Supplemental Materials: Geometric control of sliding friction

S1. Performed sliding experiments

We perform sliding experiments with geometrically controlled surfaces to monitor the influence of macroscopic periodic roughness on the friction force. A triangular tooth pattern is used which is fabricated on the surface of three materials, see Table S1. Triangular prisms with a depth of 20 mm are fabricated over a length of 60 mm. The plastic surfaces are manufactured with digital fabrication techniques. A commercially available resin, named Clear, is 3D printed with the Form 3 (Formlabs). Based on stereolithography, the liquid resin is cured into hardened plastic by photopolymerization with a print-resolution (both axis and lateral) of 25 µm \cite{1}. For the aluminium (6082-T6 aluminium alloy) and stainless steel (AISI 316) surfaces, wire electrical discharge machining is used; the patterning is fabricated by removing material with electrical discharges (sparks) from a 0.25 mm brass wire. This technique has a resolution of 5.0 µm for the fabrication of mm-size macroscopic surface roughness. However, the resolution of this technique is significantly less for the fabrication of high surface slopes in µm-size surface patterns. The microscopic surface topography — the microscopic surface roughness — of all materials is measured prior to sliding laser-scanning profilometry (Keyence VK-X1000) over an area of 208 by 208 µm with a lateral resolution of 138 nm/pixel and 20 nm resolution in the height direction. The calculated root-mean-square surface height variation $S_q$ is listed in Table S1.

Table S1: Details of the materials used for the custom-made surfaces. The listed resolutions are for the fabrication of flat surfaces. The listed roughness $S_q$ is the root-mean-square surface height variation which is quantified with a laser-scanning profilometer.

<table>
<thead>
<tr>
<th>Material name</th>
<th>Material type</th>
<th>Fabrication technique</th>
<th>Resolution (µm)</th>
<th>Roughness (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic</td>
<td>Clear</td>
<td>Stereolithography</td>
<td>25.0</td>
<td>3.454</td>
</tr>
<tr>
<td>Aluminium</td>
<td>6082-T6</td>
<td>Wire electrical discharge</td>
<td>5.0</td>
<td>0.705</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>AISI 316</td>
<td>Wire electrical discharge</td>
<td>5.0</td>
<td>0.470</td>
</tr>
<tr>
<td>kirigami</td>
<td>Mylar</td>
<td>Laser cutting (2.0” lens)</td>
<td>1.0</td>
<td>0.073</td>
</tr>
</tbody>
</table>

The sliding experiments are performed with various gravitational forces $F_G$. In Table S2 the performed experiments are listed including the quantified microscopic friction coefficient $\mu_0$ for the material without the patterning.

Table S2: Details of the sliding experiments performed for the various materials and surface patterns. The gravitational force $F_G$ is controlled by placing dead weights on the slider. The listed microscopic friction coefficient $\mu_0$ is the measured coefficient without the patterning, i.e., the plate-on-plate friction coefficient.

<table>
<thead>
<tr>
<th>Material</th>
<th>Patterning</th>
<th>Gravitational force $F_G$ (N)</th>
<th>Friction coefficient $\mu_0$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>Triangular</td>
<td>1.18, 1.67, 2.69</td>
<td>0.23 ± 0.02</td>
</tr>
<tr>
<td>Aluminium on stainless steel</td>
<td>Triangular</td>
<td>1.18, 1.67, 2.69</td>
<td>0.21 ± 0.02</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>Triangular</td>
<td>1.53, 2.03, 3.00</td>
<td>0.18 ± 0.03</td>
</tr>
<tr>
<td>Plastic</td>
<td>Triangular</td>
<td>0.61, 1.10, 2.09</td>
<td>0.12 ± 0.01</td>
</tr>
<tr>
<td>Plastic on kirigami</td>
<td>Triangular on kirigami</td>
<td>0.32, 0.61, 1.10</td>
<td>0.20 ± 0.01</td>
</tr>
</tbody>
</table>

S2. (In)commensurability of macroscopic periodic roughness

In Fig. S1 (in)commensurable triangular tooth patterned surfaces are shown; in (a) the friction coefficient is plotted for increasing angle of the top surface $\theta_T$ when sliding over a surface with a set angle $\theta_B = 45^\circ$. In the domain $\theta_T < \theta_B$, the measured friction varies with the angle $\theta_T$ and agrees with the model [Eqs. (2) to (4)]. However, for $\theta_T > \theta_B$ the friction remains constant and is set by the patterning angle of the bottom surface; the horizontal dashed lines represent the expected friction coefficient for a constant angle $\theta_B = 45^\circ$ [Eqs. (2) to (4)]. This suggests that sliding on a surface with a quasi-randomly macroscopic surface roughness can be understood based on the lowest angle of the
Figure S1: (a) Friction coefficient $\mu$ as a function of the patterning angle $\theta_T$ of the top surface which is pulled over a surface with a constant patterning angle of $\theta_B = 45^\circ$. The maximum, average and minimum friction coefficient are represented in, respectively, red, green, and blue. The observed sliding friction is set by the lowest angle of the top and bottom surface. The continuous lines and dashed lines represents the expected friction based on, respectively, $\theta_T$ and $\theta_B$ [Eqs. (2) to (4)]. (b) Friction coefficient $\mu$ for sliding on a surface with a quasi-random surface roughness as a function of the sliding distance $d$. The sliding friction can be predicted based on the lowest angle of the top and bottom surface (continuous lines).

two surfaces. As an example, we perform a sliding experiment on a quasi-random patterned surface; see Fig. S1(b). We slide a top surface with $\theta_T = 45^\circ$ over this designed surface and plot the friction coefficient as a function of the sliding distance $d$ together with the calculated friction coefficient based on the lowest angle of the two surfaces. The model is indeed in agreement with the measured sliding friction coefficient and show that the maximum tuning is limited by the top surface; although sliding over a high angle as $\theta_B = 60^\circ$, the maximum friction is set by the top surface with $\theta_T = 45^\circ$. The domain of tuning is therefore restricted by the incommensurability; the friction is set by the lowest angle of the macroscopic roughness of the surfaces.

S3. Influence of the microscopic surface roughness on the sliding friction.

In Fig. S2 the line profiles are given for three triangular tooth patterns with decreasing height which are characterised with 3D laser-scanning profilometry. Unfortunately, the resolution of the fabrication technique limits us to reach the aimed high angle $\theta$ and the low surface roughness $S_q$ on the surfaces. In Fig. S2 the actual patterning angle $\theta$ is calculated based on the slope of the measured height profiles. In addition, the surface roughness $S_q$ on the patterns

Figure S2: The line profiles, the height $Z$ as a function of the width, for three triangular tooth patterns with decreasing height. The black line represents the measured line profile where the gray line shows the profile that was attempted to be fabricated. The angle $\theta_{\text{measured}}$ can be quantized based on the slope of the line profiles.
can be quantified; see Fig. S3. The aimed surface roughness of $S_q = 0.705 \, \mu m$ can be reached for a triangular tooth with a height of at least $515 \, \mu m$. However, the surface roughness increases significantly for smaller surface patterns up to $S_q = 1.695 \, \mu m$ (measured for a limited area of $32.7 \, \mu m$ by $208 \, \mu m$) for a patterning height of $17 \, \mu m$. The gradual increase of the surface roughness for smaller patterns can be approximated with a logarithmic fit that is represented with the dashed line in Fig. S3.

In addition, we measured the influence of the increased surface roughness on the microscopic friction coefficient $\mu_0$. For flat surfaces, i.e., $\theta = 0$, with the same materials under the same conditions (see Table S2), are manufactured with various surface roughness. The measured microscopic friction coefficient $\mu_0$ as a function of the surface roughness is given in the inset of Fig. S3. Although the surface roughness domain is limited, an approximate linear dependency can be found (see the dashed line in the inset of Fig. S3).

With the use of the linear dependency of the microscopic friction coefficient on the surface roughness and, in turn, the logarithmic dependency of the surface roughness on the patterning height, the simple geometrical model can be corrected for the size of the patterning height. Eq. S1 can be rewritten as:

$$\mu_{\text{max}} = \tan \left( \theta_{\text{measured}} + \tan^{-1}(\mu_0(h)) \right).$$

Together with the characterised $\theta_{\text{measured}}$ (Fig. S2) the maximum friction coefficient can be plotted as given in black dashed line in Fig. 3(b). A reasonable agreement between the measured maximum friction coefficient and the corrected geometrical model has been found. The decrease of the friction coefficient for decreasing patterning height is therefore the result of a decreasing patterning angle $\theta$ which is partly compensated by an increase of the friction due to an increase of the surface roughness.

S4. Triangular kirigami surface patterning

We have performed sliding test on a kirigami metamaterial surface. The designed triangular patterning is inspired by Reference [2]. This patterning is based on an array of unit cells (see the black lines in Fig. S4) where the unit cell is a rhombus: a quadrilateral where all four sides have the same length $l_0$. Two straight cuts (red line in Fig. S4) along the sides of the unit cell were made which are connected at the top corner and leave a length $\delta$ intact. Inspired by Reference [2], we design the kirigami pattern with interior angles of the unit cell of $\alpha_1 = 60^\circ$ and $\alpha_2 = 120^\circ$, side length of $l_0 = 4.5 \, \text{mm}$ and cuts with a length of $3.8 \, \text{mm}$ ($\delta = 0.7 \, \text{mm}$). The patterning is made along the full width of
the 125 µm thin Mylar sheet, 50.67 mm, over a length of $L_0 = 63.08$ mm. The kirigami sheets exhibit an out-of-plane roughness when stretched uniaxially; triangular ‘scales’ are formed where the height of the formed roughness can be amplified. In the sliding experiments, we have clamped both long sides of the kirigami patterned sheet and, with the use of a micro-screw, stretch it uniaxially up to a length $L$. Consequently, we monitor the strain $\epsilon$ which is defined as

$$\epsilon := \frac{L - L_0}{L_0},$$  \hspace{1cm} (S2)$$

where $L$ and $L_0$ are, respectively, the deformed and undeformed length of the kirigami patterned part of the surface.

S5. Geometrical friction model for kirigami metamaterial surfaces

For the sliding experiments on a kirigami metamaterial surface, we slid a single triangular tooth patterned ($\theta_T = 45^\circ$) plastic surface horizontally against and along the kirigami scales. Consequently, the slider moved up and down over the kirigami patterned surface. The resulting kirigami texturing was asymmetric; as such, the horizontal sliding lengths for moving up and down the scales were not equal. In order to calculate the average macroscopic friction coefficient based on the simple geometrical model, a weighted average friction coefficient had to be quantified for the specific geometry.

In Fig. [S5] a schematic representation of the sliding experiment is given from a side view. The formed scales of length $l_0$ (thick black and red (gray) lines) point out of the plane with an angle $\theta_K$, which is set by the strain $\epsilon$. In Fig. 4(c) the angle $\theta_K$ is quantified for increasing uniaxial strain and fitted with the use of

$$\theta_K = p_1 \sqrt{\epsilon} + b \epsilon + c,$$  \hspace{1cm} (S3)$$

with $p_1 = 102.40$, $p_2 = -48.89$, and $p_3 = -0.21$ and $\theta_K$ in degrees. The kirigami patterned sheet has, as seen from the side, two overlapping rows of kirigami scales, which are represented as black and red triangles in Fig. [S5]. The top surface made the transition between sliding up and down when it was in contact with both overlaying rows (the scales as presented in Fig. [S5]). Consequently, the sliding lengths $s_1$ and $s_2$ for sliding up and down can be defined. The weighted average friction coefficient for sliding against and along the kirigami patterned surface can be defined as
Figure S5: Schematic illustration of the performed sliding experiments on a kirigami metamaterial, illustrated from the side. A slider with a triangular tooth patterned surface with angle of $\theta_T$ was pulled horizontally over the scales of the kirigami patterned surface. Consecutive lines of scales were formed which halfway overlapped in depth; see the thick lines.

To calculate the weighted average friction coefficient, we expressed the sliding lengths $s_1$ and $s_2$ in terms of the quantified strain $\epsilon$ and angles $\theta_K$ and $\theta_T$. The overlap between the kirigami patterning is half of the triangle base length $l$. Therefore, sliding only occurred on the top part of the pattern, as shown by the red- and grey-filled triangles in Fig. S5. These top triangles have side lengths of $l/2$ and $l_0/2$ and the same interior angle of $\theta_K$. Consequently, the sliding paths $s_1$ and $s_2$ can be written as

$$s_1 + s_2 = \frac{l}{2},$$

which can also be defined in terms of $\epsilon$ and $l_0$ with $l = (1 + \epsilon)l_0$ as

$$s_1 + s_2 = (1 + \epsilon)\frac{l_0}{2}.$$

The transition for the top surface between sliding up and down the kirigami patterns can be defined in the condition $\theta_K < \theta_T$ when both tips of the surface patterns are in contact with the opposite surface. The tangent points are defined as $P$ and $Q$ (see Fig. S5). With the use of the origin $O$, the $(x,y)$ coordinates for both points can be written as

$$\begin{pmatrix} x_P \\ y_P \end{pmatrix} = \begin{pmatrix} s_1 \\ l_0 \sin(\theta_K)/2 - s_1 \tan(\theta_K) \end{pmatrix}, \quad (S8)$$

$$\begin{pmatrix} x_Q \\ y_Q \end{pmatrix} = \begin{pmatrix} s_1 + s_2 \\ l_0 \sin(\theta_K)/2 \end{pmatrix}, \quad (S9)$$

In addition, the points $P$ and $Q$ can be related to the geometry of the slider as

$$\tan(\theta_T) = \frac{y_Q - y_P}{x_Q - x_P}. \quad (S10)$$

Therefore, with the use of Eqs. (S7) through (S10), the lengths $s_1$ and $s_2$ can be written as
Figure S6: The average friction coefficient as a function of the kirigami patterning angle $\theta_k$. The two dashed lines represent the weighted average friction coefficient for sliding a triangular tooth patterned top surface with an angle of $\theta_T$ of 45° horizontally against or along the formed scales of the kirigami metamaterial. The kirigami angle $\theta_K$ is calculated with Eq. (S3) for a given strain $\epsilon$, and the weighted average friction coefficient is subsequently plotted based on Eqs. (S4) and (S5). As a reference, the average friction coefficient for a symmetric sliding path, i.e. for $s_1 = s_2$, is included as the continuous line. Inset: the sliding lengths $s_1$ and $s_2$ as a function of the kirigami angle $\theta_K$. In addition, the total length $l/2 = s_1 + s_2$ is included.

\[
s_1 = \frac{(1 + \epsilon)l_0}{2[1 + \tan(\theta_K)/\tan(\theta_T)]}, \quad (S11)
\]

\[
s_2 = \frac{(1 + \epsilon)l_0}{2[1 + \tan(\theta_T)/\tan(\theta_K)]}. \quad (S12)
\]

In the inset of Fig. S6, the sliding lengths as a function of the angle $\theta_K$ are given. Here, $s_1$, $s_2$ and the total length $l/2 = s_1 + s_2$ are plotted, where the increasing strain $\epsilon$ is transferred to the found angle $\theta_K$ with Eq. (S3). Using the calculated sliding lengths $s_1$ and $s_2$, the weighted average friction coefficient for sliding against and along the kirigami surface can be derived (see Fig. S6). As a reference, the average friction coefficient for a symmetric sliding path ($s_1 = s_2$) is included as the continuous line. Due to the sharp scale when sliding against, and the long upward path $s_1$ when sliding along the kirigami surfaces, the average friction coefficients for kirigami are higher than for a symmetric commensurable case.

References
