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# Reverberation mapping of active galactic nuclei : The SOLA method for time-series inversion

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## ABSTRACT

We present a new method to find the transfer function (TF) of the broad-line region in active galactic nuclei. The subtractive optimally localized averages (SOLA) method is a modified version of the Backus-Gilbert method and is presented as an alternative to the more often used maximum-entropy method for the inversion of variability data. The SOLA method has been developed for use in helioseismology. It has been applied to the solar oscillation frequency splitting data currently available to deduce the internal rotation rate of the sun. The original SOLA method is reformulated in the present paper to cope with the slightly different problem of inverting time series. We use simulations to test the viability of the method and apply the SOLA method to the real data of the Seyfert-1 galaxy NGC 5548. We find similar TFs for these data as previous studies using the maximum-entropy method. We confirm thereby previous results while simultaneously presenting an alternative and independent inversion method. Moreover, we do not find significant negative responses in the TF. The integral of the TF, an important quantity measuring the total observed line-processing in the broad-line region, is correctly reproduced by the SOLA method with high accuracy. We investigate the effects of measurement errors and how the resolution of the TF critically depends upon both the sampling rate and the photometric accuracy of the data.

**Key words:** AGN – inversion techniques – time-series analysis – galaxies: individual: NGC 5548

## 1 REVERBERATION MAPPING

The broad-line region (BLR) in active galactic nuclei (AGNs) is too small to be resolved spatially with even the largest planned telescopes. Indirect methods must therefore be employed to obtain information about its structure and dynamics. One way of doing so is via reverberation or echo mapping, a technique possible for variable sources only.

The broad emission lines originating in the BLR are photoionized by a small central continuum source (see Netzer 1991 for a detailed review of the standard model). In many AGN, especially the low-luminosity ones, this continuum source is observed to be variable in its flux. The broad emission lines respond to the continuum variations, albeit with a time delay of several days due to light-travel-time effects through the BLR. The high velocities of the clouds comprising the BLR redistribute line photons in wavelength, resulting in the broad emission-line profiles. Therefore, the combination of flux and profile variations of the emission lines in response to the ionizing-continuum variations can be used to map the phase space of the BLR (Blandford & McKee 1982; see Peterson 1993 and references therein for an extensive review).

Mathematically, the concept of reverberation mapping reduces to the inversion problem

$$L(v, t) = \int d\tau \Psi(v, \tau) C(t - \tau), \quad (1)$$

where  $L(v, t)$  is the observed emission-line light curve,  $v$  the projected velocity with  $v = 0$  the line centre,  $C(t)$  the observed continuum light curve, and  $\Psi(v, \tau)$  the transfer function (TF) of the BLR. The TF is the quantity that holds the information about the geometry, kinematics and physics of the BLR and is thus of central importance. To obtain higher  $S/N$  in the time series, Eq. (1) is usually integrated over projected velocity  $v$  which reduces it to the total-flux case

$$L(t) = \int d\tau \Psi(\tau) C(t - \tau). \quad (2)$$

To date two inversion methods have been applied to AGN variability data to invert Eq. (2) and so deduce the TF. These are the Fourier-transform method (FTM) (e.g., Bracewell 1986; Maoz et al. 1991) and the maximum-entropy method (MEM) (Skilling & Bryan 1984; Horne, Welsh & Peterson 1991; Krolik et al. 1991). The FTM is very noise-

sensitive and cannot handle irregularly sampled light curves properly. Only for the galaxy NGC 4151 has a TF been derived via this method (Maoz et al. 1991) but without a clear result. The MEM on the other hand has been successful in finding a number of TFs for several AGN: NGC 5548 (Horne, Welsh & Peterson 1991; Krolik et al. 1991; Peterson 1993; Peterson et al. 1994), Mkn 590 (Peterson et al. 1993) and NGC 3516 (Wanders & Horne 1994). A third method, based on a linear regularization method (cf. Press et al. 1992) is currently under development (Krolik 1994).

In this paper we will outline another method for the inversion of Eq. (2), not hitherto applied to AGN. This method is a modified version of the Backus-Gilbert method (Backus & Gilbert 1967, 1968, 1970) in use in geoseismology. More recently the method has been applied to helioseismology, specifically to the solar-oscillation frequency-splitting data to deduce the internal rotation rate of the sun. A comparison of various methods in use in this field, including the Backus-Gilbert method, can be found in papers by Gough (1985) and by Christensen-Dalsgaard, Schou & Thompson (1990). The Backus-Gilbert method is also sometimes referred to as optimally localized averages (OLA) or multiplicative OLA (MOLA). The MOLA methods are generally considered not to be cost-effective since they require numerous matrix inversions of large matrices. This led Pijpers & Thompson (1992, 1994) to reformulate the OLA method into a fast method called the subtractive optimally localized averages (SOLA) method. In Sect. 2 this method will be outlined in more detail and the transformation from the helioseismological formulation to the reverberation-mapping concept will be made. Section 3 describes on which points the application to reverberation mapping differs from helioseismology, and problems that are specific to AGN time-series analysis will be addressed. Section 4 discusses simulations done on realistic (i.e. noisy and irregularly sampled) artificial data and shows that recovery of the TF by the SOLA method can be done with currently available data. In Sect. 5 we then apply the SOLA method to the real data of the Seyfert-1 galaxy NGC 5548 and present its TF for four years of monitoring data. We show that the presently derived TFs are consistent with the previously derived ones using the MEM and the observed global changes in the TF from year to year are reproduced by the SOLA method. These global changes are both changes in the position of the peak of the TF and in the integral of the TF. The evolution of the BLR in NGC 5548 on the dynamical time scale is thereby confirmed. In Sect. 6 we summarize our results.

## 2 THE SOLA METHOD

The strategy of the SOLA method is to find a set of linear coefficients which, when combined with the data, produce the unknown convolved function under the integral sign. In the application to reverberation mapping these coefficients will be multiplied with the individual line fluxes in the measured time series and then summed to obtain an estimate of the TF at a certain lag  $\tau_0$ .

Consider again Eq. (2) but now with the line fluxes explicitly written as a time series consisting of  $N$  individual measurements  $L(t_i)$  with an associated measurement error

$$\delta L(t_i)$$

$$L(t_i) - \delta L(t_i) = \int_0^{\infty} d\tau \Psi(\tau) C(t_i - \tau) \quad \forall i = 1, \dots, N. \quad (3)$$

The lower limit of the integration is set to 0 which implies that the line flux is assumed to respond to the continuum variations and not vice versa. If the continuum flux  $C$  would be known for all values of  $t$  and would be error-free this problem would be equivalent to the problem of inverting helioseismology data. In that case an *averaging kernel*  $\mathcal{K}$  would be constructed :

$$\mathcal{K}(\tau_0, \tau) = \sum_{i=1}^N c_i(\tau_0) C(t_i - \tau) \quad (4)$$

which is strongly peaked around  $\tau_0$  and which is normalized :

$$\int_0^{\infty} d\tau \mathcal{K}(\tau_0, \tau) \equiv 1. \quad (5)$$

An estimate  $\widehat{\Psi}(\tau_0)$  of the TF at time lag  $\tau = \tau_0$  is then obtained by combining the line fluxes with this same set of coefficients

$$\widehat{\Psi}(\tau_0) \equiv \sum_{i=1}^N c_i(\tau_0) L(t_i) = \int_0^{\infty} d\tau \mathcal{K}(\tau_0, \tau) \Psi(\tau) + \sum_{i=1}^N c_i(\tau_0) \delta L(t_i). \quad (6)$$

The first term on the right of Eq. (6) is a weighted average of  $\Psi$  in which  $\mathcal{K}$  is the weighting function. The other term expresses the propagation of the errors in the line fluxes. In the SOLA method the aim is to make the kernel  $\mathcal{K}$  resemble some chosen target form  $\mathcal{T}$  while at the same time moderating the effect of the errors in the line fluxes. This is achieved by minimizing

$$\int_0^{\infty} d\tau [\mathcal{K}(\tau_0, \tau) - \mathcal{T}(\tau_0, \tau)]^2 + \mu \sum_{ij} E_{ij} c_i c_j \quad (7)$$

subject to the constraint (5). Here  $\mathbf{E}$  is the error variance-covariance matrix of the observed line fluxes.  $\mu$  is a trade-off parameter which may be chosen according to the relative desirability of making the first and second terms in (7) small. The ideal target function for the averaging kernel would be a delta function but it is clear that an infinite resolution cannot be attained in practice. A convenient form of a target function with an adjustable width is

$$\mathcal{T} = \frac{1}{f\Delta} \exp \left[ - \left( \frac{\tau - \tau_0}{\Delta} \right)^2 \right]. \quad (8)$$

Here  $f$  is a normalization factor which is introduced to make the total integral of  $\mathcal{T}$  equal to unity. If  $\Delta \ll 1$  the factor  $f$  reduces to  $\sqrt{\pi}$ .

The minimization leads to the matrix equation (Pijpers & Thompson 1994)

$$\mathbf{A} \cdot \mathbf{c}(\tau_0) = \mathbf{v}(\tau_0). \quad (9)$$

Here  $\mathbf{c}(\tau_0)$  is the vector of linear coefficients which is unknown, with an extra  $(N+1)$ th element which is a Lagrange multiplier. The elements of the symmetric matrix  $\mathbf{A}$  are :

$$A_{ij} = \begin{cases} \int_0^\infty d\tau C(t_i - \tau)C(t_j - \tau) + \mu E_{ij} & (i, j \leq N) \\ \int_0^\infty d\tau C(t_i - \tau) & (i \leq N, j = N+1) \\ \int_0^\infty d\tau C(t_j - \tau) & (j \leq N, i = N+1) \\ 0 & (i = j = N+1) \end{cases} \quad (10)$$

The elements of the vector  $\mathbf{v}$  are given by

$$v_i(\tau_0) = \begin{cases} \int_0^\infty d\tau C(t_i - \tau)\mathcal{T}(\tau_0, \tau) & (i \leq N) \\ 1 & (i = N+1) \end{cases} \quad (11)$$

Note that the elements of the matrix  $\mathbf{A}$  do not depend on the point  $\tau_0$  at which an estimate of the TF will be determined. The matrix has to be inverted only once. After this the different vectors  $\mathbf{v}(\tau_0)$  can be combined with the inverse matrix with relatively little computational effort, to scan the TF over the range of  $\tau_0$  of interest. It should also be noted that if a different error weighting  $\mu$  is chosen the matrix elements of  $\mathbf{A}$  can be calculated quickly but a new matrix inversion is required.

In practice one finds that there is a trade-off between the error weighting  $\mu$  and the resolution width  $\Delta$ . Increasing  $\mu$  produces a decreasing error in the TF propagated from the line flux measurements but the averaging kernel  $\mathcal{K}$  will depart more and more from the target form  $\mathcal{T}$  unless  $\Delta$  is also increased. This departure of  $\mathcal{K}$  from the prescribed form introduces a ‘systematic error’ in the determination of the TF which is undesirable. A more detailed discussion of this trade-off can be found in the paper of Pijpers & Thompson (1994).

### 3 ASPECTS OF TIME-SERIES ANALYSIS WITH THE SOLA METHOD

#### 3.1 Adapting SOLA

It is clear that for the problem of time series analysis discussed here the original formulation of SOLA cannot work since the continuum fluxes are known only for discrete points in a finite time interval. An extra problem is that this time series for the continuum flux also has measurement errors. These problems will be dealt with in several steps leading to a working algorithm for SOLA inversion of time series.

#### 3.2 The finite extent of the continuum time series

In Fig. 1 an example is shown of a time series of measurements of the continuum flux. The index  $i$  counts the measurements. For each added measurement the whole time series is replotted with an arbitrary vertical offset. The time series is plotted as a function of delay time  $\tau$  so that the most recent measurement is always the left-most point at  $\tau = 0$ . The dashed lines represent the unknown part of the light curve before the first measurement. For this time series of 10 measurements the 10 curves shown in Fig. 1 would be the set of base functions  $\mathcal{C}$  that appear in Eqs. (10) and (11). From Fig. 1 it is clear that it is not possible to evaluate the integrals since there is always a part of the base functions  $\mathcal{C}$  which is not known since it falls before the first measurement. The solution to this problem is to decide before

**figure 1** Example of a time series with 10 measurements. The index  $i$  counts the consecutive measurements. The whole time series is re-plotted as a function of delay time  $\tau$  for each added measurement, with an arbitrary vertical offset. The dashed lines represent the flux before the first measurement in the series

starting the inversion to put the upper limit of the integrations to a certain finite time lag  $\tau_{\max}$ . This implies that the TF will be assumed to be negligible for time lags  $\tau > \tau_{\max}$ . In essence this is an assumption regarding the maximum size of the BLR. If this parameter is unknown one must experiment with various values for  $\tau_{\max}$  and re-do the inversion for each value.

The maximum time lag  $\tau_{\max}$  is indicated in Fig. 1 by the vertical dash-dotted line. Even with a finite value of  $\tau_{\max}$  there will still be some base functions that are known for only part of the range  $[0, \tau_{\max}]$  so these will have to be dropped completely. This reduces the number of independent base functions. It is important not to choose  $\tau_{\max}$  too large because then the number of independent base functions becomes very small which has an adverse effect on the error in the reconstructed TF and also on the resolution that can be attained.

It is also important not to choose  $\tau_{\max}$  too small since then each base function is determined by only a very few points. This means that the integrals in (10) and (11), which are calculated on the basis of an interpolation, become uncertain. Note that the discarding of some of the base functions does not mean that the first continuum measurements in the series are discarded. These are all used but shifted to the right in the base functions for higher  $i$ . However, if it is assumed that the measurements of continuum flux and line flux are always simultaneous there is some data that is obtained observationally which cannot be used and must be discarded. If the first  $M$  base functions are discarded the first  $M$  line-flux measurements are also discarded. Since the line flux of these first  $M$  measurements (partly) responds to variations in a part of the continuum flux that has not been measured, these measurements are of limited use for the inversion. Because the TF is only calculated at one  $\tau_0$  at a time, one could, in principle, use a floating  $\tau_{\max}$ , i.e.,  $\tau_{\max}$  is short if  $\tau_0$  is short, such that fewer emission-line data are ignored. However, this would mean the matrix inversion (9) must be done for each different  $\tau_{\max}$  which is expensive. Furthermore, with finite resolution, boundary effects play a significant rôle when  $\tau_{\max}$  becomes of the order of twice the

resolution or less. All in all, it is not advantageous to do this and in this paper we will keep  $\tau_{\max}$  fixed and drop the first  $M$  line-flux measurements.

### 3.3 The discrete sampling of the continuum time series

Even after limiting the inversion to a finite interval in  $\tau$  the base functions are not completely determined. The continuum flux is only known on a discrete set of points. The second step towards a working algorithm is to interpolate between the individual measurements. It is important to realize that this has an impact on the resolution that can be attained in the determination of the TF. It implies that there is a minimum to the width  $\Delta$  of the target function (8) even with error-free data.

At this point it is useful to discuss the ‘intrinsic’ resolution that could be attained with a series of error-free continuum-flux measurements. If the sampling rate of the continuum flux is strictly regular the base functions also are sampled strictly regularly. It is immediately clear that the minimum possible resolution is then of the order of the time interval between samplings. If the sampling is irregular the result is less obvious. It is useful to distinguish at this point between a regular time series with ‘missing data’ and a fully irregular or non-redundant time series.

In a time series of the first type the sampling can be thought of as taken on integer multiples of some minimum time interval  $\Delta t_{\min}$ . The ‘missing data’ are then all those integer multiples of  $\Delta t$  for which a measurement is not available. In this case in the diagram of Fig. 1 a set of  $K$  regularly spaced vertical lines can be drawn with  $K = \tau_{\max}/\Delta t_{\min}$ . All measured points of all the base functions will fall on those  $K$  lines. Considered in this way the minimum resolution width  $\Delta$  with this type of irregular sampling must be of the order of  $\Delta t_{\min}$ . If a fraction  $f$  of the data is missing in a time series with this imaginary sampling rate  $\Delta t_{\min}$  the length of the total time series must be larger by a factor  $1/f$  than a similar time series without missing data, in order to really attain something close to this resolution.

In a non-redundant time series there is no minimum sampling interval for which all measurements fall on integer multiples of that time interval. Note that this does not require that some samples are taken with an infinitely small time interval between them. Consider again Fig. 1, where now the interval between 0 and  $\tau_{\max}$  is rescaled so that it is mapped onto the interval  $[0, 1]$ . In the limit of an infinitely long non-redundant time series the set of  $\tau/\tau_{\max}$  for which at least one measurement is available would lie dense in the unit interval. In other words the distribution of these points would have the same properties as the rational numbers on the unit interval. For an error free time series of this type the minimum resolution width is inversely proportional to the total length of the time series.

The time series of measurements of fluxes of variable AGN as they are usually reported fall into the first category of irregularly sampled data. The minimum time interval here is 1 day. However, one should not expect to attain this limiting resolution of 1 day for two reasons. The first reason is the finite error in the measurements of the line flux. This leads to the usual trade off between errors in the TF and resolution of the TF, mentioned in the previous section. The

second reason is finite error in the continuum flux measurements. These errors give rise to an error or uncertainty in the integration kernel  $\mathcal{K}$  that is constructed. This error increases for a decreasing width of the target kernel  $\mathcal{T}$  which effectively limits the resolution width to a minimum which may lie well above the minimum of 1 day for error-free data.

There is one other reason why this limiting resolution may not be attained which has nothing to do with the sampling strategy or the measurement errors. It is clear that if the continuum does not vary, then it is impossible to deduce anything about a TF. As an extension to this it is not hard to see that if there is a finite minimum time scale on which the continuum source varies, which is larger than the sampling time interval, one should not expect to attain a better time resolution in the TF than that typical time scale of the continuum source. This should in principle be reflected in an impossibility for the algorithm to find a good kernel  $\mathcal{K}$  with a smaller width than this time scale (cf. Fig. 7).

### 3.4 Interpolation of the continuum time series

The practical problem left is to find an interpolation scheme for the continuum flux time series so that the integrals of (10) and (11) can be calculated. This interpolation must not introduce spurious variations due to the errors in the continuum flux. When tested on simulated time series a cubic spline was found to introduce spurious oscillations with amplitudes of up to four times the formal errors on the individual measurements. Such spurious structure in the continuum flux would of course have no counterpart in the line-flux measurements. In this way an artificial de-correlation would be found for some determinations of the TF, effectively forcing the TF towards values close to 0. This would happen in particular for values of  $\tau_0$  for which none or only a few base functions have a measured point.

The interpolation scheme decided upon here is a Savitzky-Golay smoothing filter for the continuum time series, adapted to an irregular sampling interval (cf. Press et al., 1992). With this algorithm a least-squares fit of a low order polynomial to a moving window of  $2N_s + 1$  points in the time series is constructed. Except at the beginning and end of the entire series these points are always distributed symmetrically around the point for which the polynomial will be used in the interpolation. An interpolating polynomial constructed for the continuum light curve around time  $t_i$  is then used only in the interval  $[(t_{i-1} + t_i)/2, (t_i + t_{i+1})/2]$ . For the  $N_s$  points at both ends of the time series the number of points used is reduced one at a time until only  $N_s + 1$  points are used to construct an interpolating polynomial for the first and last point in the time series. In this case the order of the polynomial is also reduced to make sure that the error in the coefficients of the polynomial remains under control. A choice of a moving window of 5 points (i.e.  $N_s = 2$ ) and a quadratic polynomial are found to perform satisfactorily. At the boundaries the polynomial was reduced to linear. The choice of the number of points in the smoothing window and the degree of the polynomial in general must depend on the ratio of the real point-to-point variations in the continuum light curve and the statistical measurement noise.

Note that this interpolation scheme is not continuous for all  $\tau$  in the range  $[0, \tau_{\max}]$ , contrary to a cubic spline. This is of no importance for the algorithm but it can show

up in the reconstructed kernel  $\mathcal{K}$  which then appears somewhat jagged. Note also that the polynomial is intended to produce an interpolation without spurious oscillations. It is not intended to significantly reduce the errors from the data, for which a much larger window should be chosen. Suppressing the errors from the continuum is of secondary importance since this can better be done by increasing the total length of the time series, and therefore the number of base functions, which only requires additional observations. The cost of smoothing with a window with more points is a degraded limiting resolution unless the sampling is done at a higher rate. It is certainly impossible to get a higher sampling rate in the part of a time series that is already obtained. Also there are practical limitations to the highest sampling rate in observing campaigns, whereas extending a time series is always possible in principle.

The resolution limit imposed by this scheme is not uniform since it is proportional to the length in time of the smoothing window. A guide-line for the smallest value of  $\Delta$  is found by equating the FWHM of the Gaussian target function with the width of the window:

$$\Delta_{\min} \sim \frac{2N_s + 1}{2\sqrt{\ln 2}} \Delta t_{\text{av}} > \Delta t_{\min} \quad (12)$$

Here  $\Delta t_{\text{av}}$  is the average time interval between individual flux measurements. As an example consider a time series of 100 measurements spread over one year. The limiting value for the resolution width would be  $\Delta \sim 11$  d.

### 3.5 Error propagation

Combining the steps in the previous sections leads to a working inversion algorithm. Such an algorithm is not much use without an estimate of the error in the determination of the TF, which are simply propagated measurement errors. In the original SOLA formulation the error propagation is relatively simple. The TF is obtained from linear combinations of the line fluxes and therefore the variance of the TF is simply

$$\sigma^2(\Psi(\tau_0)) = \mathbf{c}^T(\tau_0) \cdot \mathbf{E}_L \cdot \mathbf{c}(\tau_0) \quad (13)$$

Here  $\mathbf{E}_L$  is the error variance-covariance matrix of the line-flux measurements. Usually the errors in the individual line-flux measurements are assumed to be uncorrelated which means that the matrix  $\mathbf{E}_L$  is diagonal.

In the new formulation of SOLA the coefficients  $\mathbf{c}$  should also have an error associated with them due to the fact that they are calculated from the base functions  $C$  that are not error-free. Calculating the error propagation through a matrix inversion is extremely cumbersome. Furthermore the convolution in Eq. (3) implies that the error estimate should involve the integral of the unknown TF even if the errors in the individual measurements of the continuum flux are uncorrelated. Finally, adding error terms in the definition (10) for the cross-correlation matrix of the base functions  $C$  means that the matrix  $\mathbf{E}$  will not be a diagonal matrix. This is not a problem for the algorithm but the error estimate is likely to be sensitive to the interpolation scheme.

There is a simpler way to measure the uncertainty due to the measurement errors in the continuum flux. Once the coefficients  $\mathbf{c}$  have been obtained they are regarded as known with an arbitrarily high precision. The coefficients are combined with the line-flux measurements in the usual way and

the error estimate is obtained from (13). The new element in the analysis is to combine the coefficients with the errors in the base functions  $C$ . This will produce an error estimate for each point of the integration kernel  $\mathcal{K}$ , with the propagated errors obtained from the interpolation scheme and the coefficients. Calculating this error propagation is relatively simple since it results entirely from linear combinations with known coefficients of the least-squares fits of the interpolating polynomials which have associated error estimates (cf. Eq. 4).

The departure of  $\mathcal{K}$  from the target form  $\mathcal{T}$  is already used for an estimate of the error due to an imperfect match of the two (cf. Pijpers & Thompson, 1994). Instead of a strict upper limit to this error that would be obtained if the continuum flux was error free, the upper limit is now ‘fuzzy’. This arises because there is an error  $\delta\mathcal{K}(\tau)$  associated with the integration kernel  $\mathcal{K}$ . Because the kernel is only used in integrated form the effect of a variance is reduced. If by analogy to the definition used by Pijpers & Thompson (1994) a ‘systematic error’ measure  $\chi$  is defined

$$\chi \equiv \int_0^{\tau_{\max}} d\tau [\mathcal{K} - \mathcal{T}]^2, \quad (14)$$

the upper limit to the systematic error in  $\Psi$  is given by :

$$E_{\text{upp}} = \left( \frac{\Psi_{\max} - \Psi_{\min}}{2} \right) \chi^{\frac{1}{2}} \quad (15)$$

where  $\Psi_{\min}$  and  $\Psi_{\max}$  refer to the minimum and maximum value of the TF over the range  $[0, \tau_{\max}]$ . Only if this error  $E_{\text{upp}}$  is of the same order of magnitude as the  $\sigma(\mathcal{K}(\tau))$  or smaller than that, will the errors in the integration kernel significantly affect the error estimate  $E_{\text{upp}}$  due to imperfect matching of the calculated integration kernel  $\mathcal{K}$  to the target function  $\mathcal{T}$ .

In this way the errors in the data are fully taken into account. The errors in the line fluxes are propagated properly into an error estimate for the TF determination. The errors in the continuum fluxes are propagated properly into an error estimate of the integration kernel  $\mathcal{K}$ . The departure of  $\mathcal{K}$  from  $\mathcal{T}$  can be used to estimate the effect of this on the estimate of the TF at the appropriate  $\tau_0$ . It should be noted that this measure  $\chi$  is not available in the classical MOLA methods such as the original Backus & Gilbert method (1967, 1968, 1970). This is due to the fact that the concept of a target function is unique to the SOLA method.

At this point one could argue that the minimization in Eq. (7) is not optimal if only the errors in the line flux are included in the matrix  $\mathbf{E}$ . From Eq. (3) it can be seen that if there is an uncorrelated measurement error associated with the continuum measurement as well as the flux measurement, then this will introduce an additional variance of the order of  $\Phi^2 \sigma^2(C(t_i))$ . The integral  $\Phi$  of  $\Psi$  can be determined independently (cf. sec. 4.5). The combination of the errors in the continuum and emission-line light curves can be estimated very roughly by adding the two. For the matrix  $\mathbf{E}$  used in the optimization such a rough estimate is sufficient. The form that is used here is given by :

$$\begin{aligned} E_{ii} &= [\sigma^2(L(t_i)) + \Phi^2 \sigma^2(C(t_i))] / E_n \\ E_{ij} &= 0 \quad i \neq j \end{aligned} \quad (16)$$

Because of the interpolation of the continuum light curve the

errors  $\delta C$  are not really uncorrelated. It is therefore not correct to use these estimates for the final error determination. It is therefore never used for that but instead the analysis is used that is described in the first part of this section. However, for the minimization in (7) it is a sufficiently accurate approximation to ensure that the errors on the constructed kernel  $\mathcal{K}$  remain sufficiently small to produce meaningful results.

In Eq. (16) the factor  $E_n$  is introduced to produce a dimensionless quantity. In helioseismology it is customary to make the trace of the matrix  $\mathbf{E}$  equal to the number of base functions. The same is done here :

$$E_n = \frac{1}{N} \sum_{i=1}^N \sigma^2(L(t_i)) + \Phi^2 \sigma^2(C(t_i)) \quad (17)$$

### 3.6 Simultaneity of line and continuum measurements

In the previous sections it is assumed that the measurements of the line flux and of the continuum flux are simultaneous. This is not necessary for the method to work. As was mentioned before at the start of the time series the line-flux measurements are of little use so they could be omitted.

If there are continuum-flux measurements not accompanied by a measurement of the line flux these can simply be incorporated into the base functions  $C$ . This does not add a new base function but it does improve locally the accuracy of the interpolation.

If there are line-flux measurements not accompanied by a measurement of the continuum flux these can also be added to the data. In this case a new base function is added with a value at  $\tau = 0$  of this base function that is interpolated between subsequent and successive measurements of the continuum flux. It is important to note that adding such line-flux measurements can serve to improve the  $S/N$  ratio in the TF somewhat, but that they cannot improve the resolution with which the TF can be determined since the appropriate time resolution in the continuum flux is absent.

## 4 SIMULATIONS

### 4.1 The set-up

To test the SOLA method on typical AGN variability data we chose a representative continuum light curve and a representative TF to simulate real data. The representative continuum light curve was taken to be the NGC 5548 light curve as published by Peterson et al. (1994). This four-year light curve was interpolated linearly onto a regular grid and then smoothed with a moving-average filter of total width 40 d. The weights for the individual points were taken to be equal. Note that this smoothing has implications for the time resolution that can then be attained in the determination of the TF.

Previous MEM reconstructions of the TF yielded TFs with little or no response at zero lag and peaking at 10–20 d lag. We thus decided to use a TF with similar characteristics. We calculated a TF using the RAN model from Blandford & McKee (1982) where we chose an inner radius  $r_0$  of the BLR of 15 light days. Thus our artificial TF peaks at 30 d lag and has little response at zero lag. The maximum lag

**Figure 2.** The TF that was used for the simulations

$\tau_{\max}$  with a non-zero TF is 60 d. This TF was normalized to unity in order to compare the integral over the TF of the reconstructions with unity. The resulting TF is shown in Fig. 2. The sharp peak may be unrealistic but does provide us with an estimate of how well the SOLA method can reconstruct sharp features. The peak was chosen to lie at 30 d instead of 20 d because the smoothing of the continuum time series degrades the final attainable resolution and we decided to ‘stretch’ the simulated TF by 50% in order to retain approximately the same resolution relative to  $\tau_{\max}$  as in the real data where  $\tau_{\max} \sim 40$  d.

We then convolved the artificial continuum light curve with this TF and obtained an artificial emission-line light curve. The two light curves were then chopped such that only the second year of data (cf. Peterson et al. 1994) remained. This reduced data set was then resampled irregularly onto the same grid as the original observations were done. In this way two ideal (i.e., noise-free) light curves were produced.

To examine the effects of data errors, random noise from a Gaussian distribution with a standard deviation of 2%, 5% and 10% of the signal was added. The routine RAN1 from the Numerical Recipes (Press et al. 1992) was used as the random-number generator and for every continuum light curve the same seed was used, such that the noise pattern was identical for each continuum curve except for the magnitude of the noise. The same was done for the emission-line light curves albeit with a different seed to avoid correlated errors between the continuum and emission-line time series. The eight light curves (four continuum and four emission-line with 0%, 2%, 5% and 10% noise each) are presented in Fig. 3.

These light curves were then used to test the SOLA method. Ten simulation runs were devised according to the scheme in Table 1. In simulations 1–4 the noise was increased from 0 to 10% on both light curves. These simulations reflect real data. In simulations 5–7 only the noise in the emission-line light curve was increased from 2 to 10% in order to test the effect of the errors on the emission-line data only, and in simulations 8–10 only the noise in the continuum was increased from 2 to 10% whereas the emission-line light curve was taken to be error-free. These last simulations thus measure the effect of continuum-error propagation.

### 4.2 The results

We have run the simulations twice, once with a high resolution of  $\Delta = 5$  d and once with a resolution of  $\Delta = 10$  d. The

3

The eight light curves used for the simulations. The left column shows the continuum light curve and the right column the emission-line light curve. The upper row shows the noise-free curves, the second row 2% noise has been added to the data, the third row 5% noise, and the lower row 10% noise

mean sampling interval of the time series is only 3.4 d and one might argue for being able to reach this resolution but as explained in Sect. 3 the introduction of noise prohibits this. For both simulation runs, the trade-off parameter  $\mu$  between the importance of resolution on the one side and error suppression on the other side was set to 0.001. The results of the SOLA inversion on the simulated data are presented in Figs. 4 and 5. Figure 4 shows the 5-day reso-

lution solution whereas Fig. 5 shows the 10-day resolution solution. As an example, Fig. 6 displays the kernels of SIM 2 (i.e., the simulation with 2% noise on both the continuum and emission-line light curves) with  $\Delta = 10$  d.

We used  $N_s = 2$  and the expected maximum resolution for the error-free data due to the irregularity of the time series and the interpolation scheme used to compensate for that is  $\Delta_{\min} \sim 10$  d (cf. Eq. 12). We thus expect the simu-



**Figure 4.** The recovered TFs with a five-day resolution for the simulated data. The target TF is shown as a dashed line in each plot

lation with  $\Delta = 5$  d to be undersampled.

The first thing we note from Figs. 4 and 5 is that the error bars on the  $\Delta = 5$  simulation are systematically larger than on the  $\Delta = 10$  simulation. This is an obvious result due to the fact that less light-curve data are used per TF resolution element because the kernels  $\mathcal{K}$  for the  $\Delta = 5$  simulation are narrower than for the  $\Delta = 10$  simulation.

The second thing we note in both ( $\Delta = 5$  and  $\Delta = 10$ ) ideal simulations (SIM1) is that the TF is recovered to

within the error bars but shows a slightly displaced peak towards shorter  $\tau$ . This is due to the asymmetry of the used TF and the skewness of the kernels at  $\tau \lesssim 20$  d. The skewness of the kernel can be seen from Fig. 6 where the kernels of SIM2 with  $\Delta = 10$  d are displayed. The kernels that are significantly affected, at  $\tau = 15$  d and 20 d, are notably asymmetric: they have more power towards larger lag than shorter lag. Together with the asymmetric simulated TF used it is obvious that this would raise the reconstructed

**Table 1.** The simulations

continuum noise	Emission-line noise			
	0%	2%	5%	10%
0%	SIM 1	SIM 5	SIM 6	SIM 7
2%	SIM 8	SIM 2		
5%	SIM 9		SIM 3	
10%	SIM 10			SIM 4

TF above the input TF at these lags.

The  $\tau = 0$  boundary of the reconstructed TF is slightly raised compared to the input TF, especially in the  $\Delta = 10$  case, because there is no data on the negative side of the boundary and the finite width of the kernels let more-positive signal from larger  $\tau$  leak into the  $\tau = 0$  domain. If the peak of the TF would lie close to a resolution element away from  $\tau = 0$ , this boundary effect will pull the observed peak of the TF towards  $\tau = 0$ , resulting in an underestimation of the size of the BLR. The effect of the boundary is made clear by the kernels in Fig. 6. Here it is observed that with  $\Delta = 10$  d the kernel at  $\tau = 5$  d is still effectively a kernel at  $\tau = 0$  d and also the kernel at  $\tau = 10$  d is significantly affected by the boundary. This boundary effect must be taken into account when examining short lags in reconstructed TFs.

Having established these artificial effects on the ideal data set (SIM1) we can increase the noise on both continuum and emission-line light curves to 2%, 5% and 10% via simulations 2–4. The reconstructed TFs show increasing error bars but also substructure that must be entirely due to noise. In the undersampled  $\Delta = 5$  simulations we would hardly recognize the TF in the 5%-noise case (SIM3) even though the reconstruction is consistent with the input TF to within the errors. SIM3 is still acceptable in the  $\Delta = 10$  case though also here we undersample because the noise degrades the maximum attainable resolution of 10 d somewhat. The real data on NGC 5548 have error bars on the 3% level, close to SIM2 and significantly better than SIM3. For the real data, a resolution of  $\Delta \sim 10$  d would be expected with  $N_s = 2$ . Even though the sampling rate of the NGC 5548 light curves is significantly better than 10 d it is not legitimate to aim to resolve structure in the TF on scales less than  $\sim 10$  d! Actual experimenting with  $N_s$  and  $\Delta$  on the real data suggested  $N_s = 2$  and  $\Delta = 8$  is the preferred parameter setting for the NGC 5548 data.

Simulations 5–7 show the effect of increasing the noise on the emission-line light curve while using the error-free continuum data. It is clear that this will increase the error bars and substructure considerably and it merely shows that the resolving power of the inversion degrades, as expected, and we aim for a too high resolution when we set  $\Delta = 10$  d. The error bars in SIM7 are larger than in SIM4 because of the way the data errors are used in the minimization. In the minimization the variance of the continuum measurements and line measurements are added. The error bars in the TF are calculated on the basis of the line-flux measurements only whereas the errors in the continuum flux are used to obtain an error estimate for the integration kernel. So with the same weighting factor  $\mu$  the trade-off in the minimization shifts towards smaller error variance in the TF from SIM7

to SIM4. The error variance in the constructed integration kernel  $\mathcal{K}$  is larger in SIM4 than in SIM7 (cf. Fig. 7) which compensates for the decrease of  $\sigma(\text{TF})$ .

The simulations that used error-free emission-line light curves but increasingly noisy continuum light curves (simulations 8–10) show the effect of the error propagation ‘under the integral’. These errors clearly introduce spurious signals and structure in the TF on seemingly random time scales larger than the resolution. This is a problem that one must test for in order to interpret a noisy inversion correctly. The effect, however, is moderate up to the 2–3% noise level.

### 4.3 The resolution of the TF

Figure 7 presents all kernels peaking at  $\tau = 30$  d for both the  $\Delta = 5$  (left column) and  $\Delta = 10$  simulations (right column). These are the kernels responsible for picking up the TF signal at  $\tau = 30$  d. Obviously, these kernels have finite width and the full width at half maximum (FWHM) is a measure of the resolution of the reconstructed TF. Note that the FWHM is at best  $2\Delta\sqrt{\ln 2}$ . Equation (12) shows that with the present interpolation scheme, the maximum resolution attainable is  $\Delta_{\min} \sim 10$  d for the present data set, even in the absence of noise! The FWHM of the kernel for the  $\Delta = 5$  SIM1 simulation is  $\sim 30$  d, implying  $\Delta_{\min} \sim 18$  d. This is due to the fact that we cannot resolve high-frequency structure that is not present; recall that these simulations are smoothed versions of a real light curve with a smoothing window of 40 days. For ideal data, the interpolation can be done better than done at present because  $N_s$  can be decreased.

The kernels clearly show that increasing the noise on the continuum-flux data increases the uncertainty in the determination of the kernel, whereas the emission-line noise has no effect on the determination of the kernels. When increasing the noise, the kernels for the  $\Delta = 5$  simulations become narrower than the theoretical limit of  $\Delta_{\min}$ . The reason for this is that the noise is then used by the method just as real rapid variations would be used : to produce a small width of the kernel. The noise then allows a closer resemblance of the kernel to the target function. Thus, in the presence of noise, aiming for a resolution better than  $\Delta_{\min}$  means risking spurious structure to show up in the final TF, because a narrow kernel also means fewer (noisy!) data points are used for the inversion. A spurious datum can thus have a large impact on the inversion when aiming for too high a resolution. This is effectively equivalent to undersampling the data. Comparing the  $\Delta = 5$  simulations with the  $\Delta = 10$  simulations clearly shows this undersampling problem is not severe if we confine ourselves to  $\Delta \sim \Delta_{\min} \sim 10$  d.

Note that aiming for a  $\Delta$  less than  $\Delta_{\min}$  does *not* increase the error estimate of the TF. The error bars on the TF are calculated from the emission-line profiles only, whereas  $E_{\text{upp}}$  is calculated from the mismatch of the kernel and the target function. in the case of noisy data, the noise allows the kernel to closely resemble the target function even in the absence of intrinsic high-frequency variations (cf. Fig. 7), thereby keeping  $E_{\text{upp}}$  small. With noise-free data, it is not possible to construct a kernel that is narrower than the minimum variability time scale of the intrinsic variations. Aiming for  $\Delta < \Delta_{\min}$  thus equals risking to fit noise instead of signal; this leads to the spurious substructure in the TF, and

**Figure 5.** The recovered TFs with a ten-day resolution for the simulated data. The target TF is shown as a dashed line in each plot

this is evidently not a wise thing to do.

#### 4.4 The cross-correlation function

It is well-known that the position of the peak of the cross-correlation function (CCF) of the continuum and emission-line light curves is a measure of the size of the BLR (Gaskell & Sparke 1986; Gaskell & Peterson 1987; Edelson & Krolik 1988; Robinson & Pérez 1990). It is not difficult to work out

(e.g., Bracewell 1986; Penston 1991) that the CCF is the convolution of the TF with the auto-correlation function (ACF) of the continuum time series. Because the ACF is a symmetric function, asymmetric properties of the TF, like e.g. the position of the peak, must be reflected in a similar way in the CCF. Of course, the reverse is also true. The error analysis described in the previous sections, can thus also be applied, at least qualitatively, to the CCF.

Table 2 presents the peak values and the peak positions

**Figure 6.** The kernels for simulation 2 with a resolution  $\Delta = 10$  d

of the CCFs of the time series used for SIM1–SIM10. It is clear that increasing the noise on either the continuum or emission-line light curve decreases the correlation coefficient of the peak of the CCF; noise increases the width of the ACF and the CCF peak becomes less well-defined. This is confirmed by Table 2. Also, noisy data decrease the correlation coefficient at small delays (provided the noise on the continuum and emission-line light curves are not positively correlated), thereby increasing the position of the peak of

the CCF towards larger delays. This is also confirmed by Table 2. Thus, noisy data most likely overestimate the size of the BLR when only the peak of the CCF is considered. Note that theoretical modelling of BLRs show the peak of the CCF is more sensitive to the inner boundary of the BLR than to its centre, thereby underestimating the actual size of the BLR (Robinson & Pérez 1990). However, this is different from the numerical effect presented here.

Because negative lags are unphysical, with finite resolu-

**Figure 7.** The kernels for all simulations for  $\tau = 30$  d. The left column displays the 5-day resolution case; the right column the 10-day resolution case. An artificial vertical offset has been given for displaying purposes

tion not only the TF, but also the CCF will show a boundary effect at  $\tau = 0$ . The correlation coefficient at  $\tau = 0$  will thus be increased for TFs peaking away from zero delay, and this effect will be larger for worse resolutions. This pulls the peaks of the TF and CCF towards shorter lags. In the data set examined here this might be a problem, especially with the  $\Delta = 10$  resolution, where the kernels are significantly asymmetric up to  $\tau \sim 20$  d due to the boundary. Also, the asymmetric TF together with the finite resolution will shift the peak towards smaller lag. The peak of the TF is known to be at  $\tau = 30$  d but both the TF and the CCF peak at  $\tau \sim 23$  d for the noise-free data due to the low resolution of the experiment.

Noisy and incomplete data thus reduce the resolving power of both the TF and the CCF such that the peak positions of the TF and CCF are displaced.

#### 4.5 The integral of the TF

As Wanders (1994) showed, the delay integral of the TF is a physically relevant parameter  $\Phi = \int d\tau \Psi(\tau)$  that measures the total observed line reprocessing in the BLR.  $\Phi$  is not directly reproducible by the MEM; only by first solving for the TF and then integrating.

From Eq. (6) it can be seen that an estimate  $\hat{\Phi}$  of the integral  $\Phi$  can be obtained by letting the kernel  $\mathcal{K}$  be unity

**Table 2.** The cross-correlation results and the integral  $\Phi$  of the TF for the simulations. The error in the peak position of the CCF is about 2 d

	CCF( $\tau_{\text{peak}}$ )	$\tau_{\text{peak}}$	$\hat{\Phi}$	$\delta\hat{\Phi}$
SIM 1	0.91	23 d	1.000	0.1%
SIM 2	0.89	23	0.998	0.3%
SIM 3	0.79	25	0.996	0.7%
SIM 4	0.62	37	1.002	1.4%
SIM 5	0.90	23	1.000	0.3%
SIM 6	0.85	29	1.000	0.7%
SIM 7	0.74	34	1.000	1.4%
SIM 8	0.90	23	0.997	0.1%
SIM 9	0.85	30	0.993	0.1%
SIM 10	0.73	30	0.993	0.1%

over  $\tau \in [0, \tau_{\text{max}}]$ . This can be accomplished by aiming for an infinitely low resolution, i.e., by setting  $\Delta$  equal to a very large number. In this way, SOLA optimizes correctly for the integral problem. The calculation can be done at any  $\tau_0$  with this flat kernel. Similarly to the calculation of  $\hat{\Psi}(\tau_0)$ , a direct error estimate for  $\hat{\Phi}$  is obtained. The results for the ten simulations are presented in Table 2. Except for the unrealistic case of error-free line flux measurements the deviation from 1 of the integral is smaller than the error estimate from SOLA based solely on the line-flux errors. This means that the errors in the continuum flux do not prevent an accurate measurement of the integral. It is immediately clear that  $\Phi$  can be recovered to a precision better than  $\sim 1\%$  for realistic data with  $\lesssim 5\%$  noise. For the real data on NGC 5548, which have characteristics close to SIM 2, we can therefore expect an accuracy of  $\hat{\Phi}$  of  $\sim 1\%$ .

Constant contributions to the fluxes can confuse this result but will not be discussed here because they do not influence the conclusions of the present paper. See Wanders (1994) for a discussion of this.

## 5 NGC 5548

Peterson et al. (1994) published four years of monitoring of NGC 5548 and presented MEM-recovered TFs for each year of monitoring. We have taken the same data set and recovered the TF of each year using the SOLA method. We did not aim for a higher resolution than  $\Delta = 8$  d since this would introduce local structure in the TF that is due to measurement errors in the continuum and emission-line light curves. The trade-off parameter  $\mu$  was set to 0.001 for all four inversions, and the maximum  $\tau$  for which we calculated the TF was 50 d. Runs with larger  $\tau_{\text{max}}$  showed no significant structure at larger lags. Since we have to ‘throw away’ the first data of the the emission-line light curve up to  $\tau_{\text{max}}$ , decreasing  $\tau_{\text{max}}$  means including more data.

The results for the four individual years are shown in Fig. 8. These TFs resemble the TFs derived by the MEM (Peterson et al. 1994): little response at zero lag, little response at lags larger than about 40 d and peaking at approx. 12–20 d. The decreasing trend in the position of the peak of the TF from  $\sim 20$  d in 1989 to  $\sim 13$  d in 1992 as was reported by Peterson et al. is also reproduced here (cf. Fig. 8). The peak of the TF is always more than a resolution

**Table 3.** The integral  $\Phi$  of the TF for NGC 5548

year	$\Phi_{\text{SOLA}}$	$\delta\Phi_{\text{SOLA}}$	$\Phi_{\text{av}}$
1989	0.863	0.004	0.876
1990	0.797	0.004	0.825
1991	0.776	0.007	0.794
1992	0.720	0.006	0.738

element away from the boundary and therefore the boundary will not have too much influence on the position of the peak. However, the simulations have shown that a systematic underestimation of the position of the peak of  $\sim 5$ –7 d cannot be excluded. The upper limit  $E_{\text{upp}}$  to the systematic error due to the uncertainty in the kernels is significantly smaller than the statistical uncertainty propagated from the line measurements, indicating we do not run into serious trouble due to the uncertainties in the continuum measurements. Note that the TF is not significantly away from zero at zero lag in any of the reconstructions, except for the year 1989. There is no indication that the TF would be significantly negative anywhere. This does not exclude the TF to become negative on smaller time scales than the present resolution of about 10 d, but the positiveness of the TFs suggests the MEM does not run into serious trouble while using its positivity constraint, at least not with the resolutions attained with the present data. Negative values in the TF might arise e.g. due to nonlinear line responses (Sparke 1993).

Note that the SOLA method is not hampered or aided by a positivity constraint and negative responses would have been recovered as long as a convolution of a gaussian with such a negative signature in the TF is larger than the noise level. Experimenting with the resolution ( $\Delta$ ) showed that when a too high resolution was aimed for, artificial structure appeared in the TFs, similar to the noisy simulations in Sect. 4. These artificial structures can become significantly negative, but are totally due to noise. Care must be taken not to overinterpret such structure.

In all, the present SOLA results are consistent with previous MEM results for the recovery of the shape of the NGC 5548 TF. This gives us more confidence in the viability of these results and it presents the SOLA method as an alternative to the MEM.

Table 3 presents the integral of the four TFs of NGC 5548 as derived by the SOLA method ( $\Phi_{\text{SOLA}}$ ) and as published by Wanders (1994) who derived  $\Phi$  via the averaging method  $\Phi_{\text{av}} = \langle L(t) \rangle / \langle C(t) \rangle$ . The errors on  $\Phi_{\text{av}}$  are approximately 2.5% and as discussed in Sect. 4.5, the errors on  $\Phi_{\text{SOLA}}$  are on the order of 1%. It can thus be seen that the decreasing trend in  $\Phi_{\text{av}}$  is confirmed by  $\Phi_{\text{SOLA}}$ .

A decrease in  $\Phi$  is equivalent to a decrease in observed reprocessing of the emission-line within the BLR (Wanders 1994). We thus confirm the findings that both the position of the peak of the TF and the total integral of the TF are time-variable on a time scale of a few years, corresponding to the dynamical time scale of the BLR clouds. This strongly suggests evolution of the BLR takes place in NGC 5548 and the system is not stationary on these time scales.

**Figure 8.** The four transfer functions for the H $\beta$  emission-line region of NGC 5548. The TFs are for the monitoring years 1989, 1990, 1991 and 1992. Also shown are the upper limits  $E_{\text{upp}}$  to the systematic error made due to the uncertainty in the kernels (cf. Eq. 15)

## 6 RÉSUMÉ

This paper is primarily concerned with the development and testing of a new method for the inversion of time-series of AGN continuum and line-emission fluxes, to obtain the transfer function of the broad-line region. The results of the application of this method to real data of NGC 5548 are also shown. The various aspects of the method, of the tests and of the applications are discussed extensively already in sections 3, 4, and 5 respectively so we will confine ourselves to

a few summarizing statements.

The SOLA method constructs an integration kernel from a linear combination of subsections of the continuum light-curve. The TF is reconstructed from a linear combination of the emission-line measurements corresponding to the continuum subsections, using the same linear coefficients. The value of the TF thus obtained is a weighted average of the TF where the weighting function is the constructed integration kernel.

The method is not subject to any regularity constraint

or positivity constraint for the TF.

The method allows a very clear determination of the time resolution with which the transfer function can be determined. Most of the procedure depends only on the sampling strategy and the standard deviations in the flux measurements. It is therefore possible to determine the resolution before reconstructing the TF. This provides a-priori a criterion for avoiding or discarding spurious sub-structure in the TF.

The method uses the variance in the data to produce the smallest possible standard deviation in the TF at the time resolution used. Because the method is linear the calculation of error propagation is particularly straightforward.

Because only one large-matrix inversion has to be done, the SOLA method is cost-effective and a typical inversion of the NGC 5548 data (typically  $\sim 80$  base functions) takes approximately one hour of CPU time on a SUN Sparc-2 workstation. This allows for experimenting in parameter space by varying the control parameters  $\mu$  and  $\Delta$ . In the original Backus-Gilbert formulation the large number of matrix inversions prevents parameter-space exploration.

The tests of the method show that for a realistic TF and time sampling the TF can be recovered to within the formal error estimates. Because of the finite resolution of the time series some systematic shifts are found in the position of the peak of the TF. We argue that this is essentially an effect of resolution. It is therefore intrinsic to the problem and the sampling. This means that other methods to determine this peak position such as the CCF and the MEM must also suffer from these systematic effects.

The application of the method to the data obtained for NGC 5548 confirms the time evolution found previously of the TF over the four years 1989-1992: both the position of the peak of the TF and its total integral  $\Phi$  are time-variable. This evolution of the broad-line region takes place on the dynamical time scale of the clouds within the BLR.

The results of the method, the tests, and the application to real data allow us to make precise recommendations for setting up observing campaigns, based on the required resolution of the TF and the expected standard deviation in the flux measurements. The strategy of current observing campaigns cannot be expected to produce much higher resolution or better results than shown here and we strongly urge future monitoring campaigns to use a higher sampling rate and an even better photometric accuracy. Unfortunately, both will increase the pressure on telescope time.

The method can easily be adapted to a 2-D inversion for the TF as a function of time-lag and velocity. Considering that the shape of an emission-line profile can be determined more accurately than its flux this may yield a great deal of new information. For instance, the SOLA method opens the way to the determination of the  $Q$ -function as introduced by Perry, van Groningen & Wanders (1994). They split the full TF into a steady-state and a deviation term:

$$\Psi(v, \tau) = \frac{\Phi(v)}{\Phi} \Psi(\tau) + Q(v, \tau) \quad (18)$$

where  $\Phi(v) = \int d\tau \Psi(v, \tau)$  is the steady-state profile, i.e., the emission-line profile in response to a constant continuum flux.  $Q(v, \tau)$  is then a function extremely sensitive to variations in the emission-line profile and from this a great deal about the structure of the BLR can be learned. Perry et al.

present an inversion problem to directly measure  $Q(v, \tau)$  via normalized emission-line profiles. Because  $Q(v, \tau)$  is partly negative, the MEM cannot recover this function because of its built-in positivity constraint. This is no problem for the present SOLA method.

Also the steady-state profile  $\Phi(v)$  is an important quantity and can be measured from variable sources by the SOLA method similarly to the way  $\Phi$  is measured as described in Sect. 4.5. Evolution of the BLR will reflect itself in  $\Phi(v)$  also and  $\Phi(v)$  will thus be a powerful way to study evolution while simultaneously being comparatively easy to calculate by the SOLA method.

The SOLA method is thus not only an alternative to existing inversion techniques for the derivation of the TF, but can be expected to be a powerful tool in the study of emission-line profiles as well, which opens the way to a better understanding of the broad-line region in active galactic nuclei.

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