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# The Amsterdam Auction

Jacob K. Goeree and Theo Offerman\*

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## ABSTRACT

Auctions used to sell houses often attract a diverse group of bidders, with realtors and speculators out for a bargain competing against buyers with a real interest in the house. Value asymmetries such as these necessitate careful consideration of the auction format as revenue equivalence cannot be expected to hold. From a theoretical viewpoint, Myerson's (1981) mechanism design approach has identified the seller's optimal choice. The proposed mechanism entails assigning credits to weaker bidders to promote competition and setting bidder-specific reserve prices. In practice, however, sellers often lack the detailed information needed to choose credits and reserve prices optimally, nor can they always discriminate among bidders. A more practical solution to the seller's problem is suggested by the "Amsterdam auction," where a premium is offered to encourage weak bidders to compete aggressively. This auction format, which has been used to sell houses in Amsterdam for centuries, treats all bidders the same and does not rely on detailed information about their value-distributions. In this paper, we consider premium auctions like the one in Amsterdam and demonstrate their revenue-generating virtues in asymmetric situations. We report the results of an experiment, which compares the standard first-price and English formats with two premium auctions in symmetric and asymmetric settings. The introduction of a premium leads weak bidders to set an endogenous reserve price for stronger rivals, with a dramatic effect on the sales price. Awarding a premium raises revenues, especially since Bertrand competition between weaker bidders virtually dissipates the premium to be paid.

JEL Classification: C72, D44.

Keywords: Auctions, experiments, asymmetries, premium.

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## 1. Introduction

The theoretical literature on auctions mostly assumes a symmetric situation where bidders' values are drawn from the same distribution.<sup>1</sup> In contrast, many real-world auctions exhibit some degree of *ex ante* asymmetries between bidders. In auctions used to sell houses, for instance, realtors and speculators out for a bargain compete against buyers with a real interest in the house. Value asymmetries also naturally arise in "license auctions" where rights to exploit scarce resources such as the spectrum, petrol stations along highways, and vendor locations at fairs are assigned. Here incumbent firms often have important advantages over entrants. In the recent European spectrum auctions for third-generation mobile phone licenses, for example, incumbent operators could rely on their existing networks while entrants still had to incur the cost of building their own.<sup>2</sup>

With *ex ante* differences between bidders, the first-price sealed-bid auction may raise more revenue than the English auction (Klemperer, 1998; Maskin and Riley, 2000).<sup>3</sup> The intuition is that in an open ascending auction strong bidders can simply trail weaker ones and keep overbidding them by the smallest bid increment as long as this is profitable. As a result, weak bidders may be discouraged to bid competitively (or participate at all) because they anticipate that their chance of winning is negligible. In contrast, weak bidders have a positive probability of winning in first-price sealed-bid auctions, which contain an element of surprise since strong bidders will not get a second chance if they shade their bids too much. Of course, it is this uncertainty that forces strong bidders to enter competitive bids, with a positive effect on revenue.

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<sup>1</sup> Notable exceptions are Maskin and Riley (2000) and Cantillon (2000) who consider two-bidder asymmetric auctions.

<sup>2</sup> Incumbents also benefited from an established brand name. Finally, since licensees could offer services that are (partial) substitutes to existing ones, incumbents had pre-emptive motives to protect their existing market shares (Jehiel and Moldovanu, 2000). Incumbents' values reflected not only the profitability of new third-generation services but also the possible loss in the existing market if entry would occur. Due to the oligopolistic nature of these markets the possible losses for incumbents exceed the potential gains for entrants, creating further value asymmetry.

<sup>3</sup> Maskin and Riley (2000) also identify situations where the English auction yields more revenue than a first-price sealed-bid auction. We are mainly interested in the case where the strong bidder's value distribution is "shifted to the right," for which Maskin and Riley prove that the first-price auction is superior. Pezanis-Christou (2002) provides experimental evidence that the first-price auction may yield higher revenues than the second-price auction even in settings that favor the latter.

A standard first-price auction, however, generally does not raise the most revenue in asymmetric settings. From a theoretical viewpoint, Myerson's (1981) mechanism design approach has identified the seller's optimal choice. The proposed mechanism entails assigning credits to weaker bidders to promote competition and setting bidder-specific reserve prices. In some of its spectrum auctions, the FCC has assigned bidding credits (a ten to forty percent price preference) to minority-owned firms and small business. Few governments have followed this example, however, and it seems fair to say that the use of bidding credits is a relatively rare phenomenon. Moreover, as far as we know, there are no examples where bidder-specific reserve prices were used.

One reason why Myerson's theoretical mechanism design approach has not been implemented in practice is that sellers usually lack the information needed to choose bidding credits and bidder-specific reserve prices optimally. In other cases, national or international law prohibits discrimination among bidders (as is the case in Europe). In fact, governments often refrain from setting a symmetric reserve price optimally (McAfee and Vincent, 1992) since they cannot afford to have a license go unsold and the announcement of a high reserve price is therefore not credible.<sup>4</sup>

A more practical solution to the seller's problem is suggested by "premium auctions," where a reward is offered to promote aggressive bidding. The basic intuition is that the introduction of a reward or premium stimulates weak bidders, who are often better informed about others' valuations than the seller, to set an "endogenous reserve price" for stronger rivals. Unlike Myerson's optimal mechanism, premium auctions treat all bidders the same and do not rely on detailed information about bidders' value distributions.

There exist many variants of premium auctions, which have been used for centuries across Europe to sell houses, land, boats, machinery, and equipment.<sup>5,6</sup> An example of a

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<sup>4</sup> France was one of the few European countries to set a substantial reserve price for its third-generation mobile phone licenses (\$4.74 billion per license). When only two bids were received for the four licenses, the French telecommunications regulator (ART) decided to hold a second contest.

<sup>5</sup> One of the authors described the current research project to his parents and was surprised to find that his father had bid in several premium auctions, held to sell equipment and inventory of business rivals gone bankrupt. While the author viewed his father's stories as further evidence of the practical importance of premium auctions, his father dryly wondered "... why do you still need to research such auctions? I have known them all my life ..."

premium auction with a particularly long tradition is the "First Amsterdam Real Estate Auction," run bi-weekly in the center of Amsterdam to sell real estate in Holland's capital. The main features of the auction formats studied in this paper are the same as those of the one employed in Amsterdam, which is why we refer to them as "Amsterdam auctions."<sup>7</sup>

Amsterdam auctions, like all premium formats, consist of two stages. In the first stage the auctioneer raises the price until all but two bidders have dropped out of the auction. The level at which the last bidder exits in the first stage, the "bottom price," acts as a reserve price in the second stage. In this stage, the two finalists submit a sealed bid no less than the bottom price, the highest bidder gets the object, and both receive a premium proportional to the difference between the lowest sealed bid and the bottom price. Finally, the winner pays her bid in the "first-price Amsterdam auction" while she pays the lowest sealed bid in the "second-price Amsterdam auction."<sup>8</sup>

The revenue enhancing effects of awarding a premium are illustrated with a simple example in which weak bidders compete against a single strong bidder in a second-price Amsterdam auction. Suppose the highest possible value of the weak bidders,  $W_H$ , is less than the lowest value of the strong bidder,  $S_L$ . In a standard English or first-price auction without a premium, revenues will never be higher than  $W_H$ . In contrast, in the second-price Amsterdam auction, bids below value are weakly dominated for the strong bidder who therefore does not drop out before  $S_L$ . In equilibrium, weak bidders do not drop out before  $S_L$  either since each has an incentive to stay in the auction somewhat longer and win a positive premium if others drop out before. Once the price level exceeds the lowest possible strong

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<sup>6</sup> The use of premium auctions can be traced back to the Middle Ages (Sikkel, 2001). Many Dutch and Belgian towns have their own variant, which they often claim to be unique in the world. However, all variants share the feature that they offer a premium to bidders who "stir up" competition in the auction.

<sup>7</sup> In Belgium these auctions are known as "veilingen met het recht van verdienen," which is old Dutch for "auctions with the right to make more expensive." For an example of a slightly different premium auction format than the ones considered here, see [http://www.troostwijk.be/de/alg\\_neen.htm](http://www.troostwijk.be/de/alg_neen.htm).

<sup>8</sup> Klemperer (1998, 2002) also considers auctions with a two-stage structure and introduces the terminology "Anglo-Dutch" and "Anglo-Anglo" auctions, depending on whether a first or second-price mechanism is used in the final stage. Unlike the formats discussed in this paper, no premium is awarded in the Anglo-Dutch or Anglo-Anglo auction. Awarding a premium is the main feature of the auctions studied here, and we shall use the terminology first-price and second-price Amsterdam auctions to indicate the difference.

value, however, incentives for weak bidders change as there is some chance that the strong bidder drops out. For this reason, a weak bidder that makes it to the final stage may find it optimal to bid  $S_L$ , in which case no premium is awarded. To summarize, the introduction of a premium leads weak bidders to set an endogenous reserve price for stronger rivals while Bertrand competition between them may virtually dissipate the premium they are paid.

The benefits of awarding a premium are less clear in symmetric settings since part of the revenues are transferred to the second-highest bidder. We show, however, that revenues of the Amsterdam auctions are the same as those of standard auctions in the symmetric case, i.e. revenue equivalence holds. The intuition is that optimal bidding functions in the Amsterdam auctions are increasing (which ensures that the highest-value bidder wins the object) and the lowest possible type has zero expected payoffs. In other words, the conditions underlying the Revenue Equivalence Theorem hold. Interestingly, the variance of the revenue from an Amsterdam auction is lower than that of a standard English auction in symmetric settings. This may be important for license auctions since governments are often nervous about the possibility of low revenues.<sup>9</sup>

To conclude, Amsterdam auctions have the following attractive properties: they generate high revenues with low variance, and, in equilibrium, there is no loss of efficiency. These theoretical properties, however, subsume that weak bidders are willing to take risks for possibly small premiums. In practice, weak bidders may be more cautious and drop out sooner than predicted. This paper reports the results of an experiment designed to compare the efficiency and revenue-generating properties of Amsterdam auctions with those of first-price and English auctions in both symmetric and asymmetric settings.<sup>10</sup>

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<sup>9</sup> In addition, governments are often criticized when similar licenses sell for substantially higher prices in neighboring countries. For example, the recent European spectrum auctions have demonstrated that ascending auctions can lead to very different revenues: Austria and Germany used the same design to auction their spectrum, but average price per capita was 100 Euros in Austria and 615 in Germany (Klemperer, 2002).

<sup>10</sup> Standard symmetric auctions have been tested extensively in the laboratory, see Kagel (1995) for an excellent survey. Relatively few experiments concern asymmetric auctions. Corns and Schotter (1999) show that bidding credits to weak bidders in procurement auctions can be cost effective. Kagel and Levin (1999) investigate first-price common value auctions where insiders are better informed than others. Pezani-Christou (2002) shows that the first-price auction generates more revenue even in a setting that favors the second-price auction.

The paper is organized as follows. Section 2 provides a theoretical analysis of the auctions employed in the experiment. Section 3 presents the experimental design. Section 4 discusses our findings and section 5 concludes. The Appendices contain proofs, statistical tests, and a translation of the instructions for one of the treatments.

## 2. Theoretical Background

In this section we derive the optimal bids for the premium auctions employed in the experiment. We start with the symmetric case and then discuss a setting where a single strong bidder competes against weaker rivals. We consider four formats: standard first-price and English auctions as well as first and second-price Amsterdam auctions.

### 2.1. The Symmetric Case

Suppose there are  $n$  bidders with values drawn from a uniform distribution on  $[0,1]$ . The optimal bids for the two standard auctions are, of course, well known: in the first-price auction the Nash bids are  $B_{\text{FP}}(v) = (n-1)v/n$  and in the English auction it is optimal to bid one's value,  $B_{\text{E}}(v) = v$ . Since the expected values of the highest and second-highest value draws are  $n/(n+1)$  and  $(n-1)/(n+1)$  respectively, the expected revenue from both auctions is  $R = (n-1)/(n+1)$ .

The Amsterdam auctions involve two stages and their equilibria are derived via backward induction. Recall that in the first stage the auctioneer raises the price until all but two bidders drop out. The resulting price level,  $X$ , is called the "bottom price" and serves as a reserve price for the second stage. Consider, for example, the final sealed-bid stage of the second-price Amsterdam auction. Two finalists make a sealed bid no less than  $X$  and both receive a premium equal to  $\alpha$  times the difference between the lowest bid in the second stage and  $X$ . In addition, the highest bidder wins and pays the lowest sealed bid. Let  $B_i$  denote the optimal bidding function for stage  $i = 1, 2$ , and let  $v_3$  be the value of the bidder that determined the bottom price in the first stage, i.e.  $B_1(v_3) = X$ . The second-stage expected payoffs of a bidder with value  $v$  who bids as if her type is  $w \geq v_3$  equals:

$$\pi_2^e(w|v) = (1 - v_3)^{-1} \left\{ \int_{v_3}^w (v - \mathbf{B}_2(z) + \alpha(\mathbf{B}_2(z) - X)) dz + \alpha(\mathbf{B}_2(w) - X)(1 - w) \right\}, \quad (1)$$

where the integral term on the right side occurs when the bidder wins and the second term when she loses. The first-order condition for profit maximization follows by taking the derivative of (1) with respect to  $w$  and evaluating the result at  $w = v$ :

$$v - \mathbf{B}_2(v) + \alpha \mathbf{B}_2'(v)(1 - v) = 0. \quad (2)$$

The solution to this first-order condition is given by  $B_2(v) = (v + \alpha)/(1 + \alpha)$  and is independent of  $v_3$  (or  $X$ ).

The first-stage bidding function can be determined by observing that a bidder who enters the second stage with the lowest possible value,  $v_3$ , has zero expected payoffs. Indeed, if a bidder of this type had strictly positive expected payoffs then it could not have been optimal for such a bidder to drop out at  $X$  in the first stage. From (1), this zero expected payoff condition implies  $B_2(v_3) = B_1(v_3)$ . Since this holds for all possible realizations of  $v_3$  we have  $B_1(v) = (v + \alpha)/(1 + \alpha)$ . Hence, in the second-price Amsterdam auction optimal bids in the first and second stage are the same.

A similar calculation establishes the optimal bids in the first-price Amsterdam auction. In this case, the winning bidder pays her own price, and both finalists receive a premium proportional to the difference between the lowest bid in the final stage and the bottom price. A bidder's second-stage expected payoffs when her type is  $v$  and she bids as if of type  $w$  are given by:

$$\pi_2^e(w|v) = (1 - v_3)^{-1} \left\{ (v - \mathbf{B}_2(w))(w - v_3) + \alpha \int_{v_3}^w (\mathbf{B}_2(z) - X) dz + \alpha(\mathbf{B}_2(w) - X)(1 - w) \right\}, \quad (3)$$

where the first two terms apply when the bidder wins and the final term occurs when she loses. The first-order condition for profit maximization becomes:

$$v - B_2(v) + B_2'(v)(\alpha - (1 + \alpha)v - v_3) = 0. \quad (4)$$

The solution to (4) is  $B_2(v) = (v + v_3 + \alpha)/(2 + \alpha)$ , which does depend on  $v_3$ , the type that determined the bottom price  $X$  in the first stage. Since this type has zero expected payoffs when entering the second stage, equation (3) implies  $B_2(v_3) = X$  so  $v_3 = (2 + \alpha)X/2 - \alpha/2$ . Hence, the second-stage optimal bid can be written as  $B_2(v) = (v + \alpha/2)/(2 + \alpha) + X/2$ . Finally, the optimal bids for the first stage follow from the condition  $B_1(v_3) = B_2(v_3)$  for all possible realizations of  $v_3$ , which yields  $B_1(v) = (2v + \alpha)/(2 + \alpha)$ .

It is easily verified that the two premium auctions yield the same expected revenue as the standard ones when there are more than two bidders. This follows since the assumptions underlying the revenue equivalence theorem are satisfied: the highest-value bidder wins the object and a bidder with the lowest possible value has zero expected payoffs. Interestingly, awarding a premium may lower the variance of the seller's revenue, at least compared to an English auction. (See Appendix A for proofs of the propositions.)

*Proposition 1. The seller's revenue is the same for the first-price auction, English auction, and the two variants of the Amsterdam auction. The variances of the revenues in the Amsterdam auctions are less than that of an English auction but no less than that of a first-price auction.*

## 2.2. The Asymmetric Case

We next investigate the effects of a premium in the presence of value asymmetries. In particular, consider the case when  $n - 1$  "weak" bidders with values drawn from a uniform distribution on  $[0,1]$  compete against a single "strong" bidder whose value is uniformly distributed on  $[L, H]$  where  $H > L > 1$ . There exist no closed-form expression for the optimal bidding function in the first-price auction although its solution can be found using numerical techniques (see, for instance, the bottom part of Figure 2). In the English auction, the seller's expected revenue is determined by the highest of the weak types, which is  $(n - 1)/n$ . This

revenue, however, only results when weak types bid their value even though in equilibrium they have no chance of winning. There are other equilibria, e.g. all weak bidders drop out at zero while the strong bidder is willing to bid up to her value, yielding zero revenues. In a symmetric context such behavior may be ignored as it entails playing weakly dominated strategies. In the presence of a strong bidder, however, weak bidders "have nothing to gain" anyway and it is not clear that these weakly dominated strategies can be ruled out.

The introduction of a premium dramatically alters the incentives of weak bidders. Consider, for instance, the second-price Amsterdam auction and suppose all weak bidders drop out at  $X < L$ . Irrespective of a weak bidder's value, it then pays to stay in the auction somewhat longer. To see this, note that in the second stage the strong bidder has an incentive to bid her value (or above): bids below value can only cause her to lose at a price she would have liked to win, and they may lower the premium she receives. The weak bidder can therefore safely bid  $L$ , in which case she earns  $\alpha(L - X) > 0$  independent of her value. Of course, this positive expected profit will attract other weak bidders as well. In equilibrium, Bertrand competition between weak bidders will dissipate this potential profit.

*Proposition 2. The following constitutes an equilibrium outcome of the second-price Amsterdam auction when  $\alpha \leq (L - 1)/(H - L)$ . In the first stage, the strong bidder bids up to her value and weak bidders bid up to  $L$ , at which point  $n - 2$  weak bidders drop out. In the second stage, weak bidders bid  $L$  and the strong bidder bids her value. The seller's revenue is  $L$ .*

Notice how competition between weak bidders creates an "endogenous reserve price" for the strong bidder.<sup>11,12</sup>

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<sup>11</sup> Of course, the seller could have earned the same amount by requiring a minimum bid of  $L$ . In many cases, however, the seller does not possess detailed information about bidders' valuations making it impossible to set an optimal reserve price. Besides, the seller may be reluctant to use a substantial reserve price as it may be considered non-credible.

<sup>12</sup> In the experiment, the premium parameter  $\alpha$  is chosen such that it satisfies the condition in Proposition 2. We do not derive the equilibrium outcome for  $\alpha > (L - 1)/(H - L)$ , but it should be clear that a higher  $\alpha$  will result in even more aggressive bidding by the weak bidders.

Also in the first-price Amsterdam auction, weak bidders have an incentive to stay in the auction in the first stage to collect the premium. In contrast with the second-price Amsterdam auction, however, the strong bidder has an incentive to "shade" her bid in the second stage. In one type of equilibrium, the strong bidder bids slightly above  $X$  in the second stage if the bottom price is  $X \leq L$  in the first stage. When  $X > 1$ , the weak bidder is always best off bidding  $X$  in the second stage.

*Proposition 3. The following constitutes an equilibrium outcome of the first-price Amsterdam auction. In the first stage, the strong bidder bids up to her value and weak bidders bid up to  $X$ , where  $1 \leq X \leq L$ , at which point  $n - 2$  two weak bidders drop out. In the second stage, weak bidders bid  $X$  and the strong bidder bids slightly above  $X$ . The seller's revenue is  $X$ .*

### 3. Experimental Design

The computerized experiments consisted of three parts of twelve periods each (subjects also played two practice periods). Only after one part of the experiment was completed did subjects receive instructions for a new part.<sup>13</sup> In each period, subjects received information about their own private values only. They earned "points," which were exchanged into guilders at the end of the experiment at a rate of 4 points to 1 guilder (about \$0.40). Subjects were given a starting capital of 60 points, which they did not have to pay back at the end of the experiment. Table 1 summarizes the main features of the four treatments. For statistical reasons subjects were allocated to the same group of 4 bidders in each period.<sup>14</sup>

We are mainly interested in the symmetric bidding environment (periods 1-12) and the asymmetric environment (periods 25-36). In a symmetric setting, revenue equivalence holds and all four formats are predicted to perform the same, while in the asymmetric setting the Amsterdam auctions are expected to be superior. We also added a weakly asymmetric

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<sup>13</sup> The experiment was programmed using the *Rat-Image* toolbox developed by Abbink and Sadrieh (1995). Appendix B contains a translation of the instructions for one of the treatments.

<sup>14</sup> Subjects were not informed about this aspect to avoid repeated game considerations.

**Table 1.** Experimental Design

Treatment	Number of Groups	Group Size	Premium for Two Finalists
First price	9	4	0
English	8	4	0
First-Price Amsterdam	9	4	0.3 ( $b_2 - X$ )
Second-Price Amsterdam	8	4	0.3 ( $b_2 - X$ )
All Treatments			
Bidding Environment	Bidders' Types		Valuations
Symmetric (periods 1-12)	4 symmetric		U[0,60]
Weakly Asymmetric (periods 13-24)	3 weak and 1 strong		weak: U[0,60] strong: U[40,100]
Asymmetric (periods 25-36)	3 weak and 1 strong		weak: U[0,60] strong: U[70,100]

environment (periods 13-24) to let subjects get acquainted with their roles as strong or weak bidders. In addition, this environment seems to favor neither the Amsterdam nor the standard auctions and may thus provide a more complete picture of their relative performance. Value draws were independent across bidders and periods. The role of strong bidder rotated each period and subjects were informed whether they were weak or strong.<sup>15</sup> The auction rules and the procedure to generate bidders' valuations were common knowledge.

In the first-price auction, subjects simultaneously submitted their bids and the highest bidder received a profit equal to her valuation minus her bid (the three other bidders received zero payoffs). All other formats had an ascending phase that was modeled as follows: every bidder's screen displayed a "thermometer" that started to rise from zero. The thermometer's "temperature" represented the price that the active bidders were willing to pay. This price level was automatically raised point by point as long as at least two bidders were active. If a

<sup>15</sup> The instructions used the terms "small" and "large" bidders instead of weak and strong bidders.

bidder pushed the "stop" button, she dropped out of the auction without the possibility to return that period (irrevocable exit). Other bidders in the group were immediately informed that one of their rivals had dropped out (and, when applicable, whether the bidder was weak or strong). In the English auction, the thermometer's temperature stopped rising after three bidders had dropped out (who received zero profits). The remaining bidder then received a profit equal to her valuation minus the price level at which the last rival had dropped out.

In treatments that employed an Amsterdam auction, each period consisted of two stages. In the first stage of the auction, the thermometer kept rising until two of the four bidders had dropped out (who received nothing that period). The price where the thermometer stopped was called the bottom price. In the second stage of the auction, the two remaining bidders simultaneously submitted sealed bids no less than the bottom price.<sup>16</sup> Bidders knew whether their opponent was weak or strong in this final stage. Both finalists received a premium equal to 30 percent of the difference between the lowest sealed bid and the bottom price.<sup>17</sup> The highest bidder received her value and paid her own bid in the first-price Amsterdam auction and the other's sealed bid in the second-price Amsterdam auction.

At the end of each period, subjects were informed about their profits (but not about that of others). In the first-price auction, subjects were told all bids in their group (ranked from low to high) and which bid was made by the strong bidder. Similar information was automatically available to subjects in the other formats.

### *Subjects and Bankruptcy*

A total of 140 subjects were recruited at the University of Amsterdam. The experiment took between 1.5 and 2 hours and subjects' earnings ranged from 0 to 110 guilders with an average of 54.7 guilders and a standard deviation of 17.3 guilders (54.7 guilders is about \$22). A subject went bankrupt when her cash balance became negative, in which case she had to leave the experiment and group members were informed that a

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<sup>16</sup> Sealed bids were not allowed to be higher than 60 (100) points in the symmetric (asymmetric) environment. In case of tie, the computer randomly chose a winner.

<sup>17</sup> A premium of 30 percent satisfies the condition of Proposition 2 since  $\alpha = 0.3 < (70 - 60)/(100 - 70)$ .

bankruptcy had occurred (the computer took over the role of a subject that had gone bankrupt). We discarded all data of a group after a bankruptcy occurred.<sup>18</sup> Subjects were completely informed about the bankruptcy procedure before the start of the experiment.

## 4. Results

We start by comparing the efficiency and revenues of the four formats (section 4.1). Then we study weak bidders' prospects in each of the treatments and determine whether they are stimulated to bid aggressively (section 4.2). Finally, we compare the individual bids with Nash predictions (section 4.3).

### 4.1. Revenue, Variance, and Efficiency

Figures 1 and 2 show revenue histograms of all treatments for the symmetric and asymmetric case respectively.<sup>19</sup> With symmetry, revenues seem highest for the first-price auction, followed by the first-price Amsterdam auction and the English auction, which yield comparable revenues. Note that the revenue from the second-price Amsterdam auction has the lowest mean and highest variance. While the four formats are predicted to be revenue equivalent, the first-price auction appears superior in symmetric settings. Note, however, that the revenue histograms show a completely different picture in the asymmetric case. Both variants of the Amsterdam auction revenue dominate standard auctions, with the second-price Amsterdam auction raising the most. The first-price auction again outperforms the English auction, which quite frequently yields very low revenues.

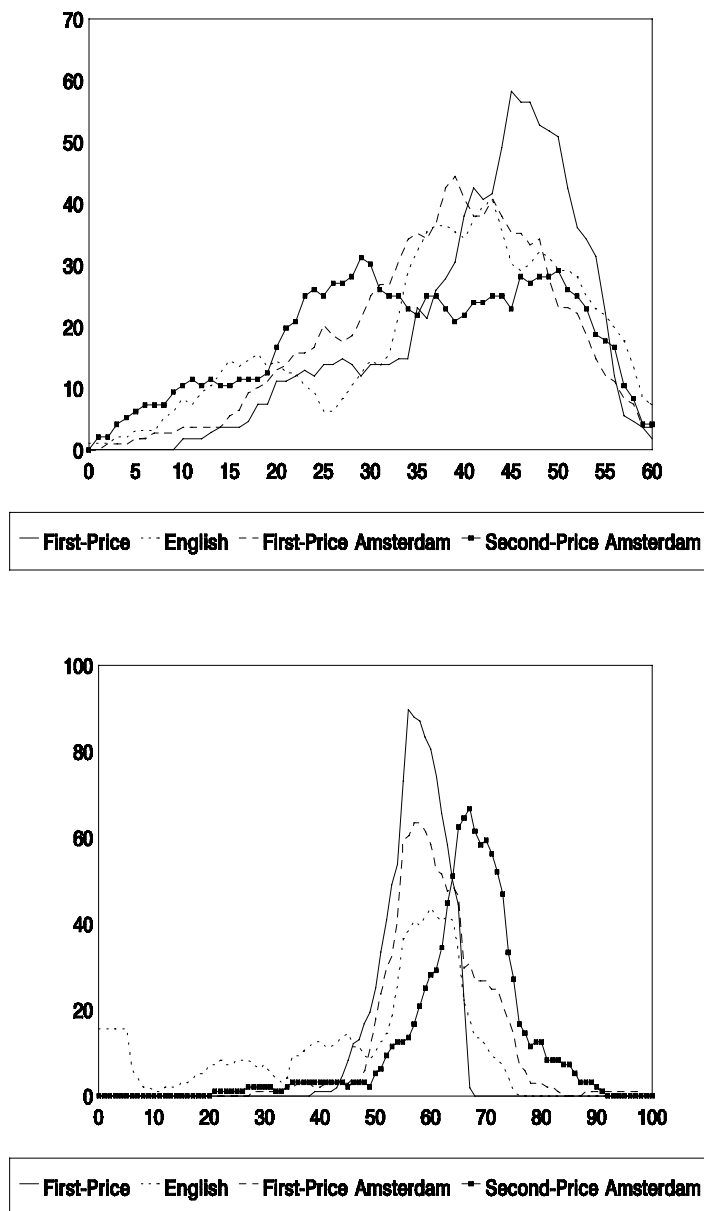
Table 2 presents statistical evidence for these claims. In symmetric settings, the first-price auction generates significantly higher revenues than other formats.<sup>20</sup> Furthermore, the revenue from a first-price auction is least variable, while that of a second-price Amsterdam

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<sup>18</sup> One of the subjects in the English auction (!) went bankrupt after only a few periods. It turned out that this subject did not (sufficiently) comprehend Dutch. All data of this group were discarded.

<sup>19</sup> We consider *net* revenues of the Amsterdam auctions, defined as the winner's payment minus the premiums paid to the bidders.

<sup>20</sup> This result is strengthened by the fact that the value draws were accidentally such that the first-price auction would have generated slightly *less* revenue than the other auctions if all bidders had used Nash strategies.



**Figure 1.** Revenue Histograms for the Symmetric and Asymmetric Case.

For every revenue level the percentage of outcomes that fall in the interval  $[\text{revenue}-5, \text{revenue}+5]$  is shown.

auction shows more variation than predicted.

Consistent with Nash predictions, the first-price auction also does well when asymmetries are introduced. Revenue rises and its variance falls. In contrast, the English auction performs much worse. First, the observed variance in revenues jumps sharply (the

**Table 2. Revenues**

		Symmetric	Asymmetric
First-Price	Actual	41.5 9.7	57.4 3.8
	Nash	35.4 8.5	58.5 0.8
English	Actual	38.2 13.2	44.1 22.7
	Nash	38.4 13.0	44.7 11.1
First-Price Amsterdam	Actual	37.7 10.8	60.1 8.2
	Nash	36.2 9.3	60.0 - 70.0 0.0
Second-Price Amsterdam	Actual	34.7 13.7	66.0 9.9
	Nash	37.1 9.3	70.0 0.0

*Notes:* The first entry in each cell displays the average revenue and the second entry (in italics) displays the standard deviation.

standard deviation of the revenue is twice as high as predicted). Second, its revenues are low (although comparable to the predicted level). In other words, the English auction is risky and unprofitable in asymmetric settings. The Amsterdam auctions perform best: they yield high revenues with low variability. Revenues from a second-price Amsterdam auction are highest and significantly different from those of standard formats (the difference with the first-price Amsterdam auction is barely significant).<sup>21</sup> It is the only format where revenues exceed the weak bidders' maximum value of 60.<sup>22</sup>

While the English auction yields low revenues it may still be preferred for efficiency reasons. Indeed, the other formats foster aggressive bidding by weak bidders with adverse effects on efficiency in case they win. Table 3 displays the average efficiency levels in all treatments for the symmetric and asymmetric settings.<sup>23</sup> As expected, the English auction is most efficient in both cases. Surprisingly, however, efficiency losses are small when other

<sup>21</sup> Tables with statistical test results are presented in Appendix C.

<sup>22</sup> With weak asymmetries (periods 13-24) the observed revenues of all formats increase while their variance decrease. The first-price auction and the first-price Amsterdam auction generate significantly more revenue than the English auction (while the difference between them is insignificant). Tables that include the results for the weakly asymmetric case can be found in Appendix C (together with test results).

<sup>23</sup> Efficiency is defined as  $(v_{\text{winner}} - v_{\text{min}})/(v_{\text{max}} - v_{\text{min}}) * 100\%$ , i.e. the winner's value minus the lowest of the bidders' values as a percentage of the difference between the highest and lowest of the bidders' values.

**Table 3. Efficiencies**

	Symmetric	Asymmetric	All
First-Price	96.5%	95.9%	96.7%
English	99.7%	96.7%	98.5%
First-Price Amsterdam	92.2%	95.6%	94.0%
Second-Price Amsterdam	92.4%	87.7%	90.3%

formats are used. Indeed, the first-price auction yields efficiency levels comparable to the English auction (although the small differences are significant), and also the first-price Amsterdam auction yields efficiency levels of 94 percent. Only in the second-price Amsterdam auction are efficiency losses somewhat more pronounced.

#### 4.2. Prospects for Weak Bidders

Sometimes it is argued that ascending auctions discourage entry in asymmetric situations because entrants anticipate they have no chance of winning (Klemperer, 2002). In our experiment, bidders did not make a formal entry decision but they could drop out at very low prices. The right-upper entry in each cell of Table 4 reports the proportion of almost zero bids in the different treatments. In the first-price auction this percentage is roughly constant across bidding environments (ranging from ten to thirteen percent). In the Amsterdam auctions, the low percentages of almost zero bids in the symmetric case fall when asymmetries are introduced. The English auction, however, shows the reverse pattern. The percentage of almost zero bids is small in symmetric settings but dramatically increases to almost forty percent in the asymmetric case.

Furthermore, weak bidders in the English auction have only a very small probability of winning, and when they win, they do so without making a profit.<sup>24</sup> (The upper-left entry in each cell of Table 4 reports the probability that a bidder who does not have the highest value

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<sup>24</sup> In the asymmetric case, weak bidders lose money on average. Some subjects chose to remain active at price levels above their values if at least two rivals were active, but to exit when only one rival was active. Such a strategy may produce losses if other bidders use a similar strategy.

**Table 4.** Chances for Weak Bidders (Highest-Value Bidder Excluded)

	Symmetric		Asymmetric		All	
First-Price	13.0%	10.4%	9.3%	13.0%	11.1%	11.3%
	0.03	2.79	0.04	0.82	0.01	2.09
English	4.1%	7.8%	5.2%	39.1%	3.5%	19.4%
	0.00	0.08	-0.27	2.18	-0.12	1.58
First-Price Amsterdam	20.3%	8.8%	5.9%	4.2%	14.2%	6.9%
	0.55	4.28	-0.08	5.48	0.32	4.60
Second-Price Amsterdam	26.0%	8.3%	18.8%	6.5%	21.2%	7.3%
	0.56	3.05	-1.18	8.35	-0.36	6.53

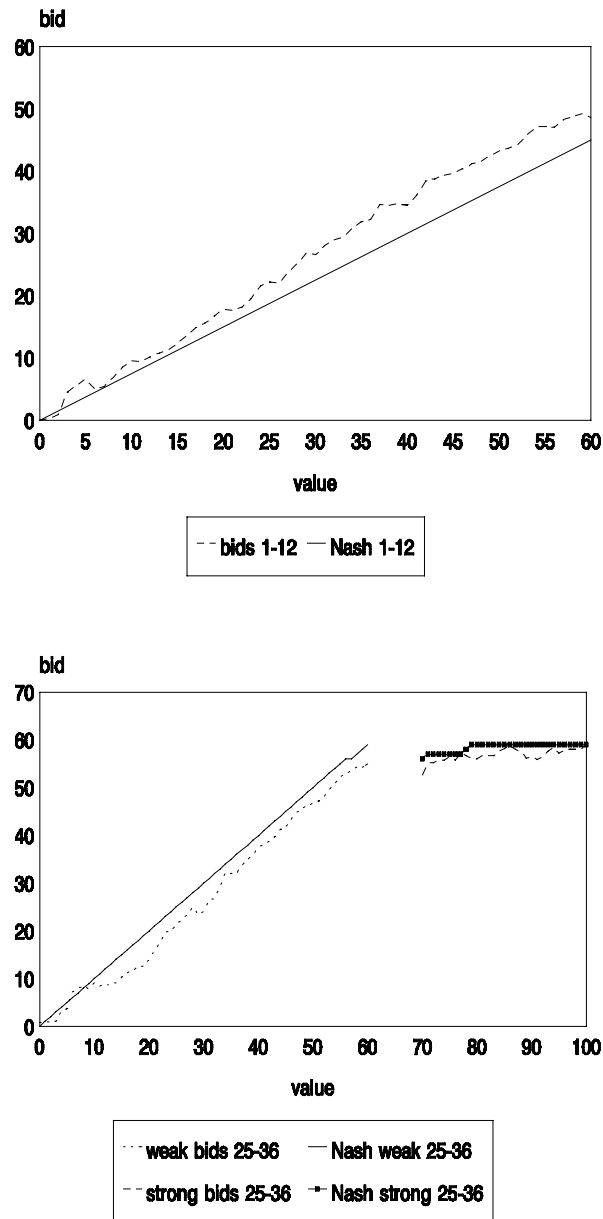
*Notes:* In each cell, the upper-left entry shows the probability that a weak bidder wins and the upper-right entry shows the observed frequency with which a weak bidder bids 5 or less. The lower-left entry displays a weak bidder's average profit and the lower-right entry displays (in italics) the standard deviation of this profit.

wins.) Weaker bidders' chances of winning are roughly three times larger in the first-price auction, and when they win, they usually make a small profit. In terms of the probability of winning, the best prospects are offered by the Amsterdam auctions and in particular by the second-price Amsterdam auction. Note, however, that this format encourages weak bidders too much since their average earnings are negative.<sup>25</sup>

### 4.3. Individual Bids

In the English auction with symmetric bidders, the observed average drop-out level was 27.3. This is only slightly higher than the Nash prediction of 26.3 (which entails bidding up to one's value). Observed bids differ sharply from Nash predictions in the asymmetric case, however. Many weak bidders realize they have no chance of winning and drop out at very low prices. At the same time, a few weak bidders remained active far above their values. The average drop-out level of weak bidders was 21.6, which is less than the predicted

<sup>25</sup> Competition between the bidders dissipates the premium to a large extent. The premium equals on average 2.20 (1.86) in the symmetric (asymmetric) first-price Amsterdam auction and 2.63 (2.20) in the symmetric (asymmetric) second-price Amsterdam auction.



**Figure 2.** Observed Bids and Nash Predictions for the First-Price Auction  
 Symmetric Bidding Environment (Top) and Asymmetric Bidding Environment (Bottom)

level of 31.2 that would have resulted if subjects had bid up to their values.<sup>26</sup>

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<sup>26</sup> This difference is weakly significant according to a Wilcoxon rank test with averages per group as the unit of observation ( $p=0.07$ ,  $n=8$ ).

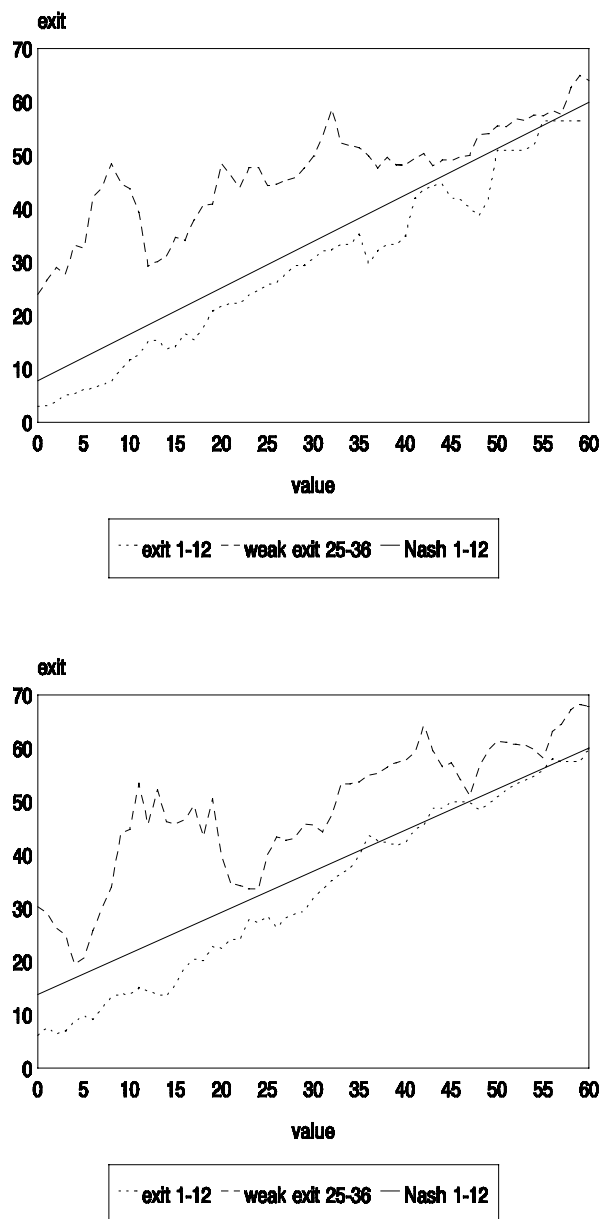
In symmetric first-price auctions, observed bids usually exceed risk-neutral Nash predictions (e.g., Cox, Smith, and Walker, 1988). The data of our first-price treatment corroborate this finding. The top panel of Figure 2 shows how observed bids in the symmetric part exceed risk-neutral Nash predictions. We were not able to derive analytic solutions for the asymmetric environment and used numerical techniques to approximate the optimal bidding functions. The bottom panel of Figure 2 displays our numerical solution together with the data. Observed bids conform nicely to theoretical predictions although the Nash equilibrium slightly overpredicts. This is intuitive as weak bidders make no profits in the Nash equilibrium and any noise in behavior will push their bids downward, which may be profitable if the strong bidder is noisy too. The data show some evidence for this as the strong bidder's observed bidding function lies slightly below the Nash benchmark.

Next we turn to the Amsterdam auctions. Figure 3 shows the (weak) bidders' drop out levels by values in the first-price (top) and second-price (bottom) Amsterdam auction. In the symmetric case, bidders tend to exit somewhat earlier than the risk-neutral Nash prediction.<sup>27</sup> When asymmetries are introduced, weak bidders correctly anticipate that they can remain active longer. In both Amsterdam auctions, bidders wait longer before dropping out, especially for lower values and the observed exit functions become flatter.

Although these figures indicate that weak bidders act strategically, there are some discrepancies with Nash predictions. Recall that in the second-price Amsterdam auction, for instance, weak bidders are predicted to bid up to 70, independent of their own value. Figure 3 shows that weak bidders quit before 70 and that their drop-out levels are increasing in their values. These behavioral corrections to the "knife-edge" Nash predictions are intuitive since the Nash equilibrium entails cut-throat competition between weak bidders. Previous experiments employing this type of Bertrand structure typically find that subjects do not exhibit such extreme behavior because, in equilibrium, they have no incentives to do so (Duwfenberg and Gneezy, 2000). In the Amsterdam auctions, weak bidders realize that their premium will be small when the price level rises to 70 and that they face a big loss in case

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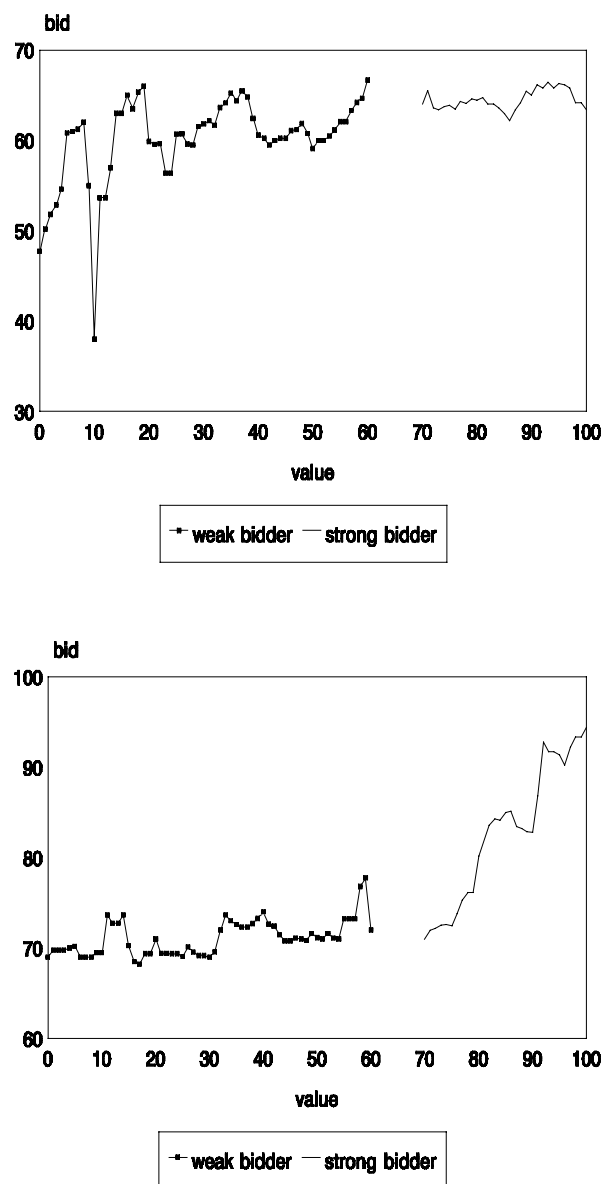
<sup>27</sup> More precisely, in the first-price Amsterdam auction the average bottom price is 26.9 compared to the Nash prediction of 30.0. In the second-price Amsterdam auction the average drop-out level is 31.2, only slightly below the Nash level of 33.3.



**Figure 3.** Exit Decisions in First-Price (Top) and Second-Price (Bottom) Amsterdam Auction  
For each value the average of exit decisions in the interval  $[value-2, value+2]$  is reported

the strong bidder drops out by mistake. As a result they tend to drop out at levels below 70.

Figure 4 shows the sealed bids of the two finalists in the asymmetric parts of the first-price Amsterdam auction (top) and second-price Amsterdam auction (bottom). In the first-price variant the observed exit functions for weak and strong bidders are nearly flat. There



**Figure 4.** Sealed Bids in the First-Price (Top) and Second-Price (Bottom) Amsterdam Auction  
For each value the average of bids that fall in the interval  $[\text{value}-2, \text{value}+2]$  are graphed

appear to be two focal outcomes for the strong bidders in this auction: a small majority of the strong bidders bid the maximum of the weak bidders' values (60) while the rest bid the minimum of the strong bidders' values (70).

In the second-price Amsterdam auction, weak bidders' behavior in the final stage contrasts with their cautious exit choices in the first stage (bottom panel Figure 3). Average sealed bids start at 70 for low values and increase to 75 at the upper end.<sup>28</sup> Observed bids of strong bidders are close to their values with a slight downward bias. This small bias may be due to random errors (which push bids down in the direction of the average bid of 50) or a conscious attempt to punish overly aggressive weak bidders. Indeed, for a strong bidder it is not very costly to bid slightly below value while the punishment for a weak bidder may be severe. In the experiment, some strong bidders with relatively low values decided to quit early (at prices between 60 and 70), making sure a weak bidder got "burned."<sup>29</sup> In a way, weak bidders are "cursed" in the second-price Amsterdam auction, which stimulates them too much and leaves them with negative earnings (-1.18 points per period).

## 5. Conclusions

In many real-estate auctions there exist *ex ante* differences between bidders. The presence of value asymmetries forces a seller to carefully consider the sales format as revenue equivalence cannot be expected to hold. Moreover, several economists have recently suggested that value asymmetries can adversely affect the performance of certain auction formats (e.g. Klemperer, 2002).

In our experiment, the English auction performed very poorly in terms of raising money in the presence of value asymmetries. Its revenues were lowest and showed the most variability of all the formats we tested. One reason is that the auction's revenue is

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<sup>28</sup> One possible explanation is that bidders who exit early in the first stage are risk averse. A weak bidder who makes it to the final stage could enter a low bid but then the resulting premium is small and the risk she took more or less in vain. To be consistent with her choice not to drop out in the first stage, the weak bidder may therefore be inclined to take even more risk and enter a high sealed bid in the final stage.

<sup>29</sup> In the actual Amsterdam auction (used to sell real estate in Holland's capital), bidders who participate only to win the premium are known as "premium hunters." In a typical session of the Amsterdam auction, which is held bi-weekly, there will be about ten premium hunters present. When a premium hunter ends up winning the house for sale, he is called a "hanger." In our experiment, hangers could incur a loss and possibly go bankrupt, which meant that they had to leave the experiment without receiving any money. In former times, the consequences of being a hanger in the actual Amsterdam auction could be much more severe: if a hanger could not pay for the house he won, he would be sent to prison for one or two months. If it happened twice, he would be tortured (Sikkel, 2001). (Incidentally, the reason for switching to masculine pronouns is that women were not allowed to bid in the Amsterdam auction until early in the twentieth century.)

determined by weak bidders who have no incentive to bid competitively as they have no chance to win at a profit. Recall that in an asymmetric English auction, weak bidders' expected payoff functions for bids above value are first constant at zero followed by a steep decline (i.e. for those bids with which the weak bidder may win). One way to stimulate more aggressive bidding is to raise weak bidders' expected payoffs for bids above value. Amsterdam auctions accomplish this by awarding a premium to the winner and the runner-up.

Our experiment clearly demonstrates the revenue-generating virtues of the Amsterdam auctions in asymmetric settings. The presence of a premium stimulates weak bidders to set an "endogenous reserve price" for their stronger rival with a dramatic effect on the final sales price. Moreover, the premium that has to be paid is often small due to Bertrand competition between weak bidders. As a result, the seller's revenue is significantly higher than in other formats while its standard deviation is small.

Value asymmetries also naturally arise in license auctions where incumbents often have advantages over entrants. In the recent European spectrum auctions, for instance, incumbents did not face the costs of building a network, they already possessed a consumer base, and they benefitted from an established brand name. The main objective of most license auctions is an efficient allocation, i.e. "putting the licenses in the hands of those that value them the most." At first glance, the English auction seems the most natural candidate to achieve such an efficient outcome. In our experiment, for example, the English auction yielded the highest efficiency levels in both symmetric and asymmetric settings. In a more dynamic setting, however, where entrants have a choice whether or not to participate, the English auction may well result in a low degree of competition with an adverse effect on efficiency. Indeed, it has been speculated that the use of the English auction (or, rather, its multi-license generalization, the simultaneous ascending auction) may have caused entrants to stay away in some of the European spectrum auctions.

Bidders in our experiment did not make a formal entry decision but they could choose to drop out at very low prices. We find that Amsterdam auctions are much more attractive for weak bidders: few of them drop out at near-zero prices unlike in the English auction where close to forty percent of the weak bidders quit right away. With more entry the Amsterdam auctions are also less prone to collusion. When weak bidders suspect the auction

proceeds will be divided among the members of a cartel they have a strong incentive to take part in an Amsterdam auction to pursue the premium.

Perhaps the most persuasive argument for its success is the century-long survival of the actual Amsterdam auction. Didi van den Elsaker, president of the First Amsterdam Real Estate auction, claims that sales prices at the auction generally exceed those in the real-estate market. Our experimental results provide further evidence for the revenue-generating virtues of the Amsterdam auctions in controlled laboratory circumstances. Amsterdam auctions are prime examples of practical mechanism design and the introduction of a premium is a robust and costless way to enhance revenues in an otherwise uncompetitive asymmetric setting.

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## Appendix A: Proofs of Propositions

*Proposition 1. The seller's revenue is the same for the first-price auction, English auction, and the two variants of the Amsterdam auction. The variances of the revenues in the Amsterdam auctions are less than that of an English auction but no less than that of a first-price auction.*

*Proof.* In the main text we already showed that the revenues of the first-price and English auctions are equal to  $R = (n-1)/(n+1)$ . For the second-price Amsterdam auction we need the joint density of the second and third-order statistic:  $f_{Y_3, Y_2}(x, y) = n(n-1)(n-2)x^{n-3}(1-y)$  for  $x < y$ , where we assume that  $n \geq 3$ . Its revenue can then be calculated as:

$$\int_0^1 \int_x^1 \{ \mathbf{B}_2(y) - 2\alpha(\mathbf{B}_2(y) - \mathbf{B}_2(x)) \} f_{Y_3, Y_2}(x, y) dy dx = \frac{n-1}{n+1},$$

where  $B_2(v) = (v + \alpha)/(1 + \alpha)$  is the equilibrium bidding function in the final stage of the second-price Amsterdam auction and the right side follows by simple integration. In the first-price Amsterdam auction the winner pays her bid, so that also the highest value draw matters. In this case we use the joint density  $f_{Y_3, Y_2, Y_1}(x, y, z) = n(n-1)(n-2)x^{n-3}$  for  $x < y < z$ . The revenue of the first-price Amsterdam auction can be written as:

$$\int_0^1 \int_x^1 \int_y^1 \{ \mathbf{B}_2(z, x) - 2\alpha(\mathbf{B}_2(y, x) - \mathbf{B}_1(x)) \} f_{Y_3, Y_2, Y_1}(x, y, z) dz dy dx = \frac{n-1}{n+1},$$

where  $B_2(v, v_3) = (v + v_3 + \alpha)/(2 + \alpha)$  is the final-stage bidding function when the third-highest type is revealed to be  $v_3$ , and  $B_1(v) = (2v + \alpha)/(2 + \alpha)$  is the bidding function of the first stage; the right side again follows by simple integration. It is cumbersome but straightforward to establish the variances of the revenues for the different formats:  $V_{\text{English}} = 2(n-1)/((n+1)^2(n+2))$ ,  $V_{\text{First-Price}} = (n-1)/(2n) V_E$ ,  $V_{\text{Second-Price Amsterdam}} = V_E - 2\alpha(2-\alpha)/((1+\alpha)^2(n+1)(n+2))$ , and  $V_{\text{First-Price Amsterdam}} = V_E - 2(1+2\alpha-\alpha^2)/((2+\alpha)^2(n+1)(n+2))$ . In the second-price Amsterdam auction the variance is minimized at  $\alpha = 1/2$ . At this level of  $\alpha$  the variance of the revenue of a second-price auction exceeds that of a first-price auction by  $(n-3)/((3n(n+1)(n+2)))$ , which is non-

negative since  $n \geq 3$ . Likewise, in the first-price Amsterdam auction the variance of the revenue is minimized at  $\alpha = 1/3$ . At this level of  $\alpha$  the variance of the revenue exceeds that of a first-price auction by  $(3n-7)/((7n(n+1)(n+2))$  which is strictly positive for  $n \geq 3$ . Q.E.D.

*Proposition 2.* *The following constitutes an equilibrium outcome of the second-price Amsterdam auction when  $\alpha \leq (L - 1)/(H - L)$ . In the first stage, the strong bidder bids up to her value and weak bidders bid up to  $L$ , at which point  $n - 2$  weak bidders drop out. In the second stage, weak bidders bid  $L$  and the strong bidder bids her value. The seller's revenue is  $L$ .*

*Proof.* First, consider the final stage where one weak and one strong bidder face a minimum price of  $L$ . When the strong bidder bids her value, the expected payoffs for a weak bidder with value  $v_w$  of bidding  $b \geq L$  are:

$$\pi_2^e(b|v_w) = (H - L)^{-1} \left\{ \int_L^b (v_w - z + \alpha(z - L)) dz + \alpha(b - L)(H - b) \right\}, \quad (7)$$

where the first (second) term in the curly brackets corresponds to the case where the weak bidder wins (loses). The derivative of the expected profit with respect to  $b$  is proportional to  $v_w - b + \alpha(H - b)$ , and since  $v_w \leq 1$  this derivative is negative all  $b > L$  if  $\alpha \leq (L - 1)/(H - L)$ . Hence it is optimal for a weak bidder to bid  $L$  in the second stage. The strong bidder can do no better than bidding her value in the second stage, since other bids either yield the same payoff (when strong wins) or may result in lower payoffs (when strong loses). For the same reason, the strong bidder can do no better than bidding up to her value in the first stage. Finally, in the first stage, weak bidders do not profit from dropping out earlier than  $L$  (since it would result in the same zero payoffs they receive in the proposed equilibrium). Furthermore, a single weak bidder cannot change the price level where the first stage ends when she is willing to bid up to levels higher than  $L$ . Q.E.D.

*Proposition 3. The following constitutes an equilibrium outcome of the first-price Amsterdam auction. In the first stage, the strong bidder bids up to her value and weak bidders bid up to  $X$ , where  $1 \leq X \leq L$ , at which point  $n - 2$  two weak bidders drop out. In the second stage, weak bidders bid  $X$  and the strong bidder bids slightly above  $X$ . The seller's revenue is  $X$ .*

*Proof.* First, consider the final stage where one weak and one strong bidder face a minimum price of  $X$ . When the strong bidder bids slightly above  $X$ , the weak bidder's expected payoffs for bids greater than  $X$  are  $v_w - b$ , which is negative since  $v_w \leq 1$  and  $X \geq 1$ . Hence it is optimal for a weak bidder to bid  $X$  in the second stage. The strong bidder can do no better than bidding slightly above  $X$  in the second stage, since higher bids would only raise the amount she has to pay. In the first stage, dropping out below value is weakly dominated for the strong bidder. Finally, in the first stage, weak bidders do not profit from dropping out earlier than  $X$  (since it would result in the same zero payoffs they receive in the proposed equilibrium). Furthermore, a single weak bidder cannot change the price level where the first stage ends when she is willing to bid up to levels higher than  $X$ . Q.E.D.

## **Appendix B: Instructions for the Second-Price Amsterdam Auction**

### INSTRUCTIONS EXPERIMENT: PART 1

Welcome to this experiment on decision making! You can make money in this experiment. Your choices and those of other participants will determine how much money you will make. Read the instructions carefully. There is paper and a pen on your table, which you can use during the experiment. Before the experiment starts, we will hand out a summary of the instructions and there will be two practice periods.

### THE EXPERIMENT

You will earn points in the experiment. At the end of the experiment your points will be exchanged into guilders, and each point will yield 25 cent. You will have a starting capital of 60 points. The experiment consists of 3 parts that each take 12 periods. Only when a part is finished will you receive the instructions for the next part. Each period you will be part of a group of 4 people, and a single product will be sold in every group.

### VALUE OF THE PRODUCT

For each participant, the product's value lies between 0 and 60 points, with each number between 0 and 60 being equally likely. The value for one participant is independent of the values for others. Therefore, your value will (very) likely differ from those of others. At the start of a period you will get to know your own value but not those of others. Likewise, other participants will not know your value.

### SALE OF THE PRODUCT: PHASE 1

Each period consists of two phases. In the first phase the product's price will be indicated by the "temperature" of a "thermometer" that rises point by point. Each participant has the possibility to push the "STOP" button to indicate that he or she is not willing to buy the product that period. When two participants have pushed the "STOP" button, the first phase is finished. The temperature level at which the second bidder pushed the "STOP" button is called the "BOTTOM PRICE." The two participants that have pushed the stop button in the first phase receive zero payoffs for that period.

If two (or more) participants push the "STOP" button at the same time then a random selection will determine which of these participants stop and which continue. The thermometer's temperature will never rise above 60 points. If the thermometer has not yet been stopped at 60, the computer will automatically push the "STOP" button for you.

### SALE OF THE PRODUCT: PHASE 2

In the second phase the two participants that are left will enter their "ultimate bids," and the one with the highest ultimate bid buys the product. The price that this participant pays is equal to the ultimate bid of the OTHER PARTICIPANT! Ultimate bids can be no less than the bottom price of phase 1 and can be no higher than 60 points. If both bidders enter the same ultimate bids, then chance will determine which of these two bidders buys the product.

The buyer does not literally receive a product: (s)he will receive an amount equal to the value of the product minus the price of the product (in points).

### SALE OF THE PRODUCT: PREMIUM

In the second phase both bidders receive a **premium** that depends on how much the lowest of the two ultimate bids exceeds the bottom price of the first phase. To calculate this premium, the difference

between the lowest *ultimate bid* and the bottom price is determined and both bidders receive 30% of this difference (in points).

The procedure to sell the product is illustrated with an example. THE NUMBERS IN THE EXAMPLE ARE ARBITRARILY CHOSEN.

#### EXAMPLE

The thermometer's temperature starts rising from 0. At a price of 22 bidder 1 pushes the "STOP" button. The temperature keeps rising until bidder 2 pushes the "STOP" button at a price of 32. This ends the first phase. The two remaining bidders (3 and 4) enter their ultimate bids (no less than 32) in the second phase. Suppose bidder 3 bids 42 and bidder 4 bids 50, then the results are as follows: bidder 4 buys the product at a price of 42 (the lowest ultimate bid). Bidders 3 and 4 receive a premium equal to 30% of  $42 - 32 = 10$  (since 42 is the lowest ultimate bid and 32 is the bottom price of the first phase). In addition, bidder 4 receives a payoff from the transaction equal to his/her value for the product minus 42, the price paid.

#### PROFIT AND LOSS

Notice that the highest bidder may incur a loss. If the highest bidder pays a price higher than his/her value for the product, and if this difference is greater than the premium, then a loss occurs. Just like a profit is automatically added to the amount earned up to that period, a loss is automatically subtracted. It is conceivable that at some point your earnings will become negative. (This is not likely and is under your control.) Since we do not want you to owe us money, you will have to leave the experiment in that case (without having earned any money). A participant with negative earnings will be replaced by the computer, and participants who are matched with the computer will know this in advance.

#### RESULTS OF A PERIOD

At the end of a period you will be told whether or not you had the highest bid and how much profit you made.

Then a new period will start. A new product will be sold and each participant receives a new value for the product. Your value for the product in one period will not depend on your values in other periods.

*Comment: Next some questions were asked to check a subject's understanding. When a subject gave a wrong answer, (s)he could read the relevant part of the instructions again. A subject could only proceed if (s)he had provided the right answer to a question.*

#### QUESTION ABOUT THE PRICE OF A PRODUCT

Suppose in the second phase your ultimate bid equals 52 while the other bidder has an ultimate bid of 46. What is the price that you will have to pay for the product?

#### QUESTION ABOUT THE PREMIUM

Suppose in the second phase your ultimate bid equals 52 while the other bidder's ultimate bid equals 46, and the bottom price of the first phase is 26. What is the premium for each of the two bidders in the second phase?

END

You have reached the end of the instructions. If you want to read some parts of the instructions again, push the "BACK" button. When you are ready, push the "READY" button. When all participants are ready, the experiment will start and you will NOT be able to return to these instructions. Before the experiment starts, a summary of the instructions will be handed out and two practice periods will be played. Your gains or losses during the practice periods will NOT be added to or subtracted from your earnings.

If you still have questions, please raise your hand!

#### SUMMARY OF THE INSTRUCTIONS: PART 1 (handout)

- Part 1 consists of 12 periods.
- Each period you will be part of a group of 4 persons.
- Your starting capital equals 60 points.
- Each point is worth 25 cent.
- In each group a product will be sold in every period.
- Each participant is assigned an (independent) value for the product, which is an integer number between 0 and 60, with every number being equally likely. Each participant only knows his/her own value and not that of others.
- Each period consists of two phases.

#### PHASE 1

In the first phase, a thermometer's temperature rises point by point to indicate the price. If a participant pushes the "STOP" button, (s)he indicates (s)he is not willing to buy the product that period. The temperature stops rising when two bidders have pushed the "STOP" button. The price level where the thermometer stops is called the "BOTTOM PRICE." The two participants that push the "STOP" button in the first phase receive zero payoffs that period.

#### PHASE 2

In the second phase, the two remaining participants enter their "ultimate bids." The participant with the highest ultimate bid buys the product at a price equal to the ultimate bid of the OTHER PARTICIPANT! The participant with the highest ultimate bid receives a "transaction profit" equal to his/her value for the product minus the price paid. Ultimate bids must be no less than the bottom price and no higher than 60 points. Both bidders in the second phase receive a premium. The premium is calculated as follows: each bidder receives 30% of the difference between the lowest ultimate bid and the bottom price. Hence, there are two kinds of payoffs:

(1) to the highest bidder:  $\text{TRANSACTION PROFIT} = \text{value} - \text{price} = \text{value} - \text{lowest ultimate bid}$

(2) to BOTH bidders in the second phase:  $\text{PREMIUM} = 0.3 * (\text{lowest ultimate bid} - \text{bottom price})$

#### INFORMATION

At the end of a period, gains and losses are automatically added to or subtracted from the points earned up to then and are communicated to the bidders. Then a new period will start, in which a new product is sold. Each participant will receive a new value for that period. The values for the product in one period are independent from the values in any other period.

## INSTRUCTIONS: PART 2

The second part of the experiment also lasts for 12 periods and the rules in this part are the same as those in the first part. The only difference is the way in which the value of a product is determined.

### VALUE OF THE PRODUCT

In each period, every group consists of 3 "small" bidders and 1 "large" bidder. For a small bidder the value of the product lies between 0 points and 60 points, and each integer number between 0 and 60 is equally likely. For a large bidder the product's value lies between 40 points and 100 points, and each number between 40 and 100 is equally likely. The value of one participant is independent of those of others. Therefore, your value will (very) likely differ from those of others. At the start of a period you will get to know your own value. You will also be told whether you are a small or a large bidder. You will not know the values of others, and other participants will not know your value.

### INFORMATION DURING PERIOD

When a participant pushes the "STOP" button in the first phase, the other participants will get to know whether this bidder is a small or large bidder. For both small and large bidders the thermometer's temperature will never rise above 100 points. If the thermometer has not yet been stopped at 60, the computer will automatically push the "STOP" button for you. In phase 2 each of the remaining bidders will know whether the other bidder is small or large. For both small and large bidders the ultimate bid in the second phase cannot be higher than 100 points (but higher than 60 if so desired).

After the results of a period have been communicated, a new period will start in which a new product is sold. Again it will be determined for each bidder whether (s)he is a small or large bidder and each participant receives a new value for the product. Your value for the product in the one period will not depend on your value for the product in any other period.

### END

You have reached the end of the instructions. If you want to read some parts of the instructions again, push the "BACK" button. When you are ready, push the "READY" button. When all participants are ready, the second part will start. Before the second part is started, a summary of the instructions will be handed out.

If you still have questions, please raise your hand!

### SUMMARY OF THE INSTRUCTIONS: PART 2 (handout)

- Part 2 consists of 12 periods.
- The rules are the same as those of part 1.
- The only aspect where parts 1 and 2 differ is the way in which the value of a product is determined.
- In each period there are 3 "small" bidders and 1 "large" bidder in every group.
- For each small bidder the value of the product lies between 0 points and 60 points, with each number being equally likely.
- For each large bidder the value of the product lies between 40 points and 100 points, with each number being equally likely.
- At the start of a period you will be told your value and whether you are a small or a large bidder.
- If a participant pushes the "STOP" button in the first phase, the other participants will know whether this bidder is a small or large bidder.
- For both small and large bidders the temperature of the thermometer rises in the first phase until at most 100 points. For both small and large bidders the ultimate bid in the second phase cannot exceed 100 points.

### INSTRUCTIONS: PART 3

The third part of the experiment also lasts for 12 periods. The rules in the third part of the experiment are the same as those of the second part. The only aspect where parts 2 and 3 differ is the way in which the value of a product for the large bidder is determined.

### VALUE OF THE PRODUCT

In each period there are again 3 "small" bidders and 1 "large" bidder in every group. For each small bidder the value of the product again lies between 0 points and 60 points, with each number being equally likely. For each large bidder the value now lies between 70 points and 100 points, with each number being equally likely. The value for one participant does not depend on the values for other participants. Therefore, your value will (very) likely differ from those of others.

When a participant pushes the "STOP" button in the first phase, the other participants will again know whether this bidder is a small or large bidder. In phase 2 each of the remaining bidders will know whether the other bidder is small or large.

### END

You have reached the end of the instructions. If you want to read parts of the instructions again, push the "BACK" button. When you are ready, push the "READY" button. When all participants are ready, the third part of the experiment will start. Before the third part is started, a summary of the instructions will be handed out.

If you still have questions, please raise your hand!

### SUMMARY OF THE INSTRUCTIONS: PART 3 (handout)

- Part 3 consists of 12 periods.
- The rules are the same as those of part 2.
- The only aspect where parts 2 and 3 differ is the way in which the value of a product is determined.
- In each period there are again 3 "small" bidders and 1 "large" bidder in every group.
- For each small bidder the value of the product again lies between 0 points and 60 points, with each number being equally likely.
- For each large bidder the value of the product lies between 70 points and 100 points, with each number being equally likely.

### Appendix C: Statistical Test Results

In this Appendix, test results regarding differences in revenues (Table C1) and efficiencies (Table C2) are presented. In addition, the tables list results for the weakly asymmetric parts of all treatments (see also Table C3).

**Table C1. Revenues**

		Symmetric	Weakly Asymmetric	Asymmetric	All
First Price	actual	41.5 <i>9.7</i>	54.2 <i>6.1</i>	57.4 <i>3.8</i>	51.0 <i>9.8</i>
	Nash	35.4 <i>8.5</i>	50.2 <i>4.6</i>	58.5 <i>0.8</i>	48.0 <i>11.1</i>
English	actual	38.2 <i>13.2</i>	45.4 <i>10.3</i>	44.1 <i>22.7</i>	42.6 <i>16.5</i>
	Nash	38.4 <i>13.0</i>	42.4 <i>11.2</i>	44.7 <i>11.1</i>	41.8 <i>12.1</i>
First-Price Amsterdam	actual	37.7 <i>10.8</i>	51.8 <i>9.7</i>	60.1 <i>8.2</i>	49.8 <i>13.5</i>
	Nash	36.2 <i>9.3</i>		70.0 <i>0.0</i>	
Second-Price Amsterdam	actual	34.7 <i>13.7</i>	48.3 <i>12.0</i>	66.0 <i>9.9</i>	49.7 <i>17.5</i>
	Nash	37.1 <i>9.3</i>		70.0 <i>0.0</i>	
			English	First-Price Amsterdam	Second-Price Amsterdam
		First-Price	1. > 0.02 2. > 0.00 3. > 0.07 all > 0.00	1. > 0.05 2. > 0.23 3. < 0.40 all > 0.57	1. > 0.00 2. > 0.02 3. < 0.00 all > 0.21
Mann-Whitney Test Results	English	-	-	1. > 0.77 2. < 0.01 3. < 0.02 all < 0.01	1. > 0.03 2. < 0.21 3. < 0.00 all < 0.01
	First-Price Amsterdam	-	-	-	1. > 0.12 2. > 0.12 3. < 0.10 all > 0.92

*Notes:* In the upper part, the first entry in a cell displays the average revenue and the second entry displays the standard deviation (in italics). In the lower part, test results are shown. In each cell, 1. refers to periods 1-12; 2. to periods 13-24; 3. to periods 25-36 and all to periods 1-36; > (<) indicates that the revenue of the treatment in the row is greater (smaller) than the revenue of the treatment in the column. After the >(<) sign the  $p$ -value of a Mann-Whitney test result is displayed. The tests use independent average data per group as observations ( $n_{\text{First Price}}=9$ ;  $n_{\text{English}}=8$ ;  $n_{\text{First-Price Amsterdam}}=9$ ;  $n_{\text{Second-Price Amsterdam}}=8$ ).

**Table C2. Efficiencies**

	Symmetric	Weakly Asymmetric	Asymmetric	All
First Price	96.5%	97.8%	95.9%	96.7%
English	99.7%	99.2%	96.7%	98.5%
First-Price Amsterdam	92.2%	94.3%	95.6%	94.0%
Second-Price Amsterdam	92.4%	90.8%	87.7%	90.3%
		English	First-Price Amsterdam	Second-Price Amsterdam
	First-Price	1. < 0.03 2. < 0.03 3. < 0.43 all < 0.07	1. > 0.13 2. > 0.11 3. > 0.96 all > 0.12	1. > 0.18 2. > 0.05 3. > 0.00 all > 0.00
Mann-Whitney Test Results	English	-	1. > 0.00 2. > 0.01 3. > 0.61 all > 0.01	1. > 0.00 2. > 0.00 3. > 0.01 all > 0.00
	First-Price Amsterdam	-	-	1. > 0.92 2. > 0.41 3. > 0.03 all > 0.15

*Notes:* In the upper part, each cell displays the average efficiency. In the lower part, test results are shown. In each cell, 1. refers to periods 1-12; 2. to periods 13-24; 3. to periods 25-36 and all to periods 1-36; > (<) indicates that the efficiency of the treatment in the row is greater (smaller) than the efficiency of the treatment in the column. After the >(<) sign the  $p$ -value of a Mann-Whitney test result is shown. Tests use independent average data per group as observations ( $n_{\text{First Price}}=9$ ;  $n_{\text{English}}=8$ ;  $n_{\text{First-Price Amsterdam}}=9$ ;  $n_{\text{Second-Price Amsterdam}}=8$ ).

**Table C3.** Chances for Weak Bidders (Highest-Value Bidder Excluded)

	Symmetric		Weakly Asymmetric		Asymmetric		All	
First-Price	13.0%	10.4%	11.1%	10.7%	9.3%	13.0%	11.1%	11.3%
	0.03	2.79	-0.04	2.15	0.04	0.82	0.01	2.09
English	4.1%	7.8%	1.0%	11.5%	5.2%	39.1%	3.5%	19.4%
	0.00	0.08	-0.10	1.65	-0.27	2.18	-0.12	1.58
First-Price Amsterdam	20.3%	8.8%	15.7%	7.4%	5.9%	4.2%	14.2%	6.9%
	0.55	4.28	0.47	3.96	-0.08	5.48	0.32	4.60
Second-Price Amsterdam	26.0%	8.3%	18.8%	7.0%	18.8%	6.5%	21.2%	7.3%
	0.56	3.05	-0.46	6.88	-1.18	8.35	-0.36	6.53

*Notes:* In each cell, the upper-left entry shows the probability that a weak bidder wins and the upper-right entry shows the observed frequency with which a weak bidder bids 5 or less. The lower-left entry displays a weak bidder's average profit and the lower-right entry displays (in italics) the standard deviation of this profit.

Table C3 reports a negative profit for the first-price auction in the weakly asymmetric setting, which is caused by an outlier. One subject entered a bid of 65 when his/her value was 29. Without this bid the average profit is 0.07 instead of -0.04).