

Appendix A: Proofs

A.1 Proof of Lemma 4

In order to find the conditions under which the market survives we use the Intermediate Value Theorem: for two feasible values r_α and r_β , where $r_\alpha < r_\beta$, if $G(r)$ is (i) a continuous function and (ii) $G(r_\alpha)$, $G(r_\beta)$ are of opposite signs, there exists an $r^* \in [r_\alpha, r_\beta]$ such that $G(r^*) = 0$.

We set $G(r) = \mathbb{E}[Pr(success)|r, \Omega] - (1+r)^{-1}$, where $\mathbb{E}[Pr(success)|r, \Omega] = \int_{\hat{p}(r)}^1 pf_H(p)dp + Pr(i = L|r, \Omega) \int_{\hat{p}(r)}^1 pf_L(p)dp$ and $\hat{p}(r) = \frac{v}{B-(1+r)}$. Note that $G(r)$ satisfies condition (i) as $\mathbb{E}[Pr(success)|r, \Omega]$ and $(1+r)^{-1}$ are continuous functions in r .¹ To show that condition (ii) is satisfied, we set $r_\alpha = 0$ where the function $(1+r)^{-1}$ reaches its maximum value, 1. Note that $G(0)$ is negative as $\mathbb{E}[Pr(success)|r = 0, \Omega] < 1$. Hence, for the market to survive, it is sufficient to show that there exists a feasible interest rate r such that $G(r) \geq 0$.

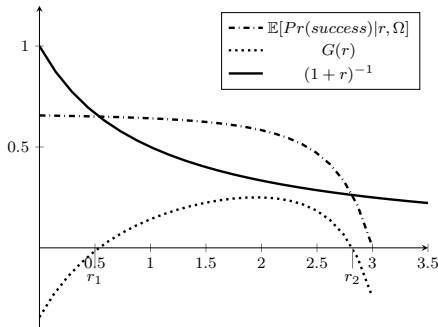


Figure 1: Case where market survives.

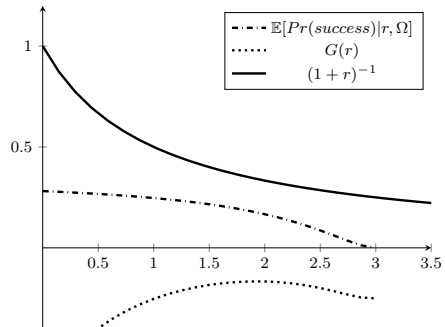


Figure 2: Case where market collapses.

A.2 Proof of Proposition 1

Manager's Problem. The manager finds it optimal to ask for a rating independently of the project type. This relates to the signaling nature of the decision to ask for a rating; the manager of an $i = H$ project always has an incentive to ask for a rating to communicate the project type. Therefore, if the manager of an $i = L$ project does not ask for a rating, her project type will be perfectly inferred by the creditors. Without loss, we assume that the manager always asks for a rating when she is indifferent.

¹Recall that, following the spirit of Nachman and Noe (1994), there is a pooling in the contracting stage, i.e., the manager of an $i = L$ project mimics the manager of an $i = H$ project by offering the same interest rate r .

The Creditors' Problem. When the CRA perfectly reveals the type of the project and the issued rating is good, the creditors' participation constraint is satisfied as long as there exists $r \in (0, r^{max})$, such that:

$$\int_{\hat{p}(r)}^1 pf_H(p)dp \geq (1+r)^{-1}, \quad (\text{A.1})$$

whereas if the rating is bad, the creditors' participation constraint is satisfied as long as there exists $r \in (0, r^{max})$, such that:

$$\int_{\hat{p}(r)}^1 pf_L(p)dp \geq (1+r)^{-1}. \quad (\text{A.2})$$

Finally, when the credit ratings are uninformative, the creditors' participation constraint is satisfied as long as there exists $r \in (0, r^{max})$, such that:

$$\lambda \int_{\hat{p}(r)}^1 pf_H(p)dp + (1-\lambda) \int_{\hat{p}(r)}^1 pf_L(p)dp \geq (1+r)^{-1}.^2 \quad (\text{A.3})$$

Equilibrium conditions. Following Lemma 4, when the ratings are perfectly informative, the equilibrium interest rate that corresponds to rating $R = GR$, denoted by r_{GR}^* , is the minimum interest rate for which (A.1) binds, whereas the equilibrium interest rate that corresponds to $R = BR$, denoted by r_{BR}^* , is the minimum interest rate for which (A.2) binds. When the ratings are uninformative, the equilibrium interest rate is the minimum interest rate for which (A.3) binds.

The reasoning behind Proposition 1 can be illustrated in Figure 3 and Figure 4 where: i) the green curve captures the creditors' beliefs about the repayment probability when they are certain that the project type is $i = H$; ii) the red curve captures the creditors' beliefs about the repayment probability when they are certain that the project type is $i = L$; iii) the yellow curve captures the creditors' beliefs about the repayment probability when the project type is hidden; iv) the blue curve captures the creditors' beliefs about the repayment probability when they believe that the project is of type $i = H$ with probability κ . Note that independently the set of admissible parameters, for each $r \in (0, r^{max})$, the green curve is always above the red curve, due to FOSD. Besides, given that $\lambda \in (0, 1)$, the yellow curve is always above the red curve and always below the green curve, as for each $r \in (0, r^{max})$, the corresponding point in the yellow

²In (A.1), (A.2) and (A.3), $\hat{p}(r) = \frac{v}{B-(1+r)}$ and $r^{max} = B - v - 1$.

curve is the linear combination of the corresponding point in the green and the red curve.

Recall that when considering whether to finance a project that promises interest rate $r' \in (0, r^{max})$, the question that the creditors ask themselves is: "for a given interest rate r' , what would be the probability of success if financing does take place?" If the answer is $s(r')$, the creditors would be willing to finance the project if $s(r')(1 + r') \geq 1$, which implies $s(r') \geq (1 + r')^{-1}$.

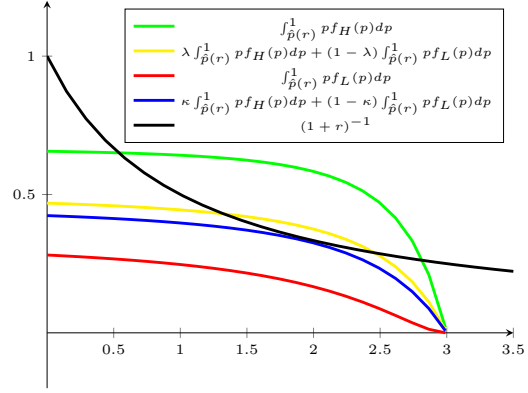
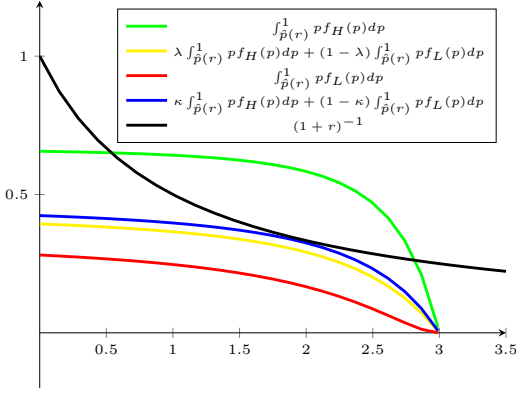


Figure 3: CRA alleviates under-financing. Figure 4: CRA leads to under-financing.
Case where: $f_H(p) = 2p$, $f_L(p) = 2(1 - p)$ Case where: $f_H(p) = 2p$, $f_L(p) = 2(1 - p)$,
 $\lambda = 0.3$, $B = 5$, $v = 1$. $\lambda = 0.5$, $B = 5$, $v = 1$.

Part one. The case where perfectly informative ratings alleviate under-financing refers to the environment where an $i = H$ project is financed only if its type is disclosed. Recall that, when the project type is hidden, the manager fails to raise capital when there is no $r \in (0, r^{max})$ that satisfies relation (A.3). In contrast, when the project type is known to be $i = H$, the manager raises capital when there exists $r \in (0, r^{max})$ which satisfies (A.1). Hence, the proof of part one boils down to showing that, given the set of feasible interest rates, the subset of interest rates that satisfies (A.1) is non-empty, whereas the subset of interest rates which satisfies (A.3) is empty.

This case is captured in Figure 3. Suppose that there is no feasible interest rate such that the corresponding point in the yellow curve is above the corresponding point in the black curve (i.e., for each feasible r , relation (A.3) is violated). This implies that there is no feasible interest rate for which the creditors would be optimistic enough about the probability of success, such that they are willing to finance the project when its type is hidden. Suppose now that the project type is disclosed to be $i = H$. This new piece of information leads the creditors to revise their

beliefs upwards, which are now illustrated by the green curve. Recall that, by assumption, the project is financed when its type is known to be $i = H$, i.e., there is at least one feasible interest rate where the creditors are optimistic enough about the corresponding success probability such that they are willing to finance the project (for this set of economic parameters, the latter is true for $r \in [0.5, 2.8]$).³

The main message of the previous paragraph is that, if the type of project remains hidden, the manager might fail to finance a socially valuable project, which would be financed if its type is disclosed. Therefore, the impact of information disclosure depends on whether there is a feasible interest rate for which creditors are optimistic enough to finance the project when its type is hidden, i.e., there is a feasible r which satisfies relation (A.3). Graphically, the latter boils down to whether there is a feasible interest rate such that the yellow curve is above the black curve. Recall that the yellow curve is given by $\lambda \int_{\hat{p}(r)}^1 p f_H(p) dp + (1 - \lambda) \int_{\hat{p}(r)}^1 p f_L(p) dp$. Thus, for any given $r \in (0, r^{max})$, the yellow curve is a linear combination of the green and red curve, with weights λ and $1 - \lambda$, respectively. Furthermore, since there is always a feasible interest rate such that the green curve is above the black curve, if the red curve is below the black curve, there is a *unique* weight κ such that the curve resulting from combination of the green and the red curve (with weights κ and $1 - \kappa$) is *tangent* to the black curve. The latter is captured by the blue curve.⁴

As a result, if $\lambda < \kappa$, the yellow curve is below the blue curve for each feasible r . The latter implies that when $\lambda < \kappa$, information disclosure alleviates the under-financing problem of socially valuable projects ($i = H$), which would not be financed if their type was hidden.

Part two. The case where perfectly informative ratings lead to under-financing refers to the environment where an $i = L$ project is financed only if its type is hidden. Recall that when the project type is hidden, the manager raises capital when there is a feasible r which satisfies relation (A.3). In contrast, when the project type is known to be $i = L$, the manager fails to

³As the capital market is competitive, the manager will offer the lowest interest rate which satisfies (A.1), i.e., $r = 0.5$.

⁴Recall that the value of κ depends on the economic parameters ($f_H(p)$, $f_L(p)$, B) and the benefit of withdrawing resources, v , which are exogenously determined. Given the value of the economic parameters and v , the value of κ is unique. See the discussion following Definition 1 in the Manuscript.

raise capital when there is no feasible value of r that satisfies relation (A.2). Hence, the proof of part one boils down to showing that, given the set of feasible interest rates, the subset of interest rates that satisfies (A.3) is non-empty, whereas the subset of interest rates which satisfies (A.2) is empty.

This case is captured in Figure 4. Suppose that there is a non-empty set of feasible values of the interest rate, such that the corresponding point in yellow curve is above the corresponding point in the black curve (for this set of parameters, the latter is true for $r \in [1.4, 2.4]$)⁵. This implies that for this non-empty set of values of the interest rate, the creditors are optimistic enough about the success probability and they are willing to finance the project when its type is hidden. Suppose now that the creditors learn that the project type is $i = L$. This new piece of information would make them update their beliefs downwards, which are now illustrated by the red curve. Notice that now, for any given interest rate, the creditors are less optimistic about the success probability, and there is no feasible value of the interest rate such that the corresponding point in red curve is above the corresponding point in the black curve (i.e., *for each* feasible r , relation (A.2) is violated). Thus, the manager fails to raise capital when the project type is disclosed to be $i = L$.

The main message of the previous paragraph is that, if the type of project is disclosed to be $i = L$, the manager fails to finance it, although it is socially valuable (under-financing). However, if the project type remains hidden, the manager might succeed in financing the socially valuable project. Therefore, the impact of information disclosure depends on whether there is a feasible interest rate for which creditors are optimistic enough to finance the project when its type is hidden, i.e., there is a feasible value of r that satisfies relation (A.3). Graphically, the latter boils down to whether there is a feasible interest rate such that the yellow curve is above the black curve. As we explained in part one of Proposition 1, if $\lambda \geq \kappa$, the yellow curve is not below the blue curve. Hence, if $\lambda \geq \kappa$, information disclosure leads to under-financing of socially valuable projects ($i = L$), that would be financed if the project type was hidden.

⁵As the capital market is competitive, the manager will offer lowest interest rate which satisfies (A.3), i.e., $r = 1.4$.

Link with Proposition A.1: One can see the similarity between Proposition 1 and Proposition A.1 in Online Appendix B, which shows the impact of informative credit ratings when the manager does not have an information advantage compared to the CRA. The underlying mechanism is the same; when the project is socially valuable independently of its type and the manager can raise capital even if when the type is hidden, information disclosure can prevent the financing of projects of type $i = L$, although it is socially valuable.

A.3 Proof of Proposition 2

Note that the CRA has two options. Either to choose a high rating fee –denoted as P_{high} – and a corresponding rating rule, such that only the manager of an $i = H$ project finds it optimal to ask for a rating (Option A), or to choose a low rating fee –denoted as P_{low} – and a corresponding rating rule, such that the manager finds it optimal to ask for a rating independently of the project type (Option B).

Option A: CRA aims to attract only $i = H$.

If such an equilibrium exists, the rating rule which corresponds to the highest willingness to pay for a rating is truthful disclosure. This is the case because truthful disclosure allows the manager of an $i = H$ project to perfectly differentiate herself from a manager of an $i = L$ project. Conditional on truthful disclosure, the optimal rating fee should leave the manager of an $i = H$ project indifferent between asking for a rating and not asking for a rating, i.e.,

$$P_{high} = \mathbb{E}U(\text{rating}) - \mathbb{E}U(\text{no rating}) \tag{A.4}$$

where the first part of the RHS is the expected utility conditional on having a rating that reveals the project type, whereas the second part is the expected utility when the manager does not have a rating. Note that this separating (regarding the decision to ask for a rating) equilibrium implies that if the manager does not ask for a rating, creditors should believe that the project type is $i = L$. Therefore, $\mathbb{E}U(\text{no rating})$ depends on whether the creditors' participation constraint is satisfied (i.e., the market survives) when the project type is believed to be $i = L$, i.e., whether there exists $r \in (0, r^{max})$ such that (A.5) holds, or equivalently $\kappa = 0$, where

$$\int_{\hat{p}(r)}^1 pf_L(p)dp \geq (1+r)^{-1} \quad (\text{A.5})$$

Thus,

$$\mathbb{E}U(\text{no rating}) = \begin{cases} \int_{\hat{p}(r^\#)}^1 f_H(p)p[B - (1+r)]dp + \int_0^{\hat{p}(r^\#)} f_H(p)vdp & \text{if } \exists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \\ 0 & \text{if } \nexists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \end{cases}$$

where $r^\#$ denotes the minimum $r \in (0, r^{max})$ for which (A.5) binds.⁶ Following that, the rating fee is given by:

$$P_{high} = \begin{cases} P'_{high} & \text{if } \exists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \\ P''_{high} & \text{if } \nexists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \end{cases}$$

where,

$$P'_{high} \equiv \int_{\hat{p}(r^{\#\#})}^1 f_H(p)p[B - (1+r)]dp + v \int_0^{\hat{p}(r^{\#\#})} f_H(p)dp - \int_{\hat{p}(r^\#)}^1 f_H(p)p[B - (1+r)]dp - v \int_0^{\hat{p}(r^\#)} f_H(p)dp \quad (\text{A.6})$$

$$P''_{high} \equiv \int_{\hat{p}(r^{\#\#})}^1 f_H(p)p[B - (1+r)]dp + v \int_0^{\hat{p}(r^{\#\#})} f_H(p)dp \quad (\text{A.7})$$

where $r^\#$ and $r^{\#\#7}$ denote the minimum $r \in (0, r^{max})$ for which (A.5) and (A.1) bind.

Option B: CRA aims to attract the manager independently of the project type.

Note that, since type $i = H$ dominates type $i = L$ in the FOSD sense, if, for a given rating rule and the corresponding rating fee, the manager of an $i = L$ project finds it optimal to ask for a rating, the same is true for the manager of an $i = H$ project. Thus, if such an equilibrium exists, the CRA's problem boils down to finding the rating rule and the corresponding rating fee which maximizes the willingness to pay of the manager of an $i = L$ project.

If there exists $r \in (0, r^{max})$ that satisfies (A.5) (or equivalently $\kappa = 0$), the optimal rating rule which maximizes the willingness to pay of the manager of an $i = L$ project coincides with the one that maximizes the degree of cross-subsidization. The rating rule that maximizes the degree

⁶Intuitively, $r^\#$ is the minimum $r \in (0, r^{max})$ that satisfies creditors' participation constraint when they believe that the project type is $i = L$.

⁷Intuitively, $r^{\#\#}$ is the minimum non-negative interest rate that satisfies creditors' participation constraint when they believe that the project type is $i = H$.

of cross-subsidization is when the CRA issues the same rating independently of the project type (babbling equilibrium). Among the rating rules which implement the babbling equilibrium, we focus on the one where the rating deflation is zero, i.e., $\alpha_G = 1$. We explain the underlying rationale behind this choice at the end of the proof.

If there is no $r \in (0, r^{max})$ which satisfies (A.5) (or equivalently $\kappa > 0$), there are two relevant cases, depending on whether $\lambda \geq \kappa$ or $\lambda < \kappa$. When $\lambda \geq \kappa$, the market survives when the project type is hidden; thus, the discussion of the case where there is $r \in (0, r^{max})$ which (A.5) satisfies, extends to this case as well.

Consider now the case where $\lambda < \kappa$, which corresponds to the setting where the manager fails to raise capital when the project type is hidden. Note that when $\lambda < \kappa$, there is no feasible rating policy that allows the manager to raise capital with certainty independently of the project type. Based on Definition 1, the creditors are willing to finance the project as long as the probability that they allocate to the project being of type $i = H$ is at least equal to κ . Conditional on a good rating, the creditors' beliefs that the project type is $i = H$ are given by

$$Pr(i = H | R = GR) = \frac{\alpha_G \lambda}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)}.$$

Recall also that, for the manager to have an incentive to ask for a rating, it must be that, conditional on receiving a good rating, she will be able to raise capital. Hence, the necessary condition for the manager to ask for a rating is

$$Pr(i = H | R = GR) = \frac{\alpha_G \lambda}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)} \geq \kappa. \quad (A.8)$$

Consider now the CRA's problem. The CRA wishes to maximize the willingness to pay of the manager of an $i = L$ project, where the willingness to pay is increasing in the probability of receiving a good rating, i.e., is decreasing in α_B .⁸⁹ Thus, effectively, the maximization problem

⁸⁹Note that lower values of α_B correspond to a higher probability of receiving a good rating (thus, being able to raise capital), but on the other hand, to higher interest rates conditional on financing. However, the first force dominates the second one. The latter is because the manager is only partly affected by the interest rate, as she withdraws resources with positive probability.

⁹⁰Note also that not financing a socially valuable project results in inefficiency that hurts the CRA, as all the other parties break even in equilibrium. Thus, among the combinations of rating rule and rating fee which incentivize the manager to buy a rating independently of her project type, the CRA prefers the combination which corresponds to the highest probability of the project financing.

of the CRA boils down to choosing α_G and α_B such that α_B takes its minimum value conditional that (A.8) holds. This constraint captures the feature that there is a low bound in the value of α_B^* , below which even a good rating will not be sufficient for raising capital. The latter is because the creditors would anticipate that it is likely that this rating corresponds to an $i = L$ project.

Note that the value of α_B is minimized when α_G reaches its maximum value, i.e., $\alpha_G = 1$, and (A.8) binds. Hence, to derive the optimal value α_B^* , we solve $\frac{\alpha_G \lambda}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)} = \kappa$ with respect to α_B , assuming that $\alpha_G = 1$. The latter implies that $\alpha_B^* = \frac{\kappa - \lambda}{\kappa(1 - \lambda)}$.

Following the intuition provided in the previous paragraphs,

$$\mathbb{E}U(\text{rating}) = \begin{cases} \int_{\hat{p}(r^S)}^1 f_L(p) p [B - (1 + r)] dp + v \int_0^{\hat{p}(r^S)} f_L(p) dp & \text{if } \exists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \\ (1 - \alpha_B^*) \left\{ \int_{\hat{p}(r^{SS})}^1 f_L(p) p [B - (1 + r)] dp + v \int_0^{\hat{p}(r^{SS})} f_L(p) dp \right\} & \text{if } \nexists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \end{cases}$$

where r^S denotes the minimum $r \in (0, r^{max})$ such that (A.9) binds, given that $\alpha_G = 1$ and $\alpha_B^* = 0$.¹⁰ Similarly, r^{SS} denotes the minimum $r \in (0, r^{max})$ such that (A.9) binds, given that $\alpha_G = 1$ and $\alpha_B^* = \max\{0, \frac{\kappa - \lambda}{\kappa(1 - \lambda)}\}$.¹¹ Recall that:

$$\frac{\overbrace{\alpha_G \lambda}^{Pr(i=H|R=GR)}}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)} \int_{\hat{p}(r)}^1 p f_H(p) dp + \frac{\overbrace{(1 - \alpha_B)(1 - \lambda)}^{Pr(i=L|R=GR)}}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)} \int_{\hat{p}(r)}^1 p f_L(p) dp \geq (1 + r)^{-1} \quad (\text{A.9})$$

Hence, the rating fee is given by:

$$P_{low} = \mathbb{E}U(\text{rating}) - \mathbb{E}U(\text{no rating})$$

The case where the CRA aims to attract the manager independently of the project type requires us to apply restrictions on the off-equilibrium beliefs, namely, the creditors' beliefs when they observe a firm without a rating. These beliefs are critical, as they affect $\mathbb{E}U(\text{no rating})$. We assume that if the manager does not have a rating, the creditors believe that the project type is

¹⁰Intuitively, r^S is the minimum $r \in (0, r^{max})$ that satisfies the creditors' participation constraint when their posterior beliefs coincide with their prior beliefs, i.e., they believe that the project type is $i = H$ with probability λ and $i = L$, otherwise.

¹¹Intuitively, r^{SS} is the minimum $r \in (0, r^{max})$ that satisfies the creditors' participation constraint when they observe a good rating, and they believe that the project type is $i = H$ with probability $\frac{\alpha_G \lambda}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)}$ and type $i = L$ with probability $1 - \frac{\alpha_G \lambda}{\alpha_G \lambda + (1 - \alpha_B)(1 - \lambda)}$, where $\alpha_G = 1$ and $\alpha_B^* = \max\{0, \frac{\kappa - \lambda}{(1 - \lambda)\kappa}\}$.

$i = L$, which implies that:

$$\mathbb{E}U(\text{no rating}) = \begin{cases} \int_{\hat{p}(r^\#)}^1 f_L(p)p[B - (1+r)]dp + v \int_0^{\hat{p}(r^\#)} f_L(p)dp & \text{if } \exists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \\ 0 & \text{if } \nexists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \end{cases}$$

where $r^\#$ defined earlier. Following that, the rating fee is given by:

$$P_{low} = \begin{cases} P'_{low} & \text{if } \exists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \\ P''_{low} & \text{if } \nexists r \in (0, r^{max}) \text{ s.t. (A.5) holds} \end{cases}$$

where,

$$P'_{low} \equiv \int_{\hat{p}(r^\$)}^1 f_L(p)p[B - (1+r)]dp + v \int_0^{\hat{p}(r^\$)} f_L(p)dp - \int_{\hat{p}(r^\#)}^1 f_H(p)p[B - (1+r)]dp - v \int_0^{\hat{p}(r^\#)} f_H(p)dp \quad (\text{A.10})$$

$$P''_{low} \equiv (1 - \alpha_B^*) \left\{ \int_{\hat{p}(r^{\$\$})}^1 f_L(p)p[B - (1+r)]dp - v \int_0^{\hat{p}(r^{\$\$})} f_L(p)dp \right\} \quad (\text{A.11})$$

where $r^\$, r^\#$ and $r^{\$\$}$ defined earlier.

Suboptimality of rating deflation. Note that rating deflation can not be optimal when the CRA aims to attract the manager of $i = H$ projects only; compared to truthfully disclosure, rating deflation would correspond to a strictly lower rating fee. In what follows, we explore the case where the CRA aims to attract the manager independently of the project type.

First, recall that, when there is no feasible interest rate such that the creditors are willing to finance the project when its type is revealed to be $i = L$, rating deflation is always suboptimal; rating deflation weakens the incentive of the manager of an $i = L$ project to ask for a rating. Thus, rating deflation decreases the rating fee that the CRA can charge.

Second, when there exists a feasible interest rate such that the creditors are willing to finance the project when its type is revealed to be $i = L$, or when its type is hidden, the optimal rating policy is uninformative, i.e., $\alpha_G = 1 - \alpha_B$ (babbling equilibrium). However, there are infinite combinations of α_G and α_B that satisfy $\alpha_G = 1 - \alpha_B$. Note that the CRA is indifferent between any rating rule that implements a babbling equilibrium, thus, it is theoretically possible to sustain rating deflation as part of the equilibrium policy. However, an equilibrium with rating deflation

would not survive a small perturbation on the model. For instance, if we consider a perturbation in the model, such as when there is an arbitrarily small fraction of the creditors who only care about the face value rather than the information value of a rating, then the equilibria with rating deflation would always be dominated. The aforementioned regime could arise when a subset of the creditors are naive, or when there are institutional investors who are obliged to invest in assets with good rating.¹²

A.4 Proof of Proposition 3

Part one corresponds to the case where the project can raise capital even when its type is hidden. Thus, even an uninformative rating rule (i.e., $\beta_G^* = 1$ and $\beta_B^* = 0$) would guarantee that the project is financed with certainty.

Part two corresponds to the case where the market collapses when the project type is hidden, which also implies that there is no rating rule where the project can be financed with certainty. Instead, the project could be financed only when it is accompanied by a good rating, where the probability of issuing a good rating is $\lambda\beta_G + (1 - \lambda)(1 - \beta_B)$. In this setting, the regulator has an incentive to increase the probability that the CRA issues a good rating, which is achieved by: i) truthfully revealing the type when it is $i = H$, i.e., $\beta_G = 1$, and ii) inflating the rating of type $i = L$ by decreasing the value of β_B from 1 to $1 - \epsilon$. However, for sufficiently large ϵ , even a good rating will not be sufficient for raising capital (i.e., (A.12) is violated): the creditors would correctly anticipate that it is likely that such a rating corresponds to $i = L$ project.

$$\frac{\overbrace{\beta_G \lambda}^{Pr(i=H|R=GR)}}{\beta_G \lambda + (1 - \beta_B)(1 - \lambda)} \int_{\hat{p}(r)}^1 p f_H(p) dp + \frac{\overbrace{(1 - \beta_B)(1 - \lambda)}^{Pr(i=L|R=GR)}}{\beta_G \lambda + (1 - \beta_B)(1 - \lambda)} \int_{\hat{p}(r)}^1 p f_L(p) dp \geq (1 + r)^{-1}. \quad (\text{A.12})$$

A.5 Proof of Proposition 4a

If the CRA issues a good rating only when the project is of type $i = H$, the expected profit is:

$$\mathbb{E} \Pi = \lambda P''_{high} \quad (\text{A.13})$$

¹²The role of naive creditors is discussed in Bolton, Freixas, and Shapiro (2012), whereas the role of institutional investors is discussed in Opp, Opp, and Harris (2013).

whereas if the CRA issues a good rating to a project of type $i = L$ with probability $1 - \alpha_B^* = \max\{1, \frac{(1-\kappa)(1+\kappa-\lambda)}{(1-\lambda)}\}$, the expected profit is:

$$\mathbb{E} \Pi = P''_{low} - \max\{1, \frac{(1-\kappa)(1+\kappa-\lambda)}{(1-\lambda)}\}(1-\lambda)\phi J. \quad (\text{A.14})$$

Simple algebra shows that the CRA prefers truthful ratings as long as:

$$J \geq \frac{P''_{low} - \lambda P''_{high}}{\max\{1, \frac{(1-\kappa)(1+\kappa-\lambda)}{(1-\lambda)}\}(1-\lambda)\phi} \quad (\text{A.15})$$

where P''_{low} , and P''_{high} are given by (A.11), and (A.7) respectively.

A.6 Analysis of the Motivating Example

Regime with credit ratings - Case where the project type is $i = H$. In this case, by assumption, the manager receives a good rating, which perfectly reveals the project type. Suppose that there exists an equilibrium where the manager does not withdraw resources when $p = 0.5$ or $p = 1$. The creditors' participation constraint would be satisfied if:

$$[f_H(0.5) \times 0.5 + f_H(1) \times 1] \times (1+r) \geq 1$$

which holds as long as $(1+r) \geq 1.33$. Note that the creditors' beliefs are consistent, i.e., for $(1+r) = 1.33$, the manager finds it optimal not to withdraw productive resources when $p = 0.5$ and $p = 1$.¹³ Thus, this scenario *could* be an equilibrium.

Now suppose that there is an equilibrium where the manager does not withdraw productive resources from the project only if the probability of success turns out to be $p = 1$. The creditors' participation constraint would be satisfied if:

$$[f_H(1) \times 1] \times (1+r) \geq 1$$

which holds if $(1+r) \geq 1.66$. However, this cannot be an equilibrium because the creditors' beliefs are not consistent, i.e., for $(1+r) = 1.66$, the manager would find it suboptimal to withdraw resources not only when $p = 1$, but also when $p = 0.5$.¹⁴ Hence, the unique equilibrium is the one

¹³As not withdrawing resources implies expected utility, $0.5(5.9 - 1.33) = 2.28$, whereas withdrawing resources always implies expected utility equal to one.

¹⁴As not withdrawing resources implies expected utility, $0.5(5.9 - 1.66) = 2.12$, whereas withdrawing resources

where the security pays $(1 + r) = 1.33$ in case of success, and the manager chooses to withdraw productive resources only if $p = 0$.

Regime with credit ratings - Case where the project type is $i = L$. In this case, by assumption, the manager receives a bad rating, which perfectly reveals the project type. Suppose that there exists an equilibrium where the manager does not withdraw resources when $p = 0.5$ or $p = 1$. The creditors' participation constraint is satisfied if:

$$[f_L(0.5) \times 0.5 + f_L(1) \times 1] \times (1 + r) \geq 1$$

which holds as long as $(1 + r) \geq 4$. This cannot be an equilibrium because the creditors' beliefs are not consistent, i.e., for $(1 + r) = 4$, the manager has an incentive to withdraw resources when $p = 0.5$.¹⁵ By following a similar reasoning, we can show that there is no equilibrium where the manager withdraws resources when $p = 0$ or $p = 0.5$. Hence, if credit ratings are truthful, there is no equilibrium where an $i = L$ project is financed, although it is socially valuable.

Regime without (or with uninformative) credit ratings. Without ratings, the creditors can not differentiate between type $i = H$ and type $i = L$. It can be shown that the unique equilibrium is the one where the manager withdraws resources only when $p = 0$. For this allocation of resources, the creditors' participation constraint is satisfied if:

$$\left[\lambda \times (f_H(0.5) \times 0.5 + f_H(1) \times 1) + (1 - \lambda) \times (f_L(0.5) \times 0.5 + f_L(1) \times 1) \right] \times (1 + r) \geq 1$$

which is true as long as $(1 + r) \geq 2$. For $(1 + r) = 2$, the manager finds it optimal not to withdraw resources when $p = 0.5$ or $p = 1$; thus, the creditors' beliefs are consistent.¹⁶

Summing up, in the regime without credit ratings, the project is financed *independently* of its type, whereas in the regime with credit ratings only an $i = H$ project is financed.

always implies expected utility equal to one.

¹⁵As not withdrawing resources implies expected utility, $0.5(5.9 - 4) = 0.95$, whereas withdrawing resources always guarantees utility equal to one.

¹⁶As not withdrawing resources implies expected utility, $0.5(5.9 - 2) = 1.95$, whereas withdrawing resources always implies expected utility equal to one.

Appendix B: Benchmark Cases

B.1 Case where the Manager can commit

Lemma A.1 explores the case where the manager can commit to not withdrawing resources. As in the benchmark setting, we focus on the case the project of type $i = H$ is socially valuable, i.e., $\mathbb{E}[p|i = H]B \geq 1$, whereas we do not restrict the value of the project when its type is $i = L$.

Lemma A.1. *If the manager can commit to not withdrawing resources from the project, disclosing information about the project type can never worsen the allocations of resources. If type $i = L$ is socially wasteful, disclosing the project type (i) alleviates under-financing of socially valuable projects when $\lambda < \bar{\kappa}$, and (ii) mitigates over-financing of socially wasteful projects when $\lambda \geq \bar{\kappa}$.*

Proof. See Online Appendix B.3. □

B.2 Case with Symmetric Information

The benchmark case of Section 2 explores the setting where the manager learns the true probability of success of the project after raising capital and starting the implementation of the project. Hence, even in the regime where the manager and the creditors share the same beliefs about the project type, the manager *eventually* gains superior information before the withdrawing decision is taken. In this Section, we shed light on the impact of the manager gaining superior information on: i) the finding that socially valuable projects might fail to raise capital; ii) the finding that information disclosure could be associated with worse allocations of resources.

In what follows, we explore the case where, by assumption, the manager does not gain superior information. This setting highlights that the manager gaining superior information is *not* essential for the main findings to go through; the key mechanism is the inability of the manager to commit to not withdrawing resources.

B.2.1 Socially valuable projects and financing opportunities

The goal of this subsection is to show that socially valuable projects might fail to raise capital when the manager cannot commit to not withdrawing resources. This is captured in Lemma A.2.

Lemma A.2. *If the benefit of withdrawing resources, v , is positive, even if the project is known to be socially valuable (i.e., $\mathbb{E}[p|i]B > 1$), it cannot be financed when $\mathbb{E}[p|i]B < 1 + v$.*

Proof. See Online Appendix B.3. □

B.2.2 Equilibrium characterization

The goal of this subsection is to show that information disclosure might hurt the financing opportunities for socially valuable projects. To do so, we first characterize the equilibrium when the project type is known (Lemma A.3) and when the project type is hidden (Lemma A.4).

Lemma A.3. Equilibrium when the project type is disclosed.

When the project is known to be of type i , it is financed when $\mathbb{E}[p|i] \geq \frac{1+v}{B}$ and the interest rate is given by $(1+r) = B - \frac{v}{\mathbb{E}[p|i]}$. When $\mathbb{E}[p|i] < \frac{1+v}{B}$, in the unique equilibrium, the manager fails to finance the project.

Proof. Follows directly from the proof of Lemma A.2. □

Lemma A.4. Equilibrium when the project type is hidden.

For $v \leq \mathbb{E}[p|i = L](B - \frac{1}{\mathbb{E}[p|i=H]})$, when $\lambda \geq \hat{\kappa}$, the manager raises capital and the unique equilibrium interest rate is given by (A.21), for which she never finds it optimal to withdraw resources. In contrast, when $\lambda < \hat{\kappa}$ the manager fails to raise capital. For $v > \mathbb{E}[p|i = L](B - \frac{1}{\mathbb{E}[p|i=H]})$, when $\lambda \geq \hat{\kappa}$, the manager raises capital and the unique equilibrium interest rate is given by (A.24), for which the manager finds it optimal to withdraw resources only when the project type is $i = L$. In contrast, when $\lambda < \hat{\kappa}$ the manager fails to raise capital.¹⁷

Proof. See Online Appendix B.3. □

B.2.3 Impact of Information Disclosure

Proposition A.1 (A.2) summarizes the impact of information disclosure on financing opportunities when the benefit of withdrawing resources is relatively small (large).

¹⁷The critical thresholds $\hat{\kappa}$ and $\hat{\kappa}$ are defined in Online Appendix B.3. Their interpretation is similar to the interpretation of κ in Definition 1. Relations (A.21) and (A.24) in Online Appendix B.3, provide the minimum interest rates for which the creditors' participation constraint binds.

Proposition A.1 (Impact of perfectly informative ratings)

Suppose that the benefit of withdrawing is relatively small.¹⁸ If type $i = L$ is socially valuable, allowing for perfectly informative credit ratings:

- (i) **alleviates under-financing** of socially valuable projects when the financing of an $i = H$ project is feasible only if its type is disclosed, i.e., when $\lambda \in (0, \hat{\kappa})$.
- (ii) **leads to under-financing** of socially valuable projects when the financing of an $i = L$ project is feasible only if its type is hidden, i.e., when $\lambda \in [\hat{\kappa}, 1]$.¹⁹

In contrast, if type $i = L$ is socially wasteful, allowing for perfectly informative credit ratings improves the allocations of resources.

Proof. See Online Appendix B.3. □

Proposition A.2 (Impact of perfectly informative ratings)

Suppose that the benefit of withdrawing is relatively large.²⁰ If type $i = L$ is socially valuable, allowing for perfectly informative credit ratings:

- (i) **alleviates under-financing** of socially valuable projects when the financing of an $i = H$ project is feasible only if its type is disclosed $\lambda \in (0, \hat{\kappa})$.
- (ii) **leads to under-financing** of socially valuable projects when the financing of an $i = L$ project is feasible only if its type is hidden, i.e., when $\lambda \in [\hat{\kappa}, 1]$.

In contrast, if type $i = L$ is socially wasteful, allowing for perfectly informative credit ratings improves the allocations of resources.

Proof. See Online Appendix B.3. □

The intuition of Propositions A.1 and A.2 is identical to the intuition of Proposition 1. When the project type is hidden, if λ is sufficiently large, the manager succeeds in raising capital. Thus, disclosing the project type would prevent the financing of type $i = L$, even when it is socially

¹⁸When $v \leq \mathbb{E}[p|i=L](B - \frac{1}{\mathbb{E}[p|i=H]})$, the manager never finds it optimal to divert resources.

¹⁹Evidently, when the project of type $i = L$ is financed when its type is disclosed, i.e., $\frac{1+v}{B} \leq \mathbb{E}[p|i=L]$, disclosing the project type does not affect financing opportunities.

²⁰When $v > \mathbb{E}[p|i=L](B - \frac{1}{\mathbb{E}[p|i=H]})$ the manager finds it optimal to divert resources when the project type is $i = L$.

valuable. In contrast, if λ is sufficiently small, the manager fails to raise capital. Therefore, disclosing the project type mitigates the problem of under-financing of type $i = H$.

The main difference between Proposition A.1 and Proposition A.2 is that the former refers to the case where the benefit of withdrawing is relatively small, which implies that the lack of commitment problem is not very severe. If this is the case, conditional on financing, the manager finds it suboptimal to divert resources independently of the project type. In contrast, Proposition A.2 refers to the case where the benefit of withdrawing is relatively large, which implies that the lack of commitment problem is severe. If this is the case, conditional on financing, the manager finds it suboptimal to divert resources only when the project type is $i = H$.

B.3 Proofs of Online Appendix B

B.3.1 Proof of Lemma A.1

First, we explore the decision of the manager to raise capital. Since the manager is protected by limited liability and her outside option is zero, she finds it optimal to raise capital as long as $B \geq (1 + r)$. As we explain in Section 2, there is a pooling equilibrium in the contracting stage, i.e., the manager offers the same contract independently of the project type. Hence, the creditors are willing to finance the project as long as there exists $r \in (0, B - 1)$ such that

$$[\lambda \mathbb{E}[p|i = H] + (1 - \lambda) \mathbb{E}[p|i = L]](1 + r) \geq 1.$$

Since the creditors' expected payment is increasing in r , a necessary and sufficient condition for the manager to raise capital is the creditors' participation constraint to be satisfied for the maximum feasible interest rate, $r = B - 1$, i.e.,²¹

$$[\lambda \mathbb{E}[p|i = H] + (1 - \lambda) \mathbb{E}[p|i = L]]B \geq 1. \tag{A.16}$$

If (A.16) is satisfied, the manager raises capital and the equilibrium interest rate is given by:

$$(1 + r^*) = \frac{1}{[\lambda \mathbb{E}[p|i = H] + (1 - \lambda) \mathbb{E}[p|i = L]]}.$$

²¹In the extreme case where $r = B - 1$, the manager's expected profit would be equal to her outside option. Without loss, we assume that when indifferent, the manager finds it optimal to raise capital.

Evidently, if the project is socially valuable independently of its type, i.e., $\mathbb{E}[p|i = H]B \geq 1$ and $\mathbb{E}[p|i = L]B \geq 1$, then (A.16) is always satisfied. In that case, information disclosure does not affect the allocation of resources; the project is financed independently of whether its type is disclosed or remains hidden.

The interesting case is when the type $i = L$ is socially wasteful, i.e., $\mathbb{E}[p|i = L]B < 1$. In this case, the manager would fail to raise capital when the project type is disclosed. However, the manager would succeed in raising capital if the project type is hidden and the expected productivity is sufficiently large, i.e., (A.16) holds. The latter is the case when $\lambda \geq \bar{\kappa}$, where

$$\bar{\kappa} \equiv \frac{1 - \mathbb{E}[p|i = L]B}{(\mathbb{E}[p|i = H] - \mathbb{E}[p|i = L])B}.$$

Therefore, the mechanism behind Lemma A.1 is the following. When $\mathbb{E}[p|i = L]B < 1$, if $\lambda < \bar{\kappa}$, disclosing the project type prevents under-financing of socially valuable projects (type $i = H$). In contrast, when $\lambda \geq \bar{\kappa}$, disclosing the project type prevents over-financing, i.e., financing of socially wasteful projects (type $i = L$). Equivalently, concealing the project type leads to either under-financing of socially valuable projects (when $\lambda < \bar{\kappa}$) or over-financing of socially wasteful projects (when $\lambda \geq \bar{\kappa}$).

B.3.2 Proof of Lemma A.2

Consider the case where the project type is known to the creditors (symmetric information). For an equilibrium where financing takes place to exist, it must be that there is a feasible interest rate, such that: i) the manager finds it optimal not to withdraw resources, and; ii) the participation constraint of the creditors is satisfied.

First, we consider the decision of the manager to withdraw resources. The manager whose project is of type i finds it optimal not to withdraw resources when

$$\mathbb{E}[p|i](B - (1 + r)) \geq v \tag{A.17}$$

whereas she finds it optimal to withdraw resources otherwise. Relation (A.17) can be re-written as

$$(1 + r) \leq B - \frac{v}{\mathbb{E}[p|i]} \quad (\text{A.18})$$

which implies that the manager does not withdraw resources when the interest rate is relatively small. Thus, the feasible interest rate is $r \in (0, B - 1 - \frac{v}{\mathbb{E}[p|i]})$.²² Since the creditors' expected payment is increasing in r , a necessary and sufficient condition for the manager to raise capital is the creditors' participation constraint to be satisfied for the maximum feasible interest rate, $r = B - 1 - \frac{v}{\mathbb{E}[p|i]}$, i.e., $\mathbb{E}[p|i](B - \frac{v}{\mathbb{E}[p|i]}) \geq 1$, which is satisfied if $\mathbb{E}[p|i] \geq \frac{1+v}{B}$. Thus, if $\mathbb{E}[p|i] < \frac{1+v}{B}$ the manager fails to raise capital even when offering the highest feasible interest rate. In contrast, if $\mathbb{E}[p|i] \geq \frac{1+v}{B}$, the manager raises capital and the equilibrium interest rate is given by $(1 + r) = \frac{1}{\mathbb{E}[p|i]}$, for which the manager finds it optimal not to withdraw resources.

B.3.3 Proof of Lemma A.4

We need to consider two scenarios. First, when the benefit of withdrawing resources is relatively small, such that there is a feasible interest rate for which the manager finds it optimal not to withdraw resources independently of the project type, i.e., $v \leq \mathbb{E}[p|i = L](B - \frac{1}{\mathbb{E}[p|i=H]})$ (Scenario A). Second, when the benefit of withdrawing is relatively large, such that it is prohibitively costly to incentivize the manager of an $i = L$ project not to withdraw resources, i.e., $v > \mathbb{E}[p|i = L](B - \frac{1}{\mathbb{E}[p|i=H]})$ (Scenario B).

Scenario A. If the manager of an $i = L$ project finds it optimal not to withdraw resources, the same is true for the manager of an $i = H$ project (due to FOSD). Thus, the relevant constraint is

$$(1 + r) \leq B - \frac{v}{\mathbb{E}[p|i = L]}$$

which implies that the feasible interest rate is $r \in (0, B - 1 - \frac{v}{\mathbb{E}[p|i=L]})$. Since the creditors' expected payment is increasing in r , a necessary and sufficient condition for the manager to raise capital is the creditors' participation constraint to be satisfied for the maximum feasible interest rate, $r = B - 1 - \frac{v}{\mathbb{E}[p|i]}$, i.e.,

²²Note that for an interest rate greater than $B - 1 - \frac{v}{\mathbb{E}[p|i]}$ the manager would always find it optimal to divert resources. Thus, the creditors would never have an incentive to finance the project as the repayment probability would be zero.

$$[\lambda \mathbb{E}[p|i = H] + (1 - \lambda) \mathbb{E}[p|i = L]](B - \frac{v}{\mathbb{E}[p|i = L]}) \geq 1 \quad (\text{A.19})$$

which is satisfied when $\lambda \geq \hat{\kappa}$, where

$$\hat{\kappa} \equiv \frac{\mathbb{E}[p|i = L](1 - B \mathbb{E}[p|i = L] + v)}{(\mathbb{E}[p|i = H] - \mathbb{E}[p|i = L])(B \mathbb{E}[p|i = L] - v)}.^{23} \quad (\text{A.20})$$

Besides, when $\lambda \geq \hat{\kappa}$, the equilibrium interest is given by

$$(1 + r) = \frac{1}{\lambda \mathbb{E}[p|i = H] + (1 - \lambda) \mathbb{E}[p|i = L]}. \quad (\text{A.21})$$

Finally, when $\lambda < \hat{\kappa}$, the manager fails to raise capital.

Scenario B. We now consider the case where it is prohibitively costly to prevent the manager of an $i = L$ project from withdrawing resources. For the manager to find it optimal not to withdraw resources only when the project type is $i = H$, there must be a feasible interest rate r which satisfies (simultaneously) the following conditions.

$$(1 + r) > B - \frac{v}{\mathbb{E}[p|i = L]}$$

$$(1 + r) \leq B - \frac{v}{\mathbb{E}[p|i = H]}.$$

Consistently with the above, for this case to be an equilibrium, the creditors' participation constraint must be satisfied for the maximum interest rate, which incentivizes the manager of an $i = H$ project not to withdraw resources, i.e.,

$$\lambda \mathbb{E}[p|i = H](B - \frac{v}{\mathbb{E}[p|i = H]}) \geq 1 \quad (\text{A.22})$$

which holds as long as $\lambda \geq \hat{\kappa}$, where

$$\hat{\kappa} \equiv \frac{1}{\mathbb{E}[p|i = H]B - v}. \quad (\text{A.23})$$

Note that if (A.22) is satisfied, the interest rate is given by

²³Note that $\hat{\kappa} \leq 1$ as long as $v \leq \mathbb{E}[p|i = L](B - \frac{1}{\mathbb{E}[p|i = H]})$. Also, there is an non-empty set of feasible parameters which satisfies (A.19), given that $\frac{1+v}{B} > \mathbb{E}[p|i = L]$, as merging the two conditions implies $B \mathbb{E}[p|i = L] - 1 < v \leq B \mathbb{E}[p|i = L] - \frac{\mathbb{E}[p|i = L]}{\mathbb{E}[p|i = H]}$.

$$(1 + r) = \frac{1}{\lambda \mathbb{E}[p|i = H]}. \quad (\text{A.24})$$

Finally, if $\lambda < \hat{\kappa}$, the manager fails to raise capital.

It is worth highlighting that condition (A.22) is a necessary, yet not a sufficient condition for Scenario B to be an equilibrium. Note that if there is a feasible interest rate for which the manager does not withdraw resources irrespective of the project type, then, Scenario B never arises in equilibrium; the manager of type $i = H$ would always have an incentive to switch to Scenario A by offering the interest rate given by (A.21), which is always smaller than the interest rate given by (A.24). Of course, the latter is possible as long as an equilibrium where financing takes place in Scenario A exists, which is true when $v \leq \mathbb{E}[p|i = L](B - \frac{1}{\mathbb{E}[p|i=H]})^{24}$ and $\lambda \leq \hat{\kappa}$.

B.3.4 Proof of Proposition A.1

Consider the case where, when the type is disclosed to be $i = L$, the manager fails to finance it although it is socially valuable, i.e., the following holds

$$\frac{1 + v}{B} > \mathbb{E}[p|i = L] \geq \frac{1}{B}.$$

The relevant question is whether not disclosing the project type could lead to the financing of type $i = L$. As we showed in Scenario A in the proof of Lemma A.4, for $\lambda \geq \hat{\kappa}$, if the project type is hidden, the manager raises capital and does not withdraw resources irrespective of the project type. Hence, disclosing the project type leads to under-financing of projects of type $i = L$ that would be financed otherwise.

Besides, when $\lambda < \hat{\kappa}$, the manager fails to raise capital when the project type is hidden, thus, disclosing the project type mitigates the problem of under-financing of projects of type $i = H$ that would not be financed otherwise.

B.3.5 Proof of Proposition A.2

Consider the case where, when the project type is disclosed, it is financed only when $i = H$.

$$\frac{1 + v}{B} \geq \mathbb{E}[p|i = L] > \frac{1}{B}$$

²⁴Recall that this condition satisfies that $\hat{\kappa} \leq 1$.

$$\mathbb{E}[p|i = H] \geq \frac{1+v}{B}$$

Now consider the case where the project type is hidden. As we showed in Scenario B of Lemma A.4, the manager succeeds in raising capital if $\lambda \geq \hat{\kappa}$. Hence, when $\lambda < \hat{\kappa}$, *not* disclosing the project type prevents the financing of $i = H$ projects which are socially valuable and would be financed otherwise. In contrast, when $\lambda \geq \hat{\kappa}$, projects of type $i = H$ are financed independently of whether their type is disclosed. It is worth emphasizing that as opposed to the case (ii) of Proposition A.1, preventing the financing of an $i = L$ project is not welfare decreasing. The latter is true because the manager of an $i = L$ project will choose to withdraw resources, which is value-destroying if $v < 1$.