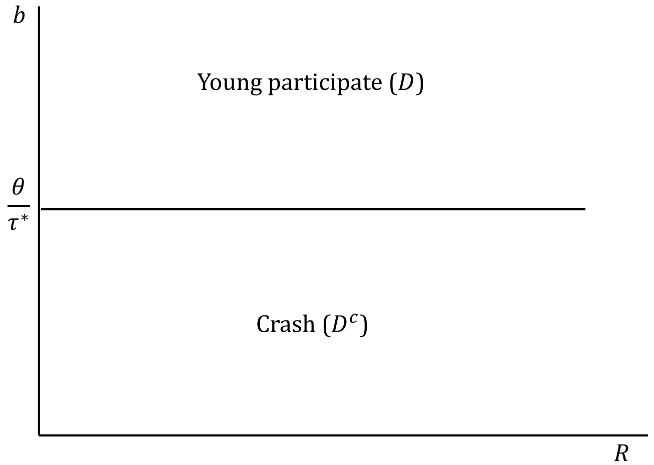

Fig. 1. Participation vs. autarky in the PAYG example.

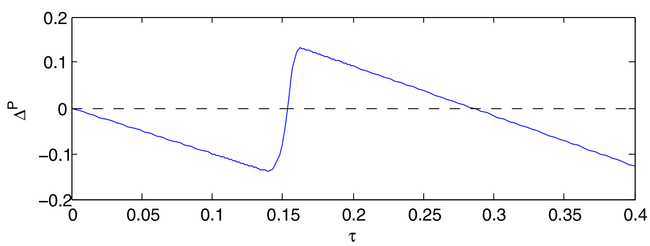

Fig. 2. $\Delta^p(\tau)$ in the PAYG system ($\theta = 0.2$).

that is, $D(\tau^*) = \{R', b' | b' \geq \theta/\tau^*\}$ and $D^c(\tau^*) = \{R', b' | b' < \theta/\tau^*\}$ – see Fig. 1. Using this area of integration we can write $\mathrm{E}\left[c_o^{1-\eta}\right]$ as

$$\mathrm{E}\left[c_o^{1-\eta}\right] = [1 - P_b(\theta/\tau^*)]\mathrm{E}_R[(R's+\theta)^{1-\eta}] + P_b(\theta/\tau^*)\mathrm{E}_R[(R's)^{1-\eta}], \tag{16}$$

where $P_b(\cdot)$ is the cumulative density function of b, and $\mathrm{E}_R(\cdot)$ is the expected value with respect to R'.

In this PAYG system we have $\Delta^p(0) = 0$, $\Delta^{p\prime}(0) < 0$, $\lim_{\tau\to 1} \Delta^p(\tau) < 0$, and $\lim_{\tau\to 1} \Delta^{p\prime}(\tau) = -\infty$ (see Appendix A.2 for details). Continuity implies that $\Delta^p(\tau) = 0$ has, besides the solution $\tau = 0$, zero roots or an even number of roots. Numerical results show that for any reasonable parameter constellation $\Delta^p(\cdot)$ has at most one such pair, which always comes as a combination of an unstable and a stable solution. At low values of τ, the negative wealth effect of an increase in τ dominates its beneficial insurance effect and utility is decreasing in τ. Raising τ leads to a decreasing probability of a collapse (the likelihood that $b' \geq \theta/\tau$ rises) and this effect may at some point start to dominate the wealth effect, leading to positive values of $\Delta^p(\cdot)$. However, for large enough τ, the wealth effect always dominates and $\Delta^p(\cdot)$ is negative.

Fig. 2 shows Δ^p for values of τ ranging from 0 to 0.4, i.e. 40% of the first period's endowment, and a pension benefit θ of 20% of the initial endowment. For this parameter constellation, the function $\Delta^p(\tau)$ has three roots: the trivial one at $\tau = 0$, an unstable root at 0.154 and a stable root at 0.290. Given our parameter values, there is a nearly-zero probability that the required contribution is higher than the threshold of 0.290. Hence, there are two possibilities: either the system collapses immediately or a collapse of this PAYG system within a reasonable time horizon is rather unlikely.

These thresholds and probabilities are mostly determined by the level of risk aversion. A lower level of risk aversion makes participation less attractive; it rotates the whole Δ^p curve in the upper panel of Fig. 2 clockwise. A higher level of financial uncertainty makes participation more attractive since the PAYG system provides some insurance against bad shocks. The demographic uncertainty only has a second-order effect. Doubling the variance of the birth rate without changing the mean birth rate has no measurable effect on the contribution thresholds.

4.2. Minimum return on pension contribution

Beetsma et al. (2012) study a system in which a pension fund guarantees a minimum return R^* on a basic pension contribution ζ. Minimum return guarantees are widely applied in defined contribution pension schemes (see Antolín et al., 2012). For example, plan providers have to offer a return level guarantee in Belgium, the Czech Republic, Germany, Japan, the Slovak Republic and Switzerland, while return guarantees relative to some benchmark apply in Chile, Denmark, Hungary, Poland and Slovenia. Minimum return guarantees may come in different guises, such as fixed or variable and being applied only at retirement date or throughout the accumulation phase. Munnell et al. (2009) and Grande and Visco (2010) explore the costs and benefits of different types of minimum return guarantees. In this subsection, we consider a minimum return in the context of a funded pension arrangement without any asset buffers. The fund starts with zero assets and, to ensure the recursiveness of the equilibrium, each period it ends with zero assets. The system is funded, because participants earn at least the market return on their basic contribution. The reason for studying the case with a zero asset buffer is that it allows us to highlight the role of guaranteeing a minimum return and that it provides a stepping stone for the next subsection, in which we introduce a buffer in our funded pension scheme. A funded system with no minimum return guarantee, a limiting case, is effectively an individual defined-contribution system. Since the financial return on pension assets is the same as the financial return on private savings, individuals are indifferent between contributing to the pension fund and saving for themselves. If mandatory contributions exceed their preferred savings, they can perfectly offset the difference by borrowing. Utility under participation in an individual defined-contribution system is always equal to utility under autarky.

With the minimum return guarantee, the pension benefit of the old generation when the system persists is

$$\theta = \begin{cases} R\zeta, & \text{if } R > R^*, \\ R^*\zeta, & \text{if } R \leq R^*. \end{cases} \tag{17}$$

Because the pension fund keeps no buffers, if the return on its financial markets investment exceeds the minimum level R^*, the fund has enough resources to pay the pension benefits. If the return on its investment is lower than the minimum return R^*, the young must make an additional contribution that depends on the deficit of the fund. Therefore, the total contribution by the young is

$$\tau = \begin{cases} \zeta, & \text{if } R > R^*, \\ \zeta + (R^* - R)\zeta/b, & \text{if } R \leq R^*. \end{cases} \tag{18}$$

Hence, the total contribution is at least the so-called "regular contribution" ζ. Like the PAYG system above, this system is also fully recursive. If the young participate, they replenish the assets managed by the pension fund to ζ since $a + \tau - \theta/b = \zeta$.[14] The

[14] Notice that, initially, $a_0 + \tau_0 - \theta_0/b_0 = \zeta$, since $a_0 = \theta_0 = 0$ and $\tau_0 = \zeta$. Hence, using (3), if $R_1 \leq R^*$, $a_1+\tau_1-\theta_1/b_1 = R_1\zeta/b_1+\zeta+(R^*-R_1)\zeta/b_1-R^*\zeta/b_1 = \zeta$, while if $R_1 > R^*$, $a_1+\tau_1-\theta_1/b_1 = R_1\zeta/b_1+\zeta-R_1\zeta/b_1 = \zeta$. This extends to all future t.

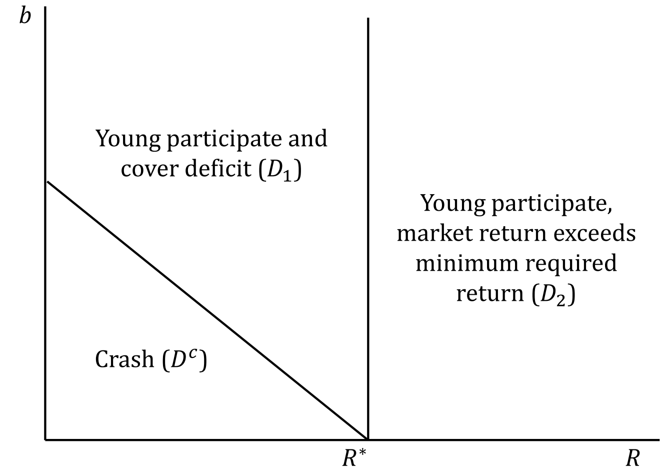

Fig. 3. Participation vs. autarky in the minimum return example.

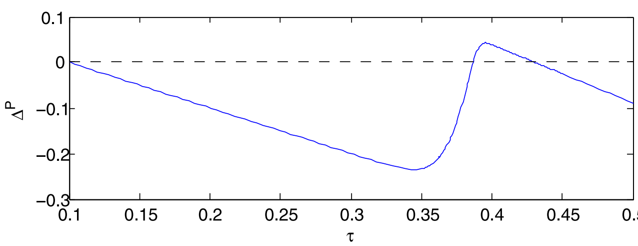

Fig. 4. $\Delta(\tau)$ for the funded system without buffers ($\zeta = 0.1$, $R^* = 4.0$).

pension benefit of an old individual if the new young decide not to participate is

$$\theta^c = R\zeta. \tag{19}$$

Given that the contribution is at least equal to ζ according to (18), we can ignore cases in which $\tau^* < \zeta$, so we have

$$\mathrm{E}\left[c_o^{1-\eta}\right] = \int_{D_1(\tau^*)} (R's + R^*\zeta)^{1-\eta} p(\phi') + \int_{D_2(\tau^*)\cup D^c(\tau^*)} (R'(s+\zeta))^{1-\eta} p(\phi'), \tag{20}$$

where $D_1(\tau^*) = \{R', b' | R' < R^*, (R^* - R')/b' \le \tau^*/\zeta - 1\}$ are the states in which the new generation has to cover a deficit, but participates nonetheless, because the total contribution of a new young person is less than the threshold τ^*. Further, $D_2(\tau^*)$ consists of the states in which $R' \ge R^*$, so the new generation only has to pay the regular contribution ζ. By definition we have $D_1(\tau^*) \cup D_2(\tau^*) = D(\tau^*)$, the set of next-period states in which the system continues to exist. Finally, $D^c(\tau^*) = \{R', b' | \left(R^* - R'\right)/b' > \tau^*/\zeta - 1\}$, i.e. all the states in which the system collapses. Fig. 3 depicts the areas in (R, b) space.

The belief $\tau^* = \zeta$ is a stable equilibrium. As in the PAYG example, the Δ^p-function is zero and downward sloping at the minimum contribution, now $\tau = \zeta$, and negative at the maximum contribution $\tau = y + \zeta$ that can still guarantee non-negative old-age consumption. This again implies that besides the trivial minimum contribution, the Δ^p-function has an even number of roots in (unstable, stable)-combinations.

Fig. 4 shows Δ^p for values of τ ranging from 0.1 to 0.5, assuming a basic contribution of $\zeta = 0.1$ and a minimum return $R^* = 4.0$. This is guaranteed by the pension system, as long as it does not collapse. This minimum gross return is 57 percentage points lower than the expected gross return over the same period. For this parameter constellation, the function $\Delta^p(\tau)$ has three roots: the trivial one at $\tau = \zeta = 0.1$, an unstable root at 0.39 and a stable root at 0.43. Given our parameters, the probability that the total contribution exceeds 0.1 is 57% and the probability that the total contribution exceeds 0.43 is very small. Depending on the selected threshold the system collapses immediately if the contribution is higher than ζ (with probability of 57%), or a collapse within a reasonable time horizon is unlikely. These thresholds and probabilities are mostly determined by the level of risk aversion, the rate of time preference and the financial uncertainty. The demographic uncertainty only has a second-order effect. As in the PAYG system, doubling the variance of the birth rate without changing the mean birth rate again has no measurable effect on the thresholds.

4.3. Minimum return with buffer

As a third example we discuss the case in which the pension fund again guarantees a minimum return on the basic contribution ζ, but it also maintains a minimum buffer equal to a fraction $\alpha \ge 0$ of the basic contribution. That is, after the cash in- and outflows have taken place in a given period, the fund's assets per young individual must be equal to $(1+\alpha)\zeta$. Such a buffer is a general feature of private pension providers that guarantee a minimum return on the contribution. This is not surprising as the buffer helps to avoid drastic increases in contributions needed to fulfil the minimum return requirement. In defined-benefit funded systems, such as the states' public-sector worker pension funds in the U.S. and the occupational pension funds in the Netherlands, participants build up entitlements, the honouring of which requires sufficiently large buffers. When buffers drop to too low levels, they need to be restored in part by raising contributions levied on the active participants.

The pension benefit is the same as in the previous example, i.e. it is given by (17). Hence, as long as the system persists, the participants again receive a minimum return R^* on their contribution. However, if the system collapses, because the young decide not to participate, every member of the old generation now receives

$$\theta^c = R(1+\alpha)\zeta. \tag{21}$$

This payout after a crash is higher than in the previous system, because the old now also receive the buffer.

To ensure the fund's asset level of $(1+\alpha)\zeta$, the total contribution must now be equal to

$$\tau = \begin{cases} \zeta + \alpha\zeta[1 - R/b], & \text{if } R > R^*, \\ \zeta + (R^* - R)\zeta/b + \alpha\zeta[1 - R/b], & \text{if } R \le R^*. \end{cases} \tag{22}$$

These contributions differ from (18) in the previous example by the term $\alpha\zeta[1 - R/b]$. This difference between the required contribution per young in a system with a buffer ($\alpha > 0$) and a system without a buffer consists of two parts. In the case of a buffer the young have to inject an additional amount $\alpha\zeta$ into the system, but they inherit $\alpha\zeta R/b$. The inheritance is the gross return on the buffer in the previous period divided by the gross population growth. If the size of the young generation is larger relative to the old generation, then the inheritance per young person falls. Whenever the inherited buffer exceeds the required buffer $\alpha\zeta$, the net effect $\alpha\zeta[1 - R/b]$ is negative and the contribution per young individual is lower than in a system without a buffer. Formally, the effect of a buffer on the required contribution (standardised by ζ) is given by

$$\frac{\partial(\tau/\zeta)}{\partial\alpha} = 1 - \frac{R}{b} \gtrless 0.$$

For realisations of R above the birth rate b the presence of buffers lowers the required contribution paid by a young individual.

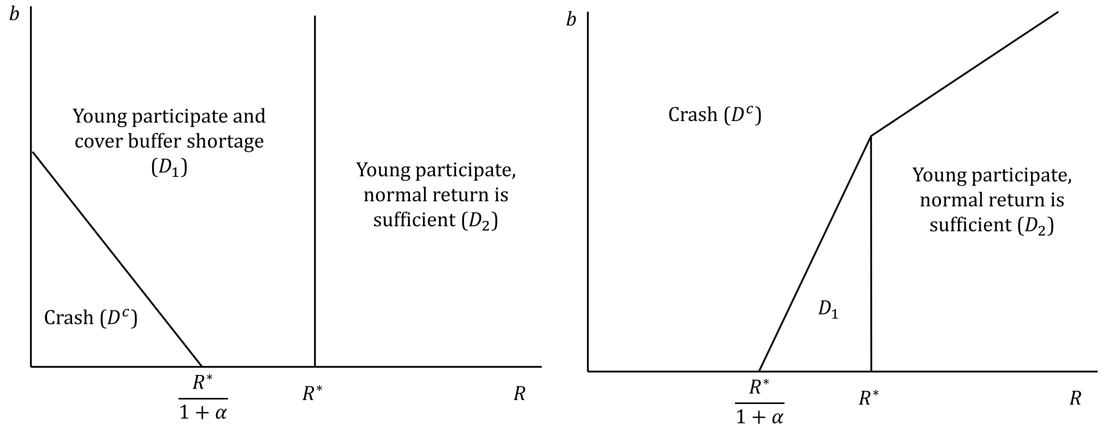

Fig. 5. Participation vs. autarky in the buffer system with $\tau^* > (1+\alpha)\zeta$ (left panel) and $\tau^* < (1+\alpha)\zeta$ (right panel).

A second effect of a buffer is that it increases the sensitivity of the contribution to the return on investments. The slope of the standardised total contribution depends on the region:

$$\frac{\partial(\tau/\zeta)}{\partial R} = \begin{cases} -\alpha/b < 0, & \text{for } R > R^*, \\ -(1+\alpha)/b < 0, & \text{for } R \leq R^*. \end{cases}$$

In the case of a buffer, a higher return always lowers the total contribution, even if $R > R^*$. This is due to the fact that any additional return on the buffer is not needed to finance the pension payouts, but can be used to lower the total contribution of the young. Higher buffer requirements reinforce this effect,

$$\frac{\partial^2(\tau/\zeta)}{\partial R \partial \alpha} = -\frac{1}{b} < 0,$$

and increase the sensitivity of the total contribution to financial shocks.

A third effect, not highlighted before in the literature, is that a larger young cohort does not necessarily lower the contribution of the young. Differentiating the standardised contribution with respect to the gross rate of population growth yields:

$$\frac{\partial(\tau/\zeta)}{\partial b} = \begin{cases} \frac{\alpha R}{b^2} > 0, & \text{for } R > R^*, \\ \frac{(1+\alpha)R - R^*}{b^2} \gtrless 0, & \text{for } R \leq R^*. \end{cases}$$

First focus on the case of $R > R^*$. The smaller is the young generation, i.e. the lower is b, the more buffer there is per young person, hence the lower the total contribution can be. If $R \leq R^*$, there are two cases. One is when $R(1+\alpha) > R^*$, hence there are "excess" buffers[15] left in the fund after the old have received their benefit and, hence, a smaller new generation is better off since they can divide these buffers over a smaller group of individuals. The other case is when $R(1+\alpha) < R^*$. In this case, the new generation prefers a big cohort (high birth rate), so that the burden of replenishing the fund can be distributed over more individuals.

In this example,

$$\mathrm{E}\left[c_o^{1-\eta}\right] = \int_{D_1(\tau^*)} (R's + R^*\zeta)^{1-\eta} p(\phi') + \int_{D_2(\tau^*)} (R'(s+\zeta))^{1-\eta} p(\phi') + \int_{D^c(\tau^*)} (R'(s+(1+\alpha)\zeta))^{1-\eta} p(\phi'),$$

15 It is easy to see that if $R = R^*/(1+\alpha)$, then the assets with which the fund enters the period exactly cover the benefits paid out to the old, so that the build up of the new required buffer is financed fully by the contributions of the young. Hence, a situation in which $R > R^*/(1+\alpha)$ could be referred to as one of an "excess" buffer.

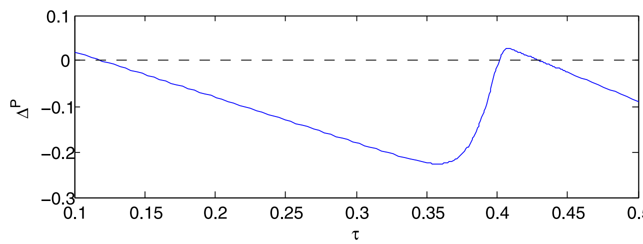

Fig. 6. $\Delta(\tau)$ for the funded system with a buffer ($\zeta = 0.1$, $R^* = 4$, $\alpha = 0.2$).

where $D_1(\tau^*)$ consists of all combinations of R' and b' such that $R' \leq R^*$ and $(R^* - R')/b' + \alpha\left[1 - R'/b'\right] \leq \tau^*/\zeta - 1$. $D_2(\tau^*)$ consists of all combinations of R' and b' such that $R' > R^*$ and $\alpha\left[1 - R'/b'\right] \leq \tau^*/\zeta - 1$. Hence, $D_1(\tau^*) \cup D_2(\tau^*)$ is the set of all combinations of R' and b' such that the contribution threshold is not exceeded and the system persists. As before, $D^c(\tau^*)$ is the complement of $D_1(\tau^*) \cup D_2(\tau^*)$ and indicates when the system collapses — see Fig. 5.

Note that for $\tau^* < \zeta(1+\alpha)$ the system may collapse even if $R > R^*$. This happens if the new cohort is relatively large, hence the existing buffers are low in per young capita terms, so that the new cohort has to contribute more than ζ to ensure that the buffer is restored to its required level. The risk to this cohort is that next period's return could be less than the guaranteed return R^*, but that the next young still decide to participate. The current young then only receive $R^*\zeta$, so they lose their contribution to the buffer.

Fig. 6 shows Δ^p for values of τ ranging from 0.1 to 0.5, assuming a buffer of 20 percent ($\alpha = 0.2$) and, as before, a basic contribution ζ of 10% of the initial endowment and a minimum gross return R^* of 400%. The basic contribution $\zeta = 0.10$ does not correspond to an equilibrium threshold since Δ^p is positive. If the system continues, the return on this contribution is at least the return on private savings. If the next generation does not participate, the return on this contribution is higher than the return on private savings since the current generation can then also claim the buffers when they are old. For this parameter constellation, the function $\Delta^p(\tau)$ has three roots: one at $\tau = 0.12$, which is equal to $(1+\alpha)\zeta$, an unstable root at 0.40 and a stable root at 0.43. There is a 46.5% chance of a higher total contribution than 0.12, while the chance that the required contribution is higher than the high threshold of 0.43 is again very low. The lower stable root corresponds to the left corner solution in the previous example without a buffer. Compared to that situation, the probability of a collapse is lower in this case.

Table 1
Thresholds and probabilities of a collapse of the buffer system for various parameter settings.

	1st stable threshold[a]	Prob. of collapse	2nd stable threshold[a]	Prob. of collapse
Base	12.0%	46.5%	43.1%	0.0%
α				
10%	11.0%	51.6%	43.1%	0.0%
30%	13.0%	41.9%	43.1%	0.0%
R^*				
300%	12.0%	30.4%	36.9%	0.0%
500%	12.0%	59.3%	–	–
η				
6.0	12.0%	46.5%	–	–
12.0	12.0%	46.5%	46.6%	0.0%

[a]as percentage of the initial endowment y

As before, we ignore unstable thresholds. Table 1 shows the numerically computed stable thresholds and probabilities of a collapse of the buffer system for various parameter constellations. We vary respectively the buffer size α, the minimum return R^* and the risk aversion η, each time keeping all the other parameters at their baseline levels given by $\alpha = 0.20$, $\eta = 10$ and $R^* = 4.0$, shown in the row labelled " Base". There are never more than two stable thresholds, while in some cases there is only one. The first block shows that increasing the buffer leads to a lower probability of a collapse associated with the lowest threshold, as expected. There is a second stable threshold of 43.1%, which is virtually unaffected by the value of α. If the current generation actually has this "optimistic" belief, a crash of the system in a given period is low. Obviously, over a longer period, the chance of a collapse accumulates. The second block shows the effect of a lower and higher minimum return R^*. The first (low) threshold is essentially unaffected by higher promised returns. The probability of a collapse increases, because for higher promised returns the next young generation is more likely to be confronted with low or even empty buffers that must be replenished. The third block shows the effect of an increase in the relative risk aversion parameter. As expected, this increases the willingness to participate as indicated by a higher possible contribution threshold. For a level of 6.0, there is only one low threshold and the probability of a crash is 46.5%. For a sufficiently higher level of relative risk aversion, there is a second, high threshold, which essentially guarantees the survival of the system over the foreseeable future.

5. Optimal pension schemes

This section analyses for the above three sample arrangements how the welfare maximising values of the pension parameters change with risk aversion and uncertainty regarding the financial return and demographic risk. Because the model is stylised, the emphasis is on the intuition behind the direction in which the welfare maximising values of the pension parameters change, rather than on the magnitude of the effects. We will also discuss the direction of the associated welfare effects.

The system designer sets the parameters of the pension system, and these parameters result in a threshold τ^* and a probability of a collapse P in each period. The payout to the current old (θ_0) and the contribution of the current young (τ_0) are known at the moment the system is started. If the system collapses, it will remain in autarky forever. For our baseline parameter constellation, all arrangements have two stable equilibrium values for the threshold τ^*, a relatively low one and a relatively high one. The question is which of the two equilibrium thresholds is the relevant one. We select the relevant equilibrium threshold by imposing that it be consistent with the participation constraint of the *first* young generation. This generation is only willing to participate if τ^* is such that $\mathcal{U}^p(\tau_0, \tau^*) \geq \mathcal{U}^a$. It turns out that this additional equilibrium selection criterion rules out the low threshold. For the PAYG arrangement the low threshold is zero, implying that the first young generation would never be willing to make a positive payment to the old, so that it would never be possible to start a system with voluntary participation. Under the funded scheme without a buffer, as argued above, $\tau^* = \zeta$ is a stable threshold; the first young are willing to just pay the regular contribution rate when this low equilibrium threshold applies, but not more than that. This implies that they do not share shocks with future generations. If subsequent generations are also faced with the low equilibrium, they also refuse to participate when their required contribution exceeds the regular contribution. Hence, for the low equilibrium $\tau^* = \zeta$, the pension system collapses when the return is lower than the promised minimum return, and the system provides no insurance at all. For the funded scheme with a buffer $\alpha > 0$, for any reasonable value of risk aversion, the low equilibrium threshold is slightly below $(1+\alpha)\zeta$, the contribution to be made by the first generation in order to build up the buffer. Summarising, if the low equilibrium threshold is the relevant one, it would be impossible to start a pension system that actually provides risk sharing.

Based on these findings, we rule out the low-equilibrium threshold and focus on the high-equilibrium threshold.[16] Our results below suggest that the high-equilibrium threshold is almost never binding and the system would very likely survive over the foreseeable future. Obviously, given the stylised nature of the model, we should be careful not to put too much emphasis on the precise values of the thresholds. Introduction of more refinement into the model likely results in more realistic values for the high-equilibrium threshold. In particular, here the length of the retirement period is assumed to be the same as the length of the working life. In reality, the former is roughly half of the latter, implying that the same total benefit would correspond to a substantially larger replacement rate of the wage during the active life or, vice versa, that the current replacement rate would be achieved with a much lower contribution rate.

The system designer chooses the pension parameters to maximise social welfare, which is the discounted (to period 0, the moment the system is started) sum of the expected utilities of all generations alive and born in period 0 and later. The first old generation is only affected by this pension system via their

16 In principle, one might try to motivate the low equilibrium threshold by events outside the model. For example, the experience of a severe crisis seems to be a good predictor of the emergence of benefit systems. In particular, the U.S. President Franklin D. Roosevelt presented the Social Security Act of 1935 as a response to the consequences of the Great Depression for vulnerable groups like senior citizens. This response may well have been made acceptable by solidarity feelings towards these groups, even though the initial contributors did not expect to obtain any net benefit from the arrangement — for more discussion, see Beetsma et al. (2016). However, we will not follow this route, but require that the participation constraint be fulfilled for all generations, including the first young generation.

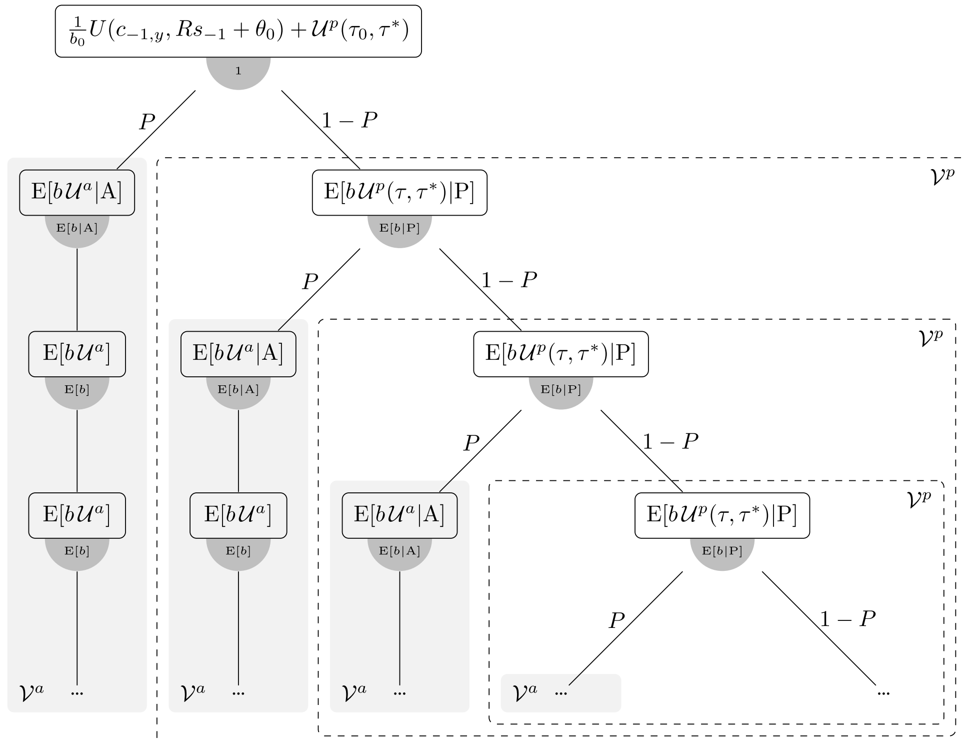

Fig. 7. Order of events to measure welfare. The blocks denote payouts (utility of a specific cohort), the grey half circles under these payouts denote the weight of the sub-tree following this payout. The light grey blocks indicate $\mathcal{V}^a$, while the blocks demarcated by dashed lines indicate $\mathcal{V}^p$.

old-age consumption. Hence, this criterion combines the ex-ante criterion applied to the current young and the future-born and the ex-interim criterion applied to the current old, which is a common approach in the related literature. We evaluate social welfare for given values of θ_0 and τ_0. The factor used to discount the utility of future generations is assumed to be the same as the individual discount factor β. Fig. 7 shows the order of events. Because we confine ourselves to the high stable equilibrium threshold τ^*, the initial young want to participate and the system can be started. By assumption, the root node is certain. Then, there are two possibilities. The first possibility, with probability P, is that the next young generation chooses autarky and the system stays in autarky forever. This gives "continuation" social welfare $\mathcal{V}^a$. The second possibility is that the next generation participates, which is valued at $\mathcal{V}^p$. Using the recursiveness of the model, we can write social welfare $\mathcal{W}^{\mathcal{P}}$ under participation in period 0 as

$$\mathcal{W}^{\mathcal{P}} = \frac{1}{b_0}U(c_{-1,y}, Rs_{-1}+\theta_0)+\mathcal{U}^p(\tau_0,\tau^*)+\beta\left\{P\mathcal{V}^a+(1-P)\mathcal{V}^p\right\}, \tag{23}$$

with

$$\mathcal{V}^a = \mathrm{E}[b\mathcal{U}^a|\mathrm{A}] + \mathrm{E}[b|A]\left((\mathrm{E}[b]\beta)^1+(\mathrm{E}[b]\beta)^2+(\mathrm{E}[b]\beta)^3+\dots\right)\mathcal{U}^a,$$
$$\mathcal{V}^p = \mathrm{E}[b\mathcal{U}^p(\tau,\tau^*)|\mathrm{P}] + \beta\mathrm{E}[b|\mathrm{P}]\left(P\mathcal{V}^a+(1-P)\mathcal{V}^p\right),$$

where $\mathcal{V}^a$ indicates the welfare value of staying in autarky (the light grey blocks in Fig. 7) and $\mathcal{V}^p$ the welfare value of continued participation (the blocks demarcated by the dashed lines in the figure). Further, $\mathrm{E}[b|\mathrm{A}]$ and $\mathrm{E}[b|\mathrm{P}]$ measure the expected value of the birth rate given that the next generation chooses autarky, respectively participation. $\mathrm{E}[b\mathcal{U}^p(\tau,\tau^*)|\mathrm{P}]$ measures utility under participation, corrected for the cohort size. Note that b and $\mathcal{U}^p(\tau,\tau^*)$ are correlated since the birth rate affects the required contribution and, hence, also the willingness to participate. Solving $\mathcal{V}^a$ and $\mathcal{V}^p$ gives,

$$\mathcal{V}^a = \frac{\mathrm{E}[b|A]}{1-\beta\mathrm{E}[b]}\mathcal{U}^a,$$
$$\mathcal{V}^p = \frac{\mathrm{E}[b\mathcal{U}^p(\tau,\tau^*)|\mathrm{P}]+\beta\mathrm{E}[b|\mathrm{P}]P\mathcal{V}^a}{1-\lambda},$$

with $\lambda = \beta(1-P)\mathrm{E}[b|\mathrm{P}]$. Welfare is defined as long as $0 < \beta\mathrm{E}[b] < 1$ and $0 < \lambda < 1$. These conditions are fulfilled for modest birth rates or a sufficiently high probability of a collapse of the system. For the examples with a funded system (Sections 4.2 and 4.3), $\theta_0 = 0$, so the current old are unaffected by the introduction of a pension system, and the first term in (23) may be ignored.

Now we will turn to the search for the optimal pension arrangements under our three sample systems. We start from our baseline parameter values and focus on two values for the relative risk aversion parameter η, namely 6 and 10. This yields two stable thresholds for each of the arrangements we consider. As argued above we always consider only the higher threshold. Relative to the baseline parameter setting we also consider variations on this baseline in which we raise the uncertainty parameter σ_r of the financial process by 50% and/or the uncertainty parameter σ_b of the birth process by 50%. To compensate for the effect on the expected return, the parameters μ_r and μ_b are correspondingly reduced, so the expected financial return and expected population growth remain unchanged.

The expected financial market return is higher than the expected population growth rate, so in the absence of uncertainty, the Aaron condition indicates that a PAYG system would lower welfare (see Table 2). For a combination of low risk aversion and a low level of financial uncertainty, this result is not overturned and it is still optimal to have no PAYG system at all. For higher levels of risk aversion and/or financial uncertainty, the

Table 2
Welfare maximising pension parameters for the PAYG system. $\mathcal{W}^{\mathcal{P}}$ and $\mathcal{W}^{\mathcal{A}}$ measure welfare in certainty-equivalent consumption units.

η	6				10			
std(R)	3.69		4.52		3.69		4.52	
std(b)	0.035	0.043	0.035	0.043	0.035	0.043	0.035	0.043
θ	0	0	0.33	0.33	0.36	0.36	0.40	0.40
τ^*	–	–	0.35	0.35	0.41	0.41	0.49	0.49
$\mathcal{W}^{\mathcal{P}}$	1.23	1.23	1.17	1.17	1.15	1.15	1.14	1.14
$\mathcal{W}^{\mathcal{A}}$	1.23	1.23	1.16	1.16	1.11	1.11	1.04	1.04

insurance provided by the PAYG system against low financial returns more than compensates for the difference between the financial market return and the expected population growth rate, and it is optimal to contribute 33%–40% to the PAYG system. This outcome is not surprising, because an increase in financial market uncertainty lowers the level of personal retirement assets when financial markets perform poorly. Along with the increase in the optimal pension benefit, we observe an increase in the equilibrium threshold for the contribution rate. The effects of an increase in the uncertainty of the birth rate on the optimal benefit level and the contribution threshold are tiny, the reason being that the birth rate does not have any direct effect on utility. It only has an effect through the probability of a collapse of the pension arrangement. An increase in the degree of risk aversion raises the penalty associated with fluctuations in resources during retirement and, hence, a larger stable source of income becomes desirable. As a result, the optimal pension benefit rises and, along with it, the maximum willingness of the young to contribute. The final two rows of Table 2 report the welfare levels under participation ($\mathcal{W}^{\mathcal{P}}$) and in autarky ($\mathcal{W}^{\mathcal{A}}$) in certainty-equivalent consumption units. We see that for relatively low risk aversion and low financial risk, the welfare difference between PAYG and autarky is negligible. When risk aversion or financial uncertainty increase, the welfare gain from PAYG instead of autarky increase. For the combination of high risk aversion and high financial risk, in order to have the same level of welfare as under the optimal PAYG system, private endowments in autarky need to increase by around 10%.

Tables 3 and 4 report the optimal arrangements for our funded pension systems. For the system with minimum returns but without buffers (Table 3), we see that the optimal promised minimum gross return R^* is about 1.5 times higher than the expected return of 4.57 over a 30 year horizon. One striking feature of all optimal arrangements is the high threshold for the total contribution, especially when compared to the regular contribution. The threshold contribution ranges from 28 to 57 percent, while the regular contribution ζ ranges from merely 4.3 to 10.2 percent. The reason is that the penalty for a system collapse is severe. Once the system collapses, the economy will stay in autarky forever and all current and future generations lose the benefits from intergenerational risk sharing. To avoid this, the optimal system has to be such that the probability of a collapse is very low. Higher risk aversion lowers the optimal minimum return R^*. The consequences of a collapse of the fund become more severe if risk aversion is higher. Hence, to reduce the chance that the young would need to make such a large contribution to the scheme that they decide not to participate, the minimum return R^* has to be reduced. A similar effect is observed when we increase financial market uncertainty. Higher risk aversion and higher financial market uncertainty also raise the contribution threshold due to the larger intergenerational risk-sharing gains forgone with a system collapse. Because the population growth rate (1.06% per annum) is lower relative to the expected financial market return (5.2%), welfare under the funded system is potentially much higher than under the PAYG system (compare the welfare figures in Table 3 with those in Table 2).

Table 3
Welfare maximising pension parameters for the funded minimum return system.

η	6				10			
std(R)	3.69		4.52		3.69		4.52	
std(b)	0.035	0.043	0.035	0.043	0.035	0.043	0.035	0.043
ζ	0.043	0.045	0.068	0.070	0.080	0.079	0.102	0.096
R^*	7.74	7.01	6.85	6.32	6.39	6.26	5.74	5.98
τ^*	0.29	0.28	0.42	0.41	0.48	0.47	0.57	0.56
$\mathcal{W}^{\mathcal{P}}$	1.29	1.28	1.29	1.29	1.30	1.30	1.29	1.29
$\mathcal{W}^{\mathcal{A}}$	1.23	1.23	1.16	1.16	1.11	1.11	1.04	1.04

Table 4
Welfare maximising pension parameters for the buffer system.

η	6				10			
std(R)	3.69		4.52		3.69		4.52	
std(b)	0.035	0.043	0.035	0.043	0.035	0.043	0.035	0.043
α	0.00	0.55	2.93	3.28	2.80	3.09	1.16	3.03
ζ	0.045	0.041	0.036	0.025	0.036	0.041	0.038	0.042
R^*	7.33	6.06	10.86	12.85	10.21	8.84	11.61	10.35
τ^*	0.29	0.24	0.35	0.34	0.41	0.41	0.51	0.51
$\mathcal{W}^{\mathcal{P}}$	1.29	1.29	1.35	1.34	1.38	1.37	1.39	1.38
$\mathcal{W}^{\mathcal{A}}$	1.23	1.23	1.16	1.16	1.11	1.11	1.04	1.04

Comparing Table 4 with Table 3 shows that the welfare maximising buffers are substantial — they can be more than three times the regular contribution. The first young generation is willing to build a buffer of up to $\alpha = 3.28$ times the regular contribution, since they receive a high guaranteed return if the next generation participates. If the next generation does not participate, then they get the realised market return on their entire contribution (including the buffer). New generations face an even more attractive deal. First, if future financial returns are sufficient, then they want to participate since their contribution is relatively low due to the return on this buffer; to them, this is a free source of money. Second, they want to participate to have a chance to receive the buffer if the next generation is not willing to participate.

In the first two columns of Table 4, so with a low level of risk aversion and low level of financial market risk, the pension system has little effect on social welfare. Buffers are relatively small (even zero in the first column), and the results are quantitatively comparable to the funded system without buffers. For the other economies, the buffer is sizeable, as is the promised minimum return. More financial uncertainty or a higher level of risk aversion makes the fund with a buffer more attractive. This allows the system to increase the minimum return, which raises welfare. The higher minimum return, which in this economy can be up to almost three times the expected financial market return even dominates the negative welfare effects of the additional financial uncertainty. Comparing the rows for $\mathcal{W}^{\mathcal{P}}$ in Tables 3 and 4 shows that the welfare gain from introducing a pension scheme with a buffer can be 5 to 9 percentage points higher than the gain from introducing a scheme without a buffer.

6. Conclusion

This paper studied the sustainability of pension arrangements based on voluntary individual participation and the cost of their collapse. The outcome of the analysis indicated whether and to what extent mandatory participation can be relaxed without a high risk of such a collapse. This question is important as there is increasing public and political pressure to relax the widespread obligation to participate in existing pension arrangements. We found that the need for making participation mandatory is limited when the promised return on the contribution to the arrangement is set optimally, because the cost of a permanent loss of the intergenerational risk-sharing benefits is sufficiently large to make a collapse of these optimal arrangements unlikely.

We applied our analysis to three important examples, namely a PAYG pension scheme, a pension fund with a minimum return on contributions, but without a buffer, and a fund with a minimum return and a buffer. Our findings have interesting practical implications for pension fund buffer policies, as financial and demographic developments put these buffers under pressure, which prompts enhanced supervisory scrutiny. On the one hand, if the asset returns are sufficiently large, the buffer creates some " free" money that can be used to lower the contributions. However, if the asset returns are relatively low, the incoming cohort not only has to guarantee the pension benefits of the retired, but it also has to replenish the buffer, implying a contribution that is actually higher than in the case without buffers. Overall, the sensitivity of the contributions to the asset returns increases when the buffer requirements are raised. In addition, contrary to common wisdom, individual contributions may be rising or falling in the size of the young cohort, depending on whether the fund has a larger or smaller buffer than required. Finally, also in contrast to common wisdom, a large young cohort in itself does not guarantee the sustainability of a funded scheme, as it may imply a small buffer per young capita and, hence, a relatively high contribution to restore the buffer.

We confined ourselves to recursive settings and explored equilibria characterised by thresholds on the contribution that young individuals are prepared to make. Our numerical analysis showed that for our baseline parameter settings each system features two such equilibria, of which only the one with the higher threshold is consistent with the initial young benefiting from starting the system. The main result of the paper was that, in contrast to the related literature, the equilibrium probability of a collapse of the system is non-zero. We also explored the optimal pension parameter settings from a social welfare perspective, and how these change in response to changes in the fundamental model parameters. Higher risk aversion and more financial market uncertainty make risk sharing more valuable and, hence, they raise the benefit from participation in a pension scheme.

The analysis in this paper may be extended into a number of directions. While we studied the choice whether or not to participate in the pension fund, i.e. the external margin of participation, one could extend the analysis by also allowing for an internal margin of choice. This would require modelling a trade-off between labour and leisure, where, conditional upon participation in the fund, the individual's contributions would be some fraction of her labour income, while her expected future pension benefit would be positively linked to her contributions, although generally this link would not be one-for-one in a system that allows for intergenerational risk sharing. For example, in the case of the Netherlands, when a pension fund is underfunded, part of the contribution is used for restoring its financial position, so that it can continue to pay the benefits promised to the elderly. Hence, the contribution would to some extent be perceived as a tax by the individual young, and participation would not only require guaranteeing the benefits of the old, but also likely cause a reduction in the supply of labour as a result of the distortion produced by the contribution. An individual who has the choice between saving for retirement at the individual level or participating in a pension fund trades off the benefit from intergenerational risk sharing against the cost of the distortion. The distortion would depend on the contribution rate and, if the latter is too high, the fund collapses. Another extension would be to relax the assumption that a collapse of the pension arrangement results into permanent autarky, but allow for the possibility to set up a new system once the old one has collapsed. One can also extend the set up to a larger number of overlapping generations. This would help to obtain more realistic quantifications of the benefits from participation. For the sustainability of a funded pension scheme it may matter whether already existing cohorts of workers have the option to quit before retirement (and on what terms) or whether new cohorts take a once and for all participation decision. We can also study more complicated pension fund policies and how these affect the risk of a collapse of the fund. In particular, the question is how the financial recovery rules for funds in financial distress could be designed, i.e. which cohort (including the retired) bears which part of the recovery burden, such that all the existing participants are willing to stay within the system and new cohorts are still prepared to enter.

Appendix

A.1. Proof of Proposition 2

To prove the last part of the proposition, we need an intermediate result:

Lemma 1. *For any $\lambda_0 \in \Lambda$, the series $\lambda_{i+1} = T(\lambda_i)$, converges to $\lambda \in \Lambda$, as $i \to \infty$, if and only if $\tau^*_{i+1} = \tau^*(\lambda_{\tau^*_i})$ converges to $\tau^*(\lambda)$.*

Proof. First assume that the series $\lambda_{i+1} = T(\lambda_i)$ converges to λ. That implies that, for each $\varepsilon > 0$, there exists an M such that $|\tau^*(\lambda_{i+1}) - \tau^*(\lambda)| < \varepsilon$, for each $m > M$. Now notice that $\tau^*_i = \tau^*(\lambda_i)$, for all $i \geq 0$. Hence, for the same ε and M we have $|\tau^*_{i+1} - \tau^*| < \varepsilon$, so that series converges to $\tau^*(\lambda)$. To prove the reverse, notice that for all $\lambda_0 \in \Lambda$, all subsequent $\lambda_i \in \mathcal{I}_\tau$, $i = 1, 2, \ldots$, where $\mathcal{I}_\tau \subset \Lambda$ is the set of simple indicator functions, and the result follows immediately from the one-to one relationship between τ^* and $\lambda \in \mathcal{I}_\tau$. □

Now, to prove Parts 1. and 2. of Proposition 2, let λ denote the decision rule associated with a threshold τ^*, so $\mathcal{U}^p_\lambda(\tau^*) = \mathcal{U}^a$. Notice that $\mathcal{U}^p(\tau(\phi), \tau^*) = \mathcal{U}^p_\lambda(\tau(\phi)) = \mathcal{U}^p_{\lambda,\tau}(\phi)$. Together with Proposition 1 we can rewrite Definition 1 as

$$\mathcal{U}^p_{\lambda,\tau}(\phi) = \mathcal{U}^p(\tau(\phi), \tau^*) \geq \mathcal{U}^a \Leftrightarrow \lambda(\phi) = 1 \Leftrightarrow \tau(\phi) \leq \tau^*. \tag{24}$$

Now, assume that a decision rule λ is valid (so λ and τ^* are such that (24) holds). Since $\mathcal{U}^p(\cdot, \cdot)$ is continuous and $\mathcal{U}^p_1(\cdot, \cdot) < 0$, we have for any $\tau > \tau^*$ that $\mathcal{U}^p(\tau, \tau^*) < \mathcal{U}^p(\tau^*, \tau^*) = \mathcal{U}^p_\lambda(\tau^*) = \mathcal{U}^a$. For $\tau \leq \tau^*$, we have $\mathcal{U}^p(\tau, \tau^*) \geq \mathcal{U}^p(\tau^*, \tau^*) = \mathcal{U}^a$. This proves that Parts 1. and 2. hold, if λ is valid.

Now take a ϕ. There are two possibilities. If $\tau(\phi) \leq \tau^*$ for this ϕ, then according to Part 1. of the proposition we have $\mathcal{U}^p(\tau(\phi), \tau^*) = \mathcal{U}^p_{\lambda,\tau}(\phi) \geq \mathcal{U}^a$. The other possibility is that ϕ is such that $\tau(\phi) > \tau^*$. Part 2. then shows that $\mathcal{U}^p(\tau(\phi), \tau^*) = \mathcal{U}^p_{\lambda,\tau}(\phi) < \mathcal{U}^a$. This proves that, if τ^* is such that Parts 1. and 2. hold, τ^* corresponds to a valid decision rule.

To prove the final part of Proposition 2, use Lemma 1. Now, define the function $\tilde{\tau} = \tilde{\tau}(\tau^*)$ with $\tilde{\tau}$ defined implicitly by $\mathcal{U}^p(\tilde{\tau}, \tau^*) = \mathcal{U}^a$. Notice that $\tilde{\tau}(\tau^*) = \tau^*(\lambda_{\tau^*})$. A series starting at τ^0 in the neighbourhood of τ^* yields the sequence $\tau^1 = \tilde{\tau}(\tau^0)$, $\tau^2 = \tilde{\tau}(\tau^1)$, etc. This series converges to τ^* if $|\tilde{\tau}'| < 1$ in $\tilde{\tau} = \tau^*$ and diverges if $|\tilde{\tau}'| > 1$. Implicit differentiation yields $\tilde{\tau}' = -\mathcal{U}^p_2/\mathcal{U}^p_1$, which is strictly smaller than 1 in absolute terms if and only if $|\mathcal{U}^p_2| < |\mathcal{U}^p_1|$ in this point, which is the desired result.

A.2. Slope under PAYG scheme

Start with the definition of Δ^p:

$$\Delta^p(\tau) = \mathcal{U}^p(\tau, \tau) - \mathcal{U}^a,$$

with $\mathcal{U}^p(\cdot,\cdot)$ for the PAYG system defined in (14) and (16). For the sake of readability define the following functions

$$\Phi(s,\tau) = [1-P_b(\theta/\tau)]\mathrm{E}_R[(R's+\theta)^{1-\eta}] + P_b(\theta/\tau)\mathrm{E}_R[(R's)^{1-\eta}] > 0,$$
$$\Omega(s,\tau) = (1-s-\tau)^{1-\rho} + \beta\Phi(s,\tau)^{\frac{1-\rho}{1-\eta}} > 0,$$
$$\Xi(\tau) = \frac{\theta}{\tau^2}p_b(\theta/\tau),$$
$$\Theta(s) = \mathrm{E}_R[(R's+\theta)^{1-\eta}] - \mathrm{E}_R[(R's)^{1-\eta}].$$

The envelope theorem gives

$$\frac{\partial\Delta^p}{\partial\tau} = -\Omega(s,\tau)^{\frac{\rho}{1-\rho}}\left\{(1-s-\tau)^{-\rho} + \beta\frac{1}{1-\eta}\Xi(\tau)\Phi(s,\tau)^{\frac{\eta-\rho}{1-\eta}}\Theta(s)\right\}. \tag{25}$$

The slope at the right endpoint is simple. To prevent negative consumption in the second period in the case that the system collapses, savings s must be positive, so any $\tau \geq 1$ is excluded. If $\tau \to 1$, the second term within curly brackets, $\Xi(\tau)\Phi(s,\tau)^{\frac{\alpha-\rho}{1-\alpha}}\Theta(s)$, is finite, while the marginal utility when young, $(1-s-\tau)^{-\rho}$, goes to infinity, so the slope at the right endpoint goes to minus infinity.

The slope at $\tau = 0$ depends on $\Xi(\tau)$ as $\tau \downarrow 0$. For the log-normal distribution we have

$$\Xi(\tau) = \frac{\theta}{\tau^2}\frac{\tau}{\theta\sqrt{2\pi\sigma^2}}\exp\left[-\frac{1}{2\sigma^2}(\log\theta - \log\tau - \mu_b)^2\right]$$
$$= \frac{e^{\mu_b}}{\theta\sqrt{2\pi\sigma^2}}\exp\left[(\log\theta - \log\tau - \mu_b) \times \left(1 - \frac{1}{2\sigma^2}(\log\theta - \log\tau - \mu_b)\right)\right].$$

For the limit $\tau \downarrow 0$ we have

$$\lim_{\tau\downarrow 0}\Xi(\tau) = \frac{e^{\mu_b}}{\theta\sqrt{2\pi\sigma^2}}\lim_{\tau\downarrow 0}\exp\left[(\log\theta - \log\tau - \mu_b) \times \left(1 - \frac{1}{2\sigma^2}(\log\theta - \log\tau - \mu_b)\right)\right] = 0.$$

This implies that the second term between curly brackets in (25) vanishes and we have

$$\left.\frac{\partial\Delta^p}{\partial\tau}\right|_{\tau=0} < 0. \tag{26}$$

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