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THE SPECTRAL RESOLVING POWER OF IRREGULARLY SAMPLED TIME SERIES

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ABSTRACT

A method is presented for investigating the periodic signal content of time series in which a number of signals is present, such as arising from the observation of multi-periodic oscillating stars in observational asteroseismology. Standard Fourier analysis tends only to be effective in cases when the data are perfectly regularly sampled. During normal telescope operation it is often the case that there are large, diurnal, gaps in the data, that data are missing, or that the data are not regularly sampled at all. For this reason it is advantageous to perform the analysis as much as possible in the time domain. Furthermore, for quantitative analyses of the frequency content and power of all real signals, it is of importance to have good estimates of the errors on these parameters. This is easiest to perform if one can use linear combinations of the measurements. Here such a linear method is described. The method is based in part on well-known techniques in radio technology used in every FM radio receiver, and in part on the SOLA inverse method.

Key words: data analysis; time series; oscillating stars.

1. INTRODUCTION

Low-amplitude solar and stellar oscillations can be considered as a linear superposition of normal modes of resonance. In theory one indexes each by a radial order n for the mode, and the degree ℓ , and order m of the spherical harmonic functions which describe the azimuthal spatial behaviour of the normal modes. For solar-like oscillations, excited stochastically by turbulent convection, there are usually many oscillation modes present at any epoch, with comparable amplitudes. If a time series, in which there are a number of periodic signals present, is sampled perfectly regularly the problem of determining the frequencies can be solved in a straightforward manner using a Fourier transform and there is a single peak for each real frequency present in the data.

If there are gaps in the data or if the data is irregularly sampled there will be more than one peak in the Fourier spectrum for each signal frequency in the data. The shape

of this response for each signal frequency that is present in the data is known as the ‘window function’. Clearly, if there are a large number of real signal frequencies in gapped or irregularly sampled data, the Fourier transform very quickly becomes very complicated and hence it becomes very difficult to determine which are true frequencies, and which are aliases generated by the sampling.

There exist a number of techniques for resolving the problem of determining amplitudes, frequencies and phases of periodic signals in irregularly sampled, gapped data. Some attempt to ‘fill gaps’ using autoregressive methods (cf. Fahlman & Ulrych (1982), Brown & Christensen-Dalsgaard (1990), Fossat et al. (1999)). Other methods attempt to do a least-squares fitting of sinusoids in the time domain (cf. Scargle (1982), Scargle (1989), Koen (1999)) or to define a best-fit spectrum in a least-squares sense, but mollified to take into account that this problem is ill-posed since it is a form of an inverse problem (Stoica et al. (2000), Vio et al. (2000)).

Apart from the problems of sampling, there is also the problem of noise in the data. In principle a star can produce ‘intrinsic noise’, through the stellar equivalent of granulation. The telescope and instrumentation can also introduce noise into time-series which in an ideal case would amount to the photon noise due to finite integration times. Such noise might be white, i.e. independent of frequency, but in some cases it could be confined to bands of frequencies. It therefore makes some sense to ‘chop up’ the full frequency range available, set by the sampling and length of the time series, into different regimes. Otazu et al. (2004) refer to this as a ‘multiresolution approach’.

2. THE METHOD

In the approach of Otazu et al. (2004) the series is convolved with every member of a set of Gaussians, with successively increasing widths $\Delta_{j+1} = 2\Delta_j$ and the differences between these successive convolutions are computed. If the data were regularly sampled each of these differences would correspond in the frequency domain to having applied a band-pass filter to the data. If all the different multiresolution levels were to be co-added the

original data would be reproduced. The convolution is carried out in the time domain by defining weights w_i for each datum Y_i measured at time t_i and then summing $w_i Y_i$ for all i .

This method of separating frequency space into broad bands has the convenience of simplicity since convolving with Gaussians in the time domain corresponds to multiplying by Gaussians in the Fourier domain and the filter parameters are easy to determine. However, the shape of the filter corresponding to the differences between levels is awkward since it is skewed and it reduces the amplitude of the signal everywhere except at one off-center frequency in the band. It is preferable to have a filter that is flat over the band, at least in the limit that errors σ_i are equal for all i , with a symmetric smooth drop-off at the lower and upper limit of the band. One example of that is a filter defined by the following convolution :

$$\begin{aligned} H(\omega) &= \int_{-\infty}^{\infty} d\omega' \frac{1}{\delta_\omega \sqrt{\pi}} \exp \left[-\frac{(\omega - \omega')^2}{\delta_\omega^2} \right] \Pi(\omega') \\ \Pi(\omega) &= 1 \text{ for } |\omega| < \Delta_\omega \\ &= 0 \text{ for } |\omega| > \Delta_\omega \end{aligned} \quad (1)$$

Band-pass filtering, i.e. multiplying in the Fourier domain by this filter function $H(\omega - \omega_c)$, centered on a frequency ω_c , corresponds in the time domain by convolving with the function :

$$h(t) = \frac{\sin \Delta_\omega t}{\pi t} \exp \left[-\frac{1}{4} \delta_\omega^2 t^2 \right] \cos \omega_c t \quad (2)$$

One can create successive bands that are orthogonal by choosing for instance :

$$\begin{aligned} \frac{\delta_\omega}{\Delta_\omega} &\equiv r \ll 1 \text{ fixed} \\ \Delta_{\omega,j+1} &= \alpha_j \Delta_{\omega,j} \\ \frac{\omega_{c,j+1}}{\Delta_{\omega,j+1}} &\equiv \gamma_{j+1} = 1 + \frac{\gamma_j + 1}{\alpha_j} \text{ with } \gamma_1 = 0 \end{aligned} \quad (3)$$

The weights $w_{m,ij}$ are therefore in this case set as :

$$w_{m,ij} = \frac{\Delta_{\omega,j}}{W_j \pi} h_j(x_{m,ij}) \quad (4)$$

where W_j is again a normalisation factor and the $x_{m,ij}$ and h_j are :

$$\begin{aligned} x_{m,ij} &= \Delta_{\omega,j}(t_m - t_i) \\ h_j(x) &= \frac{\sin x}{x} \exp \left[-\frac{1}{4} r^2 x^2 \right] \cos(\gamma_j x) \end{aligned} \quad (5)$$

The band-pass filtered data R are now calculated as :

$$R_{i,j} \equiv R_{t_i,j} = \sum_m w_{m,ij} Y_m \quad (6)$$

2.1. local oscillator step

In order to obtain the frequency content of the time series, without performing Fourier transforms on unevenly sampled data, one can employ what is known as a Hilbert transform. In radio receiver technology this would be referred to as mixing the signal with a local oscillator, and then applying a low-pass filter. The frequency f of the local oscillator is ‘tunable’ : the width of the band-pass set by the previous step is covered, sampling regularly in frequency with some sampling step Δf . This sampling distance Δf is chosen in combination with the width of the low-pass filter subsequently applied, so that some oversampling is performed.

The low pass filter to apply to the ‘mixed signal’ is chosen to be a Gaussian. This is a convenient choice because multiplying by a (narrow) Gaussian filter in the frequency domain, corresponds to convolving with a (wide) Gaussian in the time domain. The width Δ_{LPF} in the time domain of what corresponds to a low-pass filter, is linearly related to the parameter Δ of the multiresolution level within which the signal is to be frequency analysed.

One proceeds as follows. For each frequency f_k define sets of ‘local oscillator’ weights $q_{i,k}, p_{i,k}$:

$$\begin{aligned} q_{i,k} &= 2 \cos(2\pi f_k t_i) \\ p_{i,k} &= -2 \sin(2\pi f_k t_i) \end{aligned} \quad (7)$$

and also define a set of low-pass filter weights $z_{i,l}$:

$$z_{i,l} = \frac{1}{\Delta_{LPF} \sqrt{\pi}} \exp \left[-\left(\frac{t_i - T_l}{\Delta_{LPF}} \right)^2 \right] \quad (8)$$

The central times T_l are spaced by some factor of order unity times Δ_{LPF} . Δ_{LPF} is large in order to only let through signal in a narrow band, and therefore the number of times T_l is small : typically of the order of the number of nights of observations. One can define a cosine-weighted average $\langle R_{\cos} \rangle$ and a sine-weighted average $\langle R_{\sin} \rangle$ as follows :

$$\begin{aligned} \langle R_{\cos} \rangle_{lkj} &= \sum_i z_{i,l} q_{i,k} R_{i,j} \\ \langle R_{\sin} \rangle_{lkj} &= \sum_i z_{i,l} p_{i,k} R_{i,j} \end{aligned} \quad (9)$$

The three indices for these two averaged quantities refer to :

1. j the ‘resolution level’ of the time series, which corresponds to a broad range in frequencies between a lower and upper bound set by a smoothing width Δ_ω .
2. k the central frequency f_k of the tunable narrow-band filter runs from the lower to the upper limit of the band j to explore the amplitude of signal in the time series at/around each f_k .

- the central time on which a broad Gaussian is centered, providing a narrow band filter. By taking a finite value for the width of this Gaussian (rather than infinite) and stepping through the time series, a time-frequency analysis is done, since it provides a mechanism to consider the frequency content per night, or per few successive nights.

For data covering only a few nights with a large gap during the day-time, the window function has quite strong daily sidelobes which complicates analysis of a spectrum if there are a number of peaks present, as is common in multi-periodic variable stars. It is therefore useful to consider improvements on the method with the aim of reducing sidelobe structure. This may be achieved by choosing the low-pass filtering weights $z_{i,l}$ in a different manner than described by (8).

2.2. Strategy

Consider again the two summations of Eq. (9) which are defined to be functions of frequency f . The weights are now ζ_i and in principle one wishes to be free to choose weights differently depending on the frequency f_k of the local oscillator. The time T_l on which the low-pass filter is centered in Eq. (8) now enters the discussion by explicitly setting the reference time for the phase ϕ , which means replacing t_i with $t_i - T_l$. With the weights ζ yet to be determined two functions of f are defined as follows :

$$\begin{aligned} \sum_i \zeta_{i,kl} \cos(2\pi(f - f_k)(t_i - T_l)) &\equiv \Xi(f) \\ \sum_i \zeta_{i,kl} \sin(2\pi(f - f_k)(t_i - T_l)) &\equiv X(f) \end{aligned} \quad (10)$$

The method has three separate aims to achieve with its choice for the weights. These aims may well not be perfectly compatible and therefore require a form of compromise :

- The first function $\Xi(f)$ needs to be a function peaked at $f = 0$ and smoothly dropping away, with as little side-lobe structure as possible. For instance :

$$\Xi(f) \approx \frac{1}{\sqrt{\pi}\Delta_f} \exp\left[-\left(\frac{f - f_k}{\Delta_f}\right)^2\right] \quad (11)$$

with as small a width Δ_f as achievable.

- The second function $X(f)$ needs to be as close to 0 as possible everywhere
- The errors in the resulting amplitudes A^2 need to be kept as small as possible.

In the framework of the SOLA method (cf. Pijpers & Thompson (1992), Pijpers & Thompson (1994)) the best compromise between these three aims

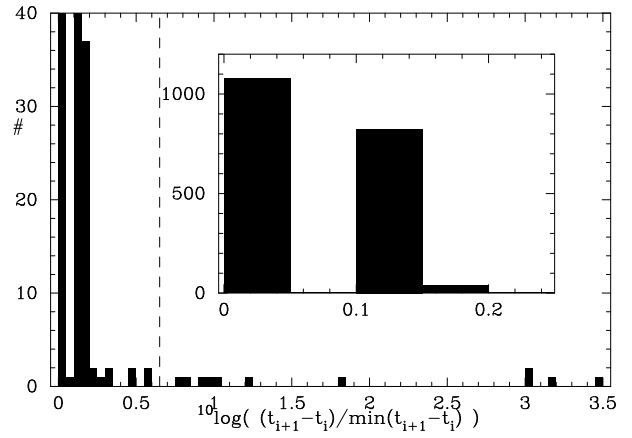


Figure 1. A histogram of the distribution of intervals between successive sampling times for an artificial time series, divided by the smallest of these intervals. The vertical dashed line indicates the mean time interval for the entire set. The inset shows the part of the distribution at small intervals which is off-scale for the larger figure.

can be achieved by minimising for $\zeta_{i,k}$ the quantity $\mathcal{W}(\zeta)$ defined by :

$$\begin{aligned} \mathcal{W}(\zeta) &\equiv (1 - \mu) \int_{f_{\text{low}}}^{f_{\text{high}}} df [\Xi(f) - \mathcal{T}(f)]^2 \\ &+ \mu \int_{f_{\text{low}}}^{f_{\text{high}}} df [X(f)]^2 + \lambda \sum_{i,j} \zeta_{i,k} \zeta_{j,k} \Sigma_{ij} \end{aligned} \quad (12)$$

while taking into account the normalisation constraint :

$$\sum_i \zeta_{i,k} \left[\int_{f_{\text{low}}}^{f_{\text{high}}} df \cos(2\pi(f - f_k)(t_i - T_l)) \right] \equiv 1 \quad (13)$$

Here the target function $\mathcal{T}(f)$ is taken to be a Gaussian as in (11). The ‘target function’ for $X(f)$ is 0. The matrix Σ_{ij} is the variance-covariance matrix of the data errors. It is usual to assume that the errors on the data are Gaussian distributed and independent, so that Σ is a diagonal matrix, but this is not essential for the algorithm. The methods described in Koen (1999) to estimate or model error properties from residuals using ARIMA models can be applied here as well. The parameters $\mu \in [0, 1]$ and $\lambda \in [0, \infty)$ weight the relative importance of the three terms. Their value needs to be set by trial and error. A very low choice of λ normally leads to unacceptably high errors on the inferred A^2 , whereas a very high value generally produces the undesirable effect of increasing width or structure in Ξ . Similar arguments hold for μ .

A point that is crucial for this version of the SOLA algorithm, is that f_{low} and f_{high} must be finite. This is necessary because the integrations in Eq. (12) are over products of cosines, which causes problems if they are to be

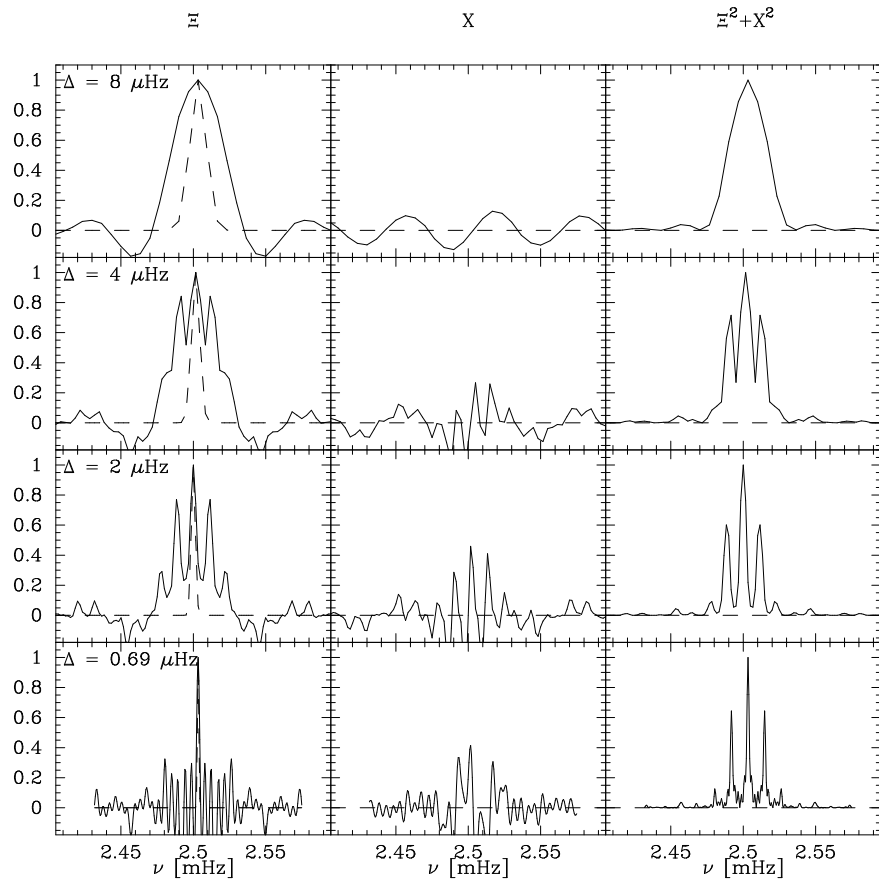


Figure 2. The response to a monochromatic signal of 2.5 mHz sampled with the sampling shown in Fig. 1, which is the equivalent of the window function for this method. The left and middle columns show the functions Ξ and X , with in addition the target functions (dashed lines). The right hand column shows the sum of the square of these which is the equivalent of the window function for power. The rows are in order of increasing spectral resolution with the $\Delta_f = 8, 4, 2, 0.69 \mu\text{Hz}$, from top to bottom. The lower and upper frequency limit of the band are set to $f_{\text{low}} = 2.5 \text{ mHz}$ and $f_{\text{high}} = 6 \text{ mHz}$.

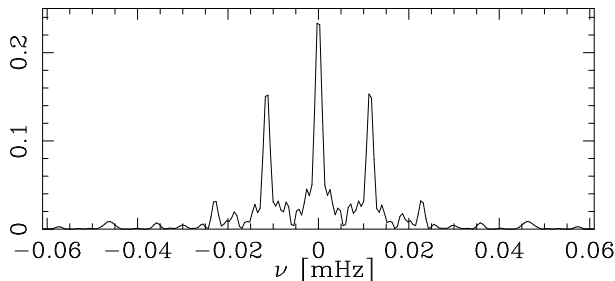


Figure 3. The window function obtained using the algorithm of section 2.1 with parameter choices corresponding to the bottom right panel of Fig. 2.

integrated over an infinite range. The algorithm is therefore limited in application to bandwidth-limited data. In practice this is not a severe problem, since by construction of the multiresolution steps, bandwidth limitation is achieved. Also, given a finite minimum interval between measurements it is known that signals with frequencies above the Nyquist frequency $\propto 1/\min(t_{i+1} - t_i)$ cannot be resolved by such data, which also sets an upper limit to the frequency.

3. RESULTS FOR THE SAMPLING OF AN ASTEROSEISMIC CAMPAIGN

To show how the methods work, the sampling times are used of a time series obtained during an asteroseismic campaign : the sampling times are nearly regularly spaced during blocks of time, with large gaps in between the blocks. This corresponds to night-time observing for a few successive nights, with day-time interruptions as well as loss of data due to adverse weather conditions as well as some technical glitches. The total number of sampling times is 1962, the median sampling rate is about ~ 70 s whereas the mean rate is $x \sim 236$ s. If the series had been regularly sampled at the median rate the Nyquist frequency would have been ~ 7.14 mHz. Fig. 1 shows the distribution of sampling time intervals between successive measurements, clearly demonstrating that the time series is not fully regularly sampled. Using these sampling times the linear coefficients ζ are determined and then used on an artificial signal with a frequency close to 2.5 mHz to obtain a window function, shown in Fig. 2. Also shown for comparison in Fig. 3 is the window function obtained from the method of Sect. 2.1. The rows in Fig. 2 are in order of increasing spectral resolution with the $\Delta_f = 8, 4, 2, 0.69 \mu\text{Hz}$, from top to bottom, where $0.69 \mu\text{Hz}$ corresponds to the resolution limit set by the full length of the time series if it were regularly sampled over this period.

Perhaps disappointingly the spectral resolution that can be attained over the band is quite poor. However, the quality of the window function depends on the ratio of the resolution over the width of the band : Δ_f/Δ_ω . By bandwidth filtering of the data in the same way as dis-

cussed in section 2, but over a narrower band, a better resolution can be achieved within the band. In Fig. 4 the resolution Δ_f is set to $2 \mu\text{Hz}$ and the limits of the band are $f_{\text{low}} = 2.456$ mHz and $f_{\text{high}} = 2.544$ mHz i.e. a factor of 40 narrower than before. The target sampling frequency in this case is 2.49 mHz. Now the FWHM of the window function in power is $3.3 \mu\text{Hz}$ (top right panel of Fig 4). Even at a target width $\Delta_f = 0.69 \mu\text{Hz}$ the side-lobe structure in the window function for power (bottom right panel of Fig 4) is much reduced compared to the same resolution for the previous case (the bottom right panel of Fig. 2).

4. DISCUSSION

The method described in this paper, for the analysis of multiperiodic, irregularly sampled time series, has several potential uses. The first point to note is that the method is a time-frequency method formulated in the time domain. This means that the frequencies of the signals in an irregularly sampled time series can be monitored as a function of epoch. This allows for the possibility to detect changes in oscillation frequencies with epoch. Such changes could arise in asteroseismology through similar mechanisms that produce changes in oscillation frequencies of the Sun over the solar cycle and are therefore of interest to determine.

The second point to note is that the method is linear. The algorithm produces a set of linear coefficients on the basis of the sampling times alone. These coefficients can then trivially be combined with data, and with measurement errors, in order to provide the power or amplitude of the time series and also an error estimate for this power, and even a phase if required. Evidently this linearity is convenient for standard asteroseismic data acquisition which provides photometry or velocity. However, it is arguably even more useful if the data obtained is more complex such as a time series of images. A particular application lies in addressing another fundamental problem in asteroseismology, which is the identification of the surface node line pattern associated with any particular frequency : the mode identification problem. For stars with a single dominant mode, long baseline optical interferometry has been suggested as a means to image the flux variation over the surface, thus allowing the identification of the mode (cf. Jankov et al. (2001) for a description of the relevant technique). In particular the closure phase of interferometric signals between three telescopes is sensitive to symmetries in the flux distribution over the surface of stars (cf. Domiciano de Souza et al. (2005)). For multiperiodic stars such a technique would suffer since the superposition of patterns would reduce the closure phase response. However, the acquisition of interferometric fringes is already done repeatedly as a function of time for operational reasons. By combining closure phase signals at the appropriate phase in time one could select a single frequency to image, thus restoring the closure phase signal strength. This technique might be referred to as ‘stroboscopic interferometry’. The closure phase

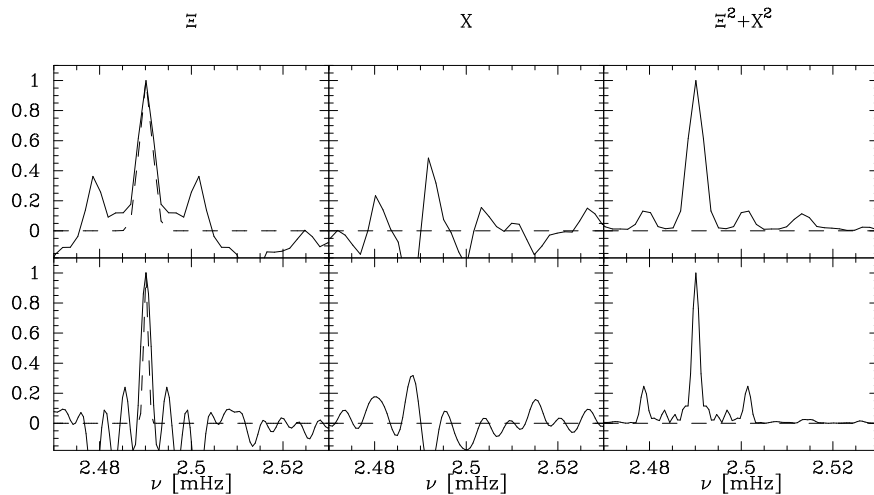


Figure 4. The effect of reducing the width of the broad band over which the frequencies are sampled on the window function. Here the lower and upper frequency limit of the band are set to $f_{\text{low}} = 2.456$ mHz and $f_{\text{high}} = 2.544$ mHz. The target sampling frequency is 2.49 mHz.

is a somewhat more complex form of data than time series of velocities or fluxes, and carrying out stroboscopic interferometry in practice is facilitated by being able to carry out the time-filtering using the coefficients generated with this SOLA based algorithm. Appropriate tests of the algorithm for this application are in progress (Pijpers, in preparation).

A third point to note is that this method can also be adapted to streamline procedures for model fitting in asteroseismology. The Gaussian form of the target function chosen here is intended to obtain measures of the oscillation power or amplitude as a function of frequency. However, in fitting models to a given star it is not just a single frequency but the entire pattern of frequencies for which a best match needs to be found. This can be achieved more directly by calculating the expected oscillation spectrum for each model and taking for the target function $\mathcal{T}(f)$ not a single Gaussian, but a sum of Gaussians centered on model frequencies f_{nl} , in which the nl are the radial mode order n and azimuthal degree l , instead of the single f_k . The coefficients obtained in this way, when combined with the data, will produce a maximal response for that model which has a set of frequencies that is the closest match to the data. Further tests of this are necessary to establish the error propagation properties of such an approach, including the influence of systematic effects (Pijpers, in preparation).

ACKNOWLEDGMENTS

The method described in section 2.2 formalises, within the framework of the SOLA algorithm (Pijpers & Thompson (1992), 1994), an idea originally proposed by H. Kjeldsen (private communication), who is thanked for suggesting to the author to pursue this.

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