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On the opacity change required to compensate for the revised solar composition
(Research Note)

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ABSTRACT

Context. Recent revisions of the determination of the solar composition have resulted in solar models in marked disagreement with helioseismic inferences.

Aims. The effect of the composition change on the model is largely caused by the change in the opacity. Thus we wish to determine an intrinsic opacity change that would compensate for the revision of the composition.

Methods. By comparing models computed with the old and revised composition we determine the required opacity change. Models are computed with the opacity thus modified and used as reference in helioseismic inversions to determine the difference between the solar and model sound speed.

Results. An opacity increase varying from around 30 per cent near the base of the convection zone to a few percent in the solar core results in a sound-speed profile, with the revised composition, which is essentially indistinguishable from the original solar model. As a function of the logarithm of temperature this is well represented by a simple cubic fit. The physical realism of such a change remains debatable, however.

Key words. Sun: abundances – Sun: interior – Sun: helioseismology

1. Introduction

The opacity in the solar interior, and hence the solar internal structure, depends sensitively on the abundances of the heavy elements (e.g., Turck-Chièze et al. 2001a; Turck-Chièze & Talon 2008). Recent analyses of the solar spectrum have led to substantial revisions of the solar abundances, particularly of oxygen, carbon and nitrogen (for a review, see Asplund 2005). Relative to previous work these studies have the advantage of being based on three-dimensional hydrodynamical models of the solar atmosphere and taking departures from local thermodynamic equilibrium into account. Also, unlike earlier analyses they result in consistent abundance determinations from different spectral lines. As a result of the revision, the ratio \( Z_s / X_s \) between the present solar surface abundances by mass of heavy elements and hydrogen is determined to be 0.0165, corresponding, in calibrated solar models, to \( Z_s = 0.0125 \). For comparison, the commonly used composition of Grevesse & Noels (1993) yields \( Z_s / X_s = 0.0245 \), resulting in \( Z_s = 0.0181 \).

As pointed out, e.g., by Basu & Antia (2004), Montalbán et al. (2004), Turck-Chièze et al. (2004), Bahcall et al. (2005) and Delahaye & Pinsonneault (2006) this revision has substantial effects on solar models, greatly increasing the difference between their internal sound speed and the sound speed inferred from helioseismology. As an example, we consider Model S of Christensen-Dalsgaard et al. (1996), using the Grevesse & Noels composition and OPAL opacities from Iglesias et al. (1992), and a corresponding model based on the new abundances and updated OPAL opacities (Iglesias & Rogers 1996). Fig. 1 shows sound-speed differences between the Sun and these two models, inferred through inversion of the ‘Best Set’ of observed frequencies of Basu et al. (1997), combining data obtained with the BISON network and the LOWL instrument (for further details on the inversion, see Christensen-Dalsgaard & Di Mauro, 2007). Large differences are also found between models based on the revised composition and the helioseismically inferred depth of the convection zone and envelope helium abundance. Chaplin et al. (2007) found that analysis of low-degree solar oscillations strongly supported the old heavy-element abundance; on the other hand, the results of the analysis by Houdek & Gough (2007), with careful inclusion of the influence of the outer layers of the Sun, indicated a heavy-element abundance somewhat lower than the Model S value, although substantially above the value obtained with the revised abundances. The effects on solar models of the new composition, as tested with helioseismology, were reviewed by Basu & Antia (2008). Guzik (2006) provided an overview of the, largely unsuccessful, attempts to modify the assumptions in the model computation to compensate for the composition change.
By far the most important effect on solar modelling of the heavy-element abundance arises through the opacity. Thus Montalban et al (2004) found that a substantial opacity increase near the base of the convection zone would help reducing the discrepancy caused by the revised abundances. Similarly, it was noted by Bahcall et al (2005) that an intrinsic change in the opacity could be used to correct the model computation, and they estimated that an opacity increase of around 11 per cent over a relatively broad range in temperature would be required. Indeed, there are undoubtedly significant uncertainties in the very complex opacity calculations. In the present note, following Bahcall et al., we make a more detailed analysis of this nature, estimating the intrinsic change in the opacity required to obtain a model structure corresponding largely to Model S, but with the revised composition.

2. Determination of the opacity change

The goal is to determine an opacity modification such that the sound-speed structure of Model S can be approximately reproduced with the revised solar surface composition. With just this constraint we can clearly only determine a modification that depends on a single variable which we take to be temperature \( T \). Thus we write the modified opacity as

\[
\log \kappa (\rho, T, X, Z) = \log \kappa^{(\text{Asp})}(\rho, T, X, Z) + f(\log T)
\]

(1)

where \( \rho \) is density, \( X \) and \( Z \) are the abundances by mass of hydrogen and heavy elements, and \( \kappa^{(\text{Asp})} \) is the opacity evaluated with the revised heavy-element composition; \( \log \) denotes logarithm to base 10. The goal is to determine \( f(\log T) \) such that \( \kappa \) evaluated for a structure corresponding to Model S, but with the revised heavy-element composition, matches the opacity \( \kappa^{(\text{GN})} \) in Model S, evaluated with the original Grevesse & Noels (1993) heavy-element composition.

![Fig. 1. Inferred relative differences in squared sound speed, in the sense (Sun) − (model), from inversion of the ‘Best Set’ of observed frequencies of Basu et al (1997). The open circles show results for Model S of Christensen-Dalsgaard et al (1996), using the old solar composition, while the filled circles show results for a corresponding model, assuming the revised composition. The horizontal bars indicate the resolution of the inversion while the vertical bars (hardly visible on this scale) show the 1−σ errors in the inferences.](image1)

![Fig. 2. The solid curve shows the intrinsic opacity change \( \Delta \log \kappa = f(\log T) \) determined from Eq. (4), on the basis of models with the old and the revised composition. The dashed curve shows the cubic fit \( f_{\text{approx}} \) to \( \Delta \log \kappa \), given by Eq. (5).](image2)

To estimate \( f(\log T) \) we consider two models of the present Sun: one is Model S, characterized by \( \{\rho_S, T_S, X_S, Z_S\} \), as functions of position in the model, and the second is Model Asp, similarly characterized by \( \{\rho_{\text{Asp}}, T_{\text{Asp}}, X_{\text{Asp}}, Z_{\text{Asp}}\} \). We determine the difference in \( \log \kappa \), at fixed \( T \), between the two models as

\[
\delta_T \log \kappa = \log \kappa^{(\text{GN})}(\rho_S(T), T, X_S(T), Z_S(T)) - \log \kappa^{(\text{Asp})}(\rho_{\text{Asp}}(T), T, X_{\text{Asp}}(T), Z_{\text{Asp}}(T))
\]

(2)

making explicit that the models are computed with \( \kappa^{(\text{GN})} \) and \( \kappa^{(\text{Asp})} \), respectively. Using the required property of \( \delta \kappa \), and linearizing in model differences, we obtain

\[
\delta_T \log \kappa \simeq f(\log T) + \left( \frac{\partial \log \kappa}{\partial \log \rho} \right)_{T,X} \delta_T \log \rho + \left( \frac{\partial \log \kappa}{\partial \log X} \right)_{T,\rho} \delta_T \log X,
\]

(3)

neglecting the effect of the different dependence of \( Z \) on position in the two models; here, e.g., \( \delta_T \log \rho = \log \rho_{\text{Asp}}(T) - \log \rho_S(T) \). A similar expression was obtained by Bahcall et al. (2005). Hence, neglecting the relatively modest effects of the differences in \( X \) and \( Z \) at fixed \( T \), we obtain the required opacity change as

\[
\Delta \log \kappa = f(\log T) \simeq \delta_T \log \kappa - \left( \frac{\partial \log \kappa}{\partial \log \rho} \right)_{T,X} \delta_T \log \rho.
\]

(4)

This procedure can be iterated, to compensate for the error in the linearization and the neglect of the composition effects. In practice we have found that two iterations are sufficient to reach a model closely matching Model S.

3. Results

Figure 2 shows the resulting opacity change, largely restricted to the radiative interior which evidently is the only region where the change is relevant. We have computed a full evolution sequence assuming the revised surface composition and applying this change to the opacity, and, as
Inference, Fig. 4 shows the sound-speed inversion using this model as before, and using the resulting Model S′′ (revised composition and the opacity change ∆ log κ) shown in Fig. 2. The dashed curve shows differences for Model S′′, similarly computed using the fit in Eq. (4). For comparison, the dotted curve shows the inferred difference for the original Model S, whereas the remaining models are essentially in accordance with observations.

4. Discussion and conclusion

The change in log κ obtained in Fig. 2 corresponds to an opacity change of around 30 per cent near the base of the solar convection zone, decreasing to a very modest level in the core. This immediately raises the question whether such a change is physically realistic. Comparisons between independent opacity calculations (e.g., Badnell et al., 2005) (see also Basu & Antia, 2008, for a review) indicate that the precision of the opacities in the relevant temperature range is better than 5 per cent, far smaller than the required change. On the other hand it seems possible that, although highly sophisticated, the present opacity calculation might neglect significant effects. Kurucz (personal communication) has noted that the neglect of a large number of elements of low abundance could have a significant effect on the Rosseland mean opacity, which is sensitive to even rather weak absorption in spectral bands not affected by lines of the more abundant elements.

It is perhaps relevant to recall the somewhat similar situation more than two decades ago when Simon (1982) made a plea for the reexamination of opacity calculations in the light of problems with the modelling of certain pulsating stars; he suggested that an increase in the opacity by a substantial factor could remove the discrepancies between models and observations. Although Magee et al. (1984) claimed that such an increase would be ‘incompatible with atomic physics’ it was in fact found in the OPAL opacity calculations (e.g., Iglesias & Rogers, 1991; Rogers & Iglesias, 1992), as a result of the inclusion of the effects of a large number of lines. Based on this experience one should perhaps be wary of excluding the possibility of substantial opacity modifications.

It is interesting that already the early helioseismic sound-speed inferences by Christensen-Dalsgaard et al.

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**Table 1. Properties of solar models (see text for a description).**

<table>
<thead>
<tr>
<th>Model</th>
<th>D_{ce}/R_☉</th>
<th>Y_e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asp</td>
<td>0.2712</td>
<td>0.2286</td>
</tr>
<tr>
<td>S</td>
<td>0.2885</td>
<td>0.2447</td>
</tr>
<tr>
<td>S'</td>
<td>0.2882</td>
<td>0.2489</td>
</tr>
<tr>
<td>S''</td>
<td>0.2875</td>
<td>0.2488</td>
</tr>
</tbody>
</table>

for Model S, calibrating the model to solar luminosity and radius as well as to the revised Z_e/X_e. Differences between the resulting Model S′ and Model S are illustrated in Fig. [3]. It is evident that this model matches Model S very closely. To test the effect on the comparison with the helioseismic inference, Fig. [4] shows the sound-speed inversion using this model as reference. Clearly it matches the helioseismic results as well as does Model S.

The simple dependence of f(log T) on log T makes it natural to approximate it by a low-order polynomial. In Fig. [2] the dashed line shows the following fit:

\[ f_{\text{approx}}(\log T) = 0.1298 - 0.1856\xi + 0.1064\xi^2 - 0.0345\xi^3, \]  

with \( \xi = \log T - 6.3 \), obtained with a least-squares fit with uniform weight to \( f(\log T) \), for \( \log T > 6.3 \). The result of using the modification in the evolution calculation, calibrating the model as before, and using the resulting Model S'' as reference in a sound-speed inversion is also shown in Fig. [4]. This is barely distinguishable from the result of using the original \( f(\log T) \).

We finally list in Table 1 the values of the depth \( D_{ce} \) of the convection zone and the envelope helium abundance \( Y_e \) for the models considered. For comparison, the helioseismically inferred value of \( D_{ce} \) is around 0.287 and \( Y_e \) has been determined to be around 0.25, although with some sensitivity to the equation of state used in the solar modelling (for a review, see Basu & Antia, 2008). It is evident that Model Asp is inconsistent with the observed values,
We have assumed that the opacity correction is a function of temperature alone. This is obviously a gross, if unavoidable, simplification which should be kept in mind if the correction obtained here is used for other stellar-model calculations. Also, we emphasize that the fit given in Eq. (5) is only valid in the range [6.3, 7.2] in log \( T \) over which it was obtained. Even so, it might be interesting to use an opacity change similar to the one obtained here in stellar computations, such as the isochrone analysis of M67 presented by VandenBerg et al. (2007).

Given the difficulties arising for solar modelling from the revised solar abundances it is obviously crucial to carry out further tests of the results. Also, the properties of the computed atmosphere models should evidently be tested against other relevant observations (e.g., Ayres et al. 2006). The complexity of the hydrodynamical modelling of the solar atmosphere makes independent calculations highly desirable. Thus it is encouraging that such calculations are now under way (Steffen, 2007; Caffau et al., 2007). Very recently, Caffau et al. (2008) made a determination of the solar oxygen abundance, resulting in a value intermediate between the old and new compositions considered here. We expect that the corresponding opacity correction required to match Model S, obtained as in the present analysis, would be roughly half the value shown in Fig. 2. Interestingly, Holweger (2001) obtained an oxygen abundance consistent with the value found by Caffau et al.; Turk-Chieze et al. (2004) showed that this did in fact result in a sound-speed difference relative to the helioseismic inferences intermediate between the results for the old and new compositions considered here.

Alternative independent determinations of the abundances of the relevant elements would obviously be very valuable. An interesting possibility, proposed by Auxil. (Gong et al. 2001) and reviewed by Basu & Antia (2008), is to constrain the heavy-element abundance from helioseismic inference of its effect on the thermodynamic, and hence the sound speed, in the solar convection zone. This is an extension of the successful helioseismic determination of the solar-envelope helium abundance (e.g., Vorontsov et al. 1991; Kosovichev et al. 1992; Antia & Basu 1994), following the suggestion of Gough (1984) and Däppen & Gough (1986). So far the results of such analyses are somewhat uncertain: Lin & Däppen (2003) found slight indications that the standard Grevesse & Noels (1993) heavy-element abundance was too high, whereas Antia & Basu (2006) and Liu et al. (2007) concluded that the helioseismic results confirmed the original abundances. Further analysis, investigations of the effects of the uncertainties in the equation of state, as well as better data including modes of higher degree, are required to obtain more definitive results.

References

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