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Logical Comparison of Cases

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Abstract. Comparison between cases is a core issue in case-based reasoning. In this paper, we discuss a logical comparison approach in terms of the case model formalism. By logically generalizing the formulas involved in case comparison, our approach identifies analogies, distinctions and relevances. An analogy is a property shared between cases. A distinction is a property of one case ruled out by the other case, and a relevance is a property of one case, and not the other, that is not ruled out by the other case. The comparison approach is applied to HYPO-style comparison (where distinctions and relevances are not separately characterized) and to the temporal dynamics of case-based reasoning using a model of real world cases.

Keywords: Case-based reasoning · Cases · Case comparison

1 Introduction

Case-based reasoning, one of the main legal reasoning types, has been discussed in the Artificial Intelligence and Law community for years. It allows for a form of analogical reasoning [4,5], and a core issue is how to make decisions for a current case by comparing cases, namely the doctrine of *stare decisis*.

Case-based reasoning has been formalized using many different approaches. For instance, abductive logic programming [15], formal dialogue games [11], context-related frameworks [6,8], dialectical arguments [14], ontologies in OWL [18], the ASPIC+ framework [12], reason models [9], abstract argumentation [7], abstract dialectical frameworks [1] and case-based argumentation frameworks [10]. These works often discuss case comparison in terms of factors, following ideas developed in HYPO [3,13].

In [19–21], a formal approach to the modeling of case-based reasoning has been discussed using a formal logical language. It can be used for evaluating the

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validity of arguments in legal reasoning [19] and the formal comparison of legal precedents [20,21]. The approach is based on the case model formalism [16,17].

The present paper is an extended version of [20,21], where we discuss the comparison between precedents that are represented as conjunctions of factors and outcomes. Here we define case comparison in the general setting of the case model formalism developed by Verheij [17]. This generalization is needed for the here newly presented application to the dynamics of case-based reasoning following the research developed by Berman and Hafner [6,8] as modeled in terms of case models [16].

In Sect. 2, we show the technical part of comparing cases using our formalism. Section 3 applies our comparison approach in a discussion of case comparison in HYPO-style case-based reasoning. Section 4 applies our approach to the development of precedential values in a series of legal cases. With these applications, we show that our approach can generalize case-based reasoning by comparing cases with general formulas and refine case-based reasoning by introducing the new notion of relevances. In this way, we show that comparing cases with respect to general properties, represented by general propositional formulas, offers a novel angle on case-based reasoning.

2 Theory: Case Comparisons

In this section, we present the case model formalism [16,17] and apply it to case comparison in case-based reasoning (also shown in [20,21]). The notions about case comparison are based on the analogies and distinctions defined in [17].

The formalism introduced in this paper uses a propositional logic language L generated from a finite set of propositional constants. We fix language L. We write $\neg$ for negation, $\wedge$ for conjunction, $\vee$ for disjunction, $\leftrightarrow$ for equivalence, $\top$ for a tautology, and $\bot$ for a contradiction. The associated classical, deductive, monotonic consequence relation is denoted $\models$.

Cases can be compared through the preference relation between cases in case models. A *case model* is a set of logically consistent, incompatible cases forming a total preorder (i.e., a transitive, total binary relation) representing a preference relation among the cases.

Definition 1. (Case models [16,17]). *A* case model *is a pair* $\mathcal{C} = (C, \geq)$ *with finite* $C \subseteq L$*, such that, for all* π, π' *and* $\pi'' \in C$*:*

1. $\not\models \neg\pi$;
2. *If* $\not\models \pi \leftrightarrow \pi'$, *then* $\models \neg(\pi \wedge \pi')$;
3. *If* $\models \pi \leftrightarrow \pi'$, *then* $\pi = \pi'$;
4. $\pi \geq \pi'$ *or* $\pi' \geq \pi$;
5. *If* $\pi \geq \pi'$ *and* $\pi' \geq \pi''$, *then* $\pi \geq \pi''$.

As customary, the asymmetric part of $\geq$ is denoted $>$. The symmetric part of $\geq$ is denoted $\sim$. Intuitively, $\geq$ means 'at least as preferred as'.

Example 1. Figure 1 shows a case model with case $\pi_0 = P \wedge Q \wedge R$ and $\pi_1 = P \wedge \neg Q$. π_0 is more preferred than π_1 as suggested by the size of boxes.

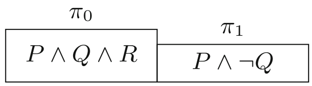

Fig. 1. A case model

We define now the notions of analogy, distinction and relevance. *Analogies* between two cases are the formulas that follow logically from both two cases. *Distinctions* are the formulas that only follow logically from one of the cases while their negation is logically implied by the other case. *Relevances* are, intuitively, formulas that are relevant to the analogies and distinctions between two cases. Relevances only follow from one of the cases but, unlike in distinctions, neither themselves nor their negation are logically implied by the other case. Intuitively, an analogy describes a shared property between two cases. Distinctions and relevances both describe unshared properties between cases, where distinctions are contradicting properties in the cases, and relevances are the properties that are not shared and not contradicted. As such distinctions and relevances clarify the two ways in which two cases may differ. Even though we present these notions in terms of cases, they can be defined in general for any given pair of propositional formulas, which is what we do now.

Definition 2 (Analogies, distinctions, relevances). *For any $\pi, \pi' \in L$, we define:*

1. *a sentence $\alpha \in L$ is an* analogy between π and π' *if and only if $\pi \models \alpha$ and $\pi' \models \alpha$. A* most specific analogy between π and π' *is an analogy that logically implies all analogies between π and π'.*
2. *a sentence $\delta \in L$ is a* distinction in π with respect to π' *(π-π'* distinction) *if and only if $\pi \models \delta$ and $\pi' \models \neg\delta$. A* most specific π-π' distinction *is a distinction that logically implies all π-π' distinctions.*
3. *a sentence $\rho \in L$ is a* relevance in π with respect to π' *(π-π'* relevance) *if and only if $\pi \models \rho$, $\pi' \not\models \rho$ and $\pi' \not\models \neg\rho$. ρ is a* proper π-π' relevance *if and only if ρ is a π-π' relevance that logically implies the most specific analogy between π and π'. A most specific π-π' relevance is a relevance that logically implies all π-π' relevances.*

Both π-π' distinctions and π'-π distinctions are called *distinctions between π and π'*. Both π-π' relevances and π'-π relevances are called *relevances between π and π'*. When a most specific analogy/distinction/relevance exists we consider it unique modulo logical equivalence, and we thus refer to it as *the* most specific analogy/distinction/relevance. Notice that when introducing relevances, we also define a special kind of this notion, namely proper relevances. These formally describe those relevances that logically imply the most specific analogy and are implied by the most specific distinction (if it exists).

Example 2. Comparing π_0 and π_1 in Fig. 1, we have:

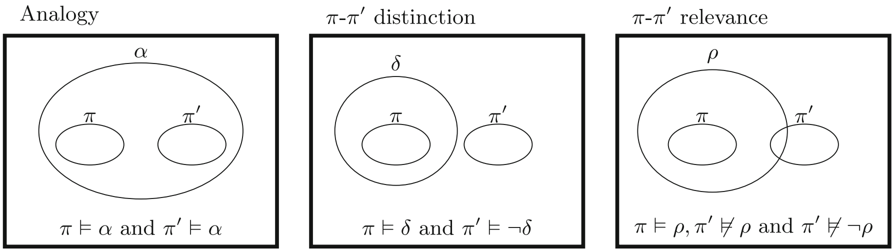

Fig. 2. Case comparison illustrated in terms of sets of worlds

- Analogies between π_0 and π_1: *e.g.*, P, $P \vee R$;
- The most specific analogy between π_0 and π_1: $(P \wedge Q \wedge R) \vee (P \wedge \neg Q)$;
- π_0-π_1 distinctions: *e.g.*, Q, $P \wedge Q$;
- The most specific π_0-π_1 distinction: $P \wedge Q \wedge R$;
- π_0-π_1 relevances: *e.g.*, R, $P \wedge R$;
- Proper π_0-π_1 relevances: *e.g.*, $P \wedge R$.

Now we further discuss the notions in Definition 2. Figure 2 illustrates analogies, distinctions and relevances using Venn diagrams representing the sets of worlds (or valuations) in which sentences are true (the so-called truth sets). As shown in Fig. 2, for any analogy α between cases π and π', the sets of π and π' worlds are subsets of the set of α worlds; for any π-π' distinction δ, the π worlds are a subset of the δ worlds, while the π' worlds and the δ worlds are disjoint; for any π-π' relevance ρ, the π worlds are a subset of the ρ worlds, while the π' worlds and the ρ worlds are not subsets of each other and the intersection of the π' worlds and the ρ worlds are always not empty. Notice that for any proper π-π' relevance ρ, not only the π worlds are a subset of the ρ worlds, but also the ρ worlds are a subset of the union of the π worlds and the π' worlds.

The following proposition shows the properties of analogies, distinctions and relevances between cases.

Proposition 1. *For any $\pi, \pi' \in L$:*

1. *The most specific analogy between π and π' always exists and is logically equivalent to $\pi \vee \pi'$.*
2. *There exists a π-π' distinction if and only if $\pi \wedge \pi' \vDash \bot$. If a π-π' distinction exists, then the most specific π-π' distinction exists and is logically equivalent to π.*
3. *A most specific π-π' relevance exists if and only if $\pi \wedge \pi' \nvDash \bot$ and $\pi' \nvDash \pi$. When it exists, the most specific π-π' relevance is logically equivalent to π.*
4. *If a π-π' distinction exists, then the most specific π-π' distinction logically implies each proper π-π' relevance. Each proper π-π' relevance logically implies the most specific analogy between π and π'.*

Proof. For any $\pi, \pi' \in L$:

$\boxed{\text{Property 1}}$ By Definition 2, for any analogy α, $\pi \models \alpha$ and $\pi' \models \alpha$. By propositional logic it follows that any analogy α is logically implied by $\pi \vee \pi'$. By Definition 2, $\pi \vee \pi'$ is therefore a most specific analogy.

$\boxed{\text{Property 2}}$ We prove the first claim first. Left to right. Assume a π-π' distinction δ exists. By Definition 2, $\pi \models \delta$ and $\pi' \models \neg\delta$. It follows by propositional logic that $\pi \wedge \pi' \models \bot$. Right to left. If $\pi \wedge \pi' \models \bot$, then by propositional logic $\pi' \models \neg\pi$. By Definition 2 and propositional logic, π is therefore a most most specific π-π' distinction. The second claim follows directly from the proof of the right to left direction of the previous claim.

$\boxed{\text{Property 3}}$ We prove the first claim first. Right to left. Assume $\pi \wedge \pi' \not\models \bot$ and $\pi' \not\models \pi$. Then we have that $\pi \models \pi$ (trivially), $\pi' \not\models \pi$ (by assumption) and $\pi' \not\models \neg\pi$ (by assumption). By Definition 2 π is therefore a relevance, and it is trivially most specific. Left to right. We proceed by contraposition and assume that either $\pi \wedge \pi' \models \bot$ or $\pi' \models \pi$. Clearly, if $\pi' \models \pi$ no relevance exists by Definition 2. So assume that $\pi \wedge \pi' \models \bot$. We show by a counterexample that a most specific relevance does not exist. Let $L = \{f_1, f_2\}$ and $\pi = f_1$, $\pi' = \neg f_1$. Consider then the two π-π' relevances $f_1 \vee f_2$, $f_1 \vee \neg f_2$. By propositional logic and Definition 2 there exists no π-π' relevance which entails both. So no most specific relevance exists in this example.

The second claim follows directly from the proof of the right to left direction of the previous claim.

$\boxed{\text{Property 4}}$ We prove the first claim first. By Property 2 if the most specific π-π' distinction exists, then it is logically equivalent to π. As to the second claim, by Definition 2, π logically implies all π-π' relevances, including proper ones, and proper π-π' relevances always logically imply the most specific analogy between π and π'. $\square$

As shown in Proposition 1, $\pi \vee \pi'$ is the most specific analogy between π and π'. In legal case-based reasoning, this may seem counterintuitive. However, by the definition of case comparison in terms of propositional logic, we can see that the sentence $\pi \vee \pi'$ characterizes the properties shared exactly by the two cases (i.e., those implied by both).

Based on Property 2 and 3 in Proposition 1, we see there always exists a distinction between any pair of cases in a case model, since they are mutually incompatible. Recall that in any two cases π and π' in a case model are either identical or logically incompatible. By Property 3 then there cannot exist a most specific relevance between two cases in a case model: when π and π' are the same formula, no relevance exists between the two by the definition of case model; when they are not they need to be incompatible by the definition of case model, and hence by Property 3 no most specific relevance can exist between them.

Property 4 in Proposition 1 shows why we have singled out proper relevances: in the formally precise sense of the proposition, they are logically 'in between' the most specific distinction (if it exists) and the most specific analogy.

Two cases can be compared with a third case using the analogy relation defined below, which is similar to what is called on-pointness in HYPO [3]. The analogy relation is based on the shared formulas between cases. When comparing cases π and π' in terms of case π'', if the most specific analogy between π and π'' logically implies the most specific analogy between π' and π'', then we say that π is at least as analogous as π' with respect to π''. We define the analogy relation as follows:

Definition 3 (Analogy relation between cases). *For any π, π' and $\pi'' \in L$, we define:*

$$\pi \succeq_{\pi''} \pi' \text{ if and only if } \pi \vee \pi'' \models \pi' \vee \pi''.$$

Then we say π is at least as analogous as *π' with respect to π''.*

As customary, the asymmetric part of the relation is denoted as $\pi \succ_{\pi''} \pi'$, which means π is *more analogous* than π' with respect to π''. The symmetric part of the relation is denoted as $\pi \sim_{\pi''} \pi'$, which means π is *as analogous as* π' with respect to π''. If it is not the case that $\pi \succeq_{\pi''} \pi'$ and $\pi' \succeq_{\pi''} \pi$, then we say π and π' are *analogously incomparable with respect to* π''.

Example 3. Comparing π_0 and π_1 in Fig. 1 in terms of case $\pi_2 = P \wedge Q$, we have $\pi_0 \succ_{\pi_2} \pi_1$; If $\pi_2 = P$, then we have $\pi_0 \sim_{\pi_2} \pi_1$; If $\pi_2 = \neg R$, then π_0 and π_1 are analogously incomparable with respect to π_2.

In the following proposition, we show some interesting properties of the analogy relation.

Proposition 2. *For any π, π' and $\pi'' \in L$:*

1. *The analogy relation is reflexive and transitive, hence a preorder;*
2. *$\pi \succeq_{\pi''} \pi'$ if and only if $\pi \models \pi' \vee \pi''$;*
3. *For any $\alpha \in L$, if $\pi \succeq_{\pi''} \pi'$, and α is an analogy between π' and π'', then α is also an analogy between π and π''.*

Proof. $\boxed{\text{Property 1}}$ The relation is reflexive, since $\pi \vee \pi'' \models \pi \vee \pi''$. The relation is also transitive because of the transitivity of entailment in propositional logic. Assume $\pi = f_1 \wedge f_2$, $\pi' = f_1 \wedge f_3$ and $\pi'' = f_1 \wedge f_2 \wedge f_3$, π and π' are analogously incomparable with respect to π'', hence the relation is not in general total.

$\boxed{\text{Property 2}}$ From left to right, by Definition 3 we obtain $\pi \vee \pi'' \models \pi' \vee \pi''$, and by propositional logic $\pi \models \pi' \vee \pi''$. From right to left, from $\pi \models \pi' \vee \pi''$ and propositional logic, we obtain $\pi \vee \pi'' \models \pi \vee \pi''$, and by Definition 3 $\pi \succeq_{\pi''} \pi'$.

$\boxed{\text{Property 3}}$ Follows directly from Definition 2 and 3. $\square$

Notice that if $\pi \succeq_{\pi''} \pi'$, then it is still possible that $\pi \not\models \pi'$ and $\pi \not\models \pi''$. For instance, if $\pi = f_1$, $\pi' = f_1 \wedge f_2$, $\pi'' = f_1 \wedge \neg f_2$, then we have $\pi \succeq_{\pi''} \pi'$, but both π' and π'' are not logically implied by π. Also notice that if $\pi \succeq_{\pi''} \pi'$, it cannot be concluded that $\pi \models \pi'$. For instance, $\pi = f_1 \wedge f_2$, $\pi' = f_3$ and $\pi'' = f_1$. In this example, $\pi \succeq_{\pi''} \pi'$ but $f_1 \wedge f_2 \not\models f_3$.

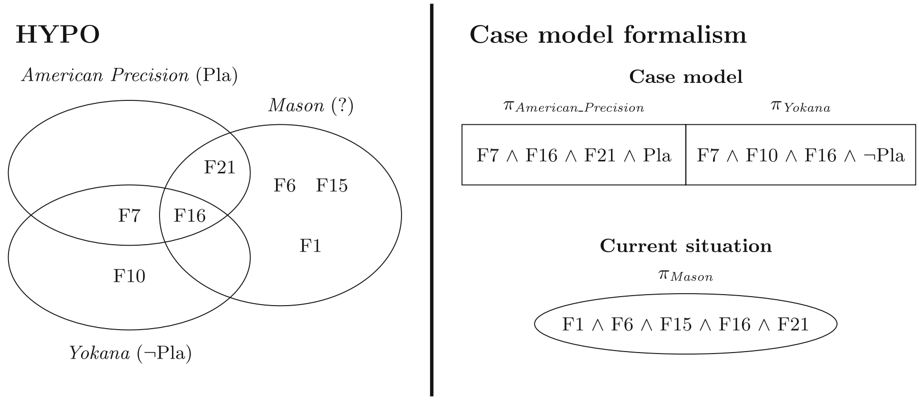

Fig. 3. The *Mason* problem in HYPO (the Venn diagram [4]) and in a case model [19,21]

3 Application: HYPO-style Comparison

In this section, we apply our formalism to case comparison in HYPO-style case-based reasoning with an example from a real legal domain.

As shown in [3,4], in HYPO, the set of shared factors between two cases are called *relevant similarity*, while the set of unshared factors are called *relevant difference*. Unshared factors can be used for pointing out the two cases should be decided differently. When comparing two cases in terms of a current situation, HYPO always makes sure that the cases are *on point* to the situation, namely the set of shared factors between any of the cases and the situation is not empty. If one of the cases shares more factors with the situation than the other one, then former case is *more on point* than the latter one with respect to the situation.

We take a set of legal cases from the United States trade secret law domain as an example. The cases has been discussed in [4,20]. As shown in Fig. 3, the *Yokana* case[1] and the *American Precision* case[2] are considered as two decided cases. *Yokana* favors for defendants (represented by ¬Pla) and *American Precision* favors for plaintiffs (represented by Pla). The *Mason* case[3] is considered as a current undecided case in this example. HYPO considers F6, F7, F15, and F21 as pro-plaintiff factors and F1, F10, and F16 as con-plaintiff factors, which suggests that they favor for different sides in the court.

HYPO represents cases as sets of factors. When comparing *Mason* with *Yokana* in HYPO, we consider:

1. {F16} as the set of the relevant similarity between *Mason* and *Yokana*, since both *Mason* and *Yokana* contain F16;

[1] Midland-Ross Corp. v. Yokana, 293 F.2d 411 (3rd Cir.1961).
[2] American Precision Vibrator Co. v. National Air Vibrator Co., 764 S.W.2d 274 (Tex.App.-Houston [1st Dist.] 1988).
[3] Mason v. Jack Daniel Distillery, 518 So.2d 130 (Ala.Civ.App.1987).

2. {F6, F15, F21, F10} as the set of the relevant difference between *Mason* and *Yokana* that against the defendant's claim, since F6, F15, F21 are in *Mason*, but not in *Yokana*, and they are favorable for the plaintiff as suggested by HYPO. F10 is in *Yokana*, but not in *Mason*, and it is favorable for the defendant as suggested by HYPO.

In the case model shown in Fig. 3, we represent cases in HYPO by logical conjunctions of factors and outcomes. We consider both factors and outcomes as literals. A literal is either a propositional constant or its negation. Unlike in CATO [2], our use of factors does not assume that factors favor a side of the decision, either pro-plaintiff or pro-defendant, as such an assumption is not needed for our logical definitions of case comparison. Unlike HYPO [3], our factors do not come with a dimension that can express a magnitude. For instance, the *Yokana* case is represented as $\pi_{Yokana} = \text{F7} \wedge \text{F10} \wedge \text{F16} \wedge \neg \text{Pla}$. Similarly for *American Precision* (as $\pi_{American\_Precision}$) and *Mason* (as π_{Mason}). The case model is with equal preference, as in HYPO's case base, all the cases are as preferred as each other. When comparing π_{Mason} with π_{Yokana} in the case model formalism, we consider:

1. F16 as an analogy between π_{Mason} and π_{Yokana} in the case model, since:
   (a) $\pi_{Mason} \models \text{F16}$; and
   (b) $\pi_{Yokana} \models \text{F16}$.
2. $\text{F6} \wedge \text{F15} \wedge \text{F21}$ as a π_{Mason} -π_{Yokana} relevance, since:
   (a) $\pi_{Mason} \models \text{F6} \wedge \text{F15} \wedge \text{F21}$;
   (b) $\pi_{Yokana} \not\models \text{F6} \wedge \text{F15} \wedge \text{F21}$ and $\pi_{Yokana} \not\models \neg(\text{F6} \wedge \text{F15} \wedge \text{F21})$.
3. F10 as a π_{Yokana}-π_{Mason} relevance, since:
   (a) $\pi_{Yokana} \models \text{F10}$;
   (b) $\pi_{Mason} \not\models \text{F10}$ and $\pi_{Mason} \not\models \neg\text{F10}$.

Notice that there is no distinction between π_{Mason} and π_{Yokana}, as these two cases are not incompatible.

Compared to the notions in Definition 2, the relevant similarity between cases (in terms of sets of factors) corresponds to an analogy between the cases (in terms of logical sentences), in the sense that the conjunction of the factors in the relevant similarity are logically implied by each of the two conjunctions of factors that represent the cases. However, the relevant difference between cases (in terms of sets of factors) can not be simply considered as distinctions or relevances between cases (in terms of logical sentences), since HYPO does not consider the negation of factors, hence it cannot separate distinctions and relevances. For those unshared factors between cases as the relevance difference, they are implied by one of the cases, but not the other one. If the negation of the factor can be applied by the other one, then it is considered as a distinction, if not, it is considered as a relevance.

The relevant similarity is not the only analogy between π_{Mason} and π_{Yokana}. We also have sentences like $\text{F16} \vee \text{F21}$, $(\text{F7} \wedge \text{F10} \wedge \text{F16} \wedge \neg\text{Pla}) \vee (\text{F1} \wedge \text{F6} \wedge \text{F15} \wedge \text{F16} \wedge \text{F21})$ as analogies between them. Notice that the latter one is the most specific analogy between π_{Mason} and π_{Yokana}. We also have $\text{F1} \wedge \text{F21}$ as a *Mason*-*Yokana* relevance and $\text{F16} \wedge \neg\text{Pla}$ as a *Yokana*-*Mason* relevance.

In the onpointness relation of HYPO, only sets of the shared factors are compared, and there is no outcome involved in the comparison. However, in the logical analogy relation, both factors and outcomes can be taken into account. Namely, when comparing cases in terms of onpointness, we compare the sets of factors in the conjunctions that represent cases, when comparing them in terms of the logical analogy relation, we use the most specific analogy between cases which can include outcomes. When comparing *American Precision* and *Yokana* in terms of *Mason*, *American Precision* is more on point than *Yokana* with respect to *Mason*, as the relevant similarity between *Yokana* and *Mason* ({F16}) is a subset of the relevant similarity between *American Precision* and *Mason* ({F16, F21}). However, according to the analogy relation, *American Precision* and *Yokana* are analogously incomparable with respect to *Mason*, which is determined by the most specific analogy between *Yokana* and *Mason* ($\pi_{Yokana} \vee \pi_{Mason}$) and between *American Precision* and *Mason* ($\pi_{American\_Precision} \vee \pi_{Mason}$):

1. $\pi_{Yokana} \vee \pi_{Mason} \not\models \pi_{American\_Precision} \vee \pi_{Mason}$; and
2. $\pi_{American\_Precision} \vee \pi_{Mason} \not\models \pi_{Yokana} \vee \pi_{Mason}$.

The above shows that, based on different comparison relations (analogy relation/onpointness), the selection of better case can be different. We observe that if two cases are onpointness comparable with respect to a third case, namely one of the two cases is either more on point or as on point than the other one with respect to the third case (otherwise, they are onpointness incomparable), the two cases are not always analogously comparable. For instance, when comparing *American Precision* and *Yokana* in terms of *Mason*.

For convenience, we now give an abstract example. We assume π_0, π_1 and π_2 are cases in HYPO, namely they are conjunctions of factors and outcomes (both are literals). When comparing π_0 and π_1 with respect to π_2, based on Proposition 2, we can further observe that:

1. If π_0 and π_1 are onpointness comparable with respect to π_2, then π_0 and π_1 are not always analogously comparable;
2. If π_0 and π_1 are onpointness incomparable with respect to π_2, then π_0 and π_1 are not always analogously incomparable;
3. If π_0 and π_1 are analogously comparable with respect to π_2, then π_0 and π_1 are always onpointness comparable:
   (a) If π_0 is more analogous than π_1 with respect to π_2, then π_0 is also more on point than π_1 with respect to π_2;
   (b) If π_0 is as analogous as π_1 with respect to π_2, then π_0 is also as on point as π_1 with respect to π_2.

The first observation has already been discussed above. For the second observation, assume that $\pi_0 = P \wedge Q$, $\pi_1 = \neg P$, and $\pi_2 = Q$, then π_0 and π_1 are onpointness incomparable with respect to π_2, but $\pi_0 \succeq_{\pi_2} \pi_1$, namely π_0 is more analogous than π_1 with respect to π_2. The last observation follows from Property 3 in Proposition 2.

4 Application: The Dynamics of Case-Based Reasoning

In our approach, we can formally distinguish the unshared part between cases in distinctions and relevances. We now apply this comparison approach to the development of precedential values in case-based reasoning by following a series of research [6,8,16]. The case model we analyzed has been studied in [16], and represents the series of New York car accident cases used in [6,8]. The focus is on the selection of the jurisdiction choice rules that applied in the cases:

1. **Smith v. Clute** 277 N.Y. 407, 14 N.E.2d 455 (1938): The claim was in tort law (driver negligence). The territorial rule applies.
2. **Kerfoot v. Kelley** 294 N.Y. 288, 62 N.E.2d 74 (1945): The claim was in tort law (driver negligence). The territorial rule applies.
3. **Auten v. Auten** 308 N.Y. 155, 124 N.E.2d 99 (1954): The claim was in contract law (enforce a child support agreement). The center-of-gravity rule applies.
4. **Kaufman v. American Youth Hostels** 5 N.Y.2d 1016 (1959): The claim was in tort law (travel guide negligence). The territorial rule applies.
5. **Haag v. Barnes** 9 N.Y.2d 554, 175 N.E.2d 441, 216 N.Y.S. 2d 65 (1961): The claim was in contract law (reopen a child support agreement). The center-of-gravity rule applies.
6. **Kilberg v. Northeast Airlines** 9 N.Y.2d 34, 172 N.E.2d 526, 211 N.Y.S.2d 133 (1961): The claim was in tort law (common carrier negligence). The territorial rule is overridden for reasons of public policy.
7. **Babcock v. Jackson** 12 N.Y.2d 473, 191 N.E.2d 279, 473 N.Y.S.2d 279 (1963): The claim was in tort law (driver negligence).

From the list of cases above, there are two kinds of cases, namely tort cases (represented as $\mathtt{TORT}$) and contract cases (represented as $\mathtt{CONTRACT}$). The jurisdiction choice rules can be: the territorial rule (represented as $\mathtt{TERRITORY}$), the center-of-gravity rule (represented as $\mathtt{GRAVITY}$), and exceptions (represented as $\mathtt{EXCEPTION}$).

A case model (also discussed in [16]) can be generated from above cases. The model assumes that the kinds of cases exclude each other pairwise ($\neg(\mathtt{TORT} \wedge \mathtt{CONTRACT})$), and similarly for the choice rules ($\neg(\mathtt{TERRITORY} \wedge \mathtt{EXCEPTION})$, etc.). We restrict the case model to the cases up and until a particular year. For instance, we write $\mathcal{C}(1945)$ for the case model with the set of cases that contains *Smith* and *Kerfoot* dating from 1945 or before. The cases in the model are represented as follows:

$$\pi_{Smith} = \mathtt{TORT} \wedge \mathtt{TERRITORY} \qquad \pi_{Haag} = \mathtt{CONTRACT} \wedge \mathtt{GRAVITY}$$
$$\pi_{Kerfoot} = \mathtt{TORT} \wedge \mathtt{TERRITORY}$$
$$\pi_{Auten} = \mathtt{CONTRACT} \wedge \mathtt{GRAVITY} \qquad \pi_{Kilberg} = \mathtt{TORT} \wedge \mathtt{EXCEPTION}$$
$$\pi_{Kaufman} = \mathtt{TORT} \wedge \mathtt{TERRITORY} \qquad \pi_{Babcock} = \mathtt{TORT} \wedge \mathtt{GRAVITY}$$

Now we compare a new, undecided tort case $\pi = \mathtt{TORT}$ with the decided cases.

Suppose π arises before 1954 when only the *Smith* case and the *Kerfoot* case have been decided (i.e., $\mathcal{C}(1945)$), we can see $\texttt{TORT} \wedge \texttt{TERRITORY}$ is a relevance between in π_{Smith} (or $\pi_{Kerfoot}$) with respect to π in $\mathcal{C}(1945)$, as:

1. $\pi_{Smith} \models \texttt{TORT} \wedge \texttt{TERRITORY}$; but
2. $\pi \not\models \texttt{TORT} \wedge \texttt{TERRITORY}$ and $\pi \not\models \neg(\texttt{TORT} \wedge \texttt{TERRITORY})$.

Similarly, $\texttt{TORT} \wedge \neg\texttt{GRAVITY}$ and $\texttt{TORT} \wedge \neg\texttt{EXCEPTION}$ are also relevances between them in $\mathcal{C}(1945)$.

In 1954, *Auten* has been added into the model $\mathcal{C}(1954)$, which generally introduce the $\texttt{GRAVITY}$ rule. If the undecided case π arises after 1954, $\top \wedge \texttt{GRAVITY}$ can be a relevance in π_{Auten} with respect to π in $\mathcal{C}(1954)$. Formerly, in $\mathcal{C}(1945)$, $\top \wedge \texttt{GRAVITY}$ cannot be considered as a relevance between the cases, as neither π nor the decided cases (*Kerfoot* and *Smith*) imply the sentence.

Not only new jurisdiction choice rules can be considered as relevances between cases, but also new exceptions of these rules. In $\mathcal{C}(1961)$, *Kilberg* gives the territorial rule an exception, and sentence $\texttt{TORT} \wedge \texttt{EXCEPTION}$ for this new exception can be considered as a relevance in $\pi_{Kilberg}$ with respect to an undecided case π that arises after 1961. Before the exception occurred, $\texttt{TORT} \wedge \texttt{EXCEPTION}$ cannot be considered as a relevance or a distinction between them.

The general introduction of the $\texttt{GRAVITY}$ rule cannot make the rule be a relevance when considering new cases in the tort law domain (represented by $\texttt{TORT} \wedge \texttt{GRAVITY}$). For instance, in $\mathcal{C}(1961)$, $\texttt{TORT} \wedge \neg\texttt{GRAVITY}$ is a relevance between an undecided case π and any of the decided cases in the model, but not $\texttt{TORT} \wedge \texttt{GRAVITY}$. This is changed when *Babcock* is added, which introduces the $\texttt{GRAVITY}$ rule into the tort law domain, as in $\mathcal{C}(1963)$, $\texttt{TORT} \wedge \texttt{GRAVITY}$ can be considered as a relevance in $\pi_{Babcock}$ with respect to an undecided case π.

As shown above, the definition of relevances can support the analysis of the development of rules in the series of cases. When a new rule or exception is introduced in the case model, there is a new relevance for the future undecided case. For instance, the general introduction of the $\texttt{GRAVITY}$ rule in 1954 by the *Auten* case makes $\top \wedge \texttt{GRAVITY}$ a relevance between *Auten* and undecided cases. For the cases that are decided after *Auten*, they need to consider whether the $\texttt{GRAVITY}$ rule should be applied or not. Formally, they need to consider whether an undecided current case should imply sentence $\top \wedge \texttt{GRAVITY}$ or its negation $\neg(\top \wedge \texttt{GRAVITY})$.

Intuitively, for the unshared parts implied by decided cases but not by the undecided ones, their implied status in the undecided cases are unknown as yet, namely, these unshared sentences need to be considered by decision makers.

In contrast, the unshared parts between decided cases are not considered as relevances, but as distinctions in our formalism, since the status of these sentences in the cases are implied. For instance, $\texttt{TORT} \wedge \texttt{GRAVITY}$ and $\texttt{TORT} \wedge \neg\texttt{GRAVITY}$ are two distinctions between *Babcock* and *Smith* in $\mathcal{C}(1963)$, since:

1. $\pi_{Smith} \models \texttt{TORT} \wedge \neg\texttt{GRAVITY}$; and
2. $\pi_{Babcock} \models \texttt{TORT} \wedge \texttt{GRAVITY}$.

Similarly, $\texttt{TORT} \wedge \texttt{TERRITORY}$ and $\texttt{TORT} \wedge \neg\texttt{TERRITORY}$ are also distinctions between them. Recall that $\texttt{TORT} \wedge \texttt{GRAVITY}$ and $\texttt{TORT} \wedge \texttt{TERRITORY}$ are considered as relevances between $\pi_{Babcock}$ and an undecided case π in $\mathcal{C}(1963)$.

Therefore, the comparison approach in our formalism is able to formally identify the difference between the unshared sentences between two decided cases and between an undecided case and a decided case. For the unshared part between two decided cases, they are considered as distinctions in our approach, as they are known differences between the cases. For the unshared part in a decided case with respect to an undecided one, the status of this part in the undecided cases is unknown, and can be turned into analogies or distinctions in the further development of the cases. In this sense, they are different from the unshared part between two decided cases. The same sentence can play different roles in different comparisons of cases. As shown above, in $\mathcal{C}(1963)$ we can see sentences like $\texttt{TORT} \wedge \texttt{GRAVITY}$ as a distinction between two decided cases ($\pi_{Babcock}$ and π_{Smith}), as a relevance between an undecided case π and a decided case $\pi_{Babcock}$. This cannot be achieved without first distinguishing distinctions and relevances. In this way, our approach refines the analysis of case-based reasoning.

5 Discussion

In this paper, we discuss case comparisons in case-based reasoning with the case model formalism, which is described in a formal propositional logic language. Unlike other case-based reasoning models, in which cases are represented as dimensions [3], sets of rules [11], sets of factors [12], combinations of rules, facts and outcomes [9] and hierarchies [1,2]. The formalism we present here represents cases using propositional logic sentences. Building on [17], we give a concrete account of the approach of comparison.

As an extension of [21], this paper defines the notions for case comparison in case models rather than in precedent models, a subclass of case models. We now discuss the comparison of cases in the general setting, not just in the precedents represented by conjunctions of factors and outcomes. In particular, the application shown in Sect. 4 is not in terms of the formal notion of precedents represented by factors and outcomes [21], and instead focuses only on the jurisdiction choice rules that applied in the legal cases modeled.

Case-based reasoning models following HYPO often discuss comparison between cases in terms of factors. In the formalism we present here, we generalize the comparison approach in case-based reasoning, namely comparing cases not only with factors, but also with more general propositional formulas.

Section 3 shows a key difference between our comparison approach and the research following HYPO [2,3,9]. Factors in HYPO-style comparison typically favor a side in the court case, showing which factors can strengthen or weaken the arguments given by the parties involved, hence constraining possible argument moves. However, the more general formulas used in our comparison approach may not favor a specific side in the court. For instance, $\mathrm{F16} \vee \mathrm{F21}$ is an analogy between π_{Mason} and π_{Yokana}, it does not favor a side in the court. The formulas we discuss are more general logical expressions than factors. It would be interesting to discuss the role of sides favored by factors in our formalism, for instance by investigating the modeling of argument moves in CATO, such as downplaying or emphasizing distinctions.

The comparison approach introduced here allows us to discuss general formulas beyond factors in case-based reasoning, such as conjunctions or disjunctions of factors, which can bring new discussion on case-based reasoning with the case model formalism. For instance, for future research we can discuss hierarchical factors shown in CATO [2], as higher level factors can be represented with compound formulas based on base-level factors. Therefore, it seems possible to compare abstract factors between cases directly in the formalism.

As shown in Sect. 3, when comparing *American Precision* and *Yokana* with respect to *Mason* in terms of the analogy relation defined in Definition 3, the result is different from the comparisons based on other relations, such as the preference relation in case models and onpointness in HYPO. By the preference relation of the case model shown in Fig. 3, *American Precision* and *Yokana* are as preferred as each other; according to onpointness, *American Precision* is more on point with respect to *Mason*; and according to the analogy relation, these two cases are analogously incomparable with respect to *Mason*. This is because the analogy relation discusses comparison in terms of the most specific analogy between cases, while other comparison relations are in terms of other notions. For instance, the onpointness is about the shared factors between cases, the conjunction of these factors is not always the most specific analogy between the cases, hence the comparison based on the analogy relation and based on the onpointness relation can have different results. In the above example we show that *American Precision* is a better precedent than *Yokana* based on the onpointness relation, however, the two cases are analogously incomparable by using the analogy relation. Things can be various in other examples. For instance, as we discussed in the observations shown in Sect. 3, cases can be onpointness incomparable but analogously comparable. Onpointness is for factor-based comparison built on sets, while the analogy relation is for logic-based comparison built on logical sentences. These two methods are from different perspectives and based on different theories. In legal case-based reasoning, users may pay more attention to the onpointness rather than the analogy relation, since the shared or absence of some factors can make the difference for winning a case. The formalism we develop is a general theory, which contributes to the further systematization of our formal understanding of case comparison in case-based reasoning. As the analogy relation can lead to a different selection of better cases, in the future, it will also be interesting to have a look at the comparison relation based on distinctions and relevances, the notions in the new refinement shown by our approach.

The representation we use can treat case-based reasoning from a perspective that is closer to logic, thereby allowing an analysis of the properties that are shared and not shared between cases, in terms of our notions of analogies, distinctions and relevances. In [17,19], we do not separate the distinctions and relevances between cases, nor do HYPO [3] and other case-based reasoning models [2,12]. In the formalism we present here, relevances between cases are distinguished from analogies and from distinctions. This refinement points to potential modification of case comparisons in case-based reasoning, in which the situation can change accordingly when new facts are found. While analogies between two cases refer to formulas that hold in both cases, and distinctions to formulas that hold in one case and are negated in the other, relevances are formulas that are not determined in a case and hence have the potential to turn out as an analogy or distinction once determined. Although both distinctions and relevances are related to unshared factors, relevances cannot be considered as distinctions directly, since if such relevant formulas are determined to hold in a situation, they will turn out as analogies rather than as distinctions between the case and the situation. For instance, in the *Mason* problem discussed in Sect. 3, we can see F7 as a relevance in *Yokana* with respect to *Mason*, since the conjunction of *Yokana* logically implies F7, but neither F7 nor ¬F7 is implied by the conjunction of *Mason*, in the sense that the status of F7 is unknown in *Mason*. Therefore, if F7 can be found in *Mason* later, then it will be considered as an analogy, if ¬F7 is found, then it will become a distinction. Therefore, our comparison approach has the potential to model heuristics for hypothetically modifying cases occurred in HYPO reasoning, such as heuristic H1 ("Make a near-miss Dimension apply" [3]).

As shown in Sect. 3, the relevant similarity and the relevant difference between cases in HYPO can be modeled in terms of analogies, distinctions and relevances defined in Definition 2. Although the relevant similarity is an analogy between cases, factors in the relevant difference are not always distinctions between cases, but can also be relevances. In this sense, our approach compares cases in a more specific way than HYPO. However, as we have not defined dimensions of factors in the formalism, it is unable to discuss the magnitude of factors in relevant differences, which means we cannot compare cases in terms of dimensions, such as finding a contrary case which has some factors with extreme magnitude. This needs further discussion in the future.

The refinement of case comparison can be further illustrated with the application in Sect. 4, which has not been discussed in [21]. The application shows that our comparison approach can support the discussion of the development of precedential values in case-based reasoning. By following [6,8,16], we have shown the difference of the unshared parts that exist between two decided cases and between an undecided case and a decided one, thereby we find the connection between the introduction of new rules and exceptions and the notion of relevances we define. When new rules and exceptions are introduced into the model, the number of relevances that need to be considered when comparing new cases with decided cases increases accordingly, which can make the decision making process harder than before.

6 Conclusion

In this paper, we have studied the logical comparison of cases. Continuing from [19–21], the comparison of cases is here not limited to precedents represented by conjunctions of factors and outcomes but is extended to the more general case model formalism. We also extend it with a discussion of the development of case-based reasoning in a temporal context.

With the formalism, we provide a way that refines comparisons in case-based reasoning. As shown in Sect. 3, we discuss not only the shared factors between cases, but also other logically compound formulas based on factors, which allows us to compare cases from a logical perspective and discuss other features among cases. We further distinguish the unshared formulas between cases into distinctions and relevances based on the implication of themselves and their negation in the cases. In this way, we show a refinement of comparisons in case-based reasoning. With the application about the dynamics of case-based reasoning shown in Sect. 4, we show how the refinement can support the analysis of the development of rules in cases.

The case model formalism has the potential to help analyze argument moves and applied status of legal rules in case-based reasoning and support the selection of good cases to cite in a court discussion. These topics could be investigated in future research.

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