SUPPLEMENTAL MATERIAL

A. Distribution of the number of muons and raw fluctuations

The distribution of the relative number of muons $(R_\mu - \langle R_\mu \rangle)/\langle R_\mu \rangle$ in the data in the six energy bins is shown in Fig. 1. The best-fit model for the data is shown in gray, the physical distribution is shown in blue. The data is well described by a Gaussian.

The relative variance in the data, $V/\langle R_\mu \rangle^2$, and the average relative resolutions of the muon and energy measurements are shown in Fig. 2.

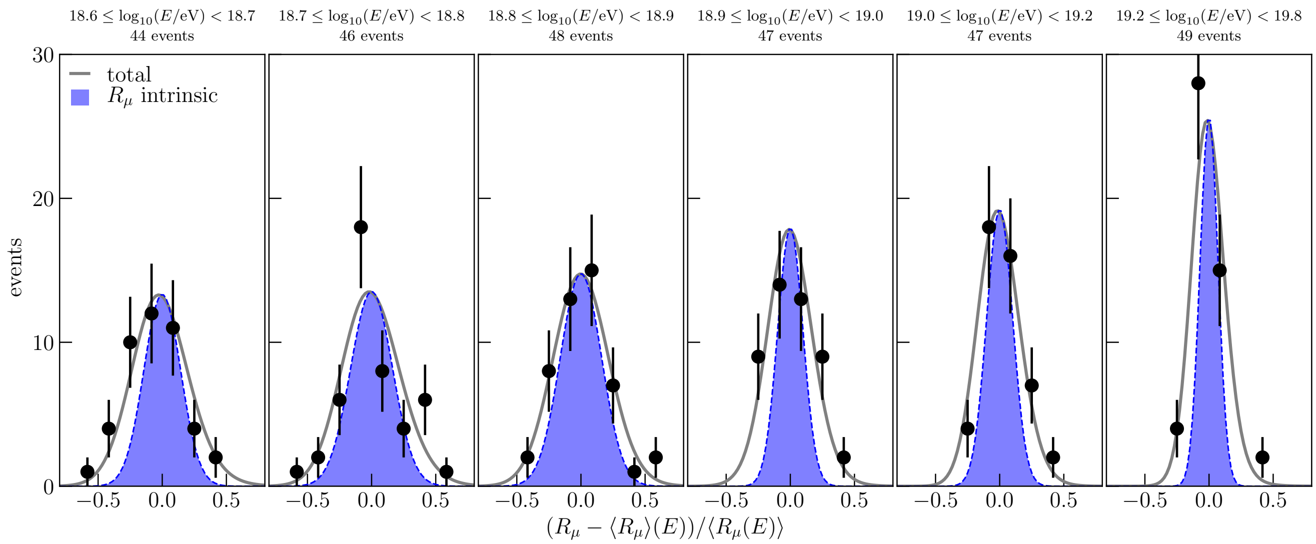

FIG. 1. Distribution of the relative number of muons in six bins of energy from $10^{18.6}$ eV to $10^{19.8}$ eV. The model for the full distribution is shown in gray, the inferred intrinsic distribution of the number of muons is shown by the filled-in curve.

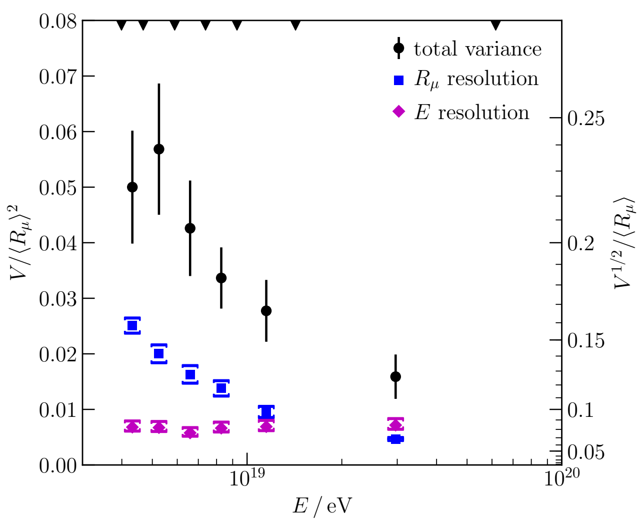

FIG. 2. Black points show the total relative fluctuations in R_μ as a function of the shower energy (left axis for the variance and right axis for the standard deviation). Blue and pink points show the average relative resolution in R_μ ($\langle s_\mu/R_\mu \rangle$) and E ($\langle s_E/E \rangle$) respectively. The error bars show the statistical uncertainty.

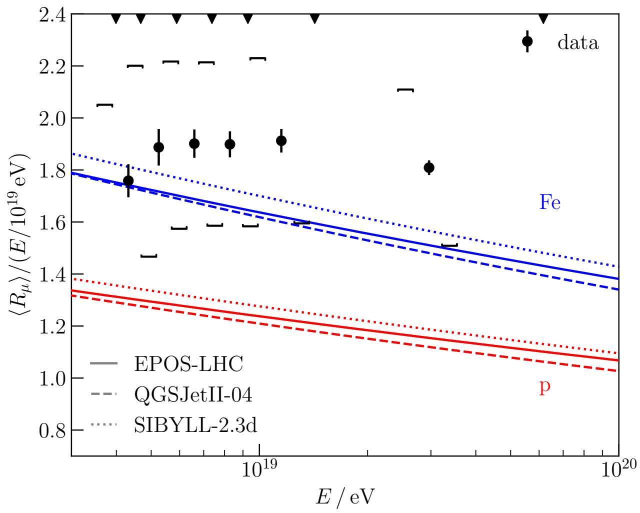

FIG. 3. Measured average number of muons as a function of the energy and the predictions from three interaction models for proton (red) and iron (blue) showers.

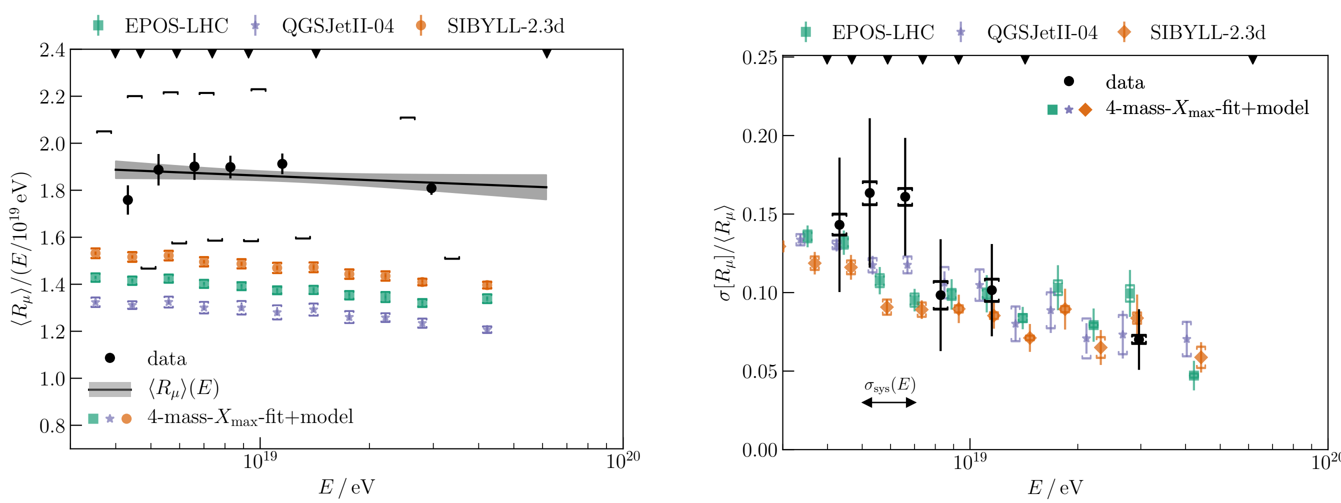

FIG. 4. Left panel: Average number of muons measured as a function of the energy together with the predictions from three interaction models given the composition measured with X_{max}. The line is the best fit of the form $\langle R_\mu \rangle [E] = a(E/(10^{19}\,\mathrm{eV}))^b$. Right panel: Relative fluctuations in the number of muons measured as a function of the energy together with the predictions from three interaction models given the composition measured with X_{max}.

B. Detailed comparison between interaction models and measurement

In Fig. 3 the average number of muons in each bin of energy is shown. The model predictions for proton and iron primaries are shown as well.

In Fig. 4 the measurement of the average number of muons (left panel) and the relative fluctuations (right panel) are shown as a function of the energy. The predictions from interaction models given the measured composition are shown for each model individually. In Figs. 5 and 6 the measurement of the average number of muons and the relative fluctuations are compared with the predictions from the interaction models separately. All models, given the measured composition, reproduce the fluctuation measurement. In case of the average number of muons none of the models yields enough muons to describe the data.

In Fig. 7 the measurement of $\langle X_{\mathrm{max}} \rangle$ and $\langle \ln R_\mu \rangle$ at $10^{19}\,\mathrm{eV}$ are compared. Both quantities scale linearly with $\langle \ln A \rangle$, meaning the predictions for different primary compositions fall on a line.

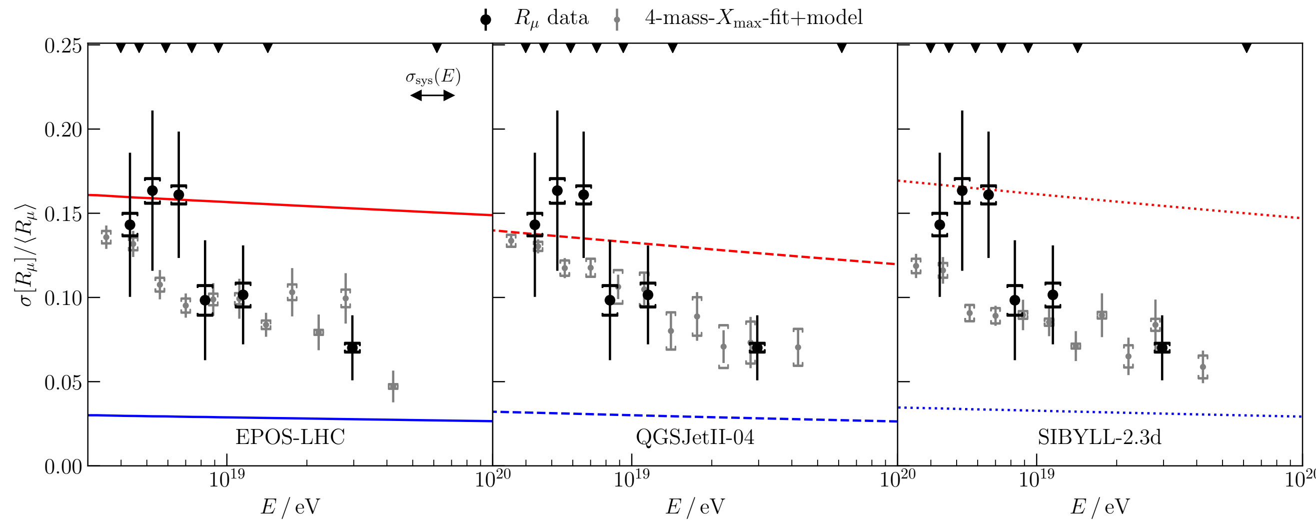

FIG. 5. Relative fluctuations in the number of muons measured as a function of the energy. The three panels show the predictions for the measured composition from EPOS-LHC (left), QGSJet II-04 (middle) and Sibyll 2.3d (right). The lines show the predictions for pure proton (red) and iron (blue). Fitting $\sigma(E)/\langle R_\mu\rangle = p_0 + p_1 \log_{10}(E/10^{19}\mathrm{eV})$ to the measurement, yields $p_0 = 0.12\pm0.01$ and the slope $p_1 = -0.10\pm0.04$. The average slope predicted for pure proton (iron) primaries is -0.01 (-0.003). The values of χ^2/n.d.f. between the trend expected from the measured composition and the measured fluctuations are 3.0/6, 2.5/6 and 4.3/6 for EPOS-LHC , QGSJet II-04 and Sibyll 2.3d, respectively.

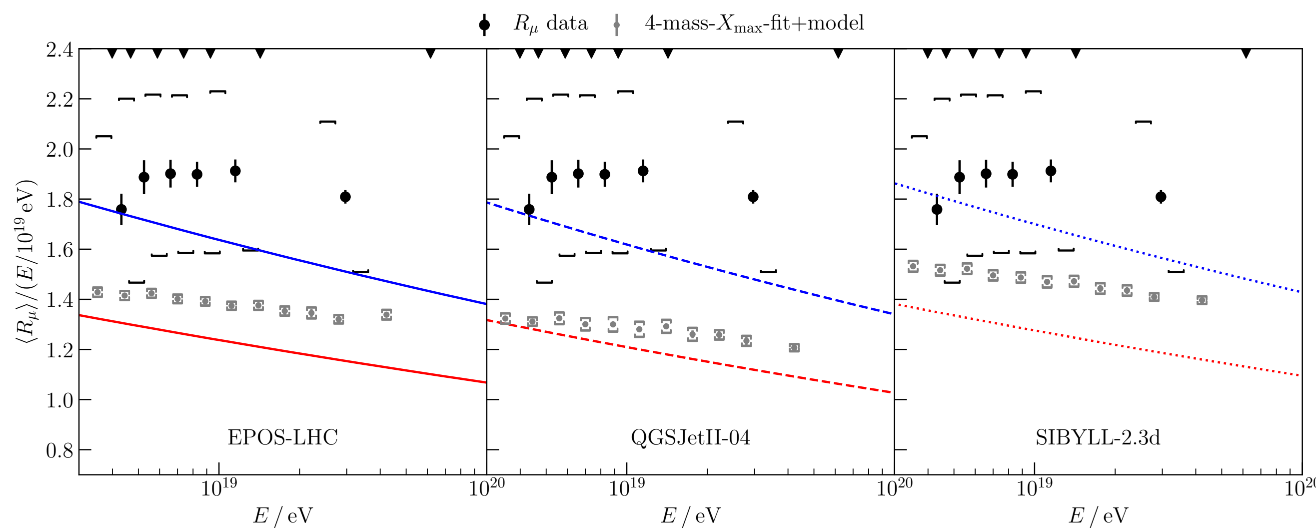

FIG. 6. Average number of muons measured as a function of the energy. The three panels show the predictions for the measured composition from EPOS-LHC (left), QGSJet II-04 (middle) and Sibyll 2.3d (right). The lines show the predictions for pure proton (red) and iron (blue).

C. Independence of the muon and energy measurements

The direct contribution from muons to the calorimetric energy through the excitation of the molecules in the air is below 5% and thus the fluctuations introduced in $E_{\rm cal}$ by muons are negligible (see Barbosa *et al.* and Risse *et al.*). For showers of a given total energy, due to the conservation of energy, $E_{\rm inv}$ and $E_{\rm cal}$ are anti-correlated on an event-by-event basis and $E_{\rm inv}$ depends on R_μ. However, the fluctuations in $E_{\rm inv}$ due to the fluctuations in R_μ are very small (around 1% at 10^{19} eV relative to E (see Pierog *et al.*)), such that in practice the determination of the two variables E and R_μ can be considered to be independent measurements. The value of 0.1 we find for the relative fluctuations at 10^{19} eV is consistent with this estimation of the fluctuations in the invisible energy.

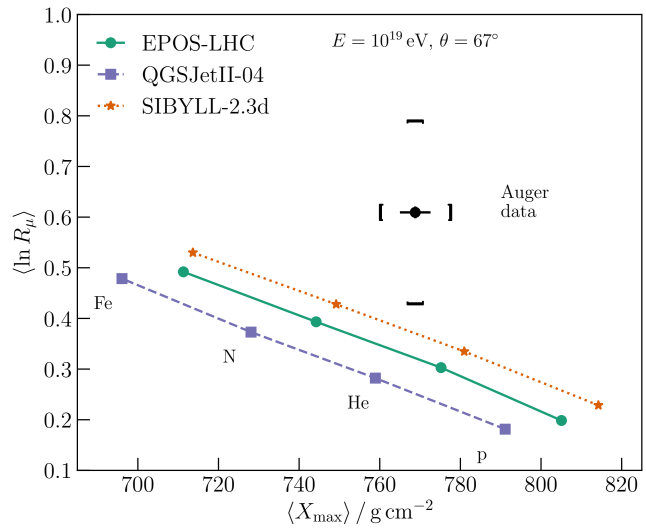

FIG. 7. Average logarithmic muon content, $\langle \ln R_\mu \rangle$, as a function of the average shower depth, $\langle X_{\rm max} \rangle$.

D. Number of muons and its fluctuations

The average number of muons in a proton shower of energy E has been shown in simulations to scale as $N_\mu^*(E) = C\ E^\beta$ where $\beta \simeq 0.9$ (see main text for references).

If we assume all the secondaries from the first interaction produce muons following the same relation as given for protons above, we obtain the number of muons in the shower as

$$N_\mu(E) = \sum_{j=1}^{m} C\ E_j^\beta = N_\mu^*(E) \sum_{j=1}^{m} x_j^\beta \ = \ N_\mu^*(E)\ \alpha_1\ , \tag{1}$$

where index j runs over m secondary particles which reinteract hadronically and $x_j = E_j/E$ is the fraction of energy fed to the hadronic shower by each. In this expression the fluctuations in N_μ are induced by α_1 in the first generation which fluctuates because the multiplicity m and the energies x_j of the secondaries fluctuate.

Consider a "toy" interaction producing only pions, all with the same energy and only a fraction f of them are charged and contribute to the hadron cascade. This model has no fluctuations and should by construction give $\alpha_1 = 1$, which follows from Eq. (1) if we identify the average number of muons for proton showers with $N_\mu^*(E)$ which coincides with our definition. This incidentally implies a condition for $\beta = \log(m)/\log(m/f)$ which is the same as that obtained by Matthews and by Kampert *et al.* ($\beta \simeq 0.90$ for $f = 2/3$ and $m \sim 50$). In a more realistic scenario α_1 fluctuates because the particles do not have the same energy and f (the ratio of charged pions) and m fluctuate.