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Metastable Helium Reveals an Extended Atmosphere for the Gas Giant HAT-P-18b

Kimberly Paragas,1 Shreyas Vissapragada,2 Heather A. Knutson,2 Antonija Oklopčić,3 Yayaati Chachan,2 Michael Greklek-McKeon,2 Fei Dai,2 Samaporn Tinyanont,4 and Gautam Vasisht5

1Astronomy Department, Wesleyan University, 96 Foss Hill, Middletown, CT 06459, USA
2Division of Geological and Planetary Sciences, California Institute of Technology, 1200 East California Blvd, Pasadena, CA 91125, USA
3Anton Pannekoek Institute of Astronomy, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, Netherlands
4Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064, USA
5Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr, Pasadena, CA 91109, USA

ABSTRACT

The metastable helium line at 1083 nm can be used to probe the extended upper atmospheres of close-in exoplanets and thus provide insight into their atmospheric mass loss, which is likely to be significant in sculpting their population. We used an ultranarrowband filter centered on this line to observe two transits of the low-density gas giant HAT-P-18b, using the 200" Hale Telescope at Palomar Observatory, and report the detection of its extended upper atmosphere. We constrain the excess absorption to be $0.46 \pm 0.12\%$ in our 0.635 nm bandpass, exceeding the transit depth from the Transiting Exoplanet Survey Satellite (TESS) by 3.9σ. If we fit this signal with a 1D Parker wind model, we find that it corresponds to an atmospheric mass loss rate between $8.3^{+2.8}_{-1.9} \times 10^{-5} M_J/\text{Gyr}$ and $2.63^{+0.46}_{-0.46} \times 10^{-3} M_J/\text{Gyr}$ for thermosphere temperatures ranging from 4000 K to 13000 K, respectively. With a $J$ magnitude of 10.8, this is the faintest system for which such a measurement has been made to date, demonstrating the effectiveness of this approach for surveying mass loss on a diverse sample of close-in gas giant planets.

Keywords: techniques: photometric – planets and satellites: atmospheres – planets and satellites: individual (HAT-P-18b)

1. INTRODUCTION

Close-in exoplanets are exposed to high-energy radiation from their host stars, which can lead to atmospheric mass loss. This atmospheric escape appears to shape the observed short-period exoplanet population (e.g. Lopez & Fortney 2013; Owen & Wu 2013; Fulton et al. 2017), but there are relatively few published measurements of present-day mass loss rates for close-in planets. Prior to 2018, most studies of atmospheric escape used the hydrogen Lyman-α line at UV wavelengths, the Hα line at optical wavelengths, and metal lines at UV and optical wavelengths (e.g., Vidal-Madjar et al. 2003, 2004; Jensen et al. 2012; Yan & Henning 2018; Cauley et al. 2019). In recent years, new theoretical and observational work on the helium (He) 1083 nm line have shown that it can also be used for atmospheric mass loss studies. For planets with a sufficient population of metastable helium atoms in their (potentially escaping) upper atmospheres, the 1083 nm line is optically thick at low pressures (high altitudes), increasing their measured transit depths in this line by a factor of a few (Oklopčić & Hirata 2018). K-type stars are favorable targets for metastable helium observations, as they emit relatively low amounts of mid-UV flux (which depopulates the metastable state), while emitting relatively high levels of EUV flux (which populates the metastable state), and thus K-type stars are likely to be the most effective targets for observing metastable helium absorption.

Excess He I absorption was first detected in the atmosphere of the sub-Saturn WASP-107b using the Hubble Space Telescope (HST; Spake et al. 2018). Since then, this line has been used to detect extended atmospheres in six other planets (5 gas giants and 1 sub-Neptune, with masses ranging from 0.044$M_J$ to 1.116$M_J$) using both space and ground based facilities (Allart et al. 2018; Mansfield et al. 2018; Nortmann et al. 2018; Salz et al. 2018; Alonso-Floriano et al. 2019; Ninan et al. 2020; Palle et al. 2020). Understanding how the metastable helium signal scales with stellar EUV flux, planetary
gravity, and stellocentric distance on a sample of large, well-characterized larger planets is a prerequisite for He I observations of smaller planets, where measurements and interpretation are inherently more difficult (Kasper et al. 2020). Additionally, detections of outflowing gas giant atmospheres in the He I line provide useful constraints on crucial radiative and collisional processes in theoretical atmospheric mass loss models (e.g. Salz et al. 2016; Oklopcic & Hirata 2018).

Using the He 1083 nm line, we study the extended atmosphere of HAT-P-18b (Hartman et al. 2011), which is a Jupiter-sized (0.947 ± 0.044\(R_J\)), Saturn-mass (0.196 ± 0.008\(M_J\)), \(T_{eq} = 841 ± 15\) K planet (Esposito et al. 2014) orbiting a K2-type star with \(J = 10.8\) (Cutri et al. 2003). The low density of the planet combined with the spectral type of the host star makes it an excellent target for metastable helium studies. Previous Rossiter-McLaughlin studies of this system have shown it to be one of the only planets around a cool main-sequence star with a retrograde orbit (Esposito et al. 2014). Additionally, optical transmission spectroscopy has revealed a Rayleigh scattering slope for this planet, potentially due to a high-altitude haze (Kirk et al. 2017). A secondary eclipse of HAT-P-18b has also been detected by the Spitzer Space Telescope, and the resulting brightness temperature for the planet suggests efficient day-night circulation and/or a nonzero albedo (Wallack et al. 2019).

In this work, we characterize the atmospheric mass loss of HAT-P-18b for the first time. We observed two transits of HAT-P-18b with the Hale 200" Telescope at Palomar Observatory using an ultranarrowband filter centered on the helium 1083 nm line (Vissapragada et al. 2020a) with the Wide-field InfraRed Camera (WIRC; Wilson et al. 2003). Additionally, we use TESS data from Sectors 25 and 26 to improve the ephemeris for this planet and determine its broadband optical transit depth as a comparison for our helium measurement. In Section 2 we describe the WIRC and TESS observations, and in Section 3 we jointly model the light curves from both instruments. We describe our results in Section 4, and offer some concluding thoughts in Section 5.

2. OBSERVATIONS
2.1. WIRC Observations

We observed transits of HAT-P-18b on UT June 6 2020 (hereafter night 1) and July 8 2020 (night 2). We used an ultranarrowband filter centered on the helium 1083.3 nm line with a FWHM of 0.635 nm (Vissapragada et al. 2020a). For these observations we typically use a custom beam-shaping diffuser, which produces a stable 3" diameter top hat point spread function (Ste-
Figure 1. (a) Results for WIRC night 1 and night 2 shown in (a) and (b), respectively. The top row shows the median-normalized raw light curves of the target (in black) and its comparison stars (in grey) as well as the water absorption proxy described in Section 2.1 (dotted blue line) throughout each respective night. The middle row shows the helium light curve with unbinned data in grey and binned data to a 15 minute cadence in black, with the best-fit joint helium model in red and the TESS model in blue, and the residuals of each fit. The bottom row shows the Allan deviation plot for each dataset.

1083.4 nm overlaps with the telluric water feature (Allart et al. 2018; Salz et al. 2018). This means that OH emission originating higher up in the atmosphere (>80 km; Bernath & Colin 2009) can be absorbed by H$_2$O while passing through the lower atmosphere. We can therefore track the telluric water variation over the
night by dividing the time-varying flux in the water-
contaminated OH emission line (as measured by our
scaling factors) by that of the uncontaminated $R_1(3/2)$
and $R_2(1/2)$ OH lines at 1075 nm and 1078 nm. If
the water variation is significant enough to impact our
photometry, we can utilize this absorption proxy as a
decorrelation parameter in our transit fits.

We performed aperture photometry on the target star
and six comparison stars (the same ones on each night)
using the package photutils (Bradley et al. 2016). We
tested different aperture sizes in one pixel steps from
3 to 13 pixels in radius. We removed 3σ outliers from
the data using a moving median filter. This process is
detailed in Vissapragada et al. (2020a). Our optimal
aperture sizes for our night 1 and night 2 observations
were 7 pixels (1'75) and 11 pixels (2'75) in radius, re-
spectively, with the difference arising from our use of a
slight defocus on the first night and a diffuser on the
second night. The raw light curves of the target and the
comparison stars are shown in Figure 1 for both nights.

2.2. TESS Observations

We used the 2-minute cadence TESS observations of
HAT-P-18b obtained during Sectors 25 and 26. TESS
observed the target for 51.5 days starting on May 14
2020 and June 9 2020 for Sectors 25 and 26, respectively,
covering 8 transits in total. We downloaded the Pre-
search Data Conditioning Simple Aperture Photometry
(PDCSAP) light curve from the Mikulski Archive for
Space Telescopes (MAST) using the lightkurve pack-
age (Lightkurve Collaboration et al. 2018). With the
transits masked, we removed low-frequency variability
from the data using the Savitzky-Golay filter from scipy
(Jones et al. 2001) and rejected 5σ outliers using a mov-
ing median filter. However, we noticed that even after
the filter was applied, there were still strong uncorrected
systematics that biased the transit depths of the first two
transits, so we omitted them from our combined fit. Al-
though these transits may be recoverable with different
detrending methods, our constraint on the TESS transit
depth from the remaining six transits were sufficiently
precise for comparison to the WIRC light curves (i.e.,
the uncertainty in the comparison is dominated by the
uncertainty on the WIRC transit depth).

3. LIGHT CURVE MODELING

We simultaneously fit both nights of WIRC data along
with the corrected TESS photometry using exoplanet
(Foreman-Mackey et al. 2020). For each WIRC light
curve, we fit an instrumental noise model consisting of a
linear baseline along with a linear combination of com-
parison star light curves, with the weights of the com-
parison stars left as free parameters in the fit. This
is an update from our previous modeling methodology
(Vissapragada et al. 2020a), where we used an ordinary
least-squares method to quickly determine comparison
star weights at each likelihood evaluation, and it is en-
abled by the rapid No U-Turn Sampler (NUTS) sampler
that exoplanet makes available for high-dimensional
light-curve fitting. We also tried including two addi-
tional parameters in our instrumental noise model for
each night: the water absorption proxy (as described in
Section 2.1), and the distance from the median centroid. We find that two comparison stars in each WIRC night have posterior probability distributions for their weights that overlap with zero, and we therefore remove them from the fit, lowering the Bayesian Information Criterion (BIC) value by 35 and 28 for night 1 and night 2, respectively. Although these stars are not the same for each dataset, the two nights had different observing strategies and weather conditions. We find that we obtain optimal fits when we only include the telluric water proxy in our fits to the night 1 data, lowering the BIC value by 17; this is not surprising, as this night had relatively poor and variable weather conditions. We opted to keep the distance from the median centroid as a decorrelation parameter for both nights, as their removal from the fits resulted in a ΔBIC < 10. Our final systematics model contained 15 parameters: two parameters for each of the linear baselines, four comparison stars for each dataset, the distance from median centroid for each dataset, and the absorption proxy for the first night.

We fit a transit model simultaneously with the systematics. We have three fit parameters that are common to all datasets: the predicted mid-transit time $T_0$, the period $P$, and the impact parameter $b$. Initially, we allowed each night of WIRC data to have its own transit depth in the joint fit. The two transit depths $(2.11_{-0.23}^{+0.25}\%$ for night 1 and $2.35\pm0.14\%$ for night 2) were within $1\sigma$ of each other, indicating that the magnitude of helium absorption appears consistent between these two epochs. We therefore fit a single transit depth for both helium light curves. We fit for the limb darkening coefficients $[u_1, u_2]_{(TESS)}$ and $[u_1, u_2]_{(WIRC)}$ and transit depths $(R_p/R_\star)_{(HE)}^2$ and $(R_p/R_\star)_{(TESS)}^2$ in each bandpass, both sampled uniformly (see Table 1). For each WIRC light curve, we fit for a jitter parameter describing the excess noise in addition to the photon noise log($σ_{\text{extra}}$). For TESS, we noticed that the error bars that came with the PDCSAP fluxes were not an accurate representation of the photon noise and in fact overestimated the observed scatter in the data. We therefore include a scaling factor $k$ for the TESS error bars.

We use the NUTS in PyMC3 (Salvatier et al. 2016) to sample the posterior distributions for our model parameters. We ran four chains, tuning each for 1500 steps (the “burn-in” period) and then taking 1000 draws in each chain, achieving good convergence with a Gelman-Rubin (Gelman & Rubin 1992) statistic of $< 1.006$ for all parameters. The priors and posteriors for the physical parameters in our model are given in Table 1 for the joint fit and the detrended light curve, residuals, and Allan deviation plot for each night of WIRC data are displayed in Figure 1. The final combined helium and TESS light curves are displayed in Figure 2, and the posterior distributions for the model parameters are visualized in Figure 4.

![Figure 3](image321x495to562x680)

**Figure 3.** Atmospheric mass loss model for HAT-P-18b. Each point is a different mass loss model corresponding to specific $T_0$ and $M$ values, and the shading indicates the compatibility between the model and our observed excess absorption (with the lighter regions indicating the most concordant models).

4. RESULTS & DISCUSSION

We measure a transit depth of $2.29_{-0.12}^{+0.13}\%$ in the helium line. The corner plot for our fit parameters is shown in Figure 4. Our measurement can be compared to the TESS transit depth measurement of $1.830_{-0.052}^{+0.049}\%$. Our measured transit depth in the helium bandpass exceeds that in the TESS bandpass by $0.46\pm0.12\%$ (3.9σ). The TESS bandpass is between 600 nm to 1000 nm, making it reasonable to use as a comparison for our measurement. Previous studies on our target have shown variation within this range to be limited to variations on the order of the scale height (Kirk et al. 2017). The difference between the two transit depths exceeds, by an order of magnitude, the expected change in transit depth for a one (lower-atmospheric) scale-height change in planet radius (0.03%) for this target. Thus, the observed excess absorption cannot be explained by broad absorption features – for instance, by water – in the lower atmosphere. The helium line is near an opacity minimum of water anyways so this explanation is disfavored a priori. We conclude that the observed absorption indeed arises from metastable helium in HAT-P-18b’s extended atmosphere.

We use the model described in Oklopčić & Hirata (2018) to convert our measured excess absorption into a joint constraint on HAT-P-18b’s mass loss rate $\dot{M}$.
and upper atmospheric temperature $T_0$. This model calculates the velocity and density profiles of a 90%/10% H/He 1D Parker wind as a function of $\dot{M}$ and $T_0$, and then calculates the level populations for helium given a UV stellar spectrum. We use the MUSCLES UV spectrum of $\epsilon$ Eridani (France et al. 2016; Loyd et al. 2016; Youngblood et al. 2016), which is another K2 type star, as a stand-in for the unknown UV spectrum of HAT-P-18. Accounting for the stellar radius and semi-major axis of HAT-P-18b, the EUV irradiance of the planet was 8 W/m$^2$ integrated between 5.5 Å and 911 Å. The results are shown in Figure 3. HAT-P-18b’s mass loss rate is likely between $8.3^{+2.8}_{-1.9}$ $\times$ 10$^{-5}$ and $2.63^{+0.46}_{-0.64}$ $\times$ 10$^{-3}$ $M_J$/Gyr for thermosphere temperatures between 4000 K and 13000 K, respectively. Using the EUV irradiance above along with an efficiency parameter $\epsilon = 0.1$, we can also calculate an energy-limited mass loss rate for HAT-P-18b (e.g. Murray-Clay et al. 2009):

$$\dot{M} = \frac{\epsilon \pi R_p^3 F_{XUV}}{GM_p} \approx 4 \times 10^{10} \text{ g/s.}$$

(1)

This estimate agrees well with the inferred mass loss rates in Figure 3 (with uncertainties of a factor of few accommodate by similar uncertainties on the efficiency parameter), suggesting that our observationally-derived constraints are energetically feasible.

Because $\epsilon$ Eridani is a relatively young, active star, with $\log(R_{HK}) = -4.51$ compared to $\log(R_{HK}) = -4.80$ for HAT-P-18, we repeated the modeling using the MUSCLES spectrum of HD 40307, a fairly inactive ($\log(R_{HK}) = -4.99$) K2.5V star. By trying proxy stars with activity levels on either side of HAT-P-18b’s activity, we can get a sense for the uncertainty on the result based on our choice of EUV proxy. With HD 40307 as a proxy, we found a best-fit $\log(\dot{M}) = 9.66^{+0.11}_{-0.12}$ at 4000 K, and $11.20^{+0.037}_{-0.036}$ at 13000 K, nearly identical to our findings for $\epsilon$ Eri. This is because the mid-UV to EUV flux ratios between the two stars are quite similar (Oklopič 2019). Although $\epsilon$ Eri is a much stronger X-ray emitter, the cross section to X-ray photoionization of helium is very small compared to the cross section in the EUV near the 504 Å threshold, so the contribution to the flux-averaged photoionization cross section is negligible (Oklopič & Hirata 2018).

We note that there is a strong degeneracy between the mass loss rate and thermosphere temperature due to the complex dependence of the outflow velocity and density on the temperature and the mass loss rate. This degeneracy could be partially resolved with a precise line shape measurement, but we do not resolve the line shape in these observations. Due to the faintness of HAT-P-18, spectrographs on all but the largest telescopes may have difficulty resolving the line shape precisely enough to break the degeneracy. Additionally, our helium light curve is symmetric across our best-fit mid-transit time. However we cannot exclude the possibility of an extended egress, as our combined light curve lacks the precision required to significantly detect a trailing helium tail for such a faint target.

### 5. CONCLUSIONS

**Table 1.** Priors and posteriors for joint fit to Palomar/WIRC and TESS data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(R_o/R_\ast)^2$ (He)</td>
<td>$U(1,25)$</td>
<td>$2.29 \pm 0.12$</td>
<td>%</td>
</tr>
<tr>
<td>$(R_o/R_\ast)^2$ (TESS)</td>
<td>$U(1,25)$</td>
<td>$1.83 \pm 0.045$</td>
<td>%</td>
</tr>
<tr>
<td>$P$</td>
<td>$N(5.5080291,0.0000042)$</td>
<td>$5.508029 \pm 0.0000042$</td>
<td>days</td>
</tr>
<tr>
<td>$T_0$</td>
<td>$U(2038.5, 2039.0)$</td>
<td>$2038.82530 \pm 0.00023$</td>
<td>BTJD$_{TDB}$</td>
</tr>
<tr>
<td>$b$</td>
<td>$N(0.352, 0.057)$</td>
<td>$0.338^{+0.047}_{-0.051}$</td>
<td>–</td>
</tr>
<tr>
<td>$u_1$ (He)</td>
<td>Kipping (2013)</td>
<td>$0.58^{+0.29}_{-0.30}$</td>
<td>–</td>
</tr>
<tr>
<td>$u_2$ (He)</td>
<td>Kipping (2013)</td>
<td>$0.14 \pm 0.39$</td>
<td>–</td>
</tr>
<tr>
<td>$u_1$ (TESS)</td>
<td>Kipping (2013)</td>
<td>$0.45 \pm 0.16$</td>
<td>–</td>
</tr>
<tr>
<td>$u_2$ (TESS)</td>
<td>Kipping (2013)</td>
<td>$0.20^{+0.30}_{-0.31}$</td>
<td>–</td>
</tr>
<tr>
<td>$\log(\sigma_{extra})$ (night 1)</td>
<td>$U(-4,-2)$</td>
<td>$-2.078^{+0.036}_{-0.038}$</td>
<td>–</td>
</tr>
<tr>
<td>$\log(\sigma_{extra})$ (night 2)</td>
<td>$U(-4,-2)$</td>
<td>$-2.40^{+0.08}_{-0.12}$</td>
<td>–</td>
</tr>
<tr>
<td>$k$ (TESS)</td>
<td>$U(0.5, 1.5)$</td>
<td>$0.856^{+0.0037}_{-0.0036}$</td>
<td>–</td>
</tr>
<tr>
<td>absorption proxy (night 1)</td>
<td>$N(0.0, 0.1)$</td>
<td>$-0.078^{+0.026}_{-0.028}$</td>
<td>–</td>
</tr>
</tbody>
</table>

Note—BTJD$_{TDB} = BJ$D - 2457000. Note that we omitted the stellar parameters and all of the detrending weights except for the absorption proxy for night 1.
In this work, we use an ultra-narrowband helium filter centered on the 1083 nm line to observe two transits of HAT-P-18b. We detect $0.46 \pm 0.12\%$ excess helium absorption in the planet’s upper atmosphere. This detection corresponds to an atmospheric mass loss rate between $8.3^{+2.8}_{-1.9} \times 10^{-5}$ and $2.63^{+0.46}_{-0.25} \times 10^{-3} M_J$/Gyr, which means HAT-P-18b is losing less than 2% of its mass per Gyr. This is typical for close-in gas giants, with other helium outflow detections having mass loss rates less than 5% per Gyr (Allart et al. 2018; Mansfield et al. 2018; Spake et al. 2018; Alonso-Floriano et al. 2019).

Of the handful of planets with detected helium outflows, WASP-107b is the most comparable to HAT-P-18b with a similar radius of 0.94 $R_J$, mass of 0.12 $M_J$, separation of 0.55 au, and equilibrium tempera-

Figure 4. Corner plot displaying the posterior probability distributions for the joint model for HAT-P-18b. Note transit depth $(R_p/R_\ast)^2$ values are in %, period $P$ is in days, and predicted mid-transit time $T_0$ is in BTJD$_{TDB}$. We omit all of the decorrelation parameters except for the absorption proxy.
ture of 770 K (Anderson et al. 2017). If we assume that HAT-P-18b has a He line shape similar to that of WASP-107b, we can invert our excess helium transit depth to obtain an estimate of the underlying predicted line depth of $4.5 \pm 1.3\%$. This is noticeably smaller than the $7.26 \pm 0.24\%$ depth measured by CARMENES and Keck/NIRSPEC for WASP-107b (Allart et al. 2019; Kirk et al. 2020). The difference may be due to the smaller gravitational potential of WASP-107b (the mass of WASP-107b has recently been suggested to be even lower by Piaulet et al. 2020), or differences in the EUV spectra of the two stars (WASP-107 is a K6 star while HAT-P-18 is a K2). Detailed comparative modeling of these two planets may make clear the primary control on the metastable helium signal.

This is the faintest system ($J = 10.8$) with detected helium absorption thus far, establishing the effectiveness of our technique for observing such targets with a mid-sized telescope. For reference, the next faintest system with detected excess helium absorption is WASP-107b with a $J$ magnitude of 9.4. Of the 11 planets identified in Kirk et al. (2020) as promising targets for observations of helium outflows, many are challenging targets with $J > 10.5$. Our photometric technique allows us to begin surveying planets around such faint stars, expanding the sample of planets with measured metastable helium absorption. Further population-level studies of extended atmospheres in the He 1083 line will greatly improve our ability to calibrate the mass loss models used to elucidate the long-term evolution of the close-in exoplanet population.

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