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Helioseismic determination of the solar gravitational quadrupole moment

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ABSTRACT

One of the most well-known tests of General Relativity (GR) results from combining measurements of the anomalous precession of the orbit of Mercury with a determination of the gravitational quadrupole moment of the Sun J_2 . The latter can be done by inference from an integral relation between J_2 and the solar internal rotation. New observational data of high quality obtained from the Solar Heliospheric Satellite (SoHO) and from the Global Oscillations Network Group (GONG), allow the determination of the internal rotation velocity of the Sun as a function of radius and latitude with unprecedented spatial resolution and accuracy. As a consequence, a number of global properties of the Sun can also be determined with much higher accuracy, notably the gravitational quadrupole moment of the Sun. The anomalous precession of the orbit of Mercury is primarily due to GR effects but there are classical corrections the largest of which is that due to J_2 . It is shown here that the data are currently consistent with the predictions of GR.

Key words: Sun : rotation - helioseismology - general relativity

1 INTRODUCTION

For observations that are well resolved in space and in time the oscillations of the Sun can be decomposed into its pulsation eigenmodes, which are products of functions of radius and of spherical harmonic functions. Each mode, and therefore each measured oscillation frequency, is uniquely identified by three numbers : the radial order n , and the degree l and the azimuthal order m of the spherical harmonic. The solar rotation produces oscillation frequencies that are split into multiplets. The relationship between the mode frequencies and the rotation is :

$$2\pi \frac{\nu_{nlm} - \nu_{nl-m}}{2m} = \int_0^1 \int_{-1}^1 dx \, d \cos \theta \, K_{nlm}(x, \theta) \Omega(x, \theta) \quad (1)$$

where $x = r/R_\odot$ is the fractional radius, R_\odot is the radius of the Sun, and θ the colatitude. The K_{nlm} are the mode kernels for rotation. The Global Oscillations Network Group (GONG) produces values for the splittings through their data-reduction pipeline which are then available for inversion of the above integral relation. The Solar Heliospheric Satellite (SoHO) SOI/MDI instrument pipeline generally produces Ritzwoller-Lavely a-coefficients (Ritzwoller & Lavely, 1991), instead of individual splittings. The relation between these a-coefficients and the rotation rate is a linear integral equation very similar to (1) although with different kernels. Explicit expressions for the kernels for both

cases can be found in e.g. Pijpers (1997).

Using these data it is possible to determine the internal rotation rate of the Sun using inverse techniques. Results of such inversions can be found in e.g. Thompson et al. (1996) and Schou et al. (1997). Apart from the resolved rotation rate, there are some global properties of the Sun of astrophysical interest, which are related to the internal rotation rate through integral equations. One of these quantities is the total angular momentum H of the Sun which is related to the internal rotation rate through :

$$H \equiv \int_0^1 dx \int_{-1}^1 d \cos \theta \, \mathcal{I} \Omega(x, \theta) \quad (2)$$

with the moment of inertia kernel \mathcal{I} :

$$\mathcal{I} = 2\pi R_\odot^5 \rho x^4 (1 - \cos^2 \theta) \quad (3)$$

where ρ is the density inside the Sun. Another is the total kinetic energy T in rotation which is given by :

$$T \equiv \int_0^1 dx \int_{-1}^1 d \cos \theta \, \frac{1}{2} \mathcal{I} \Omega^2(x, \theta) \quad (4)$$

Since the total angular momentum is related linearly to the rotation rate Ω it is possible to construct the kernel \mathcal{I} directly from a linear combination of the individual model kernels (or a-coefficient kernels) using for instance the tech-

nique of Subtractive Optimally Localized Averages (SOLA) (cf. Pijpers and Thompson, 1992, 1994) as was done using GONG data by Pijpers (1998). This avoids the circuitous route of first determining the resolved rotation rate and then re-integrating. The reasons for doing this are that it can have better properties from the point of view propagation of the measurement errors, as well as avoiding systematic errors introduced at each computational step and it is computationally much less expensive. It is not possible to do the same for the kinetic energy since this is quadratic in the rotation rate. To determine T , one has to either follow the route of determining the resolved rotation rate, taking the square and then re-integrating, or one must make use of the second order splittings which are generally much less well-determined and are affected by physical effects other than rotation such as internal magnetic fields (cf. Gough & Thompson, 1990).

Another quantity of particular interest is the gravitational quadrupole moment J_2 of the Sun, caused by its flattening due to the rotation. The gravitational quadrupole moment J_2 of the Sun is that component of the gravitational field corresponding to the second Legendre polynomial as a function of co-latitude in an expansion of the gravitational field on Legendre polynomials. It is related to the solar oblateness Δ_ϕ , the ellipticity of the visible solar disk, as $J_2 = \frac{2}{3}\Delta_\phi$. The gravitational quadrupole moment of the Sun modifies the precession of the orbits of the planets. Therefore in using for instance the precession of the orbit of Mercury for testing the prediction from GR it is necessary to know J_2 . Expressions for the integral relation between J_2 and the internal rotation rate of the Sun have been derived for special cases of a rotation rate dependent on the radius only, or on simple parameterizations with respect to latitude (cf. Gough, 1981, 1982 ; Ulrich & Hawkins, 1981). More general expressions have been given by Dziembowski & Goode (1992) who expand the rotation rate by projection onto Legendre polynomials. However it can be shown that this is a somewhat cumbersome approach and quite simple expressions can be found even for a general distribution of $\Omega(x, \theta)$.

In section 2, the integral relation between the gravitational quadrupole moment of the Sun and a general internal rotation rate is given. In section 3 the results are given of performing the direct inversion for H and the values for T and J_2 obtained by taking the square of the resolved rotation rate and re-integrating, using data from GONG and using data from SOI/MDI on board SoHO. Conclusions are presented in section 4.

2 THE GRAVITATIONAL QUADRUPOLE MOMENT

The general expressions relating the various moments of the gravitational potential of rotating stars to their rotation rate have been given by Goldreich & Schubert (1968) and by Lebovitz (1970). These lead to what is essentially Clairaut-Legendre equations for the moments. For convenience the steps will be briefly repeated here. Starting point are Poisson's equation which relates the gravitational potential to the density distribution :

$$\nabla^2 \phi = -4\pi G\rho, \quad (5)$$

and the equation of motion :

$$\rho \nabla \phi = \nabla p - \rho \Omega(r, \theta)^2 r \sin \theta \varpi \quad (6)$$

where ϕ is the gravitational potential, G is the constant of gravity, ρ and p are the gas density and pressure respectively, and Ω is the rotation rate which is a function of radius r and co-latitude θ . ϖ is a unit vector perpendicular to the rotation axis. Writing equation (6) out in components yields :

$$\begin{aligned} \rho \frac{\partial \phi}{\partial r} &= \frac{\partial p}{\partial r} - \rho r(1-u^2)\Omega(r, u)^2 \\ \rho \frac{\partial \phi}{\partial u} &= \frac{\partial p}{\partial u} + \rho r^2 u \Omega(r, u)^2 \end{aligned} \quad (7)$$

in which $u \equiv \cos \theta$. Following the treatment of Goldreich & Schubert (1968) and Lebovitz (1970) for slowly rotating stars all quantities are described in terms of perturbations of the spherically symmetric non-rotating star, i.e. Ω^2 is treated as a quantity of first order in a small parameter expansion. Subscripts 0 refer to the non-rotating configuration, and 1 to the perturbed quantities. Collecting the first order terms in the perturbation analysis of equation (7) :

$$\begin{aligned} \rho_0 \frac{\partial \phi_1}{\partial r} + \rho_1 \frac{\partial \phi_0}{\partial r} &= \frac{\partial p_1}{\partial r} - \rho_0 r(1-u^2)\Omega(r, u)^2 \\ \rho_0 \frac{\partial \phi_1}{\partial u} &= \frac{\partial p_1}{\partial u} + \rho_0 r^2 u \Omega(r, u)^2 \end{aligned} \quad (8)$$

Of interest for the quadrupole moment of the gravitational potential is the projection onto the Legendre polynomial $P_2(u) = (3u^2 - 1)/2$. In the first equation of (8) all terms are multiplied by $\frac{5}{2}P_2(u)$ and then integrated over u . The second equation would yield $0 = 0$ since all its terms are odd in u . Therefore this equation is first integrated in u and then projected. In the following the subscripts 12 refer to the part of the first order perturbed quantities corresponding to these P_2 Legendre polynomial projections.

$$\begin{aligned} \rho_0 \frac{\partial \phi_{12}}{\partial r} + \rho_{12} \frac{\partial \phi_0}{\partial r} &= \frac{\partial p_{12}}{\partial r} - \rho_0 r \int_{-1}^1 du \frac{5}{3} [1 - P_2(u)] \\ &\quad \times P_2(u) \Omega(r, u)^2 \\ \rho_0 \phi_{12} - p_{12} &= \rho_0 r^2 \int_{-1}^1 du \frac{5}{2} P_2(u) \int_{-1}^u dv v \Omega(r, v)^2 \end{aligned} \quad (9)$$

The double integral in the second equation can be re-written, using partial integration :

$$\begin{aligned} \int_{-1}^1 du \frac{5}{2} P_2(u) \int_{-1}^u dv v \Omega(r, v)^2 &= \\ &= \frac{5}{4} \left\{ \left[\int_{-1}^u dv v \Omega(r, v)^2 \right]_{-1}^1 \right. \\ &\quad \left. - \int_{-1}^1 du (u^3 - u) u \Omega(r, u)^2 \right\} \\ &= \frac{5}{4} \int_{-1}^1 du (u^2 - u^4) \Omega(r, u)^2 \end{aligned} \quad (10)$$

The second equality of equation (9) can be used to eliminate p_{12} from the first of (9). After some rearranging the result is :

$$\begin{aligned} \rho_{12} \frac{\partial \phi_0}{\partial r} &= \phi_{12} \frac{\partial \rho_0}{\partial r} - \frac{\partial}{\partial r} [\rho_0 r^2 \mathcal{G}(\Omega)] + \\ &\quad - \rho_0 r \int_{-1}^1 du \frac{5}{3} (1 - P_2(u)) P_2(u) \Omega(r, u)^2 \quad (11) \\ p_{12} &= \rho_0 \phi_{12} - \rho_0 r^2 \mathcal{G}(\Omega) \end{aligned}$$

where \mathcal{G} is defined by :

$$\mathcal{G}(\Omega) \equiv \frac{5}{4} \int_{-1}^1 du (u^2 - u^4) \Omega(r, u)^2 \quad (12)$$

The relevant equations from the perturbed Poisson's equation (5) are :

$$\begin{aligned} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi_0}{\partial r} \right) &= -4\pi G \rho_0 \quad (13) \\ \frac{\partial^2 \phi_{12}}{\partial r^2} + \frac{2}{r} \frac{\partial \phi_{12}}{\partial r} - \frac{6}{r^2} \phi_{12} &= -4\pi G \rho_{12} \end{aligned}$$

in which ρ_{12} can now be substituted using the first of equations (11) :

$$\begin{aligned} \frac{\partial^2 \phi_{12}}{\partial r^2} + \frac{2}{r} \frac{\partial \phi_{12}}{\partial r} - \frac{6}{r^2} \phi_{12} &= \frac{4\pi r^2}{\mathcal{M}_r} \left\{ \phi_{12} \frac{\partial \rho_0}{\partial r} \right. \\ &\quad \left. - \frac{\partial}{\partial r} [\rho_0 r^2 \mathcal{G}(\Omega)] \right. \\ &\quad \left. - \rho_0 r \int_{-1}^1 du \frac{5}{3} (1 - P_2(u)) P_2(u) \Omega(r, u)^2 \right\} \quad (14) \end{aligned}$$

in which use has been made of the mass within radius r :

$$\mathcal{M}_r \equiv \int_0^r dr' 4\pi \rho_0 r'^2 \quad (15)$$

Now define the linear differential operator \mathcal{L} :

$$\mathcal{L}\phi_{12} \equiv \left\{ \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) - \left(6 + \frac{4\pi r^4}{\mathcal{M}_r} \frac{\partial \rho_0}{\partial r} \right) \right\} \phi_{12} \quad (16)$$

and a function f :

$$\begin{aligned} f(r) &\equiv -\frac{4\pi r^4}{\mathcal{M}_r} \left\{ r^2 \frac{\partial}{\partial r} [\rho_0 \mathcal{G}(\Omega)] + \rho_0 r \left[2\mathcal{G}(\Omega) + \right. \right. \\ &\quad \left. \left. \int_{-1}^1 du \frac{5}{3} (1 - P_2(u)) P_2(u) \Omega(r, u)^2 \right] \right\} \\ &= -\frac{4\pi r^4}{\mathcal{M}_r} \left\{ r^2 \frac{\partial}{\partial r} [\rho_0 \mathcal{G}(\Omega)] - \rho_0 r \times \right. \\ &\quad \left. \int_{-1}^1 du \frac{5}{4} (u^2 - 1) (5u^2 - 1) \Omega(r, u)^2 \right\} \quad (17) \end{aligned}$$

so that ϕ_{12} is the solution of $\mathcal{L}\phi_{12} = f(r)$. This equation can be solved using Green's functions. For $r > R_\odot$ the density $\rho_0 \equiv 0$ and therefore $f(r) \equiv 0$. An exact solution is then

$\phi_{12} = r^{-3}$. If another solution ψ of $\mathcal{L}\psi = 0$ is constructed which is regular at $r = 0$ the general solution is :

$$\phi_{12}(R) = \int_0^\infty dz G(R, z) f(z) \quad (18)$$

with the Green's function :

$$G(R, z) = \begin{cases} \frac{\psi(R) z^{-3}}{z^2 W(z)} & 0 \leq R \leq z \\ \frac{\psi(z) R^{-3}}{z^2 W(z)} & 0 \leq z \leq R \end{cases} \quad (19)$$

where $W(z)$ is the Wronskian of the solutions r^{-3} and ψ :

$$W(z) = \begin{vmatrix} \psi & z^{-3} \\ \psi' & -3z^{-4} \end{vmatrix} = -z^{-6} \frac{d}{dz} (z^3 \psi) \quad (20)$$

Since r^{-3} is not a solution of $\mathcal{L}\psi = 0$ for $r < R_\odot$ this equation is **only** valid for $R \geq R_\odot$. Of interest here is the solution ϕ_{12} at $R = R_\odot$. If in (18) R is replaced with R_\odot it is allowed to replace $z^2 W(z)$ with $R_\odot^2 W(R_\odot)$, which can be verified by substitution of (18) into (14). Since $f(r) = 0$ for $r > R_\odot$, the expression for $\phi_{12}(R_\odot)$ simplifies to :

$$\phi_{12}(R_\odot) = -R_\odot^{-3} \left[r^{-4} \frac{d}{dr} (r^3 \psi) \right]_{r=R_\odot}^{-1} \int_0^{R_\odot} dz \psi(z) f(z) \quad (21)$$

The solar oblateness Δ_ϕ is related to ϕ_{12} as

$$\Delta_\phi = -\frac{3}{2} \frac{R_\odot}{G\mathcal{M}_\odot} \phi_{12}(R_\odot) \quad (22)$$

Substituting the expressions for ϕ_{12} and $f(r)$:

$$\begin{aligned} \Delta_\phi &= \frac{2\pi R_\odot^2}{G\mathcal{M}_\odot} \left[\frac{d}{dr} (r^3 \psi) \right]_{r=R_\odot}^{-1} \int_0^{R_\odot} dr \left\{ \frac{-3r^6 \psi(r)}{\mathcal{M}_r} \times \right. \\ &\quad \left. \frac{\partial}{\partial r} [\rho_0 \mathcal{G}] + \frac{3r^6 \psi(r)}{\mathcal{M}_r} \frac{\rho_0}{r} \left[\int_{-1}^1 du \frac{5}{4} (u^2 - 1) \times \right. \right. \\ &\quad \left. \left. (5u^2 - 1) \Omega(r, u)^2 \right] \right\} \\ &= \frac{2\pi R_\odot^2}{G\mathcal{M}_\odot} \left[\frac{d}{dr} (r^3 \psi) \right]_{r=R_\odot}^{-1} \int_0^{R_\odot} dr \left\{ 3\rho_0 \mathcal{G} \times \right. \\ &\quad \left. \frac{\partial}{\partial r} \left(\frac{r^6 \psi(r)}{\mathcal{M}_r} \right) + \frac{3r^6 \psi(r)}{\mathcal{M}_r} \frac{\rho_0}{r} \times \right. \\ &\quad \left. \left[\int_{-1}^1 du \frac{5}{4} (u^2 - 1) (5u^2 - 1) \Omega(r, u)^2 \right] \right\} \\ &= \frac{2\pi R_\odot^2}{G\mathcal{M}_\odot} \left[\frac{d}{dr} (r^3 \psi) \right]_{r=R_\odot}^{-1} \int_0^{R_\odot} dr \int_{-1}^1 du \\ &\quad \left[\frac{15}{4} \rho_0 \frac{\partial}{\partial r} \left(\frac{r^6 \psi(r)}{\mathcal{M}_r} \right) (u^2 - u^4) + \right. \\ &\quad \left. \frac{15}{4} \frac{r^6 \psi(r)}{\mathcal{M}_r} \frac{\rho_0}{r} (u^2 - 1) (5u^2 - 1) \right] \Omega(r, u)^2 \end{aligned} \quad (23)$$

in which the second equality is obtained by partial integration, and the third is a re-arranging of terms making use of the definition (12) of \mathcal{G} .

For an Ω which is a function of r only, integration over u of the second term between square brackets in (23) is identical to 0 and the first term reduces to $\rho_0 \frac{\partial}{\partial r} \left(\frac{r^6 \psi(r)}{\mathcal{M}_r} \right) \Omega(r, u)^2$. Equation (23) then reduces to equation (12) of Gough (1981). Once the density $\rho_0(r)$ of the Sun is known, it is trivial to calculate the two-dimensional kernel :

$$\mathcal{F}(r, u) \equiv \frac{15\pi R_\odot^2}{2G\mathcal{M}_\odot} \left[\frac{d}{dr} (r^3 \psi) \right]_{r=R_\odot}^{-1} \frac{\rho_0}{r} \left(\frac{r^6 \psi(r)}{\mathcal{M}_r} \right) \times \left[\frac{\partial \ln \left(\frac{r^6 \psi(r)}{\mathcal{M}_r} \right)}{\partial \ln r} u^2 - (5u^2 - 1) \right] (1 - u^2) \quad (24)$$

Determining Δ_ϕ is thus reduced to evaluating the two-dimensional integral :

$$\Delta_\phi = \int_0^{R_\odot} dr \int_{-1}^1 du \mathcal{F}(r, u) \Omega(r, u)^2 \quad (25)$$

Direct inversion would have to make use of the second order splittings in an inverse problem, so the same route is followed as with the kinetic energy T : the resolved Ω^2 is multiplied with \mathcal{F} and integrated.

Using the standard solar model S of Christensen-Dalsgaard (cf. Christensen-Dalsgaard et al., 1996) the kernels \mathcal{I} and \mathcal{F} were calculated, and normalized to have unit integral over the solar volume. Contour plots in one quadrant are shown in figure 1. The other quadrants can be obtained by reflection in the coordinate axes.

3 RESULTS AND CONCLUSIONS

Two independent data sets have been used to determine the total solar angular momentum, the total kinetic energy and the gravitational quadrupole moment. One dataset is in the form of splittings obtained with the earth-based GONG network of telescopes : 33169 splittings distributed over 542 complete multiplets with $7 \leq l \leq 150$ and $1.5 \text{ mHz} < \nu < 3.5 \text{ mHz}$ gathered from GONG months 4 to 10. The other dataset is in the form of a-coefficients gathered from 144 d out of the first 6 months of operation of the SOI/MDI instrument on board the SoHO satellite. The data consists of 414 multiplets with $1 \leq l \leq 250$ and $1.0 \text{ mHz} < \nu < 4.2 \text{ mHz}$, and the odd a-coefficients up to at most a_{35} are available.

The GONG data leads to the values :

$$\begin{aligned} H_d &= [186.3 \pm 2.4] 10^{39} \text{ kg m}^2 \text{ s}^{-1} \\ H_i &= [186.3 \pm 3.7] 10^{39} \text{ kg m}^2 \text{ s}^{-1} \\ T &= [245.5 \pm 9.8] 10^{33} \text{ kg m}^2 \text{ s}^{-2} \\ J_2 &= [2.14 \pm 0.09] 10^{-7} \end{aligned} \quad (26)$$

The MDI data leads to the values :

$$\begin{aligned} H_d &= [192.3 \pm 1.9] 10^{39} \text{ kg m}^2 \text{ s}^{-1} \\ H_i &= [192.9 \pm 3.9] 10^{39} \text{ kg m}^2 \text{ s}^{-1} \\ T &= [262.5 \pm 10.] 10^{33} \text{ kg m}^2 \text{ s}^{-2} \\ J_2 &= [2.23 \pm 0.09] 10^{-7} \end{aligned} \quad (27)$$

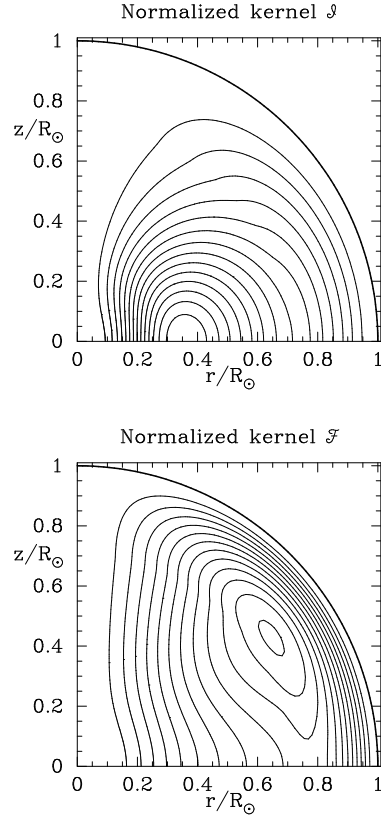


Figure 1. The normalized kernel \mathcal{I} for determination of the total solar angular momentum and kinetic energy (top panel), and the kernel \mathcal{F} for the determination of the gravitational quadrupole moment of the Sun. Contour intervals are 0.1 for both kernels.

The subscript d refers to a determination directly from the data using the freedom of the SOLA method to construct the kernel directly, the subscript i refers to the indirect method, which is re-integrating the resolved Ω . T and J_2 have been determined by re-integration only. Since the direct method should suffer much less from systematic effects, the value H_i is shown merely to demonstrate consistency between the two methods. Error weighted means for H_d , T , and J_2 are :

$$\begin{aligned} H &= [190.0 \pm 1.5] 10^{39} \text{ kg m}^2 \text{ s}^{-1} \\ T &= [253.4 \pm 7.2] 10^{33} \text{ kg m}^2 \text{ s}^{-2} \\ J_2 &= [2.18 \pm 0.06] 10^{-7} \end{aligned} \quad (28)$$

This determination of J_2 is entirely consistent with that of Paternó et al. (1996) who used direct oblateness measurements of the solar disk to infer the quadrupole moment.

One of the most well-known tests of GR results from combining measurements of the precession of the orbit of Mercury (cf. Shapiro et al., 1976 ; Anderson et al., 1987, 1991, 1992) with a determination of the gravitational quadrupole moment of the Sun J_2 . In the fully conservative parameterized post-newtonian (PPN) formalism, the predicted advance $\Delta\phi_0$ per orbital period of a planetary orbit with semi-major axis a and eccentricity e , after correcting for perturbations due to other planets, is :

$$\Delta\phi_0 = \frac{6\pi GM\lambda_p}{a(1-e^2)c^2} \quad (29)$$

where

$$\lambda_p = \frac{1}{3}(2 - \beta + 2\gamma) + \frac{R^2 c^2}{2GMa(1 - e^2)} J_2 \quad (30)$$

Here M and R are the mass and radius of the Sun, G is the gravitational constant, and c is the speed of light. The parameters β and γ are the Eddington-Robertson parameters of the PPN formalism (cf. Misner et al., 1973), which in general relativity are equal to 1. For Mercury the above relation (29) reduces to :

$$\Delta\phi_0 = 42.9794 \lambda_p \text{ ''/century} \quad (31)$$

and (30) is :

$$\lambda_p = \frac{1}{3}(2 - \beta + 2\gamma) + 2.96 \cdot 10^3 \times J_2 \quad (32)$$

Shapiro et al. (1976), using planetary radar ranging, found an anomalous precession for Mercury's orbit of 43.11 ± 0.21 . Using radar and spacecraft ranging Anderson et al. (1987) found 42.92 ± 0.20 and an update (Anderson et al., 1991) gives the value 42.94 ± 0.20 . The most recent result reported by Anderson et al. (1992) is 43.13 ± 0.14 . Combining the most recent value for the anomalous precession of Mercury's orbit and the value for J_2 given above in equations (28) yields :

$$\frac{1}{3}(2 - \beta + 2\gamma) = 1.003 \pm 0.003 \quad (33)$$

The error quoted here is entirely due to that in the planetary ranging data, since the error due to the uncertainty in J_2 is two orders of magnitude smaller.

In this paper it is thus demonstrated that the total solar angular momentum, its total kinetic energy in rotation, and the solar gravitational quadrupole moment can be determined through inverting integral equations that are linear in the rotation rate Ω or in its square, with known integration kernels. The value of the gravitational quadrupole moment (28) when combined with planetary ranging data for the precession of the orbit of Mercury yields a value for the combined PPN formalism parameters (33) which is consistent with GR in which this combination is predicted to be exactly equal to unity. More stringent tests of GR using the orbit of Mercury rely on measuring its orbital precession with much greater precision.

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