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Nonequilibrium phase transition in transport through a driven quantum point contact

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We study the transport of noninteracting fermions through a periodically driven quantum point contact (QPC) connecting two tight-binding chains. Initially, each chain is prepared in its own equilibrium state, generally with a bias in chemical potentials and temperatures. We examine the heating rate (or, alternatively, energy increase per cycle) in the nonequilibrium time-periodic steady state established after initial transient dynamics. We find that the heating rate vanishes identically when the driving frequency exceeds the bandwidth of the chain. We first establish this fact for a particular type of QPCs where the heating rate can be calculated analytically. Then we verify numerically that this nonequilibrium phase transition is present for a generic QPC. Finally, we derive this effect perturbatively in leading order for cases when the QPC Hamiltonian can be considered a small perturbation. Strikingly, we discover that for certain QPCs the current averaged over the driving cycle also vanishes above the critical frequency, despite a persistent bias. This shows that a driven QPC can act as a frequency-controlled quantum switch.

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Controlling the state of an electron gas by means of external time-dependent potentials is one of the central challenges of condensed-matter physics with immediate applications to micro- and nanoelectronics. Of particular interest is transport through quantum point contacts (QPCs) with time-dependent parameters. Many remarkable phenomena have been predicted and observed in driven QPCs, ranging from energy localization transition. The second main result is that the heating rate also vanishes above the critical frequency, despite a finite difference in particle densities and temperatures. One generally expects that in such a setting a nonequilibrium time-periodic steady state will be established in the vicinity of the QPC after initial transient dynamics. We focus on two quantities characterizing this steady regime: the heating rate and the current, and the control parameter is the driving frequency.

We assume that, initially, each chain is in its own equilibrium, possibly with different particle densities and temperatures. One generally expects that in such a setting a nonequilibrium time-periodic steady state will be established in the vicinity of the QPC after initial transient dynamics. We focus on two quantities characterizing this steady regime: the heating rate and current through the QPC, both averaged over the driving period \( \tau \). We consider system sizes large enough to avoid any finite-size distortions. The large time limit is considered after the system size is set to infinity. The first main result of the present Letter is that the heating rate experiences a nonequilibrium phase transition for an arbitrary QPC, vanishing identically when the frequency of the drive, \( \omega = 2 \pi / \tau \), exceeds a critical value equal to the single-particle bandwidth of the chain.1 An analogous effect, but for global driving, was found in a spin system [22] and in a system of coupled Kapitza pendulums [23], where it was interpreted as an energy localization transition. The second main result is that for some \( V_t \) the current also vanishes above the critical frequency, despite a finite difference in particle densities and temperatures between the chains. Given that at almost any moment of time there is a nonzero tunneling matrix element

1Nonequilibrium phase transition refers to a singular behavior of observables in the nonequilibrium steady state as a function of control parameters [19–21]. Specifically, in our case the observables are the heating rate and the current, and the control parameter is the driving frequency.
connecting two chains, this latter finding seems particularly
counterintuitive. We discuss similarities and differences be-
 tween our results and relevant prior work [22–28] at the end of
the Letter.

We substantiate our claims in three ways. First, we consider
a particular QPC—a conformal QPC—which allows for a
completely analytical treatment. We explicitly construct the
Floquet Hamiltonian and calculate the whole system is described by the Hamiltonian (1). (b) The Floquet
 sites. Wiggly lines indicate the time dependence of the QPC. The
site potentials indicated by double lines can be present on two QPC
Hamiltonian (7) of the system with the conformal QPC (4).

Finally, for a small

\[ J_t = \sin \omega t, \quad U_t^L = -U_t^R = \cos \omega t. \]  

(4)

We refer to the Hamiltonian (1) with such parameters as \( H_F^c \). A
time-independent analog of this Hamiltonian was introduced in
Ref. [29]. The transmission coefficient in Ref. [29] is con-
stant for all energies of the incoming particles (in contrast to
scattering on a generic defect), which resembles the properties
of the S matrix obtained by gluing together two conformal
field theories [30,31].

The major insight enabling a fully analytical treatment of
the conformal QPC is that \( H_F^c \) can be represented as a time-
dependent unitary transformation of \( H_0^c \),

\[ H_F^c = \sum_{i=0}^{nN} H_0^c e^{-i\omega \Sigma / 2}, \quad \Sigma = i \sum_{j=1}^{L} (c_{2L+1-j}^* c_{2L+1-j} - \text{H.c.}). \]  

(5)

As a consequence, the solution of the Schrödinger equation
\( i\hbar \partial_t \Psi_t = H_t \Psi_t \) can be recast in the form

\[ \Psi_t = e^{i\Sigma / 2} e^{-i(\hbar \omega / 2)} \Psi_0. \]  

(6)

At stroboscopic times \( t_n \) (which are integers of the period,
\( t_n = n\tau \)) the first exponent reads \( e^{i\pi N} = e^{i\pi nN} \), where \( N \)
is the particle number operator. Therefore, the stroboscopic
evolution is governed by \( \Psi_{t_n} = e^{\omega / 2} \Sigma \Psi_0 \), where the Floquet
Hamiltonian \( H_F^c \) reads

\[ H_F^c = H_0^c + \frac{\omega}{2} \Sigma - \frac{\omega}{2} N. \]  

(7)

The last term does not affect the dynamics of particle-number-
conserving quantities and is dropped henceforth. This Floquet
Hamiltonian is illustrated in Fig. 1(b). Note that the term
proportional to \( \Sigma \) introduces long-range hoppings similar to
[32].
The single-particle eigenenergies $E$ of $H_E$ can be found as zeros of the function
\[ g(E) \equiv \frac{s_+(L)}{s_+(L+1)} - \frac{s_-(L+1)}{s_-(L)}, \quad s_{\pm}(x) \equiv \sinh(z_{\pm}(x)). \]

Here the dependence of $g(E)$ on $E$ enters through the dependence of $z_{\pm}$ on $E$. The spectrum for a finite system is shown in Fig. 2. The single-particle eigenvectors of $H_E^R$ read
\[ |E\rangle = \frac{1}{\sqrt{g(E)}} \sum_{j=1}^{L} \left( \frac{s_+(j)}{s_+(L+1)} + \frac{s_-(j)}{s_-(L)} \right) c_j^\dagger(0) + \frac{i}{\sqrt{g(E)}} \sum_{j=L+1}^{2L} \left( \frac{s_+(2L+1-j)}{s_+(L+1)} - \frac{s_-(2L+1-j)}{s_-(L)} \right) c_j^\dagger(0). \]

The knowledge of the explicit form of $H_E^R$, its spectrum, and its eigenvectors allows us to perform the full analysis of the driven dynamics, which can now be reduced to an equivalent quench dynamics. Applying the form-factor expansion and summation techniques used previously for similar time-independent problems \[33,34\], we find analytical expressions for the average heating rate $\overline{\mathcal{W}} \equiv \Delta E/\tau$ and current $\overline{\mathcal{J}} \equiv \Delta N_E/\tau$, where $\Delta E$ and $\Delta N_E$ are the increase per driving cycle of the total energy and the number of fermions in the right chain, respectively.\(^2\) The result reads
\[ \overline{\mathcal{W}} = \int \frac{dE}{2\pi \tau} [\rho_L(E) + \rho_R(E)] \Gamma_\omega(E), \]
\[ \overline{\mathcal{J}} = \int \frac{dE}{2\pi} [\rho_L(E) - \rho_R(E)] T_\omega(E). \]

Here the transmission coefficient $T_\omega(E)$ and the heating function $\Gamma_\omega(E)$ are given by
\[ T(E) = \text{Re} \left\{ (1 - E^2) \left[ 1 - \frac{\sqrt{1 - E^2}}{2\omega} \left[ \sqrt{1 - (E - \omega)^2} - \sqrt{(E + \omega)^2 - 1} \right] \right]^2 \right\}, \]
\[ \Gamma(E) = 2\pi \frac{\sqrt{1 - E^2}}{\omega^2} \text{Re} \left\{ \sqrt{1 - (E - \omega)^2} (E^2 - E\omega - 1) - \sqrt{1 - (E + \omega)^2} (E^2 + E\omega - 1) \right\} + 2\pi \frac{1 - E^2}{\omega^2} \left[ (E + \omega)^2 - 1 \right] \theta(1 - \omega - E) - \left[ (E - \omega)^2 - 1 \right] \theta(E - \omega + 1). \]

They are plotted in Fig. 3. Noticeably, there are nonanalyticities present for $0 < \omega < 2$ that are associated with Floquet resonances (discussed below from a perturbative point of view in what follows). The most remarkable feature, though, is that $\Gamma_\omega(E)$ turns to zero for $\omega \geq 2$, leading to
\[ \overline{\mathcal{W}} = 0, \quad \omega \geq 2. \]

We plot $\overline{\mathcal{W}}$ and $\overline{\mathcal{J}}$ as functions of $\omega$ in Figs. 4(a) and 4(b), respectively. It can be seen that while $\overline{\mathcal{W}}$ experiences a phase transition at $\omega = 2$ in accordance with Eq. (15), this is not the case with $\overline{\mathcal{J}}$, meaning that some finite current flows through the QPC for any driving frequency.

Finally, we note that if the chains are initially filled with fermions at infinite temperature (but, possibly, with different particle densities), the heating rate $\overline{\mathcal{W}}$ is zero for any driving frequency. This immediately follows from Eq. (11) since $\Gamma_\omega(E)$ is an odd function of $E$ and $\rho_L$ and $\rho_R$ do not depend on $E$ at infinite temperature.

**Numerics.** Let us address numerically other types of QPCs. We start from a tunneling QPC given by
\[ J_t = \sin \omega t, \quad U_t^L = U_t^R = 0. \]

The average heating rate and current are calculated numerically and presented in Figs. 4(c) and 4(d), respectively. One can see that the phase transition for $\overline{\mathcal{W}}$ is there. Surprisingly,

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\(^2\)We emphasize that $\overline{\mathcal{W}}$ and $\overline{\mathcal{J}}$ characterize the nonequilibrium time-periodic steady state established in the thermodynamically large system after the initial relaxation. To calculate these quantities we first take the limit $L \to \infty$ and then $t \to \infty$. It is also possible to calculate dynamical quantities at a finite $t$. This is left for a separate publication.
in this case we discover another manifestation of the phase transition: The average current also vanishes for \( \omega \geq 2 \). In this respect the tunneling QPC is drastically different from the conformal QPC studied above.

We have further numerically explored a range of QPCs with different time dependencies (not necessarily harmonic) and various combinations of on-site and tunneling drivings. We leave a detailed description of this study for a separate publication. Here we provide a brief and qualitative account of the obtained results. We have found that the heating rate \( \overline{W} \) vanishes for \( \omega \geq 2 \) for all studied QPCs. As for the average current \( \overline{J} \), it vanishes for some but not for all QPCs. Empirically, one necessary condition for \( \overline{J} \) to vanish above the critical frequency is that the tunneling rate averages to zero,

\[
\overline{J} = \int_0^\tau J(t) \, dt = 0.
\]

This intuitively plausible condition is further supported by the perturbative analysis (see below). This condition is not sufficient, however, as can be seen from the case of the conformal QPC. We observe that whenever, in addition, \( U_{LR} \) do not depend on time, \( \overline{J} \) vanishes for \( \omega \geq 2 \). We have observed some other cases when the current vanishes above the critical frequency. However, an exhaustive list of criteria for this effect to occur remains unknown.

**Perturbative analysis.** For simplicity we focus on a particular time dependence \( V(t) = V_1 e^{i \omega t} + V_2 e^{-i \omega t} + \overline{V}_{\text{pert}} \), which covers previously considered conformal (4) and tunneling (16) QPCs. Assuming that \( V_1 \) is small, we construct the Floquet Hamiltonian perturbatively in leading order, \( H^{(1)}_{F} = H_L + H_R + W^{(1)} \). We calculate matrix elements of \( W^{(1)} \) in the basis of eigenstates \( \{|\xi p\rangle \} \) of the Hamiltonian \( H_L + H_R \) of two disconnected chains, where \( p \) is the quasimomentum and \( \xi = L, R \) discriminates between the left and the right chains. The result reads

\[
W^{(1)}_{\xi p;\eta q} = \frac{V_{\xi p;\eta q}}{\omega_{\xi p;\eta q}} \frac{E_{\eta p} - E_{q}}{E_{\eta p} - \omega + \imath} + V_{\eta p;\xi p}^\ast \frac{E_{\eta p} - E_{q}}{E_{\eta p} - \omega - \imath} + |\eta q\rangle \langle \xi p|.
\]

We remind the reader that \( E_\eta = -\cos p \) is the energy of the disconnected chain.

In the leading order the long-time behavior of observables can be addressed via Fermi’s golden rule with \( W^{(1)} \) considered the perturbation. Within this approach, the Floquet resonances at \( E_\omega = E_q \pm \omega \) in \( W^{(1)} \) are responsible for the linear growth of \( \overline{E} \) and \( \overline{N}_k \) with time. Note that the first two terms in Eq. (18) vanish for \( E_\omega = E_q \) and therefore do not cause elastic transitions between states with the same energy.

We find it convenient to parametrize \( J_1 \), \( U_{LR}^L \), and \( U_{LR}^R \) in Eq. (3) as \( J_1 = (J e^{i\omega t} + J^* e^{-i\omega t})/2 + \overline{J} \) and \( U_{LR} = (U_{LR}^L e^{i\omega t} + U_{LR}^L e^{-i\omega t})/2 + U_{LR} \). Then we obtain in the leading order

\[
\overline{W}^{(1)} = \frac{1}{2\pi} \int \frac{d\omega}{\pi} \left| \frac{|J|^2 + |U_L|^2}{2|J|^2} \rho_L + \frac{|J|^2 + |U_R|^2}{2|J|^2} \rho_R \right| \Gamma^{(1)}_\omega,
\]

\[
\overline{J}^{(1)} = \int \frac{dE}{2\pi} (\rho_L - \rho_R) T^{(1)}_\omega,
\]

with

\[
\Gamma^{(1)}_\omega = 4\pi |J|^2 \sqrt{1 - E^2} \Re \sqrt{1 - (E + \omega)^2}
\]

\[
- \sqrt{1 - (E - \omega)^2},
\]

\[
T^{(1)}_\omega = \sqrt{1 - E^2} (|J|^2 \Re \sqrt{1 - (E + \omega)^2}
\]

\[
+ \sqrt{1 - (E - \omega)^2} + 4|J|^2 \sqrt{1 - E^2}),
\]

where the argument \( E \) is dropped in functions \( \rho_{LR}(E) \), \( \Gamma^{(1)}_\omega(E) \), and \( T^{(1)}_\omega(E) \) for brevity. In Fig. 5 we compare these perturbative results with numerical calculations. One can see good agreement even for a not-so-small \( V \).

For \( \omega \geq 2 \) one obtains \( \Gamma^{(1)}_\omega = 0 \). As a consequence, the heating rate (19) vanishes for any QPC in the leading order, in agreement with our numerical findings.

![Fig. 4. (a) and (c) Average heating rate \( \overline{W} \) and (b) and (d) average current through the QPC \( \overline{J} \) vs driving frequency. Initially, the left chain is filled by fermions in the ground state with particle density \( N/L = 0.4 \) fermion per site, while the right chain is empty. (a) and (b) Conformal QPC (4). The heating rate and the current are obtained analytically from Eqs. (11) and (12), respectively. The heating rate vanishes for \( \omega \geq 2 \), while the current remains finite. (c) and (d) Tunneling QPC (16). Plots are obtained numerically for a system with \( L = 50 \) sites in each chain and \( N = 20 \) fermions. Both the heating rate and the current vanish for \( \omega \geq 2 \). Insets show the real-time dynamics of the total energy \( \overline{E} - E_0 \) [(a) and (c)] and the particle number in the right chain \( N_R \) [(b) and (d)].](image-url)

![Fig. 5. Averaged heating rate \( \overline{W} \) (left) and current \( \overline{J} \) (right) for a QPC with \( J_1 = 0.3 \sin \omega t \), \( U_{LR}^L = -U^R_3 = 0.3 \cos \omega t \) calculated numerically (solid lines; the system size and initial state are the same as in Fig. 4) and perturbatively (dashed lines; see Eqs. (19) and (20), respectively). Both the average heating rate and current vanish for \( \omega \geq 2 \) in the leading order. However, the actual current remains finite due to the higher-order corrections, as illustrated in the inset in the right panel.](image-url)
As for the current, it vanishes in the leading order above the critical frequency whenever the condition (17) is satisfied. Indeed, this condition entails $T_ω^{(1)} = 0$ for $ω \geq 2$. It should be emphasized, however, that the condition (17) alone is not sufficient to guarantee the vanishing of the current in subsequent orders. Indeed, in the considered example the current does not vanish at higher orders, as is clear from Fig. 5.

One may hope that going beyond the leading order can help to clarify under what conditions the current vanishes above the critical frequency. One should keep in mind, however, that beyond the leading order Fermi’s golden rule is inapplicable and one needs to employ more sophisticated techniques to address the transport properties. We leave this interesting question for further studies.

Let us also remark that if $J$ is finite, the Floquet Hamiltonian above the critical frequency is given by $H_L + H_R + V$ in leading order in $V$. The same result is straightforwardly obtained in the leading order of the Floquet-Magnus expansion [35]. The transport through the time-independent QPC described by this Hamiltonian has been studied in detail (see [33,34,36]).

**Summary and discussion.** To summarize, we have established a nonequilibrium phase transition in a system (1) of two fermionic chains filled (equally or unequally) by non-interacting fermions and connected by a periodically driven QPC. Namely, when the driving frequency $ω$ exceeds the bandwidth, the heating rate vanishes exactly for a generic QPC. Furthermore, for certain QPCs the current averaged over the period also vanishes, even in the presence of a filling bias between the chains. We have verified this picture by (i) calculating the heating rate (11) and the averaged current (12) explicitly for the exactly solvable conformal QPC (4), (ii) performing extensive numerical studies of various QPCs, and (iii) performing a perturbative analysis in the leading order in the limit when the QPC Hamiltonian can be considered a perturbation.

It should be emphasized that vanishing of the heating rate in periodically driven systems in the limit of infinite frequency is a well-known fact that can be proven in full generality [37]. Here we obtain a much stronger result—exact vanishing of the heating rate above a finite critical frequency.

Let us put our results in the context of prior work. First, we discuss the heating rate. It is believed that generic periodically driven many-body systems (without disorder) in the thermodynamic limit heat indefinitely [38–40]. On the other hand, it is commonly appreciated that dynamically integrable systems of various types can violate this rule [38,41]. For example, in the quantum Ising model with periodically driven transverse magnetic field the heating rate vanishes (after an initial transient dynamics) for any driving frequency [28]. This can be shown explicitly thanks to the fact that this many-body model can be factorized into a collection of decoupled driven two-level systems [28]. More intriguingly, it has been found that the heating rate in a kicked spin system [22] and a system of coupled Kapitza pendulums [23] vanishes above a critical frequency (this effect has been referred to as energy localization transition). These two systems allegedly are not dynamically integrable in any way: The vanishing of heating has been established numerically [22,23] and supported by a high-frequency expansion and a variational analysis [22]. Here we establish this energy localization transition in a very different setting of a locally driven many-body system and demonstrate its universality in this setting. We note that although we deal with noninteracting fermions, our system does not factorize into decoupled two-level systems as in Ref. [28].

Next, we discuss transport phenomena related to our findings. The most relevant one is the phenomenon of coherent destruction of tunneling [25], where the tunneling probability through a potential barrier in a driven system vanishes at certain frequencies. This phenomenon has been established, in particular, for tight-binding lattices connected by a QPC with an oscillating local potential but constant tunneling term $(J_s = \text{const})$ [26,27]. A related phenomenon is the real-space dynamical localization of a particle in a periodically tilted lattice [24,42] occurring, again, for a discrete set of frequencies. In contrast to these phenomena, the vanishing of particle flow discovered in the present work takes place for an arbitrary frequency above the critical one. Note that time dependence of the tunneling term is instrumental for this phase transition to occur.

A remark on the Floquet-Magnus expansion is in order. The Floquet-Magnus expansion is a formal expansion of the Floquet Hamiltonian in powers of $1/ω$ [43]. It is widely used to approximate Floquet Hamiltonians of few-level systems at high frequencies. However, its applicability to many-body systems is limited: In general, it has zero convergence radius for a generic many-body system in the thermodynamic limit [35,43,44] (see also [45]). In our case, the inapplicability of the Floquet-Magnus expansion can be anticipated from the fact that the exact Floquet Hamiltonian (7) is linear in $ω$. Further, it can be easily verified that the truncated Floquet-Magnus series contains only short-range hopping terms, while the exact Floquet Hamiltonian (7) contains hoppings over the entire system. As a consequence, the truncated Floquet-Magnus expansion cannot be a reliable approximation of the true Floquet Hamiltonian. In particular, the nonequilibrium phase transition is not reproduced by the Floquet-Magnus expansion.

We also note that one may attempt to get further analytical insight into the phenomena discussed here by means of the Floquet-Green’s-function formalism [46–48]. This approach remains for further work.

Finally, we briefly remark on possible ways to test our predictions experimentally. A well-developed quantum dot technology provides a necessary toolbox for this task [49]. Another option that recently emerged is to use a cold-atom simulator of a quantum point contact [50–52]. The latter platform benefits from the perfect isolation from the environment and extended control over the effective Hamiltonian. According to our findings, a QPC can act as a frequency-controlled quantum switch, and experimental observations of this effect may pave the way to its technological applications.

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