Understanding and tuning sliding friction

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CHAPTER 4

Sliding friction on wet sand

In this chapter, we present sliding experiments of a sledge on partially water-saturated sand and show that the frictional response is controlled by the penetration hardness of the granular medium. Adding a small amount of water to sand increases the hardness which results in a decrease of the sliding friction. Pouring even more water onto sand results in a decrease of the hardness and a subsequent increase of the friction. This inverse correlation between hardness of a wetted granular material and its frictional response to sliding is found to be due to ploughing of the sledge. Similar to the sphere-on-flat geometry as presented in the previous chapter, a sledge-on-flat geometry exhibits ploughing when the penetration hardness of the water-sand mixture is exceeded which is evident by a trace of the slider left after its passage. The penetration hardness sets how deep the trace of the slider is which, in turn, controls the ploughing force. Consequently, increasing the hardness of the water-sand mixtures makes pulling a sledge over it easier. In addition, we quantify the critical shear strain which sets the transition of an elastic to plastic response of (wet) granular materials which enables us to directly relate the shear modulus, in the elastic regime, to the hardness, in the plastic regime.

4.1 Introduction

In the previous chapter we have shown that water has a large influence on the mechanical response of sand. With a small amount of water, a pile of sand can be made into a sandcastle where, however, too much water results in a muddy puddle [83]. Similarly, walking on dry or very muddy sand takes effort while, on the other hand, walking on sand that has been wetted with an intermediate water volume fraction is easier.

For a sphere-on-flat geometry the penetration hardness, the critical yielding pressure for plastic flow, as a function of the water volume fraction was quantified in Chapter 3. Initially, adding water increases the hardness of the granular material due to formation of liquid bridges between neighbouring grains, this is the so-called 'pendular regime' [85, 86]. The water-sand mixture can resist a larger load before the penetration hardness is reached whereafter the material will irreversibly deform. Deformation of the granular material results in reorientation of the grains including breaking and rebuilding of capillary bridges. Pouring even more water onto the sand decreases the strength of the capillary bridges [51]; the 'funicular regime' is reached. Furthermore, a high water volume fraction eventually results in coalescence of the liquid bridges, the 'capillary regime', which decreases the plastic response of the

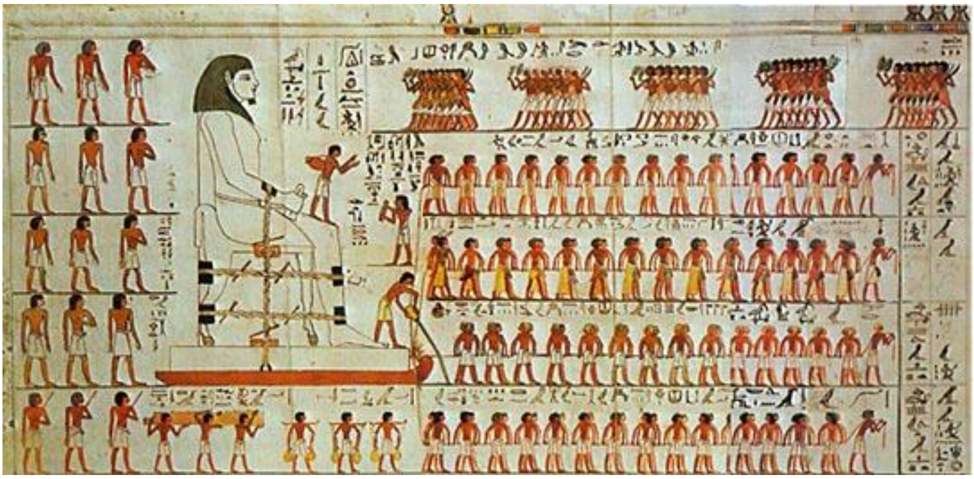

Figure 4.1: A wall painting from 1880 B.C. on the tomb of Djehutihotep [4]. A sledge with a big statue is pulled through the desert, where a person is painted who pours water onto the sand.

granular material even more.

The observed ploughing of a hemisphere through the water-sand mixture (Fig. 3.3) is controlled by the penetration hardness. For a low hardness, the hemisphere sinks into the wetted sand and leaves a deep trace into it for which a high ploughing force is observed. Increasing the hardness by adding some - but not too much - water to the sand decreases the ploughing friction as the slider remains superficial.

For a sledge sliding on sand a qualitatively similar nonmonotonic behaviour of the friction as a function of water fraction has been observed before [6]. Arguably, the ancient Egyptians, who transported statues and pyramid blocks per sledge through the desert, were aware of this as their tomb drawings show a person pouring water onto the ground in front of their sledge [4]. It therefore appears that the nontrivial relation between sliding friction on a granular material and water content of a granular material has been exploited for thousands of years. It was previously shown that the friction decreases roughly linearly with the increase in shear modulus, which suggests that when the sand is 'stiffer', the friction force decreases [6]. The shear modulus G' of a water-sand mixture quantifies the plastic response on deformation which, similar to the penetration hardness, behaves nonmonotonically as a function of the water fraction. Adding some water increases the shear modulus of the wetted sand which, with continuously increasing the water fraction, decreases again [51,87].

However, if the slider leaves a trace in the sand after its passage, the granular medium is responding plastically rather than elastically and the (linear) elastic modulus is not the pertinent quantity to consider. Sand grains irreversibly move when a critical pressure, the penetration hardness P_h, is reached. Therefore, the sliding friction is controlled by the hardness of the material instead of the linear shear modulus. In this chapter, we present sliding experiments coupled with hardness measurements for increasing water volume fractions and show that the sliding friction is a result of the plastic response of the granular material. Furthermore, the transition from the elastic to the plastic regime is unravelled which enables us to explain the link between hardness and stiffness.

4.2 Experiments

The sliding experiment is performed by pulling a wooden sledge (10.3 cm by 6.8 cm with a total mass 273 g) horizontally (as described in Section 2.1.1, although performed with a different stepper motor) at a sliding speed of 2 cm/s over a well-mixed water-sand mixture of known composition. The 'Iranian sand', which contains mainly grains in the $212 - 500$ μm range, is first dried in an oven (150 °C) and cooled down to room temperature. Demineralised water is gradually added up to a given water volume fraction and the thoroughly mixed water-sand mixture is instantly used. The sledge is pulled over a distance of 10 cm; after 2 cm a stable friction force is reached, see Figure 4.2(a). The friction force is monitored and the friction coefficient is calculated.

After sliding, the penetration hardness P_h of the water-sand mixture is measured on the same sample with an indentation-experiment. The indenter, a cone with apex angle $\alpha = 75°$ and base-radius $R = 5.05$ mm, is pushed vertically at an imposed speed of 0.1 mm/s in the water-sand mixture and indentation depth δ for increasing loading force N is monitored [see Fig. 4.2(b)]. The penetration hardness is quantified as described in Section 2.2.

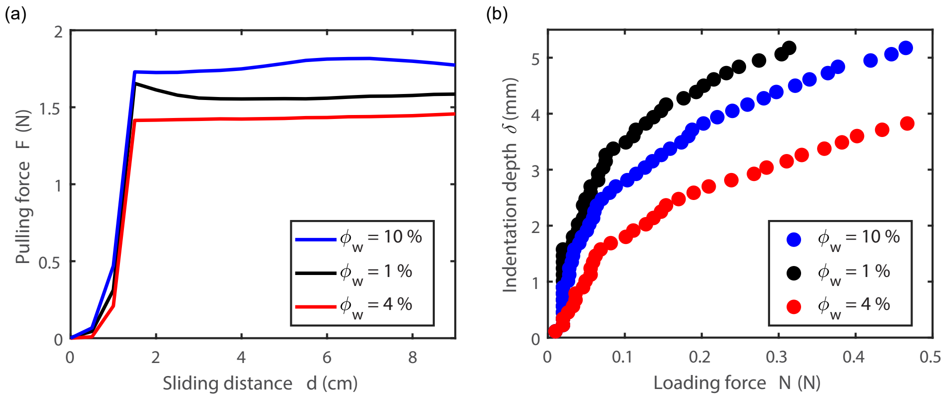

Figure 4.2: Typical results for sliding on and indentation of the water-sand mixtures. (a) The monitored pulling force F as a function of the sliding distance d for water volume fractions ϕ_w of 10%, 1%, and 4% in sand. (b) The measured indentation depth δ as a function of the loading force N measured directly after the sliding experiment. In this hardness test a conical indenter with apex angle $\alpha = 75°$ and base-radius $R = 5.05$ mm is used.

4.3 Results and discussion

4.3.1 A sledge on wet sand

The friction coefficient of a sledge sliding over a water-sand mixture is measured for increasing water volume fractions ϕ_w, see Figure 4.3 (blue squares). Adding some water initially decreases the friction coefficient where, when more water is added, the friction increases again. This nonmonotonic behaviour is qualitatively in agreement with earlier measurements where the optimum volume fraction, here 4%, is set by the grain size distribution of the used granular material [6,88]. After each sliding experiment, the slider leaves a trace in the water-sand mixture marking the width and sliding distance of the sledge. This so-called ploughing track indicates that, indeed, during sliding, the water-sand mixture is plastically deformed.

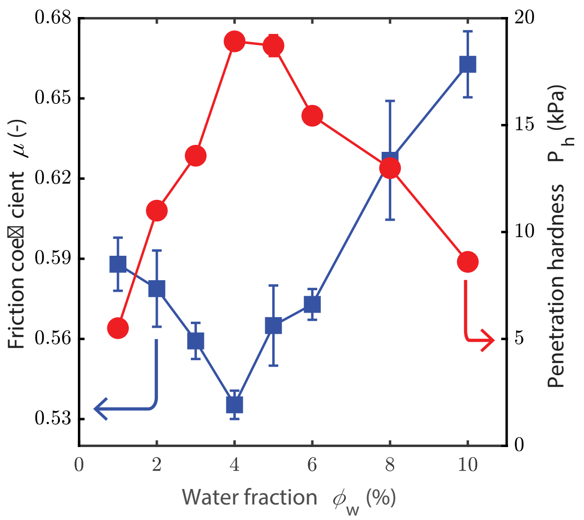

Figure 4.3: Evolution of the friction coefficient μ (blue squares) and the penetration hardness P_h (red circles) for the water volume fraction ϕ_w. The friction coefficient obtained from the measured friction force (for a fixed normal force of 2.7 N) displays a nonmonotonic behaviour for increasing water volume fractions. A similar nonmonotic behaviour for increasing water volume fractions is found for the obtained penetration hardness where the minimum in friction corresponds to the maximum in hardness at the water volume fraction of $\phi_w = 4\%$. The error bars represent the standard deviation.

The deformation of the water-sand mixture is a result of the pressure exerted by the sledge along the normal direction which exceeds the critical pressure of the granular material. In Chapter 3 the penetration hardness P_h is calculated based on the observed ploughing track and normal force along sliding. By performing an indentation-experiment with a conical indenter, a qualitatively similar dependence of the penetration hardness with the water volume fraction is observed (Fig. 4.3 red circles). Note that both the magnitude and water volume fraction domain is significantly smaller compared to the observed penetration hardness in Chapter 3. We interpret both observed decreases based on the smaller average grain sizes of the used sand here and the less firm packing of the wet sand prior to the tests [51,88,89]. The nonmonotonic behaviour of the sliding friction and the penetration hardness for increasing water volume fractions indicates a direct relation between the two; for increasing hardness of the water-sand mixture, pulling a sledge over it becomes

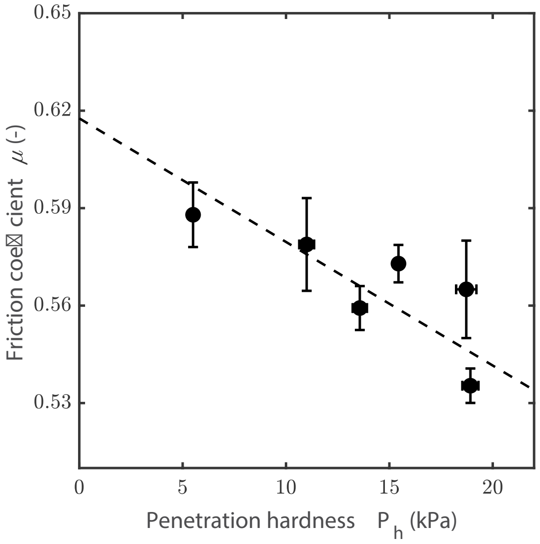

Figure 4.4: Parametric plot of the friction coefficient μ as a function of the penetration hardness P_h, both obtained as a function of the water volume fraction ϕ_w as given in Fig. 4.3. A roughly linear decrease of the friction coefficient for increasing hardness is found; it becomes easier to slide over a hard water-sand mixture. Note that water volume fractions higher than 7% are excluded from the analysis, high water fractions ('slurry sand') results in a very heterogeneous water and air distribution in the sand packing.

easier. This is quantified in Figure 4.4 where the sledge-on-sand friction coefficient is shown to decrease approximately linearly with increasing penetration hardness.

4.3.2 The transition from elastic to plastic response of wet sand

The sliding friction coefficient therefore decreases linearly with increasing shear modulus G' [6]; here we show that this linearity is also retrieved for the penetration hardness P_h of the water sand-mixture. The question then remains how the elastic and plastic response relate to each other. The water-sand mixture can hold a finite

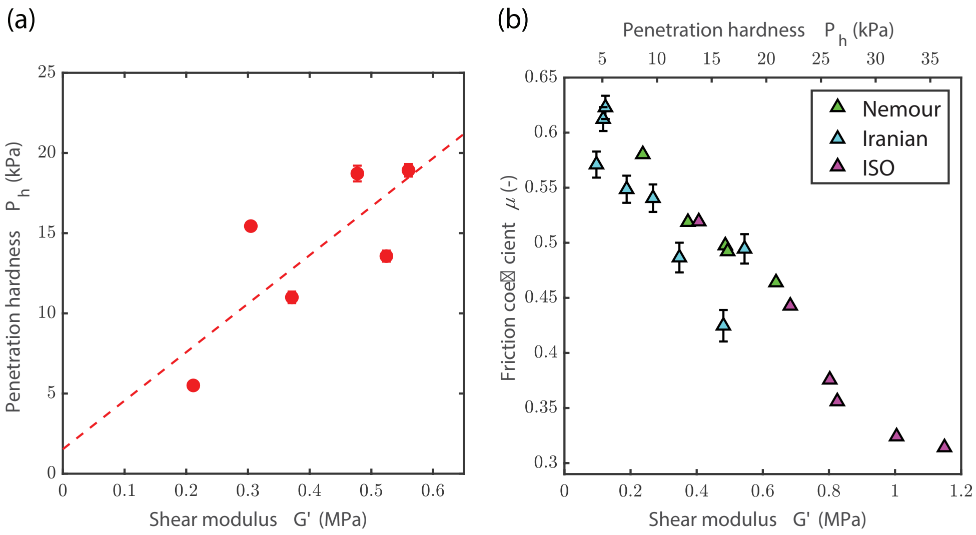

Figure 4.5: **a)** The penetration hardness P_h as a function of the shear modulus G' for different water volume fractions in sand. The shear modulus, reused from Fig. 4*b* in Ref. [6], and the penetration hardness are both obtained with so-called 'Iranian sand' (grain sizes of $212 - 500$ μm). A roughly linear dependence is found: $P_\mathrm{h} = \gamma_\mathrm{c} G' + 1528$ with $\gamma_\mathrm{c} = 0.030$ the critical shear strain. **b)** The friction coefficient μ as a function of the shear modulus G' for three sand types, reused from Fig. 5 in Ref. [6]. Furthermore, on the upper x-axis the penetration hardness P_h is given derived from the linear dependency with the shear modulus. The three sand-types 'Nemour', 'Iranian' and 'ISO', with, respectively, grain sizes in the ranges of $150 - 300$ μm, $212 - 500$ μm and $100 - 1000$ μm, were mixed with varying amounts of water.

deformation in the elastic regime before it fails whereafter a plastic (irreversible) deformation occurs. This finite deformation, the critical shear strain γ_c, defines the transition between elastic and plastic behaviour. To be able to quantify this critical shear strain, we can use the measured penetration hardness and shear modulus as a function of the water volume fraction. The latter was published previously for 'Iranian' sand for the same range of water volume fractions, see Figure 4 from Fall et al. [6]. The penetration hardness increases linearly with the shear modulus, see Figure 4.5(a), as $P_\mathrm{h} \sim \gamma_c G'$ with $\gamma_c = 3\%$ the critical shear strain. The critical strain of yielding is a constant for the given system and does not depend notably on the water volume fraction. For soft solids in general, it has indeed been shown that the critical shear strain for a given system (emulsion, microgel suspensions, foams, gels) is reasonably constant [90].

The puzzle from the data of Fall et al. is that the friction coefficient decreases linearly for increasing shear modulus for different types of sand. Now, using the relationship between penetration hardness and shear modulus, we can calculate the penetration hardness for various types of sand as shown in Figure 4.5(b). The observed linear dependence of the friction coefficient on the hardness is a result of the geometry, in this case a flat slider over a flat sand surface. For a sphere-on-flat geometry, as presented in Chapter 3, a quantitative relation is found for the hardness dependency of the friction coefficient ($\mu \sim P_\mathrm{h}^{-1/2}$). Due to the simple geometry, the relevant contact areas in the normal- and tangential direction can be calculated based on the measured ploughing track which, in the end, controls the friction coefficient for sliding. A peculiar pressure dependent friction was shown before by Crassous et al. [73] for sliding along a sandy slope. Objects on an inclined granular surface close to the avalanche threshold only slide for a narrow range of applied pressures. Sliding occurs only when the granular surface may be slightly deformed by the slider weight, but not enough to create a rim able to stop the object.

These results therefore show that the relation between the sliding friction and the (linear) shear modulus shown in Fall et al. [6] can be understood by considering the (non-linear) penetration hardness that turns out to be roughly proportional to the shear modulus [Fig. 4.5(a)]. The penetration hardness then allows for a quantitative explanation of ploughing friction.

4.4 Conclusion

In conclusion, we have performed sliding experiments of a sledge on wetted sand and measured the penetration hardness of this granular material. A roughly inverse correlation between the penetration hardness and sliding friction coefficient is obtained, which is due to ploughing. During sliding, the sledge irreversibly moves the grains, as is evident by a trace in the sand after the passage of the sledge. This trace is less pronounced if the hardness is increased by adding a small amount of water to the sand which, subsequently, results in less friction. Therefore, increasing the hardness of the water-sand mixtures makes pulling a sledge over it easier. This seems already to be experienced by the ancient Egyptians, where by pouring water in front of a sledge the sliding friction is decreased.

Pulling a sledge over sand results in irreversible movement of the grains and therefore the water-sand mixture is responding plastically. However, the mixture can hold a finite deformation elastically where the ‘stiffness’ can be quantified by the shear modulus G'. We found that the penetration hardness increases linearly with increasing shear modulus as $P_{\mathrm{h}} \sim \gamma_{\mathrm{c}} G'$ with $\gamma_{\mathrm{c}} = 3\%$ the critical shear strain which sets the transition from elastic to plastic response. Therefore, after the critical shear strain, the granular material will respond plastically and ploughing will occur. The transition from elastic to plastic deformation and the amount of ploughing during sliding is of interest in many applications, where sliding of a sledge over sand is one of them.