Appendix of Chapter 6

The LQL model has a linear-quadratic shaped log-survival curve \( ad + \beta d^2 \) below a threshold dose \( d_r \) and a linear shaped log-survival curve \( \lambda d + \delta \) above \( d_r \). The LQL model and its derivative are continuous at the threshold dose:

\[
\lambda d_r + \delta = ad_r + \beta d_r^2 \\
\lambda = a + 2\beta d_r
\]  

Substituting equation 2 into equation 1 yields:

\[
\delta = ad_r + \beta d_r^2 - ad_r - 2\beta d_r^2 = -\beta d_r^2
\]  

The effect \( E_{LQL} \) of the total dose \( D \) given in \( n \) fractions of dose per fraction \( d \) exceeding \( d_r \) is thus given by:

\[
E_{LQL} = n(\lambda d + \delta) = n(a + 2\beta d_r)d d_r^2 = D \alpha + 2\beta d_r - \frac{\beta d_r^2}{d}
\]  

With the NTD [15] defined as the total dose given in 2-Gy fractions having an equivalent effect (as determined by the LQ model) as predicted by the LQL model, i.e., \( E_{LQL} = E_{NTD} = NTD(\alpha + 2\beta) \), the NTD can be calculated as:

\[
NTD = \frac{E_{LQL}}{\alpha + 2\beta} = D \frac{\alpha \beta + 2d_r - \frac{d_r^2}{d}}{2 + \alpha \beta}
\]