Mechanistic description of population dynamics using dynamic energy budget theory incorporated into integral projection models

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Summary

1. Integral projection models (IPMs) provide a powerful approach to investigate ecological and rapid evolutionary change in quantitative life-history characteristics and population dynamics. IPMs are constructed from functions that describe the demographic rates – survival, growth and reproduction – in relation to the characteristics of individuals and their environment. Currently, however, demographic rates are estimated using phenomenological regression models that lack a mechanistic representation of the biological processes that give rise to observed demographic variation. This lack of mechanistic underpinning limits the ability of the model to predict future dynamics under novel environmental conditions because the model ingredients pertain to current environmental conditions only.

2. Here, we use dynamic energy budget (DEB) theory to construct DEB-IPMs based on a mechanistic representation of individual life-history trajectories. We derive the demographic functions describing growth and reproduction from a simple DEB growth model. The functions describing mortality and the association between parent and offspring characteristics do not follow DEB theory and hence are estimated from individual-level observations.

3. We apply the DEB-IPM to two contrasting systems: the small, fast-reproducing bulb mite Rhizoglyphus robini and the large, slow-reproducing reef manta ray Manta alfredi. In both cases, predictions of population growth rate, lifetime reproductive success and generation time agree with empirical observations. In case of the bulb mite, predictions and observations even agree across different feeding conditions.

4. If the DEB energetics model is accepted as describing growth and reproduction, DEB-IPMs can be parameterised using easy-to-collect life cycle information (growth rate, length at birth, maturation and old age) making them suitable for data-deficient species. Because species differ only in these DEB parameters, comparative studies of character and population dynamics between species are straightforward, particularly since DEB-IPMs can be extended to include population feedback on resources, of which we give an example. Most crucially, because DEB theory specifies growth and reproduction rates as explicitly dependent on environmental conditions such as food availability or temperature, DEB-IPMs provide a mechanistic platform to investigate the biological processes that determine joint change in phenotypic characters, life-history traits, population size and community structure.

Key-words: add-my-pet, cohort generation time, competition, consumer-resource dynamics, vital rates, von Bertalanfly growth rate, k-rule growth model

Introduction

Biologists increasingly face the challenge of accurately predicting how individuals, populations and communities respond to the ever greater changes in the environment. One way of tackling this challenge is to use an approach based on the characteristics of individuals and examine how the environment affects the change of individuals in these characteristics, thereby generating the dynamics of population structure (Webb et al. 2010). Different types of such approaches exist, including physiologically structured population models (PSPMs; Metz & Diekmann 1986), delay-differential equation models (Nisbet & Gurney 2003), individual-based models (IBMs; Grimm & Railsback 2005), Matrix Population Models (MPMs; Caswell 2001), and Integral Projection Models (IPMs) (Easterling, Ellner & Dixon 2000). These approaches differ less in conceptual structure than in their mathematical details. MPMs and IPMs are closely and easily linked to data, but, whereas MPMs assume that individuals occupy discrete stages, IPMs accommodate both discrete and continuous state variables (e.g. Smallegange, Deere & Coulson 2014). IPMs have emerged as a powerful tool to investigate population-level processes from an...
individual-level perspective, partly because the demographic processes of growth, survival and reproduction are estimated using flexible and easy-to-use phenomenological methods such as regression models (Ellner, Childs & Rees 2016). The downside of these regression models, however, is that they lack a mechanistic representation of the biological processes that give rise to observed survival, growth and reproduction. In this article, we discuss an alternative, more mechanistic approach to parameterisation, which explicitly incorporates an energetic description of growth and reproduction into IPMs. This will allow researchers who are not comfortable using the more mathematically challenging PSPMs, to investigate ecological and evolutionary patterns such as population dynamics, geographic distributions or evolution of life-history strategies, from an energy budget perspective on demographic rates, which has traditionally been tackled by using the mathematically more challenging PSPMs.

The character that is most commonly used to describe individuals in IPMs is body size (Ellner, Childs & Rees 2016). However, body size is determined by the process of growth, which is critically affected by food availability; the more energy is allocated to growth, the less will be available to allocate to other processes such as reproduction. When estimating demographic rates for growth and reproduction using phenomenological models, these processes are modelled independently from each other so that the latter trade-off (and hence the principle of energy conservation) is ignored. Because of the central importance of growth to any model structured by body size, it is attractive to base the model on a mechanistic representation of the individual energy budget. Dynamic energy budget (DEB) theory (Kooijman 2000) provides a well-tested framework for modelling the acquisition and use of energy for organisms during the entire life cycle. DEB models have mainly been incorporated as mechanisms in PSPMs to study density-dependent feedback effects between a population and its environment, and resulting patterns of life-history evolution (de Roos & Persson 2013). However, the analysis of PSPMs requires rather complicated methodology and PSPMs typically are more mathematical representations of biological systems. In contrast, IPMs are data-driven, provide a way of synthesising complex life-history information and can be analysed using more straightforward mathematical techniques (Smallegange & Coulson 2013). What is more, work is in progress to develop a more general approach for computing dynamic properties of nonlinear IPMs (Day & Kalies 2013). This means that, in the near future, as with PSPMs, it should be possible to analyse complex dynamics, for example by conducting bifurcation analyses of attractors (Ellner, Childs & Rees 2016).

Here, we use a simple version of the standard model of Kooijman’s DEB theory, also known as the Kooijman–Metz model (Kooijman & Metz 1984), to derive the demographic functions that describe growth and reproduction. We parameterise the resulting DEB-IPM for two species that are opposite in terms of body size and life-history speed: the small, fast-reproducing bulb mite *Rhizoglyphus robini* and the large, slow-reproducing reef manta ray *Manta alfredi*. We use the parameterised DEB-IPMs to examine for mites and rays how population growth rate, lifetime reproductive success and generation time vary with feeding conditions, and assess model performance by comparing the resulting predictions against empirical observations.

**Materials and methods**

**GENERAL STRUCTURE OF THE DEB-IPM**

The DEB-IPM, like any IPM, describes the dynamics of the length number distribution *N(L, t)* from time *t* to *t* + 1, with *N* as the number of females, by:

\[
N(L', t + 1) = \int_0^L [D(L', L(t))R(L(t)) + G(L', L(t))S(L(t))]N(L, t)dL,
\]

where the survival function \(S(L(t))\) is the probability that an individual of length *L* survives from time *t* to *t* + 1, and \(G(L', L(t))\) is the probability that an individual of length *L* grows to length *L' at *t* + 1, conditional on survival. The reproduction function \(R(L(t))\) gives the number of offspring produced between time *t* and *t* + 1 by an individual of length *L* at time *t*. The probability density function \(D(L', L(t))\) gives the probability that the offspring of an individual of length *L* are of length *L'* at time *t* + 1, and hence describes the association between parent and offspring character values. The closed interval \(\Omega\) denotes the length domain. The Kooijman–Metz model is a model for the energy allocation and growth of an individual. It implies a structure for the functions \(G(L', L(t))\) and \(R(L(t))\); when those functions are incorporated into the IPM, the resulting model describes the dynamics of a population of individuals, each of which follows the energy budget model. Below we describe how we derive the functions \(G(U, L(t))\) and \(R(L(t))\) from the Kooijman–Metz model. DEB theory does not describe the rates \(S(L(t))\) and \(D(U, L(t))\). Parameterisation of these remaining parts of the DEB-IPM therefore in principle proceeds as in the standard IPMs, namely through regression analysis of data on survival and offspring size as a function of (parent) body size.

**Growth**

We assume that an individual consumes a food resource *X* following a scaled Holling type II functional response (Kooijman 2000), which we indicate with *Y*. The constant quantity *Y* is also referred to as the experienced feeding level as it indicates the feeding rate as a fraction of the maximum feeding rate of an individual of a particular size, ranging from zero (no feeding when *X* = 0) to one (when feeding rate equals maximum feeding rate at very high levels of *X*). There is no population feedback on the resource (the Supporting Information contains an example consumer-resource DEB-IPM showing how feedback between consumers and resource could be included). The Kooijman–Metz model assumes that individual organisms are isomorphic (body surface area and volume are proportional to squared and cubed length, respectively). The rate at which individuals ingest food, *I*, is assumed to be proportional to the maximum ingestion rate \(I_{max}\), the current feeding level *Y* and body surface area, and hence to the squared length of an organism: \(I = I_{max}YL^2\). Ingested food is assimilated with a constant efficiency \(c\). A constant fraction \(\kappa\) of assimilated energy is allocated to respiration; this respiration energy equals \(\kappa I_{max}YL^2\) and is used to first cover maintenance costs, which are proportional to body volume following \(LV^2\) (\(\xi\) is the proportionality constant relating maintenance energy requirements to cubed length), while the remainder is allocated to somatic growth. The remaining fraction \(1 - \kappa\) of assimilated energy,
the reproduction energy, is allocated to reproduction in case of adults and to the development of reproductive organs in case of juveniles, and equals \((1 - \xi)I_{\text{max}}Y^2\). From the above, the change in length \(L\) over time can be derived as (Kooijman & Metz 1984; Supporting Information):

\[
\frac{dL}{dt} = \dot{r}_B(L_m - Y - L),
\]

where \(\dot{r}_B\) is known as the von Bertalanffy growth rate and \(L_m = \kappa \xi I_{\text{max}}Y^2\) is the maximum length under conditions of unlimited resource. Both \(\kappa\) and \(L_m\) are assumed to be constant across experienced feeding levels, and therefore, \(L_m\) is also assumed constant. Integrating the differential eqn 2 from time \(t\) to \(t + 1\) results in the function describing length at time \(t + 1\) as a function of length at time \(t\) for a single individual (Supporting Information):

\[
L(t + 1) = L(t)e^{-\alpha} + (1 - e^{-\alpha})L_mY.
\]

Underlying eqn 3 is the assumption that growth in body length is negative, that is \(L(t + 1) < L(t)\) and individuals shrink, when maintenance requirement exceed respiration energy; that is when \(\xi L^2 > \kappa I_{\text{max}}Y^2\), which occurs when \(L > \kappa \xi I_{\text{max}}Y \Rightarrow L > L_m\). Shrinking in response to starvation conditions occurs in many soft-bodied invertebrates and even in vertebrates (Wikelski & Thom 2000). If organisms do not shrink under starvation conditions, additional conditions are required for eqn 3 to incorporate the mechanism that, to avoid shrinking, individuals rechannel energy from reproduction to maintenance (Supporting Information):

\[
L(t + 1) = \begin{cases} 
L(t)e^{-\alpha} + (1 - e^{-\alpha})L_mY & \text{for } L \leq L_mY \\
L(t) & \text{otherwise}
\end{cases}
\]

Individuals die from starvation at a length at which maintenance requirements exceed the total amount of assimilated energy, that is the sum of respiration and reproduction energy: \(\xi L^2 > \kappa I_{\text{max}}Y^2 + (1 - \kappa)\xi I_{\text{max}}Y^2\), which occurs when \(L > \xi I_{\text{max}}Y \Rightarrow L > L_m\).

**Introducing interindividual variability in feeding: stochastic growth**

Growth of individuals as described by eqns 3 and 4 is deterministic; for example, two individuals born at the same size will be of identical size throughout their life. Implicitly underlying the population-level model of eqn 1, however, is a stochastic, IBM, in which individuals follow Markovian growth trajectories that depend on an individual’s current state (Easterling, Ellner & Dixon 2000). This individual variability is in standard IPMs modelled using a probability distribution, typically Gaussian, where the mean length and variability (model residuals) are regression functions of length (Easterling, Ellner & Dixon 2000). Parameterizing such a distribution requires an extensive data set containing pairs of body lengths measured on the same individual at time \(t\) and \(t + 1\). As we aim for users to be able to also apply the DEB-IPM to data-deficient species, we take a data-independent approach. We introduce variability as arising from how individuals experience the environment. We assume that the experienced feeding level \(Y\) follows a probability distribution \(f(Y)\), which means that individuals within a cohort of length \(L\) do not necessarily experience the same feeding level due to demographic stochasticity (e.g. individuals, independently of each other, have good or bad luck in their feeding experience). The expectation of this probability distribution is related to the resource density in the environment: \(E(Y) = \lambda(K + X)\) where \(X\) represents the resource density in the environment and \(K\) is the half-saturation constant. Because the growth rate of an individual is a linear function of its experienced feeding level \(Y\) (eqn 3), our assumptions about the interindividual variability in experienced feeding level imply that the growth realised by a cohort of individuals with length \(L(t)\) equals (assuming individuals can shrink):

\[
E(L(t + 1)) = \int \{L(t)e^{-\alpha} + (1 - e^{-\alpha})L_mY\}f(Y)\,dY
\]

\[
= L(t)e^{-\alpha} + (1 - e^{-\alpha})L_m\int f(Y)\,dY
\]

\[
= L(t)e^{-\alpha} + (1 - e^{-\alpha})L_m\cdot E(Y).
\]

The variance in length at time \(t + 1\) for a cohort of individuals of length \(L\) is then given by:

\[
\sigma^2(L(t + 1)) = (1 - e^{-\alpha})^2L_m^2\sigma^2(Y).
\]

If individuals cannot shrink, eqn 5 and eqn 6, respectively, become (Supporting Information):

\[
E(L(t + 1)) = \begin{cases} 
L(t)e^{-\alpha} + (1 - e^{-\alpha})L_mE(Y) \text{ for } L \leq L_mE(Y) \\
L(t) \text{ otherwise}
\end{cases}
\]

and

\[
\sigma^2(L(t + 1)) = \begin{cases} 
(1 - e^{-\alpha})^2L_m^2\sigma^2(Y) \text{ for } L \leq L_mE(Y) \\
0 \text{ otherwise}
\end{cases}
\]

We assume that the experienced feeding level \(Y\) follows a Gaussian distribution with mean \(E(Y) = \lambda(K + X)\) and standard deviation \(\sigma(Y)\), which means that the probability density function describing growth equals:

\[
G(L', L(t)) = \frac{1}{\sqrt{2\pi\sigma_L(L(t + 1))}} e^{-\frac{(L' - E(L(t + 1)))^2}{2\sigma_L^2(L(t + 1))}},
\]

where \(G(L', L(t))\) is given by eqn 5 and \(\sigma^2_L(L(t + 1))\) by eqn 6 for individuals that can shrink, and by eqn 7 and eqn 8, respectively, for individuals that do not shrink under starvation conditions. Following standard practice in the construction of IPMs (Ellner, Childs & Rees 2016), eqn 9 describes a Gaussian probability distribution of body lengths at time \(t + 1\) for any given body length at time \(t\). In standard IPMs, statistical regression functions are used to estimate \(E(L(t + 1))\) and \(\sigma^2_L(L(t + 1))\).

**Reproduction**

According to the Kooijman–Metz model, reproduction, that is the number of offspring produced by an individual between time \(t\) and \(t + 1\), equals \(Y_{\text{Re}}L^2/L_m^2\) (Supporting Information). The parameter \(R_m\) is the maximum reproduction rate of an individual of maximum length \(L_m\). Note that \(R_m\) is proportional to \((1 - \kappa)\) (Supporting Information), whereas \(Y_{\text{Re}}\) is proportional to \(\kappa\), which controls energy conservation. However, the role of \(\kappa\) in the DEB-IPM is mostly implicit, as \(\kappa\) is used as input parameter only in the starvation condition \(L > L_mE(Y)/\kappa\) whereas \(R_m\) and \(L_m\) are measured directly from data. Like \(L_m\), \(R_m\) is also proportional to \(I_{\text{max}}\) (Supporting Information); as both \(\kappa\) and \(I_{\text{max}}\) are assumed to be constant across experienced feeding levels, \(R_m\) is also assumed constant. Individuals are mature when they reach puberty at length \(L_p\) and only surviving adults reproduce; thus, only individuals within a cohort of length \(L_p \leq L \leq L_mE(Y)/\kappa\) reproduce. As with the growth rate, the reproduction rate of an individual is a linear function of its experienced feeding level \(Y\) with proportionality constant \(R_mL^2/L_m^2\). Hence, the reproduction realised by a cohort of individuals of length \(L\) between time \(t\) and \(t + 1\) that can shrink under starvation conditions equals:


If individuals do not shrink under starvation conditions, eqn 10 becomes (Supporting Information):

\[
R(L(t)) = \begin{cases} 
0 & \text{if } L_b \leq L < L_p \\
E(Y)R_mL(t)^2/J_m & \text{if } L_p \leq L \leq L_mE(Y)/\kappa
\end{cases} \quad \text{eqn 10}
\]

Survival

For simplicity, we assume a constant, size-independent background mortality rate \(\mu\). Additionally, an individual can only survive if it is able to cover its maintenance costs (i.e. when \(L \leq L_m\)). Assuming large numbers of individuals of length \(L\), the fraction of individuals in a cohort of length \(L\) that survive from time \(t\) to \(t+1\) equals:

\[
S(L(t)) = \begin{cases} 
e^{-\mu} & \text{for } L \leq L_mE(Y)/\kappa \\
0 & \text{otherwise}
\end{cases} \quad \text{eqn 12}
\]

Parent–offspring association

The same rationale that we used to construct the growth function \(G(L', L(t))\) (eqn 9) is used to construct the function \(D(L', L(t))\):

\[
D(L', L(t)) = \begin{cases} 
0 & \text{for } L < L_p \\
\frac{-dL(t)^2}{2L(t)^2}e^{-\frac{1}{2}L(t)^2} & \text{otherwise}
\end{cases} \quad \text{eqn 13}
\]

where \(E_{L_b}(L(t))\) is the expected size of offspring produced by a cohort of individuals with length \(L(t)\), and \(\sigma^2_{L_b}(L(t))\) the associated variance. Here, for simplicity, we set \(E_{L_b}(L(t))\) constant and \(\sigma^2_{L_b}(L(t)) = 0\) (see below). Users that wish to relate \(E_{L_b}(L(t))\) to, for example, parental body length could adopt the standard IPM approach and regress offspring body length at time \(t+1\) against parental body length at time \(t\) to obtain \(E_{L_b}(L(t))\), and the associated variance, \(\sigma^2_{L_b}(L(t))\), from the squared residuals (Easterling, Ellner & Dixon 2000).

PARAMETRISATION AND MODEL PERFORMANCE

Our two applications below illustrate how the DEB-IPM can be parameterised for a data-rich species (bulb mite) and data-poor species (reef manta ray). Importantly, if users do not have access to data to estimate all DEB parameters for their study species, one could check the add-my-pet data collection (Add-my-pet 2016), which contains DEB parameter estimates for >300 species, and use estimates of (closely) related species.

Parameterisation for the bulb mite

Bulb mites (Acaridae) live in the soil and feed on bulbs and tubers and are pests of many crops and ornamentals (Díaz et al. 2000). Bulb mites are small (100–1000 \(\mu\m\)) and live for up to a few months. From egg to adult, they go through a larval and two to three nymph stages, which takes between 11 and 40 days depending on food quality (Smallegange 2011). To parameterise the DEB-IPM, we use data on female bulb mite life-history trajectories observed at a low (ad lib access to filler paper), and high (ad lib access to yeast), feeding-level diet (Smallegange 2011). We assume that bulb mites can shrink in response to starvation [note that model output in case of the bulb mites does not differ between whether individuals can shrink or not (Fig. S1)]. Egg length at birth is independent of feeding level and maternal length (Smallegange, Deere & Coulson 2014); hence, we set \(E_{L_b}(L(t)) = L_p = 0.166\) mm (Table 1), and \(\sigma^2_{L_b}(L(t)) = 0\). The maximum length observed for an adult female is \(L_m = 1.008\) mm (Table 1). Reproduction of this female was not measured and, instead, we estimated \(R_m\) by taking the maximum, observed average daily egg production rate of adult females on the high feeding level, which equals \(R_m = 32\) (Supporting Information). Ultimate length \(L_{\text{uo}}\) is the asymptote of the von Bertalanffy growth curve in length and represents the largest length an individual can achieve at a particular feeding level. The length of the largest individual observed at the low feeding level equalled 0.642 mm, and hence, we set \(L_{\text{uo}} = 0.642\) mm (Table 1). The length of the largest individual observed on the high feeding level equalled 1.008 mm, and hence, \(L_{\text{uo}} = L_m = 1.008\) mm (Table 1). Length at puberty, \(L_p\), within species is not affected by growth and maturation rate in the Kooijman–Metz growth model (even if these vary depending on food conditions). However, bulb mites are very plastic in their size at maturity in response to food conditions (Table 1). We therefore chose to relate \(L_p\) to \(L_m\) using the linear relationship (Kooijman 2000, p. 270):

\[
L_p = 0.4L_m.
\]

We took the observed \(L_p\) and \(L_{\text{uo}}\) values for both feeding levels (Table 1) and, using linear least-squares regression, estimated \(d = 0.539\) [95% confidence interval (CI): 0.134–0.945]. The basic Kooijman–Metz model assumes a constant value for \(\beta\), which, here, results in poor predictions on mite body growth and the population biology descriptors (Figs S1 and S2). Because mites show very plastic growth in response to different feeding levels (Fig. S1), we stepped away from this assumption and used a statistical approach where we linearly related \(\beta\) to feeding level and ultimate length following:

\[
\beta = 1/(\beta + \alpha L_{\text{uo}}),
\]

where the coefficient \(\alpha\) and \(\beta\) are related to parameters from the full, scaled standard DEB model: energy conductance (\(\sigma\)) and somatic maintenance (\(\beta\)) (Kooijman et al. 2008). To estimate \(\alpha\) and \(\beta\), we first estimated \(\hat{\beta}\) for each feeding level using a nonlinear least-squares estimation procedure in \(\kappa\) (nls function) (R Development Core Team 2013) to fit the equation \(L_t = L_{\text{uo}} - (L_{\text{uo}} - L_b)e^{-\frac{t}{\kappa}}\) for \(\hat{\beta}\) (see also eqn 2) against observed growth curves (Smallegange 2011), with \(L_{\text{uo}}\) and \(L_b\) fixed at the values observed for each feeding level (Table 1). We estimated \(\hat{\beta}\) at 0.083 ± 0.001 SE \((t = 74.67, P < 0.001)\) and 0.016 ± 0.0003 SE \((t = 42.36, P < 0.001)\) for the low and high feeding levels, respectively. These values, along with the observed values for \(L_{\text{uo}}\) for each feeding level (Table 1), were next used in a linear least-squares regression to estimate \(\alpha = -137.8\) and \(\beta = 151.9\) [95% (CI): 0.03–0.05 (no variation)] (eqn 15; Supporting Information). Mortality rate \(\mu\) was set constant (eqn 12), which is for the most part justified as, in juveniles, mortality is unrelated to body length and only limitedly affected by food quality, although mortality varies with body length and food quality in adults (Smallegange, Deere & Coulson 2014). Using observed female mortality rates (per day) [coded 0 (alive) or 1 (dead)], we estimated \(\mu = 0.02\) day\(^{-1}\) [95% CI: 0.01–0.03] and \(\mu = 0.04\) day\(^{-1}\) [95% CI: 0.03–0.05] for the high and low feeding levels, respectively.
The reef manta ray (Mobulidae) is one of the largest rays in the world (Marshall et al. 2011a). Mortality rate \( \mu \) is estimated every year (Marshall et al. 2011a). Based on these observations, we set \( L_0 = 130 \text{ cm} \), \( L_p = 380 \text{ cm} \), \( L_m = 550 \text{ cm} \) and \( R_m = 1 \text{ year}^{-1} \). The survival rate of juveniles and adults is estimated at 0.95 year\(^{-1} \) (Kashiwagi 2014), and we set the mortality rate constant at \( \mu = -\log(0.95) = 0.05 \text{ year}^{-1} \) (eqn 12). The von Bertalanffy growth rate is estimated for females at \( r_B = 0.18 \) (Kashiwagi 2014). We assumed \( E_Y(L(t)) = L_0 = 130 \text{ cm} \) and \( \sigma^2_Y(L(t)) = 0 \). For the starvation condition, we assume the commonly used value of \( \kappa = 0.80 \) (Add-my-pet, 2016). Again, we explored a range of expected feeding level of 0.65 ≤ \( E(Y) \) ≤ 0.95 for three values of \( \sigma(Y) \): 0.1, 0.3 and 0.5. All data are summarised in Table 1.

### Model performance

We first compared the bulb mite functions for growth and reproduction predicted for both feeding levels against independent, empirical data. We could not do test this for reef manta rays as no independent, empirical observations are available. We next used eqn 1 to create predictions of population growth rate (\( \lambda \)), lifetime reproductive success (\( R_0 \)) and generation time (\( T \)) for each species. To this end, we discretised the IPM (eqn 1) and divided the length domain \( \Omega \) into 200 very small-width discrete bins, defined as ‘mesh points’ (a higher number of bins did not produce different results). The result is a matrix that maps a vector of 200 size classes from time \( t = 0 \) to \( t + 1 \). The dominant eigenvalue of this matrix equals \( \lambda \). \( R_0 \) was calculated as the dominant eigenvalue of the matrix \( F = V(I - GS)^{-1} \), where \( I \) is the identity matrix and \( V = DR \), where \( D \) is a matrix that approximates the parent–offspring association kernel and \( R \) is a matrix that approximates the reproduction kernel (Caswell 2001). \( G \) and \( S \), respectively, are the matrix that approximates the growth function and the survival function. In the Supporting Information, we provide a detailed explanation and example of how to construct these matrices. Generation time was approximated as \( T = \log(R_0)/\log(\lambda) \), which represents the time it takes a population to increase by a factor \( R_0 \) (Caswell 2001).

We used the DEB-IPM to investigate for both species the relationship between expected feeding level and the population biology descriptors \( \lambda, R_0 \) and \( T \) for the three values of \( \sigma(Y) \): 0.1, 0.3 and 0.5. In case of the mites, we compared these relationships against observed values for mites raised on the low and high feeding levels, but also on a diet of intermediate feeding level: ad lib access to oats, for which \( E(Y) = L_{oo} \) (oats)/1000 = 0.857 (Deere, Coulson & Smalegange 2015). In case of the rays, we compared these relationships

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(Y) )</td>
<td>Expected feeding level at which DEB parameters are measured</td>
<td>0.64</td>
<td>–</td>
<td>NA</td>
<td>–</td>
</tr>
<tr>
<td>( L_0 )</td>
<td>Body length at birth</td>
<td>0.167</td>
<td>mm/cm</td>
<td>130</td>
<td>cm</td>
</tr>
<tr>
<td>( L_p )</td>
<td>Body length at puberty (maturity)</td>
<td>0.314</td>
<td>mm</td>
<td>380</td>
<td>cm</td>
</tr>
<tr>
<td>( L_m )</td>
<td>Ultimate length</td>
<td>0.642</td>
<td>mm</td>
<td>NA</td>
<td>cm</td>
</tr>
<tr>
<td>( L_{oo} )</td>
<td>Maximum length at ( E(Y) = 1 )</td>
<td>–</td>
<td>mm</td>
<td>550</td>
<td>cm</td>
</tr>
<tr>
<td>( R_m )</td>
<td>Maximum reproduction rate at ( L_m )</td>
<td>0.016</td>
<td>#/day</td>
<td>0.18</td>
<td>year^{-1}</td>
</tr>
<tr>
<td>( r_B )</td>
<td>von Bertalanffy growth rate</td>
<td>0.082</td>
<td>–</td>
<td>0.80</td>
<td>–</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Energy allocation fraction to somatic maintenance and growth</td>
<td>0.03</td>
<td>#/day</td>
<td>0.05</td>
<td>year^{-1}</td>
</tr>
<tr>
<td>( \sigma(Y) )</td>
<td>Standard deviation of expected feeding level used in simulations</td>
<td>0.1: 0.3; 0.5</td>
<td>–</td>
<td>0.1: 0.3; 0.5</td>
<td>–</td>
</tr>
</tbody>
</table>

NA, not available.


## Table 1. Dynamic energy budget (DEB) parameters for female bulb mites raised individually at low and high feeding levels (Smallegange 2011), and for female reef manta rays in a population of Yaeyama Islands, Japan (Kashiwagi 2014)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(Y) )</td>
<td>Expected feeding level at which DEB parameters are measured</td>
<td>0.64</td>
<td>–</td>
<td>NA</td>
<td>–</td>
</tr>
<tr>
<td>( L_0 )</td>
<td>Body length at birth</td>
<td>0.167</td>
<td>mm/cm</td>
<td>130</td>
<td>cm</td>
</tr>
<tr>
<td>( L_p )</td>
<td>Body length at puberty (maturity)</td>
<td>0.314</td>
<td>mm</td>
<td>380</td>
<td>cm</td>
</tr>
<tr>
<td>( L_m )</td>
<td>Ultimate length</td>
<td>0.642</td>
<td>mm</td>
<td>NA</td>
<td>cm</td>
</tr>
<tr>
<td>( L_{oo} )</td>
<td>Maximum length at ( E(Y) = 1 )</td>
<td>–</td>
<td>mm</td>
<td>550</td>
<td>cm</td>
</tr>
<tr>
<td>( R_m )</td>
<td>Maximum reproduction rate at ( L_m )</td>
<td>0.016</td>
<td>#/day</td>
<td>0.18</td>
<td>year^{-1}</td>
</tr>
<tr>
<td>( r_B )</td>
<td>von Bertalanffy growth rate</td>
<td>0.082</td>
<td>–</td>
<td>0.80</td>
<td>–</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Energy allocation fraction to somatic maintenance and growth</td>
<td>0.03</td>
<td>#/day</td>
<td>0.05</td>
<td>year^{-1}</td>
</tr>
<tr>
<td>( \sigma(Y) )</td>
<td>Standard deviation of expected feeding level used in simulations</td>
<td>0.1: 0.3; 0.5</td>
<td>–</td>
<td>0.1: 0.3; 0.5</td>
<td>–</td>
</tr>
</tbody>
</table>

NA, not available.
against the estimated value of $\lambda = 1.02$ for the Japan population (Kashiwagi 2014), the observed value of cohort generation time $T_c = 25$ years [$T_c$ is defined as defined as the mean age at which adult reproduce and calculated, very roughly, by taking the mean of the minimum and maximum adult age: $(10 + 40)/2 = 25$ years], and $R_0 = \exp [T_c \times \log(\lambda)] = 1.6$. Note that, for both species, we compared predicted generation time $T$ (calculated from $\lambda$ and $R_0$ estimated from the DEB-IPM) against observed cohort generation time $T_c$ as no observations on $T$ exist. However, these two measures are often similar (Caswell 2001).

Results

MODEL PERFORMANCE

It is rewarding to see that the von Bertalanffy growth function that is derived from the Kooijman–Metz DEB model fits the empirical observations on bulb mite growth well for each feeding level (Fig. 1a,b). The function describing bulb mite mean reproduction overestimates independent observations (independent because these data were not used to estimate any of the DEB parameters) on reproduction rates at the low feeding level (Fig. 1c). However, for the high feeding level, predicted mean reproduction rates match independent observations on mean bulb mite reproduction (Fig. 1d: solid and dotted lines are similar).

Unsurprisingly, both $\lambda$ and $R_0$ increased with increasing feeding level for both bulb mites and reef manta rays (Fig. 2a–d). For the mites, predicted increases in $\lambda$ and $R_0$ followed empirical observations (Fig. 2a,c). Even though individual reproduction rates were overestimated for large mites at $E(Y) = 0.64$ (Fig. 1c), this did not result in an overestimation of $\lambda$ at $E(Y) = 0.64$ (Fig. 2a), but did overestimate $R_0$ (Fig. 2c) and thereby $T$ (Fig. 2e) at $E(Y) = 0.64$. Predicted $R_0$ at $E(Y) = 0.95$ did not significantly differ from observed $R_0$ for the mites (inferring from the fact that the predicted value is within the 95% CI of the observed value) (Fig. 2c). In case of the rays, predicted $\lambda$ and $R_0$ overlapped with observed values at feeding levels $E(Y)$ between 0.7 and 0.8 (Fig. 2b,d). Generation time for both species decreased with increasing feeding level (Fig. 2e,f). In case of the mites, only at the highest feeding level did the predicted value match observed cohort generation time $T_c$ (Fig. 2c). In case of the rays, predicted $T$ matched observed

![Fig. 1. Bulb mite demographic functions for growth and reproduction at time $t + 1$ in relation to body length at time $t$ for the low and high feeding levels. Lines are predictions and symbols are observations (Smallegange 2011). In panels (a) and (b), filled symbols and solid lines, respectively, denote when observed growth and growth predicted by the dynamic energy budget-integral projection model (DEB-IPM) function for growth is greater than zero; open symbols and dotted lines, respectively, denote when observed and predicted growth is equal to or lower than zero. In panels (c) and (d), solid lines represent DEB predictions on reproduction rates, and a linear regression fit is also plotted (dotted lines) to show how mean reproduction rate relates to body length.](image)

![Fig. 2. Population growth rate $\lambda$ (a (day$^{-1}$), b (year$^{-1}$)), lifetime reproductive success $R_0$ (c, d) and generation time $T$ [e (days), f (years)] in relation to feeding level $E(Y)$ as predicted for three values of $\sigma(Y)$: $\sigma(Y) = 0.1$ (solid lines); $\sigma(Y) = 0.3$ (dashed lines) and $\sigma(Y) = 0.5$ (dotted lines) for bulb mites (left-hand panels) and reef manta rays (right-hand panels). Grey symbols in (a), (c) and (e) are observed values for mites on low $[E(Y) = 0.64]$; intermediate $[E(Y) = 0.85]$ and high feeding levels $[E(Y) = 0.95]$. Observed values in (a) are calculated by taking the exponent of $\log(R_0)/T_c$ (Caswell 2001), where $T_c$ is the observed cohort generation time for bulb mites (Smallegange 2011; Deere, Coulson & Smallegange 2015). Vertical lines through the symbols in (b) and (e) are 95% CIs. Grey lines in (b), (d) and (f) are observed values for reef manta rays (Kashiwagi 2014): $\lambda = 1.02$, $T_c = 25$ years, and $R_0 = \exp[T_c \times \log(\lambda)] = 1.6$. See Table 1 and text for parameter values.](image)
at feeding levels of $E(Y) \approx 0.75$ (Fig. 2f). Variation in feeding level, $\sigma(Y)$, had little effect on how each population descriptor varied with changing feeding level in case of the mites; in case of the rays, however, an increase in $\sigma(Y)$ decreased $\lambda$ and $R_0$ across the range of explored feeding levels (Fig. 2).

**Discussion**

**LESSONS FROM THE BULB MITE AND REEF MANTA RAY APPLICATIONS**

Our application of the DEB-IPM to bulb mites and reef manta rays demonstrates the ease with which the framework can be parameterised and analysed. In case of the mites, there was a satisfactory match between predicted and observed individual growth and reproduction rates, except for reproduction by large individuals at low feeding levels. The latter mismatch was of little consequence to $\lambda$, but did result in a predicted $R_0$ that was much higher than what we observed. The latter discrepancy carried over to create a mismatch at the same low feeding level between observed and predicted generation times $T_c$ and $T$, as $T$ was calculated from the values of $R_0$ (and $\lambda$) predicted by the model. Possibly, because bulb mites are very plastic in their life-history trajectories, the assumption of a constant $\kappa$ and $L_{\text{max}}$, and hence constant $R_m$ and $L_m$ (both of which are used to calculated reproduction rates), is violated. The DEB-IPM did capture the patterns of how $\lambda$, $R_0$ and $T$ changed with changing feeding conditions in bulb mites, but predictions did not always fall within the CIs of data observations. To create such precise population forecasts across all feeding levels, alternative (DEB) models could be explored.

In case of the reef manta ray, we had few life-history data and no information on experienced feeding levels in the wild. However, we can glean some clues from the model output regarding experienced feeding levels. Most strikingly, values of $\lambda$ were much higher than what we observed. The latter discrepancy carried over to create a mismatch at the same low feeding level between observed and predicted population descriptors $\lambda$, $R_0$ and $T$ each overlapped between feeding levels of 0.7–0.8; this overlap increases confidence in the model’s performance. A feeding level of around 0.7–0.8 suggests that reef manta rays experience feeding levels whereby their gut is considered ‘just filled’, on a scale between empty and bursting (Piet & Guruge 1997). We purposefully applied the DEB-IPM to a stable population of reef manta rays, but many populations across the world are in decline, mainly due to overfishing and bycatch (Marshall et al. 2011a). Parameters in the DEB-IPM all have biological definitions (Table 1); an elasticity analysis targeting lower feeding levels could therefore aid in identifying which life-history processes should be prioritised in conservation research and management.

**MODEL EVALUATION: STRENGTHS AND WEAKNESSES**

The key strength of models based on DEBs is that they incorporate trade-offs in energy use through the $\kappa$-rule of the DEB core. This mechanistic underpinning means that the DEB-IPM is a promising tool to forecast population responses to novel or stochastically varying environmental conditions that predominantly affect growth and reproduction. To do so, using standard IPMs forces one to assume that the phenomenological demographic functions estimated for the original (constant or stationary) environment apply to individuals in the novel environment. If this assumption does not hold, for example when reproduction rates and/or size distributions differ greatly between the original and novel environment or when they vary greatly as environmental conditions vary over time, then extrapolation errors could create a substantial mismatch between predicted and actual population responses to environmental change. This is particularly likely to occur in organisms with plastic life histories; a recent stochastic, standard IPM applied to bulb mites indeed did not meet the assumption that demographic functions apply across all environments (Smallegange, Deere & Coulson 2014). A similar rationale holds for modelling density dependence under temporally varying or novel conditions. In standard IPMs, density is included as a term in the regression models describing demographic rates (Ellner, Childs & Rees 2016), again under the assumption that density dependence acts the same in the original and novel environment. Extending the DEB-IPM to include resource dynamics (one example of which is in the Supporting Information) introduces a mechanistic underpinning to density dependence that operates through resource limitation due to feedback between consumer and resource dynamics. Another strength of DEB-IPMs is that differences between species in their energy budget are mostly due to differences in their set of DEB parameters, facilitating comparative studies of character and population dynamics between species. Here, for example, we found that varying the variance in expected feeding level, $\sigma(Y)$, had little effect on predicted population growth rate and lifetime reproductive success in mites (lines in Fig. 2a,c are very similar and sometimes overlap), but did result in different values for each expected feeding level $E(Y)$ in case of the reef manta ray (lines in Fig. 2b,d are widely spaced). What drives these differences remains to be explored. Furthermore, unlike standard IPMs, no long-term individual-level data scored from birth to death are required for model parameterisation; instead, to run the DEB-IPM, one needs only seven parameter estimates ($L_m$, $L_p$, $L_{\text{max}}$, $R_m$, $R_0$, $f_0$, $\kappa$ and $\mu$). This is of particular benefit to study systems where it is difficult to track the life history of the same individual (e.g. micro-organisms, small (soil-dwelling) animals). Even if limited or no DEB data are available, one can resort to the add-my-pet data collection and use DEB parameter estimates of related species as a starting point (Add-my-pet 2016).

For the purpose of this study, we chose to use the most simple DEB growth model, but more complex alternatives exist. For example, within the ‘standard’ animal DEB model (Sousa et al. 2010), individuals can build up reserves and use them when starving (in the Kooijman–Metz model, individuals die instantaneously when they cannot cover maintenance costs). This model has been simplified to the ‘scaled standard model’ (Kooijman et al. 2008), but the use of scaling and compound parameters hampers the interpretation of equations (Jager, Martin & Zimmer 2013). In turn, Jager, Martin &...
individual-based DEB model that includes reserve dynamics. However, in a test of how well an element of the moment they cannot meet their energy requirements for starvation by using reserves. Here, in a test of how well an individual-based DEB model that includes reserve dynamics describes population patterns, Martin et al. (2013) concluded that the dynamics of starvation and recovery are too poorly understood to accurately predict population fluctuations. Only by including extra assumptions on food-dependent mortality were they able to match model predictions to independent observations from population experiments. Here, we have set a first step towards developing a more mechanistic IPM based on DEB theory. Although extending our framework is complex, we emphasise that structured population models, of any mathematical form, may benefit from the incorporation of mechanisms. For example, Alver et al. (2016) include food, energetics and temperature dependence in a partial differential equation model for marine copepods, coupled to ocean circulation. The possibilities for inclusion of mechanism are potentially endless.

OUTLOOK: EVOLUTIONARY CHANGE

The DEB-IPM, like most IPMs, does not include genes and hence does not explicitly incorporate evolution. Simple evolutionary insight can still be obtained through the estimation of evolutionary quantities such as the strength of viability and fertility selection (Smallegange & Coulson 2013), which are calculated as the difference in population-level mean body length after and before survival, or after and before reproduction, respectively. When evolutionary change in character values takes many generations, analysis of a DEB-IPM will likely provide insight into current population, life history and character dynamics. To study long-term evolutionary change, one could apply evolutionary game theory to a DEB-IPM to study the endpoints of trait evolution (e.g. Childs et al. 2011), but, then, the ecological and evolutionary time-scales are decoupled. Coulson et al. (2016) recently developed a framework that explicitly couples evolution and ecology by linking quantitative genetics and IPMs. For systems where the rate of evolutionary change in character distributions is likely to be (fairly) close to the rate of change observed on ecological time-scales, the latter framework would be highly suited to study change in phenotypic character distributions. If, in such cases, one is interested in the ecological and evolutionary dynamics of body size and its role in population and community dynamics, one would profit from incorporating the DEB-IPM into this framework. Such an endeavour would be particularly insightful in studies that focus on the eco-evolutionary dynamics of parameters that play an important role in individual life histories, such as size at birth or at maturity. This would require a multivariate DEB-IPM where the dynamics of each of these characters is modelled alongside the dynamics of body size. The accompanying challenges in doing so, however, at the moment, makes this a distant goal.

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Data accessibility

Uploaded as supporting information is a pdf-file explaining how to model growth and reproduction, the DEB-IPM (including a toy example) and a consumer-resource DEB-IPM. MatLab scripts to run the DEB-IPM and r-scripts and data files to estimate the bulb mite DEB parameters can be downloaded from: https://figshare.com/s/3432986dfl8id8b431ee.

References


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**Supporting Information**

Additional Supporting Information may be found online in the supporting information tab for this article:

**Appendix S1.** Which includes:

1 Modelling growth and reproduction from the Kooijman–Metz model.
2 Consequences of variable von Bertalanffy growth rate in mites.
3 Additional assumptions of the DEB-IPM: when individuals do not shrink.
4 Building the DEB-IPM: a detailed explanation and toy example.
5 Consumer-resource DEB-IPM: construction and a bulb mite application.