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The non-linear impact of data handling on network diffusion models

Highlights
- An overview of basic data handling and common pitfalls
- A framework for analyzing the impact of data handling on network diffusion models
- Results showing the impact of data handling on various standard network models

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In brief
We identify a critical step in the application of data-driven computational models—data handling. We suggest a framework for testing the impact of data handling and apply it to assess the impact of data handling on various types of network diffusion models. We use our results to highlight the effect that data handling may have on the conclusions researchers draw from models.
The non-linear impact of data handling on network diffusion models

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SUMMARY

Many computational models rely on real-world data, and the steps required in moving from data collection, to data preparation, to model calibration and input are becoming increasingly complex. Errors in data can lead to errors in model output that might invalidate conclusions in extreme cases. While the impact of errors in data (e.g., in sampling) have been analyzed in the literature, here we highlight the importance of data handling in the modelling and simulation process, and how particular data-handling errors can lead to errors in model output. We develop a framework for assessing the impact of potential data-handling errors for models of spreading processes on networks, a broad class of models that capture many important real-world phenomena, such as epidemics. We apply this framework to test data-handling errors on various standard models. Our results demonstrate that data-handling errors can have significant impact on model conclusions, especially in critical regions of a system.

INTRODUCTION

The modern computing revolution has led to data science techniques, and in particular computational modeling, being applied in a wide range of fields including sociology,1 psychology,2 chemistry,3 and physics.4 The influence of computational models can also be felt in society, for example, with the ongoing COVID-19 crisis. In the last year, epidemiologists and modelers have used models to predict the spread of this virus, as well as to test the effectiveness of various measures, such as vaccinations and isolation, on containing this spread. Governments are relying on models to help make decisions that affect every citizen’s way of life.5

Inherent in the modeling process is the principle of abstraction, a process that aims to condense processes and phenomena into their most basic ingredients. The dangers and risk of modeling have long been understood6 and it is vital that modelers explicitly describe their assumptions and reasoning so that models are used in the right way to understand the right phenomena.7

It is not just the underlying assumptions that modelers need to take into account. The modeling process has inherent uncertainties and errors that can influence the model and its subsequent predictions. In general, errors can originate at all stages in the modeling and simulation dataflow process. Figure 1 shows a general data pipeline for the process of executing a data-driven
computational model. First, the relevant data are produced through measurement—this can be done using, for example, surveys or instruments. Frequently, measurement may not be performed by the modeler—instead they may use existing data. Second, the various data sources need to be brought together, cleaned, integrated, and processed. We refer to this as data handling. These data can then be used as input to, and to build, a model, which can be run to produce its output results. Clearly, the quality of a model’s output requires the data to be as accurate as possible and hence a minimization of error is desirable. However, total elimination of error is not a realistic or worthwhile objective; instead, it is important to understand and quantify how error propagates through this pipeline to provide some indication of the certainty or reliability of the model output.

Such quantification is a well-studied topic in particular for the measurement and modeling steps. There are a wide variety of techniques for determining the influence of data measurement errors on models and their outputs. This also holds for errors introduced at the modeling stage. Techniques, such as uncertainty quantification, sensitivity analysis, and methods for verification and validation, provide ways to analyze these errors in a quantitative way. These methods describe systematic approaches to understand how accurately a model describes the phenomena of interest, how that accuracy changes with changes in parameter values, and how errors propagate from errors in the model input (data) to errors in the model output.

That being said, the impact of the data handling step on models and their output has not been explicitly considered in the literature. This despite the fact that data handling errors can influence model output can be introduced in the data handling step, and how they can influence model output. We argue that explicitly treating errors in data handling is important for two reasons. First, data handling and the common pitfalls therein, lead to particular types of errors (e.g., incorrect aggregation) that may generate errors that are more systematic compared with measurement errors. Second, by understanding these errors explicitly, we may be able to develop or adapt existing techniques to minimize their impact on the eventual model output.

We consider these errors within the context of models of networks, and, in particular, models of spreading processes (epidemics) and opinion formation (i.e., network diffusion models). While this represents a subset of potential models, it is one where the impact of these data handling errors is more easily understood.

The contributions of this paper are as follows:

1. A framework for quantitative analysis of the impact that data handling errors can have on network diffusion models.
2. Results showing the impact of data handling errors on several standard network models. The results demonstrate that models can be impacted in a non-linear fashion and that data handling errors’ impact on network structure is not necessarily an indication of subsequent changes in model output.

The remainder of this paper is structured as follows: “background and related work” highlights work on the impact of measurement error on networks; “basics of data handling and common pitfalls” introduces the basics of data handling and how it applies to networks; “methodology” describes our framework for evaluating the effect of data handling on network models, and the parameters and models used in our simulations; “results” details the results of our simulations. Finally, in “conclusions”, we summarize our findings.

Background and related work

Before describing the related work, we briefly introduce the network terminology used in this paper.

Network terminology

A network is a graph, G, made up of a pair (V, E), where V is a set of vertices (nodes), and E is a set of paired vertices called edges (links). The links in a network can be directed or undirected. When applied to real-world systems, nodes represent entities and edges represent links between those entities. For example, if modeling the spread of a virus transmitted through contact, a network can be constructed with nodes representing persons and edges between them representing contact.

We list here some key basic network properties:

- We can define the number of nodes N, and the number of links L. Generally, one might label one’s nodes as i = 1, 2, 3, ..., N
- The degree of a node is the number of links it has to other nodes. For directed links, we can differentiate between the in-degree (number of links that point to the node) and out-degree (number of links that start at the node and point to another node). Henceforth, we limit our definitions and study to networks with only undirected links, known as undirected networks
- We can label the degree of the ith node as ki. The average degree of a network, \( \langle k \rangle \), is the average degree of all the nodes: \( \frac{2L}{N} \).\( \sum_{i=1}^{N} k_i \)
- Degree distribution, p(k), is the probability that a random node in the network has degree k
- The neighborhood of a node i, N(i), is the set of nodes to which i has a link

When considering network models and data handling, the network structure is often constructed from the data itself. Thus, errors in measurement and handling can propagate through incorrect network structure.

Measurement error and networks

The impact of measurement error (mistakes in the collecting or coding of a network dataset) on network properties has been
well studied. These measurement errors can result in false-negative (missing) nodes and edges, false-positive (should not be there) nodes and edges, and falsely aggregated and disaggregated nodes.

Wang et al. simulated different measurement errors on real-world networks and analyzed their impact on the properties of the networks, such as degree centrality, clustering coefficient, and eigenvector centrality. Smith and Moody performed similar analyses, also simulating measurement errors and studying their effect on network parameters. These studies concluded that measurement errors can have significant impact on network parameters, and that this impact can be dependent on the structure and properties of the network.

All of these studies focused on the impact of data measurement errors on network properties and not on the data handling step. In addition, these papers consider only the impact of the errors on metrics of network structure and not model output.

We differentiate from this related work in two ways. First, we consider errors arising during the data handling step of executing a data-dependent model. When viewed exclusively through the lens of impact on the network, some of these errors might look identical to measurement errors (false-positive/negative nodes and edges); however, the systematic nature of the errors can be vastly different. Often, modelers will have limited agency in how the data measurement is done, while they may have more control of the data handling step (for example, by changing how they clean and integrate data).

Second, we focus on how these errors can impact models run on the network, rather than simple structural metrics of the network. To do this, we execute simulations using synthetic and real-world data, creating networks of various topologies. The impact of these errors will differ according to the network’s topology, model parameters, and the model itself.

**Basics of data handling and common pitfalls**

In this section, we introduce the key approaches to data handling, focusing on data cleaning and data integration in particular. We detail some of the most common issues that arise in the data handling process and describe how data issues can affect networks.

**Data cleaning**

There are numerous data issues that need to be dealt with between the collection of the data and supplying it to a model.

One of these primary issues is data cleaning. Abedjan et al. offer an overview of the state of the art in data cleaning methods, and an analysis of various techniques on numerous real-world datasets.

Data cleaning is the process of identifying and correcting data errors. Data errors are defined as deviations from the ground truth: given a dataset, a data error is an atomic value (or a cell) that differs from its given ground truth. Four types of error are identified:

1. Outliers: data values that deviate from the distribution of values in a column of a table
2. Duplicates: distinct records that refer to the same real-world entity. If attribute values do not match, this could signify an additional error
3. Rule violations: values that violate any kind of integrity constraints, such as “not null” or uniqueness requirements
4. Pattern violations: values that violate synthetic and semantic constraints, such as alignment, formatting, misspelling, and semantic data types

The outlier issues are quantitative in nature, while the other three are qualitative. Numerous data cleaning tools exist to address these concerns. They can generally be categorized as: rule-based detection algorithms (such as NADEEF); pattern enforcement and transformation tools (OPENREFINE, Data Wrangler); quantitative error detection algorithms; and record linkage and de-duplication algorithms (Data Tamer). However, these tools tend to have their own problems:

- Most algorithms are evaluated on synthetic data or real-world data with synthetically injected errors. This makes it difficult to assess how effective they are in the real-world.
- Real-world data often have multiple types of error. The tools often focus on particular types of error, and there are many tools that address each error, making the process of tool-selection more difficult.
- Many enterprises involve humans in the process of data cleaning. Since human capital is more expensive/limited than computational capital, it is becoming increasingly important to automate the data cleaning process as much as possible.

Abedjan et al. highlight how these tools and problems impact real datasets. There is no dominant tool to solve all data cleaning issues; different tools work well on different datasets, and typically a combination of tools is necessary to achieve good coverage of errors. In addition, the distribution of errors can vary significantly from one dataset to another, meaning the optimal combination of tools will not necessarily always be the same. Adding to this challenge, the ordering in which the tools are applied also has an impact on data cleaning effectiveness, and this will also vary based on dataset. Even with all of these factors optimized for a particular dataset, the coverage of errors is still well below 100%, with the algorithms missing errors that could be detected by humans. Tools such as “openclean” are currently being developed to address these issues.

**Data integration**

Dong and Srivastava and Doan et al. offer overviews of data integration. Data integration is the process of creating a single, unified view of multiple data sources. There are three common difficulties, which arise sequentially:

- Semantic ambiguity: differences arise in the attributes modeled in various sources. An example of this may be flight takeoff times versus flight gate departure times modeled by different airlines
- Instance representation ambiguity: the same instance or attribute is represented differently by different sources, such as modeling an ID with only digits versus with alphanumeric characters
In this section we describe some issues that arise in real-world data fusion.

Real-world examples

To perform data integration, Dong and Srivastava identify three basic steps: schema alignment, record linkage, and data fusion.

Schema alignment is the process of creating a mediated scheme of all relevant attributes and a mapping from each source to this mediated scheme.

Record linkage identifies which entries from one or many sources correspond to the same entity. It does so by comparing two records and deciding whether they refer to the same entity (termed pairwise-matching). This technique could be rule based (such as some ID attribute needing to be the same), classification based (training a classification algorithm using examples), or distance based. Importantly, with large amounts of data it becomes computationally infeasible to perform matching between all possible pairs of records; thus, blocking algorithms are employed to partition the records before comparison.

Data fusion aims to address differences in reported values for records that refer to the same entity. In brief, a common approach is to allow sources to “vote” on what the true value for the attribute is, with voting repeated iteratively until convergence, or until a certain number of iterations have passed.

Again, we emphasize that in all these steps errors may arise.

Data issues on networks

These same data issues can arise when dealing with networks. In general, one wishes to study real-world networks; that is, networks whose nodes and links represent real-world entities and their relationships. For example, people and their friendships. Creating these types of networks requires real-world data. However, as detailed previously, a number of issues can arise in the handling and application of data. These data issues can impact how accurately the network that one constructs reflects reality.

Advani and Malde provide an overview of network data collection and the possible resultant measurement error. They identify six possible sources of measurement error:

1. Missing data due to sampling method
2. Mis-specification of the network boundary
3. Top-coding of the number of edges
4. Miscoding and misreporting of errors (respondent or interviewer error; may forget nodes or become interview fatigued, may report desired rather than true edges)
5. Spurious nodes (spelling mistakes or multiple names may lead to spurious nodes, especially a concern when reusing data)
6. Non-response

They note that different sampling measures can result in different direction and magnitude of error in network statistics. Parameters in economic models on the network can have substantial bias due to mismeasurement of network statistics, and this error is not independent of the true network statistics.
Networks are fully defined by their links and nodes; these are the two things that can be inaccurate as a result of data issues (Table 1).

Different data handling errors could result in different network errors. For example, incorrect record linkage in data integration could lead to duplicate nodes. In this paper, we focus on the resultant issues in the network, rather than the data-specific cause, but note that all of these potential network issues can be mapped back to a data handling issue.

Another important consideration is how the same network error can be a product of a data handling error or a product of a data measurement error.

For example, one might miss nodes in the network due to poor sampling. Alternatively, nodes might be missed due to poor data cleaning leading to the data parser being unable to read the data correctly. The root cause of the issue is different, but the change observed in the network of concern is the same. In both of these cases, the network built using the data is incorrect—it does not accurately reflect the real network (ground truth). It is important that one accounts for these possible errors when one draws conclusions about the network/system as a whole based on the network constructed using the data.

Another consideration is how the cumulative effect on the network, from multiple data issues, could be the same as intentional interventions made to the network. As an example, in modeling the spread of a disease on a network, vaccinating or quarantining a person/node may be modeled through the removal of a node in the network—this node is no longer at risk of being infected, and thus also no longer poses a risk of spreading infection. Likewise, limiting or removing the contact between certain nodes (through, for example, not allowing certain nodes/persons to interact) is the same as removing links in the network. These interventions will often be applied to the network to understand the impact of different interventions. The resultant changes to the network may be equivalent to those caused through data handling errors. The impact of measures, such as immunization and isolation, on networks and diffusion models, particularly for epidemic modeling, have been well studied, and so we can utilize some of these results and analyses, while focusing on a different root cause (data handling errors).

An important part of a network is its robustness. That is, its ability to withstand random failures and attacks in the network. As one removes nodes or edges from a network, one risks “damaging” it—the network may change from connected to disconnected, or some of its properties, such as average degree, diameter, degree distribution, etc., may change drastically. This could have an impact in particular on our understanding of real-world networks, and how processes on them play out.

Percolation theory studies how clusters form on a lattice as one places pebbles with some probability. It shows that there tend to be critical thresholds—points at which the system transitions from many small clusters to one large cluster. What this means for networks, is that removing nodes will initially not break the network up; however, there will be some critical point at which removing just one/a few more nodes will drastically change the network. The greater this threshold (i.e., the greater proportion of nodes that need to be removed for the network to break up), the more robust the network.

Finally, it is important to consider cascading failures. Nodes/entities tend not to fail completely at random; their failure can be influenced by their neighbors. For example, a failure in one link in a power grid will put more strain on the neighboring links, which may lead to more failure. We often want to study how these failures can propagate through the network, which can be done through diffusion models.

Methodology
Figure 2 outlines a framework for testing diffusion model sensitivity to data handling issues. First, graphs are obtained to test with; these graphs could be real-world or synthetic graphs. Following this, a model of data handling is applied to the graph, which results in a set of degraded/inaccurate graphs. Finally, diffusion models can be run on the degraded graphs. The model output on the degraded graphs can then be compared with the output on the original graph, which produces a measure of the sensitivity of the model to the data handling issues.

The framework can be applied to testing the effect of data handling on any diffusion model. One can source synthetic or real-world networks of varying topologies and apply data handling models on top of them before running the diffusion model. This allows for the identification of which data handling errors a diffusion model is most susceptible to, and how network structure interacts with these data handling errors within the context of the diffusion model being considered.

Here, we employ this framework to test the sensitivity of two diffusion models: a susceptible-infected-recovered (SIR)
model and a Threshold model. For our data handling models, we focus in particular on the loss of nodes in the network and the aggregation of nodes based on their neighborhoods. As noted previously, the loss of nodes is something that has been studied in some detail within the context of immunization/isolation and data mismeasurement, but can still help illustrate the importance of dealing with data correctly. The aggregation of nodes has also been analyzed; however, this analysis has been limited to random aggregation, rather than systematic aggregation based on node attributes or properties as is done in this paper.

**Graph sourcing**
In the first step of the framework, graphs are sourced to test on. These graphs could be created using real-world data, or created synthetically. Here, we focus primarily on synthetic graphs generated using three different algorithms: Erdős-Rényi (ER) graphs, Watts-Strogatz (WS) graphs, and Barabási-Albert (BA) graphs. These synthetic networks aim to reflect a broad class of networks seen frequently in real-world scenarios. The synthetic networks are generated to mimic particular properties of real-world networks, while being easier to study/manipulate. In our analysis, we highlight how data errors and decisions can impact diffusion models in different ways for different network topologies, and the implications this has when deciding upon a data handling approach.

- The ER graph has \( N \) nodes, and the possible link between each pair of nodes exists with probability \( p \). Alternatively, the number of nodes and edges is specified, and the graph chosen at random from all possible graphs with this number of nodes and edges. Both definitions generate graphs with similar properties. We focus on ones generated through the latter definition.
- The WS graph is a graph with the small-world property, meaning random pairs of nodes are not often connected, but will usually share connections with other nodes, and thus have short paths (routes) between them.
- The BA graph has the scale-free property, meaning its degree distribution follows a power law, with many low-degree nodes and few high-degree nodes. This results in “hubs” forming in the network—nodes with significantly larger degree than the average.

Our experiments follow the design found in Carro et al.: we generate networks with 2,500 nodes, and generate 20 networks per test. The parameters for the networks are chosen so as to have a similar average degree. The ER graphs are generated with 312,500 edges. The WS graphs are generated with each node initially attached to its 250 nearest neighbors, and a rewiring probability of 0.1. The BA graphs are generated with new nodes being attached to 132 other nodes. All networks thus have an average degree of roughly 250, or a normalized average degree of approximately 0.1. The networks are generated using the python package “networkx” (https://networkx.org/).

**Data handling model**
Once the graph(s) has been sourced, we apply a model of data handling that simulates data handling errors or decisions, resulting in an adjusted network. We simulate two data handling errors/decisions: node removal and node aggregation.

### Algorithm 1. Shared neighbor proportion of nodes \( i \) and \( j \)

1: `neighbours_i = array of neighbors of node i, not including node j`
2: `neighbours_j = array of neighbors of node j, not including node i`
3: `shared_neighbours = array of elements in both neighbours_i and neighbours_j`
4: `Return max(len(shared_neighbours) / len(neighbours_i), len(shared_neighbours) / len(neighbours_j))`
Node removal. To test the effects of data mishandling, we delete nodes at random from the networks. We remove nodes in batches, running our diffusion model on the network after each batch removal. Nodes are removed until 125 nodes remain (this is 5% of the total initial number of nodes, as in Wang et al.\textsuperscript{13}), with batch sizes chosen based on our applied diffusion model. We use 35 and 50 batches for the SIR and Threshold models, respectively, with these sizes chosen to balance smoothness of resulting curves and running time.

We note that node removal is also used in the control of epidemics. However, in that case the removals are intentional, whereas here we have unintentional removal due to a data handling error. In addition, the intentional removal is usually not at random; rather, nodes are removed so as to minimize the spread of a virus. As we are focusing on nodes missing due to error, we limit our simulations to missing at random node removal in “edge removal” of the supplemental experimental procedures, where the results and interpretation are fairly similar to what is observed in the node removal case.

Aggregation. To test the effects of data integration decisions, we simulate the aggregation of nodes using node neighborhoods and randomized attributes. We note that the algorithm detailed below is not intended to replicate perfectly how this might be done in practice, only to illustrate the differences that can arise when one aggregates, whether in some systematic way or with random aggregation.

For each node, we sample a random variable from the skew normal distribution, with probability density function (pdf) given by:

\[
f(x) = 2\varphi(x)\int_{-\infty}^{x} \varphi(t)dt,
\]

with \(\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\) (the standard normal pdf). We set \(\alpha = 4\), and bin the realizations into 50 bins of equal length. Figure 3 shows an example of this. This distribution and binning emulates entities (nodes) having similar attribute values, where one might check whether they refer to the same real-world entity. We choose a skew distribution because many real-world attribute distributions are skewed.\textsuperscript{38} This is a simplification of the entity similarity testing process, but serves to illustrate the effect it may have on network diffusion models. The exact magnitude of the impact made by the aggregation decisions can vary, but we simply aim to show that these decisions can have an impact.

Similarly to the methodology used in Bilgic et al.,\textsuperscript{39} we compare the neighborhoods of nodes sharing similar attribute values to the decide whether they refer to the same entity. Nodes are binned together based on their simulated attribute values, with 50 bins of equal length, resulting in bins having between 1 and 160 nodes in each from our 2,500 node networks. The neighbors of any two nodes in the same bin are compared; if the proportion of neighbors in common exceeds some shared neighbor threshold, \(p\), we assume that the nodes refer to the same entity (we ignore direct edges between the two nodes being compared when considering neighbors). Nodes assumed to refer to the same entity are aggregated by deleting one of the nodes and attaching all of its edges to the other node. Algorithm 2 describes this shared neighbor aggregation algorithm, with Algorithm 1 calculating the proportion of neighbors shared.

The proportion of neighbors shared between two nodes may differ based on the degree of the nodes; in this case, we always consider the larger proportion. Figure 4 shows an example of this: suppose we are deciding whether to aggregate nodes 1 and 2 based on their shared neighbors. From the perspective of node 2, 100% of its neighbors (nodes 3 and 4) are shared with node 1; however, from node 1’s perspective, only 50% of its neighbors (nodes 3, 4, 5, and 6) are shared with node 2. We will always consider the greater of the two values in these scenarios, and so would aggregate nodes 1 and 2.

We do not address the transitivity issue: pairwise comparisons in each bin are made systematically, with each node being compared once with every other node, and aggregations are done as soon as the conditions are met, without consideration of previous or future pairwise comparison results. Using Figure 4 again for an example, suppose nodes 1, 2, and 5 all belong to the...
same bin, and we use a shared neighbor threshold of 80%. When we compare nodes 1 and 2, we would conclude that they refer to the same entity (since the maximum of their shared neighbors proportion is 100% > 80%); likewise with nodes 1 and 5. However, we would not conclude that nodes 2 and 5 refer to the same entity, since they have no neighbors in common. Thus we have node 1 = node 2, node 1 = node 5, but node 2 \neq node 5.

While this is an issue that needs to be dealt with in reality, we avoid addressing it here, and instead simply make aggregation decisions based on the order in which the pairwise comparisons are made. So if we compare nodes 1 and 2 first, we would consider them the same entity and immediately aggregate them (this would essentially delete node 2 and/or relabel node 1 as node 2). We would then compare node 1 with node 5, and would conclude that they are the same and aggregate them. Again, this is a simplification of the entity resolution process but still serves the purpose of emulating how it might affect the network. For each test, we create multiple graphs, which helps address variance concerns arising from this issue.

When running our models on each simulated network, we also consider the network resulting from random node aggregation, as was studied in Smith and Moody.\textsuperscript{14} We perform the same number of aggregations as was observed by applying the algorithm above, only now at random.

**Diffusion model**

Once we have sourced a network and applied a data handling model to it, a diffusion model is run on it. Network diffusion models represent a broad class of models that can be used to model some form of spreading phenomenon. As opposed to traditional diffusion (e.g., on a lattice), the network structure provides a specification of connectivity between the entities and hence defines the pathways by which the spreading (or diffusion) may progress. This spread could represent many different real-world processes: the spreading of failures on a power grid; the spread of a rumor at a party; the spread of a disease through a community.

We are interested in the impact of data on network-based models and how data handling may impact conclusions drawn from these models. While there are other classes of models that can be applied on networks, we restrict our analysis to diffusion models (spreading processes) as they represent a broad class of relevant applications (e.g., epidemics, information spreading, rumor spread, etc.). For succinctness, we primarily use terminology for the spreading of a disease, but the models addressed could equally be applied to any spreading phenomenon.

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**Algorithm 3. SIR diffusion model**

1: Let State\(_j\) denote the state of node \(j\)
2: Set State\(_j\) = s for nodes \(j = 1, 2, \ldots, n\)
3: Let S = \{\(j : \text{State}_j = s\)\}
4: Define infection rate, \(\beta\), and recovery rate, \(\gamma\)
5: Set State\(_i\) = i for initially infected nodes \(i\)
6: Let I = \{\(j : \text{State}_j = i\)\}
7: for Number of iterations do
8:   for \(j \in I\) do
9:     Let \(N(j)\) be the set of neighbors of \(j\)
10:    for \(k \in (N(j) \cap S)\) do
11:       Set State\(_k\) = i with probability \(\beta\)
12:    end for
13:   end for
14:   Set State\(_j\) = r with probability \(\gamma\)
15: end for

**Algorithm 4. Threshold diffusion model**

1: Let State\(_j\) denote the state of node \(j\)
2: Set State\(_j\) = s for nodes \(j = 1, 2, \ldots, n\)
3: Let S = \{\(j : \text{State}_j = s\)\}
4: Let Threshold\(_j\) denote the threshold of node \(j\)
5: Set State\(_i\) = i for initially infected nodes \(i\)
6: Let I = \{\(j : \text{State}_j = i\)\}
7: for Number of iterations do
8:   for \(j \in S\) do
9:     Let \(N(j)\) be the set of neighbors of \(j\)
10:    if \(\frac{|N(j) \cap I|}{|N(j)|} > \text{Threshold}_j\) then
11:       Set State\(_j\) = i
12:    end if
13:   end for
14: end for
There is a vast array of spreading processes used to model epidemics. As stated previously, we focus on two common diffusion processes: the SIR model and the Threshold model. The SIR model was introduced by Kermack and McKendrick. In this model, each node can be in one of three states: susceptible, meaning they have not yet been infected; infected; and removed, meaning they were previously infected. We assume that removed nodes cannot be infected again, i.e., they become immune to reinfection. The only transitions allowed for nodes is from susceptible to infected, and from infected to removed. Susceptible nodes can become infected when they have contact with other infected nodes. The model is a Markovian process run with discrete time steps, and can be executed on a network as per Algorithm 3.

The Threshold model was introduced by Granovetter. In this model, each node/individual has a binary choice/state: either they are infected or not, they believe and spread the rumor or not, they have failed or not. In most cases, we will consider one of the states as positive and one as negative, and study how the number of positive nodes changes over time. For example, we may say an infected node is positive, and study how many nodes become infected in the equilibrium. This model has a susceptible-infected form, with nodes only able to move from the susceptible (negative) to infected (positive) states. Each node has a threshold: the proportion of their neighboring nodes that need to have a positive state for them to switch from the negative state to the positive state. In the power grid example, we might say that a power station fails if 50% of its neighboring stations fail.

Algorithm 4 describes the implementation of this model on a network.

Diffusion model parameters. To illustrate the impact of the data handling model, we run the diffusion models on both the adjusted and the initial networks, and compare the diffusion model outputs. We again follow Carro et al. by using 10 random initializations for every graph and diffusion model tested. Simulations are always run until stability is reached (up to 30 iterations for the Threshold model, and up to 2,500 iterations for the SIR model). All parameter value choices are made for illustrative purposes; we simply wish best to highlight the impact that data handling can have on downstream model output. The impact of the data handling seen when using these models in reality may be greater or lesser than shown here, depending on the data handling performed and the diffusion model chosen.

When testing node removal, we use the following parameters: for the SIR model, $b$ is set to 0.001, $g$ to 0.01, and the initial fraction infected to 0.05. For the Threshold model, the threshold for each node is set to 0.5, with 0.36 of the population being initially infected. These parameters are chosen simply to illustrate the impact missing nodes might have; a different set parameter values could result in the data mishandling having a greater or lesser impact.

When testing node aggregation, we choose model parameters that result in a roughly 50% spread on the non-aggregated network. We choose these target infection rates as they best illustrate the differences that can arise in networks or regions where there is extremely high or extremely low spread, how one aggregates is likely to have little to no impact. The 50% case represents scenarios where one’s data handling decisions may have the most significant impact. For the SIR model, we adjust $\gamma = 0.2$, and leave $\beta = 0.001$ and 5% initial infection. This results in approximately 50% of the population becoming infected in the non-aggregated network. For the Threshold model, we change to 42% of the nodes initially infected, and leave each node’s threshold at 50%. This results in a full spread of the infection in roughly 50% of trials on the WS non-aggregated network.

<table>
<thead>
<tr>
<th>Diffusion Model</th>
<th>Node removal diffusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIR</td>
<td>$\beta = 0.001, \gamma = 0.01$ (edge/node removal) or 0.2 (aggregation), $\text{frac}_{-\text{inf}} = 0.05$</td>
</tr>
<tr>
<td>Threshold</td>
<td>node_threshold = 0.5, $\text{frac}_{-\text{inf}} = 0.36$ (edge/node removal) or 0.42 (aggregation)</td>
</tr>
</tbody>
</table>

**Algorithm 5. Node removal diffusion**

1: Set parameters of diffusion model, M
2: for Desired number of graphs do
3: Generate graph, G, with desired topology
4: Run M on G with $n$ random initializations and compute average infection
5: for Number of removal batches do
6: Remove nodes at random from G
7: Run M on G with $n$ random initializations and compute average infection
8: end for
9: end for
Figure 5 summarizes all models, graphs, and parameter values tested in our simulations on synthetic networks. Algorithms 5 and 6 describe our algorithms used for testing diffusion models with node removal and node aggregation respectively.

Génois et al.\textsuperscript{44} note that missing nodes correspond to immunization in the network, since a virus cannot spread through, for example, a vaccinated person. Recognizing this, they emphasize that, while the results from the deleting of nodes are the same regardless of why those nodes are deleted, they represent different realities. Nodes removed due to immunization change the real-world network, and thus the diffusion model results on this network are accurate to reality. Nodes removed due to the data being missing, however, lead to an inaccurate network; hence, one can have inaccurate diffusion model results. Génois et al.\textsuperscript{44} argue that this difference in cause justifies studying these missing nodes. Because of this, we also include the study of missing nodes, but with a view to understanding missing elements due to data mishandling, as opposed to data mismeasurement as in Génois et al.\textsuperscript{44}

**Real-world networks**

To verify the results on synthetic networks, we also study the impact of node removal on two real-world networks. We use the same networks as were chosen by Wang et al.\textsuperscript{13}: a friendship network from Slashdot.org\textsuperscript{45} (http://snap.stanford.edu/data/soc-sign-Slashdot081106.html); and a citation network from the e-print repository ArXiv\textsuperscript{46,47} (http://snap.stanford.edu/data/cit-HepTh.html). As described by Wang et al.,\textsuperscript{13} these two networks represent different types of commonly analyzed networks, with similar size but different structural properties, making them good candidates for testing. Similarly to their experiments, we treat the data as ground truth and error free, and transform the networks to be undirected.

For succinctness, we limit our analysis to the SIR model, and provide results for the “threshold model on real-world networks” of the supplemental experimental procedures. When using the model parameters tested on synthetic networks, the spread on the real-world networks without any data handling errors applied is much lower than in the synthetic networks (with approximately 0.3 and 0.6 of the population becoming infected in the Slashdot.org and ArXiv networks, respectively). As the region of most concern when applying epidemic models in reality is when one is in a critical region of full spread, we also test SIR model parameters that place the model in these critical regions. We set $\gamma = 0.01$ and the initial proportion of the population infected to 5%. We then adjust the $\beta$ value to reach regions of critical spread. We note that the two networks used are not connected, so in many cases it may be impossible to infect 100% of the population. Hence, we choose the minimum $\beta$ value (using 0.005 increments) so that at least 95% of the population becomes infected on average. For the Slashdot.org network, this $\beta$ value is 0.085; for the ArXiv network, it is 0.015. These values correspond to $R_0$ values of 8.5 and 1.5, respectively.

Our node removal simulations in the real-world networks proceed similarly as described in “node removal”: nodes are removed in batches until 5% of nodes remain. After each batch removal, the model is run with 10 different random initializations. We follow the experimental design of Wang et al.\textsuperscript{13} and perform this procedure with 10 trials on each of the real-world networks. Algorithm 7 describes this process.

**Algorithm 7. Node removal diffusion (Real-world network)**

1: Set parameters of diffusion model, M
2: for Desired number of trials do
3: Load real-world graph, G
4: Run M on G with n random initializations and compute average infection
5: for Number of removal batches do
6: Remove nodes at random from G
7: Run M on G with n random initializations and compute average infection
8: end for
9: end for
**RESULTS**

We report the results for node removal and node aggregation using the two diffusion models discussed above. As the effects of these types of errors on network properties are well understood ("measurement error and networks"), we focus on the impact on model output, with brief additional results in the "network property changes" of the supplemental experimental procedures. We also note that we consider primarily high-level model metrics, such as proportion of population infected. The conclusions drawn here may differ when studying more detailed metrics.

### Node removal

For each of the 20 initial networks of each topology, we ran the model 10 times after each batch of random node removal. In the SIR case, we first removed 1,250 nodes, since the model always had full spread when there were more nodes.

**SIR model**

Figure 6A shows the proportion of the population becoming infected as one removes nodes at random. We track this relative to the number of nodes remaining in the network. As we remove nodes, the proportion of the population that becomes infected decreases. The plots in Figure 6B show the change in proportion of the population becoming infected for varying nodes remaining.

**Node aggregation**

We also consider node aggregation, which is performed by merging all nodes in the network into one, equivalent to summing the populations of all nodes into one. The results for node aggregation using the two diffusion models discussed above are similar to those for node removal. The plots in Figure 7A and 7B show the peak proportion of the population infected at the same time and the change in peak proportion of the population infected at the same time, respectively. The plots in Figure 7C and 7D show the number of time steps to stability and the number of time steps to peak infection, respectively.
nodes, there is a reduction in the spread of the infection. The rate at which this change in spread happens is almost identical between the ER and WS networks, but differs in the BA network; in the BA network, the spread starts to reduce earlier as one removes nodes, but does not change as rapidly as in the ER and WS networks. In addition, this change in spread is not linear, beginning more slowly as one removes nodes and gradually increasing. This can be seen in Figure 6B, where the change in proportion of the population infected is plotted for varying nodes remaining. We can see that this change is highly non-linear, with an initially slow change that grows as nodes are removed. Again, we note that there are slight differences between the BA network and the ER and WS networks; the BA change graph starts increasing sooner as nodes are removed, but peaks later.

This illustrates an important point: the influence of removing nodes on the model output differs depending on the number of nodes in the network. If one is in a region with many nodes, missing nodes will not have a significant impact on the model output; however, in a region with few nodes, these same missing nodes could have a large impact on the model. This highlights the need for careful data handling in these critical regions. It is also likely that one might find oneself in and around these regions in reality—the usefulness and interest in these models is typically when there is some uncertainty about the potential outcome, since they do not provide much value when one has a large number of nodes and the virus always spreads fully.

In Figure 7, we look at the effect of removing nodes on the peak proportion of the population being infected at the same time and the times taken to reach stability and this peak infection rate. As nodes are removed, the peak proportion of the population infected decreases. Once again, this change is not linear, with a generally sharper decrease at fewer nodes. Both the time to stability and the time to peak infection follow a similar trend, with both generally increasing as nodes are removed, peaking at around 300 nodes; once nodes are removed past this point, the times start to decrease. In both the time measurements and the peak infection rates, we see the trend of the BA network results differing from the ER and WS results. Once again, the BA shows lower peaks and less steep changes, but the impact of node removal begins sooner.

Noticing the non-linear nature of the peak infection rate, we again see the importance of data handling in certain regions. Modelers need to mindful of all steps and possible problems in the modeling process, but especially in their data handling, as this is where they have the most freedom to make changes. The Threshold model

Figure 8 shows the proportion of the population above the initial infected becoming infected as we remove nodes. The colored plotted points represent the average proportion infected over the multiple trials performed at each number of nodes remaining, with the gray points representing individual trial outcomes; in each individual trial, usually either close to none or close to all nodes become infected. In each plot, a sigmoid function of the form

$$f(x) = \frac{1}{1 + \exp(-k(x-x_0))}$$  (Equation 2)

is fitted, where $k$ and $x_0$ are least-squares estimates.

All three scatterplots have the same form, with a high number of nodes resulting in little to no spreading of the disease past the initial infections, and low number of nodes having spread to the entire population. In the WS networks, a very low number of nodes leads to a decrease in the number of infected. In all topologies, there is a transition zone, where one changes from no nodes being infected to all nodes being infected; the length and steepness of this zone varies according to topology. In the ER network, this transition takes place over a small change in number of nodes, while in the WS network it happens over a much larger change.

We see here how the network structure influences the approach modelers need to take to their data handling. In the WS network, there is a large region of nodes remaining where
Small changes can influence the outcome; however, the rate of this change is relatively low. In contrast, in the ER network it is a small region where changing the number of nodes has an impact. In this small region, a change in nodes can drastically shift model outcome. Thus we see that the impact of data handling on model output can be highly network dependent.

**Node aggregation**

Entity resolution, and thus node aggregation, form critical parts of the data handling process. The modeler is able to make their own decisions in this, with fewer limitations as might be used in data measurement. Here, we show how decisions made can have a greater impact on the downstream model than might be expected based on simply the change in parts of the network structure. We do this through two different decisions: choosing to aggregate nodes using bins and a shared neighbor threshold, and choosing to aggregate nodes at random. This is different from a data measurement issue; we have the full network data, but may have beliefs about nodes representing the same entities, and can decide how or whether to combine them.

We first tested the results of performing the aggregation algorithm prior to running models on the resultant networks. Figure 9 shows the number of aggregations for varying shared neighbor thresholds required to aggregate. The points indicate the results for individual networks, with the lines showing the average over four networks for the two different topologies. For the ER network, aggregations only happened when we set the threshold very low (<0.25), and so we limited our analysis to WS and BA networks only.

The WS and BA networks show dramatically different behavior for the given aggregation algorithm. The WS network has a significantly higher threshold (0.8 versus 0.6) where it starts to have aggregations. In addition, there is less variance in the number of aggregations in different graphs for the same shared neighbor threshold, and the change in number of aggregations as one varies the shared neighbor threshold is smoother. When running models on the aggregated networks, we choose a shared neighbor threshold of 0.8 for the WS networks, and 0.57 for the BA networks. These shared neighbor thresholds were chosen to have a moderate number of aggregations. We do not say which of the aggregated or unaggregated networks are the ground truth, if either. We only wish to illustrate how aggregation (with this algorithm in particular) can affect model output.

Table 2 shows the means and standard deviations in the number of aggregations for the shared neighbor thresholds described above, as well as how the aggregation affects the normalized average degree of the networks. These results are for the total of 40 networks used to test the SIR and Threshold models (20 networks each). The chosen shared neighbor thresholds give a similar average number of aggregations, but the BA networks have a much larger variance. Both networks start with a normalized degree of approximately 0.1 (meaning that the average node has 10% of all its possible edges); when one aggregates the nodes using the shared neighbor algorithm, the normalized degree increases slightly (more so for the WS networks); when one performs the same number of aggregations at random, the normalized average degree increases by significantly more. The shared neighbor aggregation does not drastically change the network structure in terms of the average degree measurement, but the random aggregation does. However, both have a significant impact on the outcome of models on the network.

![Figure 9. Number of aggregations for various shared neighbor thresholds](image-url)
The SIR model

When testing the SIR model with the aggregation algorithm, we choose parameters that result in approximately half of the population becoming infected on the unaggregated network: $\beta = 0.001$, $\gamma = 0.2$, and 5% of the population initially infected. Figure 10 shows the boxplots of the proportion of the nodes becoming infected for different aggregation techniques, while Table 3 shows the means and variances for these proportions. As in the edge removal section, 20 networks were generated for each topology, and the SIR model run 10 times on each with different random initializations. Performing the neighborhood-based aggregation generally leads to a lower overall infection; however, in the BA network there is now far more variance—this can be attributed to the higher variance in the number of aggregations that one has with this network topology. This is similar in the case where one aggregates at random, only now the proportion of the population becoming infected increases.

We see that the neighborhood-based aggregation has a far larger effect on model output than one may suspect based on the changes in the normalized average degree. The directions of the change in the BA and WS networks are the same, but roughly half of trials on the initial WS networks would have no spread, and half would have full spread. This corresponded to each node having a threshold of 50%, and 42% of nodes being infected at initialization, which resulted in roughly 15% of trials on the BA networks having full spread. Figure 11 shows the additional proportion (above the initial infected) becoming infected for different aggregation techniques, with Table 4 showing the means and variances. With the Threshold model, there is more variance in the output when compared with the SIR. The impact of the two aggregation techniques is reversed from the SIR model, with the shared neighbor aggregation resulting in a greater proportion of the population becoming infected, and the random aggregation resulting in a smaller proportion becoming infected.

The results from the Threshold model show that not only does the aggregation technique have an impact on model output but that this impact can also be dependent on the model. In the SIR case, the shared neighbor aggregation reduces the spread, while in the Threshold model it increases the spread.

Real-world networks

Figure 12 shows the proportion of the population becoming infected as we remove nodes from the network. Here, the SIR parameters used are the same as in the synthetic networks: $\gamma = 0.01$, $\beta = 0.001$, and 5% of the population initially infected. In the synthetic networks, this resulted in the population becoming fully infected when no nodes had been removed; instead, in this case either 30% or 60% become infected. The impact of the data handling errors is now fairly linear, unlike what was observed in the synthetic networks. This suggests there may be a certain level of interaction between network properties and model parameters when considering data handling's impact on a diffusion model.

Figure 13 replicates Figure 6 for the two real-world networks with adjusted $\beta$ values. Shown in Figures 13A and 13C are the

![Figure 10. Proportion of population infected for different aggregation techniques (SIR model)](image)

Figure 10 shows the boxplots of the proportion of the nodes becoming infected for different aggregation techniques, while Table 3 shows the means and variances for these proportions.
proportions of the population infected as one varies the number of nodes remaining in the network; the change/approximate derivatives of these function are shown in Figures 13B and 13D. As intended, approximately 95% of the population becomes infected when no nodes are removed from the network. We see similar behavior as was observed in the synthetic networks: the change in the proportion of the population becoming infected as one removes nodes is non-linear, with more drastic changes taking place as more nodes are removed. The level of non-linearity differs in the two networks, with the ArXiv network showing greater non-linearity. The derivatives of the proportion infected are also different orders of magnitude, with far greater changes in the ArXiv network. Part of this can be attributed to the difference in size of the networks (the Slashdot.org network has roughly 2.5 times the number of nodes as the ArXiv network), but there are still unexplained differences. Another possible factor in these differences is the density of the two networks: the ArXiv network has a normalized average degree 6.5 times that of the Slashdot.org network. The density of both real-world networks is significantly lower than the synthetic networks tested, where we also saw much larger derivative values. While these results do not constitute a proof, they do suggest that network properties, such as density and size, play a role in the effect that data handling errors can have on models.

These results further confirm what was observed in the synthetic networks: the impact of data handling on network diffusion models is dependent on network structure and properties. Again we see that the impact of data handling can be non-linear, and the level of non-linearity is also dependent on the network. This non-linearity also seems more pronounced in regions where modelers may be most concerned in reality, i.e., where there is full spread. While further experiments are required to confirm this, properties such as network size and density may have an effect on the impact of data handling errors. We also see an interaction between diffusion model parameters and data handling error impact.

**DISCUSSION**

Having highlighted the complexity and concerns of data handling, we provide a framework for testing whether a diffusion model is susceptible to these handling errors. Our framework shows how a modeler can insert data handling errors into networks and run the diffusion model of concern on them, testing the impact on model output. We provide examples by applying this framework to both synthetic and real-world networks, with different data handling and diffusion models applied.

Our results highlight the importance of effective data handling when using data-driven models. In particular, the simulated examples show the quantifiable impact that poor data handling can have. For example, we see that one’s perception of the risk-level of a disease turning into a pandemic can be highly dependent on the proportion of the nodes in a network correctly modeled. The outcome of the model changes in a non-linear way as one removes nodes, where removing even small numbers of
nodes in regions with a few nodes can lead to vastly different predictions for total infections. How severe these changes are is dependent on both the infectiousness of the diseases, and the underlying network structure—both of these need to be taken into account when making decisions based on the available data.

In addition, we show how data integration decisions can have a far-reaching impact on a model’s output, and that this impact may not be immediately recognizable based on changes in some network structure properties alone. When aggregating using a shared neighbor approach, there can be very limited changes in the average degree of the network; however, the results of a diffusion model run on this aggregated network can differ significantly from those seen on the unaggregated network. We also show that using random aggregations can have a more noticeable effect on the average degree, but with a similar sized impact on model output. This reinforces the point: data handling decisions can have meaningful impact on model output, and this impact can differ from data mismeasurement impact.

Conclusions
With the increase in the number, diversity, and size of datasets being used for computational modeling, data handling has become increasingly important. This importance will only grow as researchers increasingly reuse data. However, the impact of data handling on computational models is, as of yet, a poorly studied area.

Hence, we introduced a framework for quantitatively analyzing the impact that data errors can have on network diffusion models. We used this framework to examine a class of standard network models. We find that:

- the impact of data errors (missing nodes) can create similar impact to simulated interventions (e.g., vaccinations or quarantine in epidemics)
- data handling algorithms, such as data integration, can have non-trivial impact on model output and in some cases conclusions of model runs
- network models (in particular) are highly sensitive to errors in certain connectivity regimes and these regimes are where models transition between qualitatively different outcomes (e.g., full infection versus no infection), hence, the network structure produced by integrating and cleaning data is crucial in these transition situations

There are many avenues for potential research arising from these findings. The devised framework can be applied in various additional ways: on a greater variety of real-world datasets; using more sophisticated, industry-level data integration tools; and on more complex diffusion models. There is also the possibility to develop a set of guidelines for how different types of diffusion models may be affected by data handling, and provide recommendations for how to address potential impacts.

We believe that these findings have important implications for data-driven modelers and again emphasize the important role that data management plays in simulation-based science.

EXPERIMENTAL PROCEDURES
Resource availability
Lead contact
James Nevin is the lead contact of this study and can be reached at: j.g.nevin@uva.nl.

Materials availability
The Jupyter notebook used for running the simulations can be obtained via Mendeley Data repository: https://doi.org/10.17632/fpjznrbd2.2.

Data and code availability
All original code has been deposited at Mendeley Data: https://doi.org/10.17632/fpjznrbd2.2, and is publicly available as of the date of publication. This paper analyses existing public data. These data are available at: http://snap.stanford.edu/data/cit-HepTh.html, and http://snap.stanford.edu/data/soc-sign-Slashdot081106.html

Figure 12. SIR model on real-world network with old parameters
(A and B) Real-world networks’ proportion of population becoming infected for varying nodes remaining, using the model parameters tested in the synthetic networks; ArXiv network (A); Slashdot.org network (B).
Figure 13. Total infection for SIR model on real-world networks with random node removal (A–D) Proportion of population becoming infected for varying nodes remaining on ArXiv network and Slashdot.org network (A) and (C), respectively; change in proportion of population becoming infected for varying nodes remaining on ArXiv network and Slashdot.org network (B and D), respectively.

SUPPLEMENTAL INFORMATION
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AUTHOR CONTRIBUTIONS

DECLARATION OF INTERESTS
The authors declare no competing interests.

REFERENCES