Understanding and tuning sliding friction

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APPENDIX B

Sliding friction of geometrically controlled surfaces

B.1 Geometrical friction model for Kirigami metamaterial surfaces

For the sliding experiments on a Kirigami metamaterial surface, we slid a single sawtooth-patterned ($\theta_T = 45°$) plastic surface horizontally against and along the Kirigami scales. Consequently, the slider moved up and down over the Kirigami patterned surface. The resulting Kirigami texturing was asymmetric; as such, the horizontal sliding lengths for moving up and down the scales were not equal. In order to calculate the average macroscopic friction coefficient based on the simple geometrical model, a weighted average friction coefficient had to be quantified for the specific geometry.

In Figure B.1, a schematic representation of the sliding experiment is given from a side view. The formed scales of length l_0 (thick black and red lines) point out of the plane with an angle θ_K, which is set by the strain ϵ. In Figure 6.11(c) the angle θ_K is quantified for increasing uniaxial strain and fitted with the use of

$$\theta_K = p_1\sqrt{\epsilon} + b\epsilon + c\ , \tag{B.1}$$

with $p_1 = 102.40$, $p_2 = -48.89$, and $p_3 = -0.21$ and θ_K in degrees. The Kirigami patterned sheet has, as seen from the side, two overlapping rows of Kirigami scales, which are represented as black and red triangles in Figure B.1. The top surface made the transition between sliding up and down when it was in contact with both overlaying rows (the black and red scales as presented in Fig. B.1). Consequently,

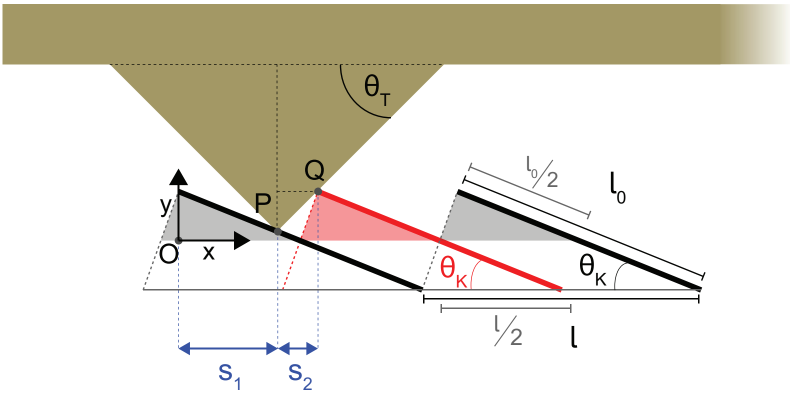

Figure B.1: Schematic illustration of the performed sliding experiments on a Kirigami metamaterial, illustrated from the side. A slider with a sawtooth-patterned surface with angle of θ_T was pulled horizontally over the scales of the Kirigami patterned surface. Consecutive lines of scales were formed which halfway overlapped in depth; see the thick black and red lines.

the sliding lengths s_1 and s_2 for sliding up and down can be defined. The weighted average friction coefficient for sliding against and along the Kirigami patterned surface can be defined as

$$\mu_{\text{against}} = \frac{s_2}{s_1 + s_2} \tan(\theta_T + \theta_0) + \frac{s_1}{s_1 + s_2} \tan(-\theta_K + \theta_0) \tag{B.2}$$

$$\mu_{\text{along}} = \frac{s_1}{s_1 + s_2} \tan(\theta_K + \theta_0) + \frac{s_2}{s_1 + s_2} \tan(-\theta_T + \theta_0) \,. \tag{B.3}$$

To calculate the weighted average friction coefficient, we expressed the sliding lengths s_1 and s_2 in terms of the quantified strain ϵ and angles θ_K and θ_T. The overlap between the Kirigami patterning is half of the triangle base length l. Therefore, sliding only occurred on the top part of the pattern, as shown by the red- and grey-filled triangles in Figure B.1. These top triangles have side lengths of $l/2$ and $l_0/2$ and the same interior angle of θ_K. Consequently, the sliding paths s_1 and s_2 can be written as

$$s_1 + s_2 = l/2 \,, \tag{B.4}$$

which can also be defined in terms of ϵ and l_0 with $l = (1+\epsilon)l_0$ as

$$s_1 + s_2 = (1+\epsilon)l_0/2 \,. \tag{B.5}$$

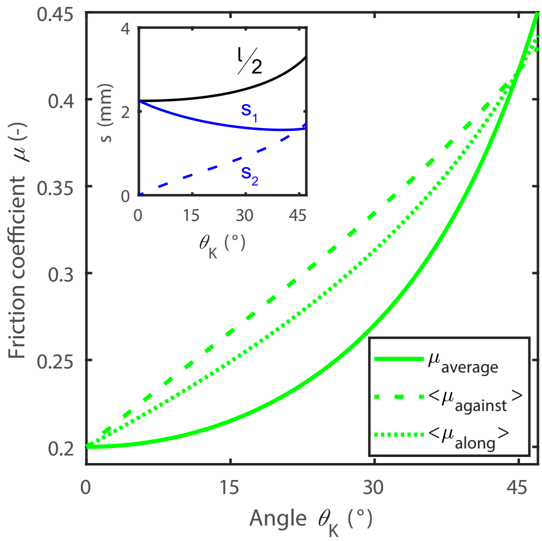

Figure B.2: The average friction coefficient as a function of the Kirigami patterning angle θ_K. The two dashed lines represent the weighted average friction coefficient for sliding a sawtooth patterned top surface with an angle of θ_T of 45° horizontally against or along the formed scales of the Kirigami metamaterial. The Kirigami angle θ_K is calculated with Eq. (B.1) for a given strain ϵ, and the weighted average friction coefficient is subsequently plotted based on Eqs. (B.2) and (B.3). As a reference, the average friction coefficient for a symmetric sliding path, i.e. for $s_1 = s_2$, is included as the continuous line. Inset: the sliding lengths s_1 and s_2 as a function of the Kirigami angle θ_K. In addition, the total length $l/2 = s_1 + s_2$ is included.

The transition for the top surface between sliding up and down the Kirigami patterns can be defined in the condition $\theta_K < \theta_T$ when both tips of the surface patterns are in contact with the opposite surface. The tangent points are defined as P and Q (see Fig. B.1). With the use of the origin O, the (x,y) coordinates for both points can be written as

$$\begin{pmatrix} x_P \\ y_P \end{pmatrix} = \begin{pmatrix} s_1 \\ l_0 \sin(\theta_K)/2 - s_1 \tan(\theta_K) \end{pmatrix} \tag{B.6}$$

$$\begin{pmatrix} x_Q \\ y_Q \end{pmatrix} = \begin{pmatrix} s_1 + s_2 \\ l_0 \sin(\theta_K)/2 \end{pmatrix} \tag{B.7}$$

In addition, the points P and Q can be related to the geometry of the slider as

$$\tan(\theta_{\mathrm{T}}) = \frac{y_{\mathrm{Q}} - y_{\mathrm{P}}}{x_{\mathrm{Q}} - x_{\mathrm{P}}} \,. \tag{B.8}$$

Therefore, with the use of Equations (B.5) through (B.8), the lengths s_1 and s_2 can be written as

$$s_1 = \frac{(1+\epsilon)l_0}{2[1+\tan(\theta_{\mathrm{K}})/\tan(\theta_{\mathrm{T}})]} \tag{B.9}$$

$$s_2 = \frac{(1+\epsilon)l_0}{2[1+\tan(\theta_{\mathrm{T}})/\tan(\theta_{\mathrm{K}})]} \,. \tag{B.10}$$

In the inset of Figure B.2, the sliding lengths as a function of the angle θ_{K} are given. Here, s_1, s_2 and the total length $l/2 = s_1 + s_2$ are plotted, where the increasing strain ϵ is transferred to the found angle θ_{K} with Equation (B.1). Using the calculated sliding lengths s_1 and s_2, the weighted average friction coefficient for sliding against and along the Kirigami surface can be derived (see Fig. B.2). As a reference, the average friction coefficient for a symmetric sliding path ($s_1 = s_2$) is included as the continuous line. Due to the sharp scale when sliding against, and the long upward path s_1 when sliding along the Kirigami surfaces, the average friction coefficients for Kirigami are higher than for a symmetric commensurable case.