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Constrained Registration of the Wrist Joint

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Abstract—Comparing wrist shapes of different individuals requires alignment of these wrists into the same pose. Unconstrained registration of the carpal bones results in anatomically nonfeasible wrists. In this paper, we propose to constrain the registration using the shapes of adjacent bones, by keeping the width of the gap between adjacent bones constant. The registration is formulated as an optimization involving two terms. One term aligns the wrist bones by minimizing the distances between corresponding bone surfaces. The second term constrains the registration by minimizing the distances between adjacent sliding surfaces. The registration is based on the Iterative Closest Point algorithm. All bones are registered concurrently so that no bias is introduced towards any of the bones. The proposed registration method delivers anatomically correct configurations of the bones. The registration errors are in the order of the voxel size of the acquired CT data ($0.3 \times 0.3 \times 0.3 \text{ mm}^3$). The standard deviation in the widths of gaps between adjacent bones is in the order of 10% with an insignificant bias. This is a large improvement over the standard deviations of 30%–80% encountered in unconstrained registration. The value of this method is its capability of accurately registering joints in varying poses resulting in physiological joint configurations.

Index Terms—Articulated registration, constrained optimization, image processing, intersubject registration, surface registration, wrist.

I. INTRODUCTION

THE wrist is a complicated and vulnerable joint. It consists of eight carpal bones and numerous articulating surfaces and ligaments. Carpal instability and arthrosis reduce the wrist functionality of a large part of the population [1], [2]. An early diagnosis of the pathology enables an early treatment, leading to a better chance of recovery and a reduction in social costs due to lost labor productivity [3].

It is generally acknowledged that motion patterns of the carpal bones are useful for diagnosis and classification of

carpal instability. Currently, motion patterns are visually assessed by video fluoroscopy. Unfortunately, the complexity of the wrist joint makes it often hard for radiologists and hand surgeons to visually identify pathological motion patterns. The basis of this problem is that an objective reference of healthy carpal bone motions is unavailable. Previous studies [4]–[6] hypothesize correlations between the motion patterns of bones and the shapes of adjacent bone surfaces in ankles and knees. We intend to study this hypothesis in the wrist by building a multibody Statistical Shape Model (SSM) [7]–[9] that represents simultaneous variations in carpal bone shapes and motion patterns. As a first step, we will build a model that represents the simultaneous shape variations of the eight carpal bones, while preserving the physiological spatial relations. A crucial step in creating such a model is the registration of a series of example wrists. We will show that the circumstance of having several objects in close proximity complicates a trivial solution.

A. Multiple Object Registration

Several registration strategies were proposed for a multibody SSM. The most straightforward approach is to apply rigid registration to each body individually. While doing so, pose differences of all objects are minimized, but the initial spatial relations between the bodies are inherently lost. Another obvious approach is to use a single transformation to simultaneously register all bodies in a rigid fashion [7], [10]–[12]. The latter method implicates that spatial relations are fixed. This is not valid for the wrist, however. A combination of these two solutions was applied to vertebrae [13] and brain structures [14]. Both references assume that posture variations between adjacent objects are negligible, but objects further apart can move. In a previous study [15], we showed that the carpal bones (wrist bones), have nonnegligible relative motions. Applying registration of each bone separately may lead to a violation of physiological relations, e.g., bones overlapping after registration or gaps between bones becoming too narrow, too wide, or skewed [15]. This is a common problem in the registration of bony structures. It has been proposed [16] to approach a joint as a non-rigid structure where bone shapes are preserved by rigidity constraints [16], [17] and incompressible structures by volume preserving constraints [18]. A volume preserving constraint, however, cannot prevent skewed volumes between adjacent bones. In [15] and [19], an articulated model, a model of rigid bodies that are linked by a set of rotations and translations, was used for the registration of wrist and mouse bones, respectively. Earlier, Du Bois d'Aische [20] used a comparable kinematical model for the registration of a string of vertebrae, combined with a stiffness constraint. The methods in [15] and [19] comprise a user-specified registration order that (inherently) influences the

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registration result. What is more, the articulated models assume accurately defined axes of rotation. Due to irregularly shaped bone surfaces, not all axes may be estimated reliably. This could result in an *a priori* introduced systematic error.

B. Restricted Movement in Multibody Registration

In this paper, we propose a novel way of constraining the piecewise rigid registration of multiple objects in close proximity. Pair wise sliding surfaces are defined along which object movement is restricted. These sliding surfaces are determined on the basis of a single image and form an implicit kinematical model. We mathematically formulate the solution as an optimization problem that includes the physical constraints as a penalty function. The method is applicable to any multiple object registration problem in which the object shapes determine the permitted motion patterns. We will show that our solution applies well to the wrist. We will experimentally study the accuracy and precision of the proposed method.

II. METHODS

Our method registers all n bones in a source wrist to those of a “target” wrist. We approach constrained registration as an optimization problem with a cost function that consists of two terms, balanced by a parameter w to determine the influence of the constraints on the final registration

$$J_{tot}(s, T_{1,...,n}) = J_{ext}(s, T_{1,...,n}) + wJ_{int}(s, T_{1,...,n}) \quad (1)$$

where both J_{ext} and J_{int} are functions of a global scaling s and the rigid transformations $T_{1,...,n}$ for all n bones in the registered wrist.

- 1) The term J_{ext} for the external relations that align corresponding bone surfaces in the source and target wrists. J_{ext} is minimal when the posture differences between the bones in the source and target wrist are minimal.
- 2) The term J_{int} for the internal relations that restrict the joint bone movement within the source wrist. An essential assumption is that adjacent bones slide along each other. The cost function J_{int} preserves anatomical feasibility and is minimal when the sliding surfaces touch (see Section II-C). The registration method minimizes (1) by taking the following steps (see Fig. 1).

- 1) Detection of the bone surfaces in both the source and target wrists.
- 2) Definition of the sliding boundary surfaces, based on the detected bone surfaces.
- 3) Estimation of external relations in order to construct J_{ext} .
- 4) Estimation of internal relations to build J_{int} .
- 5) Minimization of (1) using Levenberg–Marquardt.
- 6) Steps 3 to 5 are repeated until J_{tot} decreases by less than a user-defined threshold.

In Sections II-A–E, we describe wrist anatomy and consider the steps of the constrained registration method in detail. Steps 3 to 6 involve an extension of the Iterative Closest Point (ICP) algorithm [21] by implementing concurrent registration of all bones.

Fig. 1. Flow chart of the constrained registration method.

Fig. 2. Wrist bones of a right hand, depicted in a CT image (a), an anatomical slice of a detail of the wrist (b), and the rotation axes of the wrist (c). The abbreviations of the bone names are explained in Table I. The Pisiforme and Metacarpals 1 and 5 are not visible in (a). The rotations are named Flexion (F), Extension (E), Radial deviation (R), Ulnar deviation (U), Pronation (P), and Supination (S).

A. Wrist Anatomy

The wrist joint consists of eight carpal bones, located between the radius and ulna in the arm and the metacarpals in the hand. Fig. 2(a) shows a CT image of the bones in a wrist joint and the names of the carpal bones. In the anatomical slices of the wrist in Fig. 2(b) it may be observed that the bones are separated by two cartilage layers of approximately equal thickness and an intra-articular layer of fluid [22]. The cartilage–fluid boundaries are the structures that slide along each other when the bones assume different postures. Ligaments keep the cartilage surfaces

TABLE I
WRIST BONE NAMES

Abbreviation	Name
Ra	Radius
Ul	Ulna
Sc	Scaphoid
Lu	Lunate
Tr	Triquetrum
Pi	Pisiforme
Td	Trapezoid
Tz	Trapezium
Ca	Capitate
Ha	Hamate
Mc1-Mc5	Metacarpal 1-5

Fig. 3. Bone gap is shown as the light shaded volume in which the gradient vectors of the signed distance transforms are approximately oppositely oriented. The surface S is the defined sliding surface.

adjacent [23]. Cartilage and ligaments are invisible in a CT scan without contrast agents. To describe wrist postures we follow the orthopaedic literature [24] and use the three dominant rotation axes in Fig. 2(c).

B. Detection of Bone Surfaces (Step 1)

We segment the bones using a Geodesic Active Contour Level set segmentation [25], [26]. Using marching cubes [27] we extract the triangulated bone surface as the zero isovalue level of the resulting signed distance transform. All segmented bone surfaces were labeled semi-automatically according to Table I.

Definition of Sliding Surfaces (Step 2)

We assert that a “sliding surface” may be defined for each bone pair and that these sliding surfaces move along each other during registration. The sliding surfaces are defined as the surfaces that are initially at equal distance between adjacent bones. Consider, for instance, the lunate and capitate (see Fig. 3 for a schematic representation). Initially, the sliding surfaces of the two bones coincide and are right in the middle of the joint gap. It is assumed that the two sliding surfaces remain in close proximity as movement is imposed by the registration process.

Practically, the sliding surfaces between two adjacent bones are defined by the zero level set after subtracting the two distance transforms of the bone surfaces. Initially these coinciding surfaces are delimited on the inside of the bone gap, which is the volume between the bone surfaces where these surfaces are approximately parallel. Effectively, the bone gap is determined by the light shaded volume in Fig. 3 in which the direction of

the gradient vectors of the distance transforms deviate less than an angle α from 180° . Separate copies of the detected surface S are associated to each of the adjacent bones.

C. Estimation of External Relations (Step 3)

Consider two triangulated bone surfaces of the same bone type (e.g., hamate or capitate), but from different wrists: source surface A and target surface B . We approximate the distance between A and B as follows [21]. Surface A is approximated by a set of N_a points $\{\mathbf{a}_j\}$, which are uniformly distributed over the surface A . For all points $\{\mathbf{a}_j\}$, we find the points $\{\mathbf{b}_{a,j}\}$ on surface B for which the Euclidean distance d between $\mathbf{a}_j$ and $\mathbf{b}_{a,j}$ is minimal and take these as corresponding points

$$d(\mathbf{a}_j, B) = \min_{\mathbf{b}_{a,j} \in B} \|\mathbf{b}_{a,j} - \mathbf{a}_j\|. \quad (2)$$

Finding closest points on a triangulated surface is computationally expensive. We have used a kd-tree based parameterization of the triangulated target surface [28] to reduce the computational cost to $O(N \log N)$.

To avoid a possible bias introduced by asymmetry, an additional set of N_b points $\{\mathbf{b}_j\}$ on B and a corresponding set of closest points $\{\mathbf{a}_{b,j}\}$ on surface A are determined. Similar to [29] and [30] we define the symmetric cost function

$$J(s, T, A, B) = \frac{1}{2N_a} \sum_{j=1}^{N_a} d(sT(\mathbf{a}_j), B)^2 + \frac{1}{2N_b} \sum_{j=1}^{N_b} d(\mathbf{b}_j, sT(A))^2 \quad (3)$$

in which each bone in the source wrist is transformed with a rigid transformation T and a scaling s .

The cost function J_{ext} is the sum of the cost functions of all $n = 8$ bones in the source wrist

$$J_{\text{ext}}(s, T_1, \dots, T_n) = \sum_{i=1}^n J(s, T_i, A_i, B_i) \quad (4)$$

where T_i is the rigid transformation of surface A_i in the source wrist and B_i is the corresponding bone in the target wrist. Note that the scaling s is global. It may be noticed that due to the normalizations in (3) (i.e., the division by N_a and N_b), all relations between corresponding bones are weighted equally in (4).

D. Estimation of Internal Relations (Step 4)

The internal relations are used as a penalty function in (1) to restrict the bone movement by minimizing the distances between two sliding surfaces and to preserve a physiological configuration. In a procedure equivalent to the one in the previous section, we approximate the sliding surfaces U and V by the sets of points $\{\mathbf{u}_j\}$ and $\{\mathbf{v}_j\}$ and find corresponding points $\{\mathbf{v}_{u,j}\}$ and $\{\mathbf{u}_{v,j}\}$ for which the distance $d(\mathbf{u}_j, U)$ from (2) is minimal.

The sliding surfaces are “open.” As explained in Fig. 4, this can result in multiple points pairing with points at the edges, yielding bad convergence due to overweighted edge points [31]. We follow Turk [32] by discarding all point pairs (the dashed arrows) of which the estimated closest point corresponds to an edge point. After the removal of these point pairs, $\tilde{N}_u$ points

Fig. 4. Rejection of point pairs where one of the points lies on the edge of the surface. The points on U are the points that are matched with the closest points on surface V . Closed points lie on the surface edge. Arrows show closest point correspondence relations. Dashed arrows show rejected point pairs.

Fig. 5. Function β (continuous line) for $\tau = 2.0$ and $s = 1$ as a function of d . The dashed line is d and shows that β approximates d for $d \rightarrow 0$.

$\{\mathbf{u}_j\}$ paired with $\{\mathbf{v}_{u,j}\}$ and $\tilde{N}_v$ points $\{\mathbf{v}_j\}$ paired with $\{\mathbf{u}_{v,j}\}$ remain.

The maximum distance between two sliding surfaces U and V is limited to a threshold τ to guarantee a maximum widening or narrowing of a bone gap. This is realized by a penalty function β [33]. The strongly nonlinear penalty function depends on the distance between U and V , the parameters s and T , and the threshold τ

$$\beta(sT(\mathbf{u}_j), V, \tau) = -\tau \ln \left(1 - \frac{d(sT(\mathbf{u}_j), V)}{s\tau} \right). \quad (5)$$

At $d(sT(\mathbf{u}_j), V) \rightarrow 0$, $\beta(sT(\mathbf{u}_j), V)$ approximates $d(sT(\mathbf{u}_j), V)$ (see Fig. 5). The symmetric function that gives the cost of the distances between surfaces U and V is

$$J_\beta(s, T, U, V, \tau) = \frac{1}{2\tilde{N}_u} \sum_{j=1}^{\tilde{N}_u} \beta(sT(\mathbf{u}_j), V, \tau)^2 + \frac{1}{2\tilde{N}_v} \sum_{j=1}^{\tilde{N}_v} \beta(\mathbf{v}_j, sT(U), \tau)^2 \quad (6)$$

with scaling s and rigid transformation T . The cost function of all internal relations, J_{int} is the sum of the functions (6) for all m sliding surface pairs.

$$J_{\text{int}}(s, T_1, \dots, T_n) = \sum_{l=1}^m J_\beta(s, T_l, U_l, V_l, \tau_l) \quad (7)$$

where $T_l \in \{T_1, \dots, T_n\}$ is the rigid transformation of the bone to which the sliding surface U_l is attached. The parameter τ_l is the maximum distance between the sliding surfaces U_l and V_l . Note that the scaling s is global.

Fig. 6. Point-to-plane distance between points on A and planes tangent to B .

TABLE II
PARAMETERS IN THE CONSTRAINED REGISTRATION ALGORITHM

Parameter	Description
α	Maximum angle between gradient vectors of distance transforms at sliding surface
σ_d	Scale of the Gaussian kernel on which the gradient vectors of the distance transform are determined
N_a, N_b, N_u, N_v	Numbers of points sampled on bone and sliding surfaces
w	Weighting between internal and external relations
τ	Maximum bone gap width deviation

E. Minimization of Total Cost Function (Step 5)

At each iteration of the ICP algorithm, (1) is minimized by Levenberg–Marquardt (LM) optimization. All transformations $T_{1,\dots,n}$ and the scaling s are optimized simultaneously.

As shown in [31] and [34], approximating surface B with the points $\{\mathbf{b}_{a,i}\}$ and minimizing the distances between $\{\mathbf{a}_i\}$ and $\{\mathbf{b}_{a,i}\}$ as in the original ICP algorithm leads to slow convergence and susceptibility to local minima. Instead, we follow Chen and Medioni [34] and approximate surface B using planes tangent to B in the paired closest points $\{\mathbf{b}_{a,i}\}$. Equation (2) is replaced by the point-to-plane distance between the points $\{\mathbf{a}_i\}$ and the tangent planes at $\{\mathbf{b}_{a,i}\}$ to approximate the distance between points $\{\mathbf{a}_i\}$ and surface B during the LM optimization. Fig. 6 illustrates the first order surface approximation. In [34] and [31] it is shown that a first order approximation is reasonable for smooth surfaces. We approximate both the distances between bone surfaces as well as the sliding surfaces using the point-to-plane distance.

F. Parameters

The preprocessing of the data and the constrained registration algorithm depend on the parameters listed in Table II. The segmentation parameters are chosen, based on visual inspection, as is the maximum angle $\alpha = 5^\circ$ between the gradient vectors of the distance transforms in the sliding surface definition (Section II-C). The chosen values for the segmentation parameters and α are not critical to our method. The distance transforms are smooth images and therefore the scale of the Gaussian kernel on which its gradient vectors are computed is chosen to be small, $\sigma_d = 1$.

The choice of numbers of points N_a and N_b on bone surfaces and N_u and N_v on sliding surfaces is not critical either. Experimentally, we determined in Section III-B the optimal values as a tradeoff between registration accuracy and computation time. The constraining weight w is chosen as the maximum value for which the algorithm converges in Section III-F.

The maximum bone gap width deviation of τ is experimentally determined for each wrist and each gap in Section III-E. For each value of τ , a normalized value $\hat{\tau}$ is computed, as τ

divided by the corresponding mean bone gap width. These normalized deviations allow comparison between different wrists. The values $\bar{\tau}$ are the averaged normalized values of $\hat{\tau}$ over a set of wrists. The maximum bone gap width deviation in a newly acquired wrist is estimated as the bone gap width times $\bar{\tau}$.

III. EXPERIMENTS

The validity of our approach was tested by means of several experiments.

- 1) The accuracy and the precision of an unconstrained registration as a function of the number of points C was assessed (Section III-B). Based on this experiment, the number of points C was set for the rest of the experiments.
- 2) The influence of the noise in the data on the segmentation and unconstrained registration result was determined (Section III-C).
- 3) The assumption of constant bone gap width was verified by measuring the width of bone gaps in different postures of the wrist (Section III-D).
- 4) The validity of the defined sliding surfaces as descriptions of motion patterns was studied. The parameter $\bar{\tau}$ (see Section II-G) was also determined in these experiments (Section III-E).
- 5) The accuracy of constrained registration of different poses of a single wrist as a function of the constraining weight w in [(1)] was determined (Section III-F).
- 6) The accuracy of constrained registration of different wrists in different poses was assessed and it was verified if anatomical feasibility was preserved for a range of constraining weights w (Section III-G).

Each surface was sampled with $N = 5000$ uniformly distributed points, solely to perform distance measurements. N was chosen arbitrarily large, compared to the number of points C .

A. Data

The proposed method was tested on CT data of five wrists. Scans were made of each wrist in four different postures: neutral, 10° extension, 20° extension and 40° extension (and 0° rotations about the other axes. See Fig. 2 for the definition of the rotation axes.) A special posture device was designed to impose the aforementioned angulations [24]. The images were acquired on an Mx8000 Quad CT scanner (Philips Medical Systems, Best, The Netherlands). The acquisition parameters were: collimation 2×0.5 mm, tube voltage 120 kV, effective mAs 75, rotation time 0.75 s per 360°, pitch 0.875; the scans were made in “ultra high resolution” mode (i.e., small focal spot size). Reconstructions were made with convolution kernel E, a field of view of 154 mm, a slice increment of 0.3 mm and a matrix of 512×512 pixels, resulting in isotropic voxels of $0.3 \times 0.3 \times 0.3$ mm.

B. Accuracy and Precision of Registration as a Function of C

A single wrist was selected and a copy of each bone surface was transformed with a random, rigid transformation T . Thus, these two surfaces are identical except for the transformation. Subsequently, unconstrained registration was performed

involving C randomly generated points by minimizing (1) with $w = 0$ and $N_a = N_b = C$. Eventually, each bone delivered a transformation T' . The accuracy and precision are assessed by computing the mean Euclidean distances d_C and corresponding standard deviations between the points $P' = T'TP$ and P , in which P is a sampling of the input bone shape of $N = 5000$ points. For all values of $C \in \{200, 400, \dots, 2000\}$, the mean distance d_C in the order of 0.05 mm was well below the voxel size of 0.3 mm. In the next experiments, we choose $C = 1000$ as a safe value above which d_C improved negligible while maintaining an acceptable computation time.

C. Accuracy and Precision of Segmentation and Unconstrained Registration

One wrist was scanned five times in a single posture (neutral) and all the bone surfaces were segmented. Subsequently, each segmented wrist was registered in an unconstrained fashion (minimize (1) with $w = 0$) to the same bone in the other images, resulting in 10 ($= 4 + 3 + 2 + 1$) such registrations. Finally, the mean Euclidean distances d_s (see (2)) and corresponding standard deviations between the registered surfaces were computed. The values found for d_s were in the order of 10^{-4} mm, with standard deviations of about 0.02 mm. This indicates that the segmentation and registration method is unbiased. Moreover, the standard deviations are also well below the voxel size. One may conclude that the standard deviation due to the image noise is approximately 10% of the voxel size. Note that d_C and d_s cannot be compared directly as d_C is a ground truth measurement between corresponding points, where d_s is a precision measurement between adjacent surfaces.

D. Variations in Bone Gap Width

Remember that the sliding surface S was defined in the middle of the gap between adjacent bones (see Fig. 3). The adjacent bones are denoted by A and B . S was sampled with $N = 5000$ uniformly distributed points $\{s_i\}$ and the sets of corresponding closest points on both adjacent bone surfaces $\{a_{s,i}\}$ on A and $\{b_{s,i}\}$ on B were determined. The point sets $\{a_{s,i}\}$ and $\{b_{s,i}\}$ contain pairs of points that lie on the opposing bone surfaces that adjacent to the gap. The mean Euclidean distance d_{ab}^j between corresponding points in $\{a_{s,i}\}$ and $\{b_{s,i}\}$ was computed for each gap of wrist $j \in \{1, \dots, 5\}$ in each pose (neutral, 10°, 20° and 40° extension). To compensate for anatomical variations, all distances were normalized to the mean width of the gap in the same wrist in the neutral pose $\hat{d}_{ab}^j = d_{ab}^j / d_{ab}^{j, \text{neutral}}$. For each bone gap in each pose, the average gap width was computed over $n = 5$ wrists as $\bar{d}_{ab} = 1/n \sum_{j=1}^n d_{ab}^{j, \text{neutral}}$ and the standard deviation as $1/n \sum_{j=1}^n d_{ab}^{j, \text{neutral}}$ times the standard deviation of $\hat{d}_{ab}^j$. Fig. 7 shows the mean Euclidean distance $\bar{d}_{ab}$ as well as the corresponding standard deviation for several gaps.

The increasing standard deviation for larger angles of extension demonstrates that differences in bone gap width increase slightly with increasing extension angle. Still, except for the Triquetrum–Hamate gap, none of the changes is statistically significant. This shows that the assumption of constant bone gap width is reasonable. One might notice that there may be some

Fig. 7. Means $\bar{d}_{ab}$ as a function of the wrist posture: neutral, 10°, 20°, and 40° extension.

width variation within a bone gap as a function of posture. Effectively, the latter influence is studied in the next experiment.

E. Validation of the Sliding Surfaces Assumption

This experiment aims to validate the assumption that the sliding surfaces correctly describe the motion paths. This is shown by measuring the distances between the sliding surfaces after an unconstrained registration. For aligning different poses of a *single* wrist, the unconstrained registration is optimal, as bone shapes do not vary between poses and no error is introduced by applied constraints.

The bone surfaces were segmented in a CT scan of the wrist in the neutral posture (A) and a CT scan of the same wrist in some extension (A'). Sliding surfaces were defined in A' . Subsequently, A' was registered to A without constraints [effectively using $w = 0$ in (1)]. This experiment was repeated for five wrist under three extensions, 10°, 20°, and 40°.

Then, the mean point-to-plane distances d_{uv}^j were computed between the adapted sliding surfaces as well as the corresponding standard deviation for each gap of wrist $j \in \{1, \dots, 5\}$ in each pose. This point-to-plane distance is a signed distance: a positive distance corresponds to widening of the bone gap and a negative distance to narrowing. The “sliding surface” distance d_{uv}^j was normalized by the mean bone gap width of the wrist in neutral pose: $\bar{d}_{uv}^j = d_{uv}^j / d_{ab}^{j, \text{neutral}}$ to remove anatomical variations. Fig. 8 shows the average normalized sliding surface distance $\bar{d}_{uv} = 1/n \sum_{j=1}^n \bar{d}_{ab}^{j, \text{neutral}} \cdot 1/n \sum_{j=1}^n \bar{d}_{uv}^j$ over five wrists for several extensions and joints as well as the corresponding standard deviations.

Comparing Figs. 7 and 8, it can be observed that the mean deviation between sliding surfaces is less than 10% of the mean gap width, except for the Triquetrum–Pisiforme and the Triquetrum–Hamatum gaps. This shows that the deviation between the sliding surfaces is truly minimal.

The parameter $\hat{\tau}$ (defining the maximum distance between sliding surfaces) was set to the 95th percentile of the observed normalized sliding surface distance. The 95th percentile value was selected to decrease the sensitivity to outliers. The average $\bar{\tau}$ of the values $\hat{\tau}$ of five wrists is shown in Table III.

Fig. 8. Mean signed distances $\bar{d}_{uv}$ between pairs of sliding surfaces, averaged over five wrists and their standard deviations. Each line denotes a different pair of postures, e.g., E10-n denotes the registration of a wrist under 10° extension to a wrist in the neutral pose. See Fig. 7 for a legend.

TABLE III
VALUES OF $\bar{\tau}$

Gap	$\bar{\tau}$ (dimensionless)
Sc-Lu	0.14
Sc-Td	0.15
Sc-Ca	0.14
Lu-Tr	0.10
Lu-Ca	0.10
Tr-Pi	0.27
Tr-Ha	0.42
Td-Tz	0.08
Tz-Ca	0.12
Ca-Ha	0.17

F. Intra-Wrist Registration

One pose A' was registered to a different pose A of the same wrist by minimizing (1) with an unconstrained registration ($w = 0$) and with a constrained registration ($w > 0$). The results were A_u and A_c , respectively. As in Section III-E, the unconstrained registration is optimal.

A_c and A_u are based on the same surface representation A' and thus the correspondences are known. The norm d_u of mean vectorial differences is computed, averaged over all bones, wrists and extension angles together with the standard deviation. The constrained registration is performed for $w \in \{0.1, 0.2, 0.5, 1, 2, 5, 10\}$. The experiment is performed for five wrists, where the wrist under extensions of 10, 20, and 40° is registered to the neutral pose. The parameter $\bar{\tau}$ is estimated on four wrists, leaving the registered wrist out of the training set.

The influence of w is shown in Fig. 9. Notice that the bias is not significantly different from zero in all cases. The constraining weight w should remain smaller than two for the standard deviation to be significantly smaller than the voxel size.

G. Inter-Wrist Registration

Scans of the same wrist in two poses A and A' are involved in this experiment, as well as a scan of a different wrist imaged in the same pose B' as A' . Three registrations were performed as schematically shown in Fig. 10.

- 1) A constrained registration of A to B' , resulting in A_c .
- 2) An unconstrained registration of A to B' , resulting in A_u .
- 3) A constrained registration of A' to B' , resulting in A'_c .

Fig. 9. Norm of the mean vectorial difference and standard deviations between corresponding surface points after constrained ($w > 0$) and unconstrained ($w = 0$) registration as a function of w . The average is taken over all bones, wrists, and extension angles.

Fig. 10. Scheme of the inter-wrist registration experiment.

Fig. 11. Mean signed distance between bone surfaces of A_c (constrained registration $A \rightarrow B'$) and A'_c (constrained registration $A' \rightarrow B'$) and standard deviation as a function of w .

The error introduced by the model in the registration of A' to B' (registration 3, different wrists in the same pose) was found to be negligible, as the mean translation between points on the sliding surfaces was 0.6 mm (2 voxels, standard deviation 0.14 mm). These translations are approximately tangent to the bone surfaces, as the mean distance between transformed sliding surfaces was 0.06 mm (0.2 voxels, standard deviation 0.092 mm). Accordingly, A'_c is considered the reference upon registering A to B' in a constrained fashion. The experiment is performed for five wrists, where the wrist under extensions of 10, 20, and 40° is registered to the neutral pose of a different wrist. The parameters $\bar{\tau}$ were computed using a leave-one-out methodology. They were computed on four wrists, leaving the data of the registered wrist out.

Fig. 11 shows the norm of the mean point-to-plane distance d_c between points on A_c and A'_c and the corresponding standard deviations as a function of the weighting w . The figure was generated by averaging over all bones, wrists and postures. Apparently, the standard deviation approximates the voxel size for high values of w . The norm is not significantly different from zero in all cases.

The anatomical feasibility of the result of an unconstrained registration was also assessed. Fig. 12 compares the joint gap widths in A_u after unconstrained (g_u) and in

Fig. 12. Mean bone gap widths and standard deviations after constrained registration (*) and unconstrained registration (x). The mean gap widths as measured in the original wrist are denoted by (o). The deviations are averaged over five wrist and three postures and the weights $w \in \{0.1, 0.2, 0.5, 1, 2\}$ for the constrained registration.

A_c after constrained registration (g_c) with the gap widths in the original wrists (g_0). The results in Fig. 12 are averaged over five registered wrists, three postures and the weights $w \in \{0.1, 0.2, 0.5, 1, 2\}$. The results with $w = \{5, 10\}$ were excluded because of the results in Section III-F.

Fig. 12 shows that the results of the constrained registration have a significantly lower bias and variance than those of the unconstrained registration. After unconstrained registration both widened, narrowed and skewed bone gaps were observed. Improved registration results when applying constraints were also clear from visual inspection of the individual registration results of which an example is shown in Fig. 13. The wrist (a) is registered to wrist (b). The result of the unconstrained registration (c) shows overlapping bones, while the result of the constrained registration ($w = 1$) shows well preserved bone gaps.

IV. DISCUSSION

We developed a robust method for multibody registration, in which the motion patterns are governed by object shapes. A cost function was minimized in which one term reflected how well objects from different configurations were registered; another term preserved the physiologic, internal relations inside a configuration. The chosen application was the registration of the bones in the wrists from different individuals. However, the approach may apply equally well to other joints such as the ankle.

We have shown that an unconstrained registration resulted in wrist configurations that were not physiological, due bones that overlapped and bone gaps that were widened or skewed. This emphasizes the necessity of incorporating constraints.

It was assumed that the gap width between pairs of bones was constant over varying postures. Moreover, it was asserted that a sliding surface could be defined in the middle of the bone gap along which adjacent bones would slide during movement. We demonstrated that both assumptions hold for all but one gap. The method of defining sliding surfaces (Section II-C) shows its limitations in the Triquetrum–Hamate gap, where the gap width between parallel bone surfaces is not constant. The significant fluctuation in Triquetrum–Hamate gap width is explained by looking at Fig. 2(a). The part of the gap close to Metacarpal 4 widens significantly over different extensions, while the lower part of the bone gap can be considered to have a constant width.

Fig. 13. Example of a constrained registration versus an unconstrained registration. The wrist (a) is registered to wrist (b) using an unconstrained registration (c) and a constrained registration (d). The sliding surfaces are shown in gray.

The method in Section II-C, however, defines sliding surfaces in the entire bone gap.

When registering wrists of different individuals, the mean registration error is less than two voxels, for all tested values of the weight w , which balances the internal and external relations in the constraint registration. However, a minimum weighting of 0.5 is needed to reduce the standard deviations of the mean gap widths below the voxel size. For an appropriately chosen weight, i.e., between 0.5 and 2, the standard deviation in bone gap width after a constrained registration is approximately 10% with a negligible bias.

As all bones are registered concurrently and all cost functions of individual bones are normalized, the method contains no bias towards any of the registered bones.

The validation of the model is limited to four poses of five wrists since the goal of this paper is to validate the constrained registration method as opposed to building a kinematical wrist model. We expect similar results for other motions. Although the method uses a considerable amount of parameters, only the constraining weight w was found to have a substantial influence on the registration result.

In future research we will concentrate on building a statistical shape model of the healthy wrist that preserves physiological spatial relations between adjacent wrist bones. As training data for this model, 50 wrists were registered using the described constrained registration method. All registered wrists were deemed physiologically correct by an expert. Using this statistical shape model we aim to detect pathologies that do not alter the bone shapes, such as ligament ruptures and cartilage damage. Differences in bone positions and orientations between a constrained and an unconstrained registration of the fitted statistical shape model to a pathological target wrist will be at the basis to do so.

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