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Technical Appendix to

BUBBLE FORMATION AND (IN)EFFICIENT MARKETS IN LEARNING-TO-FORECAST AND OPTIMISE EXPERIMENTS

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Appendix A. Experimental Instructions

A.1. *Learning to Forecast (LtF) Treatment*

A.1.1. *General information*

In this experiment, you participate in a market. Your role in the market is a professional Forecaster for a large firm, and the firm is a major trading company of an asset in the market. In each period the firm asks you to make a prediction of the market price of the asset. The price should be predicted one period ahead. Based on your prediction, your firm makes a decision about the quantity of the asset the firm should buy or sell in this market. Your forecast is the only information the firm has on the future market price. The more accurate your prediction is, the better the quality of your firm's decision will be. You will get a payoff based on the accuracy of your prediction. You are going to advise the firm for 50 successive time periods.

A.1.2. *About the price determination*

The price is determined by the following price adjustment rule: when there is more demand (firm's willingness to buy) of the asset, the price goes up; when there is more supply (firm's willingness to sell), the price will go down. There are several large trading companies on this market and each of them is advised by a forecaster like you. Usually, higher price predictions make a firm to buy more or sell less, which increases the demand and vice versa. Total demand and supply is largely determined by the sum of the individual demand of these firms.

A.1.3. *About your job*

Your only task in this experiment is to predict the market price in each time period as accurately as possible. Your prediction in period 1 should lie between 0 and 100. At the beginning of the experiment you are asked to give a prediction for the price in period 1. When all forecasters have submitted their predictions for the first period, the firms will determine the quantity to demand, and the market price for period 1 will be determined and made public to all forecasters. Based on the accuracy of your prediction in period 1, your earnings will be calculated. Subsequently, you are asked to enter your prediction for period 2. When all participants have submitted their prediction and demand decisions for the second period, the market price for that period, will be made public and your earnings will be calculated, and so on, for all 50 consecutive periods. The information you can refer to at period t consists of all past prices, your predictions and earnings. Please note that due to liquidity constraint, your firm can only buy and sell up to a maximum amount of assets in each period. This means although you can submit any prediction for period 2 and all periods after period 2, if the price in last period is p_{t-1}, and you prediction is p^e_t: the firm's trading decision is constrained by $p^e_t \in [p_{t-1} - 30, p_{t-1} + 30]$. More precisely, the firm will trade as if $p^e_t = p_{t-1} + 30$ if $p^e_t > p_{t-1} + 30$, and trade as if $p^e_t = p_{t-1} - 30$ if $p^e_t < p_{t-1} - 30$.

A.1.4. *About your payoff*

Your earnings depend only on the accuracy of your predictions. The earnings shown on the computer screen will be in terms of points. If your prediction is p^e_t and the price turns out to be p_t in period t, your earnings are determined by the following equation:

$$Payoff = \max\left[1,300 - \frac{1,300}{49}\left(p^e_t - p_t\right)^2, 0\right].$$

The maximum possible points you can earn for each period (if you make no prediction error) is 1,300, and the larger your prediction error is, the fewer points you can make. You will earn 0 points if your prediction error is larger than 7. There is a payoff table on your table, which shows the points you can earn for different prediction errors. We will pay you in cash at the end of the experiment based on the points you earned. You earn 1 euro for each 2,600 points you make.

A.2. *Learning to Optimise (LtO) Treatment*

A.2.1. *General information*

In this experiment you participate in a market. Your role in the market is a trader of a large firm, and the firm is a major trading company of an asset. In each period, the firm asks you to make a trading decision on the quantity D_t your firm will BUY to the market. (You can also decide to sell, in that case you just submit a negative quantity.) You are going to play this role for 50 successive time periods. The better the quality of your decision is, the better your firm can achieve her target. The target of your firm is to maximise the expected asset value minus the variance of the asset value, which is also the measure by the firm concerning your performance:

$$\pi_t = W_t - \frac{1}{2}\text{Var}(W_t)^2. \tag{A.1}$$

The total asset value W_t equals the return of the per unit asset multiplied by the number of unit you buy D_t. The return of the asset is $p_t + y - Rp_{t-1}$, where R is the gross interest rate which equals 1.05, p_t is the asset price at period t, therefore $p_t - Rp_{t-1}$ is the capital gain of the asset, and $y = 3.3$ is the dividend paid by the asset. We assume the variance of the price of a unit of the asset is $\sigma^2 = 6$, therefore the expected variance of the asset value is $6D^2_t$. Therefore we can rewrite the performance measure in the following way:

$$\pi_t = (p_t + y - Rp_{t-1})D_t - 3D^2_t. \tag{A.2}$$

The asset price in the next period p_t is not observable in the current period. You can make a forecast p^e_t on it. There is an asset return calculator in the experimental interface that gives the asset return for each price forecast p^e_t you make. Your own payoff is a function of the value of target function of the firm:

$$Payoff_t = 800 + 40 \times \pi_t. \tag{A.3}$$

This function means you get 800 points (experimental currency) as basic salary, and 40 points for each 1 unit of performance (target function of the firm) you make. If your trades are unsuccessful, you may lose points and earn less than your basic salary, down to 0. Based on the asset return, you can look up your payoff for each quantity decision you make in the payoff table. You can of course also calculate your payoff for each given forecast and quantity using (A.2) and (A.3) directly. In that situation, you can ask us for a calculator.

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A.2.2. *About the price determination*

The price is determined by the following price adjustment rule: when there is more demand than supply of the asset (namely, more traders want to buy), the price will go up; and when there is more supply than demand of the asset (namely, more people want to sell), the price will go down.

A.2.3. *About your job*

Your only task in this experiment is to decide the quantity the firm will buy/sell. At the beginning of period 1 you determine the quantity to buy or sell (submitting a positive number means you want to buy, and negative number means you want to sell) for period 1. After all traders submit their quantity decisions, the market price for period 1 will be determined and made public to all traders. Based on the value of the target function of your firm in period 1, your earnings in the first period will be calculated. Subsequently, you make trading decisions for the second period, the market price for that period will be made public and your earnings will be calculated, and so on, for all 50 consecutive periods. The information you can refer to at period t consists of all previous prices, your quantity decisions and earnings. Please notice that due to the liquidity constraint of the firm, the amount of asset you buy or sell cannot be more than five units. Which means you quantity decision should be between −5 and 5. The numbers on the payoff table are just examples. You can use any other number such as 0.01, −1.3, 2.15 etc., as long as they are within [−5, 5]. When you want to submit numbers with a decimal point, please write a '.', NOT a ','.

A.2.4. *About your payoff*

In each period, you are paid according to (A.3). The earnings shown on the computer screen will be in terms of points. We will pay you in cash at the end of the experiment based on the points you earned. You earn 1 euro for each 2,600 points you make.

A.3. *Mixed Treatment*

A.3.1. *General information*

In this experiment, you participate in a market. Your role in the market is a trader of a large firm, and the firm is a major trading company of an asset. In each period the firm asks you to make a trading decision on the quantity D_t your firm will BUY to the market. (You can also decide to sell, in that case you just submit a negative quantity.) You are going to play this role for 50 successive time periods. The better the quality of your decision is, the better your firm can achieve its target. The target of your firm is to maximise the expected asset value minus the variance of the asset value, which is also the measure by the firm concerning your performance:

$$\pi_t = W_t - \frac{1}{2} Var(W_t)^2. \tag{A.4}$$

The total asset value W_t equals the return of the per unit asset multiplied by the number of units you buy D_t. The return of the asset is $p_t + y - Rp_{t-1}$, where R is the gross interest rate which equals 1.05, p_t is the asset price at period t, therefore $p_t - Rp_{t-1}$ is the capital gain of the asset, and $y = 3.3$ is the dividend paid by the asset. We assume the variance of the price of a unit of the asset is $\sigma^2 = 6$, therefore the expected variance of the asset value is $6D_t^2$. Therefore we can rewrite the performance measure in the following way:

$$\pi_t = (p_t + y - Rp_{t-1})D_t - 3D_t^2. \tag{A.5}$$

The asset price in the next period p_t is not observable in the current period. You can make a forecast p_t^e on it. There is an asset return calculator in the experimental interface that gives the

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asset return for each price forecast p_t^e you make. Your own payoff is a function of the value of target function of the firm:

$$Payoff_t = 800 + 40 \times \pi_t. \tag{A.6}$$

This function means you get 800 points (experimental currency) as basic salary, and 40 points for each 1 unit of performance (target function of the firm) you make. If your trades are unsuccessful, you may lose points and earn less than your basic salary, down to 0. Based on the asset return, you can look up your payoff for each quantity decision you make in the payoff table.

You can of course also calculate your payoff for each given forecast and quantity using (A.5) and (A.6) directly. In that situation you can ask us for a calculator. The payoff for the forecasting task is simply a decreasing function of your forecasting error (the distance between your forecast and the realised price). When your forecasting error is larger than 7, you earn 0 points:

$$Payoff_{forecasting} = \max\left[1,300 - \frac{1,300}{49}\left(p_t^e - p_t\right)^2, 0\right]. \tag{A.7}$$

A.3.2. *About the price determination*

The price is determined by the following price adjustment rule: when there is more demand than supply of the asset (namely, more traders want to buy), the price will go up; and when there is more supply than demand of the asset (namely, more people want to sell), the price will go down.

A.3.3. *About your job*

Your task in this experiment consists of two parts: (*i*) to make a price forecast; (*ii*) to decide the quantity the firm will buy/sell.

At the beginning of period 1 you submit your price forecast between 0 and 100, and then determine the quantity to buy or sell (submitting a positive number means you want to buy, and negative number means you want to sell) for period 1, and the market price for period 1 will be determined and made public to all traders. Based on your forecasting error and performance measure for the trading task, in period 1, your earnings in the first period will be calculated. Subsequently, you make forecasting and trading decisions for the second period, the market price for that period will be made public and your earnings will be calculated, and so on, for all 50 consecutive periods. The information you can refer to at period t consists of all previous prices, your past forecasts, quantity decisions and earnings. Please notice that due to the liquidity constraint of the firm, the amount of asset you buy or sell cannot be more than 5 units. Which means you quantity decision should always be between −5 and 5. The numbers on the payoff table are just examples. You can use any other numbers such as 0.01, −1.3, 2.15 etc. as long as they are within [−5, 5].

A.3.4. *About your payoff*

In each period you are paid for the forecasting task according to (A.7) and trading task according to (A.6). The earnings shown on the computer screen will be in terms of points. We will pay you in cash at the end of the experiment based on the points you earned for either the forecasting task or the trading task. Which task will be paid will be determined randomly (we will invite one of the participants to toss a coin). That is, depending on the coin toss, your earnings will be calculated either based on (A.6) or (A.7). You earn 1 euro for each 2,600 points you make.

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Appendix B. Payoff Tables

Table B1

Payoff Table for Forecasting Task

Payoff table for forecasting task
Your Payoff=$\max[1,300 - \frac{1,300}{49}(\text{Your Prediction Error})^2, 0]$
3,000 points equal 1 euro

Error	Points	Error	Points	Error	Points	Error	Points
0	1,300	1.85	1,209	3.7	937	5.55	483
0.05	1,300	1.9	1,204	3.75	927	5.6	468
0.1	1,300	1.95	1,199	3.8	917	5.65	453
0.15	1,299	2	1,194	3.85	907	5.7	438
0.2	1,299	2.05	1,189	3.9	896	5.75	423
0.25	1,298	2.1	1,183	3.95	886	5.8	408
0.3	1,298	2.15	1,177	4	876	5.85	392
0.35	1,297	2.2	1,172	4.05	865	5.9	376
0.4	1,296	2.25	1,166	4.1	854	5.95	361
0.45	1,295	2.3	1,160	4.15	843	6	345
0.5	1,293	2.35	1,153	4.2	832	6.05	329
0.55	1,292	2.4	1,147	4.25	821	6.1	313
0.6	1,290	2.45	1,141	4.3	809	6.15	297
0.65	1,289	2.5	1,134	4.35	798	6.2	280
0.7	1,287	2.55	1,127	4.4	786	6.25	264
0.75	1,285	2.6	1,121	4.45	775	6.3	247
0.8	1,283	2.65	1,114	4.5	763	6.35	230
0.85	1,281	2.7	1,107	4.55	751	6.4	213
0.9	1,279	2.75	1,099	4.6	739	6.45	196
0.95	1,276	2.8	1,092	4.65	726	6.5	179
1	1,273	2.85	1,085	4.7	714	6.55	162
1.05	1,271	2.9	1,077	4.75	701	6.6	144
1.1	1,268	2.95	1,069	4.8	689	6.65	127
1.15	1,265	3	1,061	4.85	676	6.7	109
1.2	1,262	3.05	1,053	4.9	663	6.75	91
1.25	1,259	3.1	1,045	4.95	650	6.8	73
1.3	1,255	3.15	1,037	5	637	6.85	55
1.35	1,252	3.2	1,028	5.05	623	6.9	37
1.4	1,248	3.25	1,020	5.1	610	6.95	19
1.45	1,244	3.3	1,011	5.15	596	*error* $\geq$ 0	
1.5	1,240	3.35	1,002	5.2	583		
1.55	1,236	3.4	993	5.25	569		
1.6	1,232	3.45	984	5.3	555		
1.65	1,228	3.5	975	5.35	541		
1.7	1,223	3.55	966	5.4	526		
1.75	1,219	3.6	956	5.45	512		
1.8	1,214	3.65	947	5.5	497		

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Table B2

Payoff Table for Trading Task

	Your profit																				
	Asset quantity: positive number means to buy, negative to sell																				
Asset return	–5	–4.5	–4	–3.5	–3	–2.5	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
–15	800	1,070	1,280	1,430	1,520	1,550	1,520	1,430	1,280	1,070	800	470	80	0	0	0	0	0	0	0	0
–14	600	890	1,120	1,290	1,400	1,450	1,440	1,370	1,240	1,050	800	490	120	0	0	0	0	0	0	0	0
–13	400	710	960	1,150	1,280	1,350	1,360	1,310	1,200	1,030	800	510	160	0	0	0	0	0	0	0	0
–12	200	530	800	1,010	1,160	1,250	1,280	1,250	1,160	1,010	800	530	200	0	0	0	0	0	0	0	0
–11	0	350	640	870	1,040	1,150	1,200	1,190	1,120	990	800	550	240	0	0	0	0	0	0	0	0
–10	0	170	480	730	920	1,050	1,120	1,130	1,080	970	800	570	280	0	0	0	0	0	0	0	0
–9	0	0	320	590	800	950	1,040	1,070	1,040	950	800	590	320	0	0	0	0	0	0	0	0
–8	0	0	160	450	680	850	960	1,010	1,000	930	800	610	360	50	0	0	0	0	0	0	0
–7	0	0	0	310	560	750	880	950	960	910	800	630	400	110	0	0	0	0	0	0	0
–6	0	0	0	170	440	650	800	890	920	890	800	650	440	170	0	0	0	0	0	0	0
–5	0	0	0	30	320	550	720	830	880	870	800	670	480	230	0	0	0	0	0	0	0
–4	0	0	0	0	200	450	640	770	840	850	800	690	520	290	0	0	0	0	0	0	0
–3	0	0	0	0	80	350	560	710	800	830	800	710	560	350	80	0	0	0	0	0	0
–2	0	0	0	0	0	250	480	650	760	810	800	730	600	410	160	0	0	0	0	0	0
–1	0	0	0	0	0	150	400	590	720	790	800	750	640	470	240	0	0	0	0	0	0
0	0	0	0	0	0	50	320	530	680	770	800	770	680	530	320	50	0	0	0	0	0
1	0	0	0	0	0	0	240	470	640	750	800	790	720	590	400	150	0	0	0	0	0
2	0	0	0	0	0	0	160	410	600	730	800	810	760	650	480	250	0	0	0	0	0
3	0	0	0	0	0	0	80	350	560	710	800	830	800	710	560	350	80	0	0	0	0
4	0	0	0	0	0	0	0	290	520	690	800	850	840	770	640	450	200	0	0	0	0
5	0	0	0	0	0	0	0	230	480	670	800	870	880	830	720	550	320	30	0	0	0
6	0	0	0	0	0	0	0	170	440	650	800	890	920	890	800	650	440	170	0	0	0
7	0	0	0	0	0	0	0	110	400	630	800	910	960	950	880	750	560	310	0	0	0
8	0	0	0	0	0	0	0	50	360	610	800	930	1,000	1,010	960	850	680	450	160	0	0
9	0	0	0	0	0	0	0	0	320	590	800	950	1,040	1,070	1,040	950	800	590	320	0	0
10	0	0	0	0	0	0	0	0	280	570	800	970	1,080	1,130	1,120	1,050	920	730	480	170	0
11	0	0	0	0	0	0	0	0	240	550	800	990	1,120	1,190	1,200	1,150	1,040	870	640	350	0
12	0	0	0	0	0	0	0	0	200	530	800	1,010	1,160	1,250	1,280	1,250	1,160	1,010	800	530	200
13	0	0	0	0	0	0	0	0	160	510	800	1,030	1,200	1,310	1,360	1,350	1,280	1,150	960	710	400
14	0	0	0	0	0	0	0	0	120	490	800	1,050	1,240	1,370	1,440	1,450	1,400	1,290	1,120	890	600
15	0	0	0	0	0	0	0	0	80	470	800	1,070	1,280	1,430	1,520	1,550	1,520	1,430	1,280	1,070	800

Note. 3,000 points of your profit corresponds to € 1.

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Appendix C. Rational Strategic Behaviour

Our experimental results are clearly different from the predictions of the rational expectation equilibrium (REE). However, we also found that subjects typically earn high payoffs, implying some sort of profit seeking behaviour.

In this Appendix, we discuss whether rational strategic behaviour can explain our experimental results, in particular in the LtO and Mixed treatments. Based on the assumption on the subjects' perception of the game and information structure, three cases are discussed:

(*i*) agents are price takers;
(*ii*) agents know their market power and coordinate on monopolistic behaviour; and
(*iii*) agents know their market power but play non-cooperatively.

We show that under price-taking behaviour, the LtF and LtO treatments are equivalent. If the subjects behave strategically or try to collude, the economy can have alternative equilibria, where the subjects collectively 'ride a bubble', or jump around the fundamental price. Nevertheless, these rational equilibria predict different outcomes than the individual and aggregate behaviour observed in the experiment.

Without loss of generality, we focus on the one-shot game version of the experimental market to derive our results. More precisely, we look at the optimal quantity decisions $z_{i,t}$ that the agents in period t (knowing prices and individual traded quantity up to and including period t) have to formulate to maximise the expected utility in period $t+1$. This is supported by two observations. First, by definition agents are myopic and their payoff in $t+1$ depends only on the realised profit from that period and not on the stream of future profits from period $t+2$ onward. Second, the experiment is a repeated game with a finite number of repetitions; subjects knew it would end after 50 periods. Using the standard backward induction reasoning, one can easily show that a sequence of one-period game equilibria forms a rational equilibrium of the finitely repeated game as well.

C.1. *Price Takers*

Realised utility of investors in the LtO treatment is given by (7) and is equivalent to the following form:

$$U_{i,t}(z_{i,t}) = z_{i,t}(p_{t+1} + y - Rp_t) - \frac{a\sigma^2}{2} z_{i,t}^2, \tag{C.1}$$

where $z_{i,t}$ is the traded quantity and $U_{i,t}$ is a quadratic function of the traded quantity. As discussed in Section 2, assuming the agent is a price taker, the optimal traded quantity conditional on the expected price $p^e_{i,t+1}$ is given by:

$$z^*_{i,t} = \arg\max_{z_{i,t}} U_{i,t} = \frac{p^e_{i,t+1} + y - Rp_t}{a\sigma^2}. \tag{C.2}$$

Note that this result relies on the assumption that the subjects do not know the price generating function. We argue that the subjects also have an incentive to minimise their forecasting error when they choose the quantity and are paid according to the risk adjusted profit. To see that, suppose that the realised market price in the next period is p_{t+1} and the subject makes a prediction error of ε, i.e. the prediction is $p^e_{i,t+1} = p_{t+1} + \epsilon$. The payoff function can be rewritten as:

$$\begin{aligned} U_{i,t}(z_{i,t}) &= z_{i,t}(p_{t+1} + y - Rp_t) - \frac{a\sigma^2}{2} z_{i,t}^2 \\ &= \frac{(p_{t+1} + \epsilon + y - Rp_t)(p_{t+1} + y - Rp_t)}{a\sigma^2} - \frac{(p_{t+1} + \epsilon + y - Rp_t)^2}{2a\sigma^2} \\ &= \frac{(p_{t+1} + y - Rp_t)^2}{2a\sigma^2} - \frac{\epsilon^2}{2a\sigma^2}. \end{aligned} \tag{C.3}$$

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This shows that utility is maximised when $\varepsilon = 0$, namely, when all subjects have correct belief. Assuming perfect rationality and price-taking behaviour (perfect competition), the task of finding the optimal trade coincides with the task of minimising the forecast error. Subjects have incentives to search for and play the REE also when they choose the quantity. We summarise this finding below.

FINDING 1. *When the subjects act as price takers, the utility function in the learning to optimise treatment is a quadratic function of the prediction error, the same (up to a monotonic transformation) as in the learning to forecast treatment. The subjects' payoff is maximised when they play the rational expectation equilibrium regardless of the design: the REE of LtF and LtO treatments are equivalent.*

C.2. *Collusive Outcome*

Consider now the case when agents realise how their predictions/trading quantities influence the price and are able to coordinate on a common strategy. This resembles a collusive (oligopoly) market, e.g. similar to a cobweb economy in which the sellers can coordinate their production.

In the collusive case, all agents behave as a monopoly that maximises joint (unweighted) utility; thus the solution is symmetric, that is for each agent i, $z_{i,t} = z_t$. In our experiment the price determination function is:

$$p_{t+1} = p_t + 6\lambda z_t, \tag{C.4}$$

and so the monopoly maximises:

$$\begin{aligned} U_t = \sum_{i=1}^{6} U_{i,t}(z_t) &= 6\left[z_t(p_{t+1} + y - Rp_t) - \frac{a\sigma^2}{2} z_t^2\right] \\ &= 6\left[z_t^2\left(6\lambda - \frac{a\sigma^2}{2}\right) + z_t(y - rp_t)\right]. \end{aligned} \tag{C.5}$$

Here we assume that a rational agent has perfect knowledge about the pricing function (C.4). Notice that when $\lambda = 20/21$, $a\sigma^2 = 6$, as in the experiment, the coefficient of z_t^2 is positive, $6\lambda - (a\sigma^2/2) = (19/7) > 0$, and thus the profit function is U shaped, instead of inversely U shaped.[1] This means that a finite global maximum does not exist (utility goes to $+\infty$ when z_t goes to either $+\infty$ or $-\infty$). The global minimum is obtained when $z_{i,t} = 7/38(rp_t - y) = 7r/38(p_t - p^f)$.

In our experiment, the subjects are constrainted to choose a quantity from $[-5, +5]$ and the price is bound to the interval [0, 300]. Collusive equilibrium in the one-shot game implies that the subjects coordinate on $z_{i,t} = 5$ or $z_{i,t} = -5$, depending on which is further away from $7(rp_t - y)/38$ (as (C.5) is a symmetric parabola). Since $7(rp_t - y)/38 > 0$ when the price is above the fundamental ($p^f = y/r$), we can see that the agents coordinate on -5 if the price is higher than the REE ($p_t > y/r$). Similarly, rational agents coordinate on +5 if the price is lower than the REE ($p_t < y/r$). If the price is exactly at the fundamental, rational agents are indifferent between -5 and 5. Notice that in such a case trading the REE quantity ($z_{i,t} = 0$) gives the global minimum for the monopoly.

As a consequence, the collusive outcome predicts that the subjects will 'jump up and down' around the fundamental. When the initial price is below (above) the

[1] If $6\lambda - (a\sigma^2/2) < 0$, this objective function is inversely U shaped. The maximum point is achieved when $z_{i,t} = (rp_t - y)/(12\lambda - a\sigma^2)$. This means when $p_t = y/r$, namely when the price is at the REE, the optimal quantity under collusive equilibrium is still 0. When the price is higher or lower than the REE, the optimal quantity increases with the difference between the price and the REE. This means there is a continuum of equilibria when the economy does not start at the REE.

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fundamental, the monopoly will buy (sell) the asset until the price overshoots (undershoots) the fundamental and so forth. Then the subjects start to 'jump up and down' as described before.

FINDING 2. *When the subjects know the price determination function and are able to form a coalition, the collusive profit function in the LtO treatment is U shaped. Subjects would buy under-priced and sell an over-priced asset. In the long-run, rational collusive subjects will alternate their trading quantities between* -5 *and 5 and so the price will alternate around the equilibrium.*

Such alternating dynamics would resemble coordination on a contrarian type of behaviour, but has not been observed in any of the experimental groups. Instead, our subjects coordinated on trend-following trading rules, which resulted in smooth, gradual price oscillations. Moreover, quantity decisions equal to 5 or -5 rarely happened in the experiment (7 times in the LtO and 44 times in the mixed treatment). Typical subject behaviour was much more conservative: 97% and 91% traded quantities in the LtO and mixed treatments respectively were confined in the interval $[-2.5, 2.5]$.

C.3. *Perfect Information Non-cooperative Game*

Consider a scenario, in which the subjects realise the experimental price determination mechanism but cannot coordinate their actions and play a symmetric Nash equilibrium (NE) instead of the collusive one. There is a positive externality of the subjects' decisions: when one subject buys the asset, it pushes up the price and also the benefits of all the other subjects. The collusive equilibrium internalises this externality, while the non-cooperative NE does not. What will rational subjects do in this situation?

In the case of a non-cooperative one-shot game, we again focus on a symmetric solution. Consider agent i, who optimises his quantity choice believing that all other agents will choose z_t^o. This means that the price at $t+1$ becomes:

$$p_{t+1} = p_t + 5\lambda z_t^o + \lambda z_{i,t}. \tag{C.6}$$

Agent i maximises therefore:

$$\begin{aligned} U_{i,t} &= z_{i,t}\big(\lambda z_{i,t} + 5\lambda z_t^o + y - rp_t\big) - \frac{a\sigma^2}{2} z_{i,t}^2 \\ &= z_{it}^2 \frac{2\lambda - a\sigma^2}{2} + z_{i,t}(5\lambda z_t^o + y - rp_t). \end{aligned} \tag{C.7}$$

Notice that $2\lambda - a\sigma^2 = -86/21 < 0$. This is an inversely U shaped parabola with the unique maximum given by the best response function:

$$z_{i,t}^*(z_t^o) = \frac{5\lambda z_t^o + y - rp_t}{a\sigma^2 - 2\lambda}. \tag{C.8}$$

A symmetric solution requires $z_{i,t}^*(z_t^o) = z_t^o$, which implies:

$$z_t^* = \frac{rp_t - y}{7\lambda - a\sigma^2} = \frac{3}{2}(rp_t - y). \tag{C.9}$$

Furthermore the reaction function $z_{i,t}^*(z_t^o)$ is linear with respect to z_t^o, with a slope $5\lambda/(a\sigma^2 - 2\lambda) = 100/86 > 1$. Thus, $z_t^o > z_t^*$ ($<$ and $=$) implies $z_{i,t}^* > z_t^o$ ($<$ and $=$), or in words, if agent i believes that the other players will buy (sell) the asset, he has an incentive to buy (sell) even more. Then as a best response, the other agents should further increase/decrease their demand; this is limited only by the liquidity constraints. The strategy (C.9) thus defines the threshold point between the two corner strategies, i.e. the full NE best response strategy is defined as:

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$$z_{i,t}^{NE} = \begin{cases} 5 & \text{if } z_t^o > z_t^* \\ z_t^* & \text{if } z_t^o = z_t^* \\ -5 & \text{if } z_t^o < z_t^*. \end{cases} \tag{C.10}$$

The boundary strategies can be infeasible if the previous price is too close to zero or 300.[2] To sum up, as long as the price p_t is sufficiently far from the edges of the allowed interval [0,300], there are three NE of the one-shot non-cooperative game, which are defined as fixed points of (C.10), namely all players playing $z_{i,t} = -5$, $z_{i,t} = z_t^*$ and $z_{i,t} = +5$ for all $i \in \{1, \ldots, 6\}$.

A simple interpretation is that, given the parameterisation, our model is an example of a (Nash) coordination game. As long as $5\lambda/(a\sigma^2 - 2\lambda) > 1$, the best response (C.8) is to amplify the average trade of the other players. This is not a surprising result, as it merely exhibits the strength of the positive feedback present in this market.[3]

If the agents coordinate on the strategy $z_{i,t} = z_t^*$, the price evolves according to the following law of motion:

$$p_{t+1} = \frac{10p_t - 60y}{7}. \tag{C.11}$$

In contrast to the collusive game, in the non-cooperative game the fundamental price is a possible steady state but only if it is an outcome in the initial period. Additional equilibrium refinements may further exclude it as a rational outcome, since it is the least profitable one. Recall that the subjects earn 0 when they play z_t^* with price at the fundamental (because there is no trade). On the other hand, they may earn a positive profit by coordinating on −5 or 5. For example, when all of them buy five units of the asset, the utility for each of them will be $[p_{t-1} + y + 6\lambda z_{i,t} - (1 + r)p_{t-1}]z_{i,t} - (\alpha\sigma^2/2)z_{i,t}^2 = (33.3 - 0.05p_{t-1}) \times 5 - 75$. This equals 76.5 when $p_{t-1} = 60$, 16.5 when $p_{t-1} = 300$ and 75 when the previous price is equal to the fundamental, $p_{t-1} = 66$. This explains why the payoff efficiency (average experimental payoff divided by payoff under REE) is larger than 100% in some markets in the LtO or mixed treatments where prices have large oscillations.

Notice that the linear equation (C.11) is unstable, so the NE of the one-shot game leads to unstable price dynamics in the repeated game even if the agents coordinate on $z_{i,t} = z_t^*$, as long as the initial price is different from the fundamental price. Indeed, if the initial price is 67 or 65 (fundamental price plus or minus one), the price will go to the upper cap of 300 or the lower cap of 0 respectively. Furthermore, the agents can switch at any moment between the three one-shot game NE defined by (C.10). This implies that in the repeated non-cooperative game, many rational price paths are possible. This includes many price paths where agents coordinate on 5 or −5, including the alternating collusive equilibrium discussed in the last Section.

FINDING 3. *In the non-cooperative game with perfect information, there are two possible types of NE. The fundamental outcome is a possible outcome only if the initial price is equal to the fundamental price. Otherwise, the agents will coordinate on unstable, possibly oscillatory price dynamics, with traded quantities of* −5 *or* 5. *When they coordinate on a non-zero quantity, their payoff can be higher than their payoff under the REE under the price-taking beliefs.*

[2] Notice that we can interpret z_t^o as the average quantity traded by all other agents, beside agent i, and the reasoning for NE strategy (C.10) remains intact. This implies that NE has to be symmetric.

[3] In practice, such an equilibrium could not be sustained in the long run, since the market maker would incur accumulating losses every period.

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C.4. *Summary*

To conclude, the perfectly rational agents can coordinate on price boom-bust cycles and earn positive profit.[4] However, this would require even stronger assumptions than the fundamental equilibrium, namely that the agents perfectly understand the underlying price determination mechanism.

Furthermore, such rational equilibria with price oscillations predict that the subjects coordinate on homogeneous trades at the edge of the liquidity constraints. The subjects from the LtO and mixed treatments behaved differently. Their traded quantities were highly heterogeneous, and rarely reached the liquidity constraints.

Therefore, the alternative rational equilibrium from the perfect information, non-cooperative games provide some useful insights into why subjects 'ride the bubbles' in the LtO and mixed treatment. However, since the rational solution cannot explain the heterogeneity of the individual decisions and non-boundary trading quantities, the mispricing in the experimental data is more likely a result of the joint forces of rational (profit seeking) and boundedly rational behaviour with some coordination on trend-following buy and hold and short sell strategies.

Appendix D. Earnings Ratios

Table D1
Earnings Efficiency

Treatment	LtF (%)	LtO (%)	Mixed forecasting (%)	Mixed trading (%)
Market 1	96.35	102.54	87.62	100.89
Market 2	94.47	95.25	67.27	87.33
Market 3	96.03	98.21	75.63	79.61
Market 4	96.18	100.43	77.41	114.63
Market 5	95.15	97.39	87.07	99.03
Market 6	94.06	99.64	91.94	97.24
Market 7	96.18	98.58	81.20	94.55
Market 8	96.54	98.41	60.80	132.01
Average	95.62	98.81	78.62	100.66

Notes. Earnings efficiency for each market. The efficiency is defined as the average experimental payoff divided by the payoff under REE, which is 26.67 euro for the forecasting task and 18.33 euro for the trading task. LtF, learning to forecast; LtO, learning to optimise.

[4] Note that the subjects earn more in a collusive and non-cooperative Nash setting, because we pay them according to the book value of the asset and the tâtonement process ensures the price movement is relatively smooth. In real life, people may not be able to realise the full book value of their asset holdings because the asset price will fall when a large fraction of them start to sell; without the marker maker in the tâtonements process absorbing all these losses, they may suffer huge losses when the asset price declines sharply.

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Appendix E. Estimation of Individual Forecasting Rules

Table E1

Estimated Individual Rules for the LtF Treatment

	Rule coefficients					
Subject	Cons.	Past price	AR(1)	Past trend	R^2	Type
Group 1						
1		0.288	0.756	0.680	0.995	
2	−1.952		1.090	0.448	0.996	
3		1.000		0.744	0.734	TRE
4	−1.349		0.982	0.427	0.998	
5	−2.080	0.307	0.725	0.362	0.997	
6		1.000		0.770	0.648	TRE
Group 2						
1			1.014		0.998	
2		0.626	0.347	0.519	0.998	
3	−2.110	0.346	0.697		0.996	
4			1.013		0.992	
5			1.013		0.997	
6	−1.857	0.475	0.561	0.391	0.996	
Group 3						
1		0.463	0.522	0.707	0.993	
2		0.513	0.495	0.655	0.994	
3		0.476	0.660	0.395	0.993	
4		1.000		0.302	0.310	TRE
5		1.000		0.364	0.390	TRE
6		0.471	0.544	0.579	0.998	
Group 4						
1		0.596	0.568	0.482	0.988	
2		1.000		0.679	0.320	TRE
3		1.000		0.161	0.025	TRE
4	−2.553	0.418	0.621	0.405	0.992	
5		0.389	0.608	0.539	0.996	
6		1.000		0.341	0.385	TRE
Group 5						
1		0.260	0.715	0.729	0.990	
2			1.021		0.895	
3	−53.068	−0.369	2.125	−1.591	0.655	
4		0.178	0.902	0.836	0.980	
5		0.452	0.587	0.791	0.993	
6		0.281	0.719	1.245	0.985	
Group 6						
1			0.993		0.880	
2		1.000		0.921	0.507	TRE
3		1.000		0.712	0.761	TRE
4		1.000		0.827	0.804	TRE
5		0.452	0.411	0.977	0.986	
6		1.000		0.804	0.809	TRE

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Table E1

(*Continued*)

	Rule coefficients					
Subject	Cons.	Past price	AR(1)	Past trend	R^2	Type
Group 7						
1	6.914		0.902		0.910	
2			1.010		0.998	
3			0.926		0.924	
4		0.359	0.590	0.399	0.966	
5			0.990		0.973	
6		0.308	0.536	0.545	0.960	
Group 8						
1		1.000		0.451	0.293	TRE
2		1.000		0.370	0.502	TRE
3	2.778		0.822	0.470	0.984	
4	7.958		0.884	0.783	0.911	
5		0.316	0.701	0.471	0.992	
6		1.000		0.342	0.081	TRE

Table E2

Estimated Individual Rules for the LtO Treatment

	Rule coefficients			R^2	Rule	Stability
Subject	Cons.	AR(1)	Past return			
Group 1						
1		−0.447	0.203	0.904	Mixed	S
2			0.175	0.819	Return	U
3			0.167	0.804	Return	U
4			0.111	0.856	Return	S
5	−0.125		0.168	0.833	Return	U
6			0.159	0.854	Return	S
Group 2						
1				0.0451	Random	S
2				0.168	Random	S
3				0.00997	Random	S
4				0.106	Random	S
5		0.478	−0.0473	0.24	Mixed	U
6				0.0473	Random	S
Group 3						
1	−0.188	−0.291	0.221	0.836	Mixed	U
2			0.16	0.272	Return	S
3	−0.26		0.16	0.645	Return	S
4			0.0781	0.124	Return	S
5		0.283	0.105	0.676	Mixed	S
6			0.152	0.879	Return	S

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Table E2

(*Continued*)

Subject	Rule coefficients: Cons.	AR(1)	Past return	R^2	Rule	Stability
Group 4						
1		0.811		0.677	AR(1)	N
2			0.174	0.549	Return	U
3			0.113	0.69	Return	S
4			0.14	0.824	Return	S
5			0.174	0.798	Return	U
6			0.119	0.346	Return	S
Group 5						
1				0.0975	Random	S
2				0.0695	Random	S
3		0.579		0.333	AR(1)	N
4				0.00356	Random	S
5				0.0238	Random	S
6			0.0487	0.183	Return	S
Group 6						
1				0.0496	Random	S
2			0.135	0.588	Return	S
3			0.125	0.854	Return	S
4		0.566		0.663	AR(1)	N
5			0.108	0.468	Return	S
6			0.148	0.595	Return	S
Group 7						
1		0.29	0.0795	0.741	Mixed	S
2		0.743		0.551	AR(1)	N
3		−0.3	0.177	0.759	Mixed	S
4		0.44	0.0893	0.675	Mixed	S
5	0.136	0.269	0.0521	0.59	Mixed	S
6			0.156	0.884	Return	S
Group 8						
1			0.2	0.258	Return	U
2			0.118	0.439	Return	S
3	0.118		0.207	0.757	Return	U
4	0.0522		0.0482	0.546	Return	S
5				0.131	Random	S
6			0.143	0.703	Return	S

Table E3

Estimated Individual Rules for the Mixed Treatment

	Quantity rule coefficients				Price prediction rule coefficients					
Subject	Cons.	AR(1)	Exp. return	Past return	Cons.	AR(1)	Past price	Past trend	Type	Stability
11	0.112		0.161		−2.344		1.214	0.694		U
12			0.167			0.347	0.650	0.946		S
13		−0.485		0.373	5.654		0.936	1.210		U
14			0.167		6.335		0.929	0.842		S

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Table E3

(*Continued*)

	Quantity rule coefficients				Price prediction rule coefficients					
Subject	Cons.	AR(1)	Exp. return	Past return	Cons.	AR(1)	Past price	Past trend	Type	Stability
15	0.092	0.714					1.000	0.884	TRE	N
16				0.129			1.000	1.089	TRE	S
21			0.037		48.425		0.216			S
22		0.580			14.590		0.759	−0.285		N
23			0.167		22.056		0.649			S
24			0.167		17.683		0.712	−0.649		S
25						0.251	0.749		ADA	N
26		0.327								N
31			−0.807			0.212	0.788		ADA	U
32					−13.408	0.535	0.661			N
33					9.092		0.867			N
34				−0.319			1.004			U
35			0.167		−12.399	0.302	0.878			U
36		0.635				0.402	0.598		ADA	N
41			0.167				0.931			S
42		0.256	0.115		9.748		0.887			S
43			0.167		3.696	−0.421	1.381	0.994		S
44			0.167				1.000	0.669	TRE	S
45		0.870	0.112				1.000	1.022	TRE	U
46		0.916					1.000	0.996	TRE	N
51						−0.831	1.795			N
52			0.167		6.572		1.257	0.814		U
53				0.081			1.000	0.860	TRE	S
54	0.093		0.065	0.096			1.000	1.158	TRE	U
55			0.149		7.634		0.886	1.231		U
56		0.564	0.220		6.024		1.103			U
61	−0.129		0.216		23.173		0.539			S
62				0.005			1.119			S
63		0.657		−0.447						U
64					15.897	0.685		0.618		N
65					28.577		0.431			N
66							1.044			N
71			0.048				1.000			S
72			0.167				1.000	0.543	TRE	S
73				0.085			0.926			S
74							1.054			N
75			0.167				1.000	0.471	TRE	S
76			−0.202	0.203		0.585	0.424	0.599		U
81				0.135			1.014			S
82			0.131		5.329	0.953		1.057		S
83			0.195				1.000	0.898	TRE	U
84		0.816			2.947		0.983	0.902		N
85		−0.572		0.281			1.000	0.801	TRE	U
86		0.815					1.000	0.944	TRE	N

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