Observational signatures of quantum gravity in interferometers

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A B S T R A C T
We consider the uncertainty in the arm length of an interferometer due to metric fluctuations from the quantum nature of gravity, proposing a concrete microscopic model of energy fluctuations in holographic degrees of freedom on the surface bounding a causally connected region of spacetime. In our model, fluctuations longitudinal to the beam direction accumulate in the infrared and feature strong long distance correlation in the transverse direction. This leads to a signal that could be observed in a gravitational wave interferometer. We connect the positional uncertainty principle arising from our calculations to the 't Hooft gravitational S-matrix.

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1. Introduction

The quantum mechanical description of gravity together with the other forces remains one of the most important questions in physics. While general relativity can be quantized as an effective field theory valid at low energies, and there has been significant theoretical progress in understanding other aspects of quantum gravity, signatures of the quantum nature of gravity have so far remained stubbornly immune to observation.

An important clue towards the ultimate theory of quantum gravity is provided by the holographic principle [1,2]. One of its implications is the covariant entropy bound [3], which states that the entropy associated to region bounded by null geodesics does not exceed $A/4\pi N$, where $A$ denotes the area of the surface and Newton’s constant is identified with the square of the Planck length via $8\pi G_N \equiv l_p^2$, with $l_p \simeq 10^{-35}$ m. The holographic principle suggests that the total number of microscopic degrees of freedom associated to a given region of space (defined by the maximal entropy) is given by the area of the surrounding surface in Planck units. In this form the holographic principle is known to be realized in spacetimes with negative cosmological constant [4], and is firmly incorporated in the framework of the AdS/CFT correspondence [5].

Motivated by the holographic principle, one is tempted to postulate that the microscopic spacetime degrees of freedom, also in flat spacetime, can be identified with Planck size pixels on the surface bounding a causally connected part of space. The spacetime volume would then emerge in the infrared from these holographic spacetime quanta. One of the intriguing aspects of the holographic principle is that, to ensure the validity of the entropy bound, the spacetime degrees of freedom are necessarily correlated in the infrared. This raises the question of whether Planck scale physics could appear at much longer, potentially observable, length scales.

Our goal in this Letter is to investigate whether fluctuations due to the graininess of spacetime can potentially lead to observable signatures. Normal intuition would say that, since the natural length and time scale associated with the quantum nature of spacetime is Planckian, that no feasible experiment exists that could measure its effects. We will argue, however, that when combined with important infrared effects naturally expected from holography, the accumulative effect of Planck scale fluctuations can be transmuted to observable time and length scales. Because of their sensitivity to exquisitely short distance scales, gravitational wave interferometers are an ideal testing ground for these ideas.

We will identify the required theoretical conditions that need to be satisfied to obtain observable effects, and construct a concrete holographic model that obeys those conditions. After showing that uncorrelated Planckian fluctuations are not macroscopically observable, we demonstrate that fluctuations with sufficient transverse correlations in the infrared do lead to observable effects. The appearance of transverse correlations is crucial and suggests a holographic description in which the longitudinal and transverse behavior of the spacetime degrees of freedom are treated on a different footing.

We will then build an explicit holographic model in terms of Planck-size pixels that saturate the holographic bound and have energy fluctuations that cause the spatial length $L$ of a causally connected region of space to fluctuate. The transverse correlations are generated through the Newtonian potential of these energy fluctuations,
allowing us to make a concrete prediction for the spectrum of length fluctuations in an interferometer. These fluctuations imply a spacetime uncertainty relation in the longitudinal direction, which we connect, albeit in a modified form, to the gravitational S-matrix approach of ’t Hooft (see Refs. [6,7]).

While there have been several previous studies seeking to heuristicly connect holography to interferometry (e.g. [8–12]), our theoretical description is structurally unique in its holographic set-up. And although the first steps we take—employing a Planckian random walk—shares commonalities with these works, our approach differs in the sense that we present a concrete theoretical model leading to length fluctuations along the longitudinal direction with a distinctive signature for strong transverse correlations, which is, as a consequence, macroscopically observable in an interferometer. A phenomenological result is that constraints from the images of distant astrophysical sources derived for uncorrelated fluctuations in Refs. [13,14] do not apply to our model.

2. Length fluctuations with Planckian white noise

In this paper we consider a toy experimental set-up, shown in Fig. 1, in which the arm length $L$ of an interferometer is measured after a single light crossing. In this idealized scenario the length fluctuations $\delta L$ due to quantum fluctuations in the metric are given by

$$\delta L(t) = \frac{1}{2} \int_{0}^{L} dz h(t + z - L)$$  \hspace{1cm} (1)

where $h \equiv h_{zz}$ is the metric component along the light beam propagation (see e.g. [15]). The magnitude of these length fluctuations is normally expressed in terms of the power spectral density (PSD)

$$S(\omega, \tau) = \int_{-\infty}^{\infty} d\tau \left( \frac{\delta L(t) \delta L(t - \tau)}{L} \right) \delta(\tau - \frac{h_{zz}}{\omega^2}) \int_{-\infty}^{\infty} d\tau \left( \frac{\delta L(t) \delta L(t - \tau)}{L} \right) e^{-i\omega \tau}.$$  \hspace{1cm} (2)

Let us first consider a simple model with a white noise signal of Planckian amplitude

$$\langle h(t + z_1 - L) h(t + z_2 - L) \rangle = C_{\ell_p} \delta(t + z_1 - z_2),$$  \hspace{1cm} (3)

where $C_{\ell_p} = \sqrt{8\pi G_N}$. This leads to a PSD of the form

$$S(\omega) = \frac{C_{\ell_p} \sin^2 \omega L}{4 \omega^2 L^2}.$$  \hspace{1cm} (4)

In this simple model the length fluctuations $\langle \delta L^2 \rangle$ obey

$$\left\langle \frac{\delta L^2(t)}{L^2} \right\rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega S(\omega) = \frac{C_{\ell_p}}{8L}.$$  \hspace{1cm} (5)

and thus grow linearly with $L$ [8–12]. This signal could in principle be observable, since the peak sensitivity for gravitational wave interferometers is right around the Planck scale: $S(\omega, \tau) \leq C_{\ell_p}$. Over the next sections our goal will be to show how some of the generic behavior in Eqs. (4), (5) can arise from a holographic model, motivating the size of the constant $C$, with crucial observational effects arising from angular correlations. In addition, in experiments like LIGO and Virgo a typical photon traverses the interferometer arm multiple times before being measured. In this paper we continue to focus on our simple set up and defer the detailed discussion of multiple crossings to future work.

3. Holographic scenario and basic postulates

Our aim in the following is to derive a result similar to Eq. (5) from a holographic scenario, in which the holographic surface is fixed by the light path of a photon, as depicted in Fig. 1. In order to clearly delineate between theoretical input and observational consequences, we will state here our basic postulates:

1. Holographic principle in flat spacetime. We postulate that the holographic principle also applies to Minkowski spacetime. It states that the maximal entropy carried by the microscopic degrees of freedom associated with a finite region of flat spacetime bounded by null geodesics is $S = A/4G_N$. This bound is saturated for a region of space whose null boundary coincides with a horizon.

2. Universality of metric fluctuations at horizons. We postulate, as a corollary of the first postulate, that metric fluctuations near null surfaces associated with the boundary of a finite region follow from the entropy and temperature using standard thermodynamic considerations. This postulate implies that metric fluctuations near a Rindler-type horizon are identical to those near a black hole horizon with the same temperature and entropy.

Note that we are treating the metric fluctuations at the holographic surface separating the inside of the causal diamond from the outside as if it were a black hole horizon (see Ref. [16]), even though we are considering the vacuum of Minkowski space. The basic reason we believe these are reasonable postulates is that a finite causal diamond in Minkowski space, when suitably foliated, can be recast in the metric of a so-called topological black hole [17]. Furthermore, a conformal field theory restricted to the diamond behaves as a thermal field theory [17], and the quantized Einstein-Hilbert metric in the infrared behaves as a conformal field theory. In related work [18], we show that these postulates are justified in the context of AdS/CFT. That they hold for the Einstein-Hilbert metric in Minkowski space must, at the present time, be ultimately verified by experiment. Fortunately, we show that the experimental signatures associated with a spacetime obeying these postulates are within reach with current interferometer technology.

4. Towards macroscopic effects in interferometers

The results in Eqs. (3)–(5), that were derived from the simple 1D-model, are by themselves not sufficient to show an effect. In order to be observable in a realistic experimental set up, the fluctuations must be coherent at macroscopic spacetime distances. To examine the conditions under which such coherent fluctuations occur, we extend our model by including the two spatial directions transverse to the beam direction. Anticipating our holographic description, we consider metric fluctuations that depend on only three coordinates, one longitudinal null direction and two transverse directions, corresponding to the outside boundary of the causal diamond in Fig. 1.

$$\left( \frac{\delta L}{L} \right)_1 \left( \frac{\delta L}{L} \right)_2 = \frac{1}{16L^2} \int_{-L}^{L} \int_{-L}^{L} du_1 du_2$$

$$\int d^3k_1 \frac{d^3k_2}{(2\pi)^3} \langle h(k_1)h(k_2) \rangle \delta(k_1 \cdot x_1) \delta(k_2 \cdot x_2).$$

We first consider an Ansatz corresponding to uncorrelated white noise in those three dimensions:
dependence on the directions where $\langle \ldots \rangle_{\text{RMS}}$This boundary. causal

\[
\langle \delta L \rangle_{\text{1}} \left( \frac{\delta L}{L} \right)_{\text{2}} = \frac{\theta}{16\pi L(\Delta x_f^2/L_p + 1)^{3/2}}. \tag{8}
\]

In the limit $\Delta x_f \to 0$, we recover a signal of an amplitude that is in principle within the observable range and is consistent with Eqs. (4)-(5). However, for a realistic macroscopic interferometer, with the beam size centimeters across such that $\Delta x_f/L_p \gg 1$, this signature would be unobservable.

Let us consider an alternative Ansatz for the metric fluctuations in which the transversal directions are treated differently:

\[
\langle h(k_1)h(k_2) \rangle = (2\pi)^3 \delta^3(k_1 + k_2) C_{\theta P}/(k_1^2 + k_2^2). \tag{9}
\]

where $k_{\text{IR}}$ acts as a regulator. Then Eq. (8), in the limit that $k_{\text{IR}}\Delta x_f \ll 1$, becomes

\[
\left( \frac{\delta L}{L} \right)_{\text{1}} \left( \frac{\delta L}{L} \right)_{\text{2}} \sim \frac{\theta}{16\pi L} \log[1/\Delta x_f k_{\text{IR}}]. \tag{10}
\]

Already this result shows important features that the underlying theory must give, notably that the longitudinal and transverse directions appear on a different footing. The metric fluctuation in the transverse direction must be correlated, while the metric fluctuations in the longitudinal direction accumulate, as in a random walk, and are transmuted to a low-energy, long-distance signature. We will show over the next sections how these features arise naturally from energy fluctuations on a holographic surface.

5. From Minkowski to Schwarzschild-like metric

The central part of our argument involves utilizing a correspondence between any horizon and a black hole horizon. To show concretely how this applies to the case at hand, we make two metric transformations, which are described below. First we define light cone coordinates $u = r + t$ and $v = r - t$ so that metric becomes

\[
ds^2 = du dv + \delta R^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{13}
\]

in such a way that the light paths of the photon are mapped onto the event horizon located at $f(R) = 0$. This is achieved by making the coordinate transformation

\[
(u - L)(v - L) = 4L^2 f(R), \quad \log \frac{u - L}{v - L} = L \tag{14}
\]

where the function $f(R)$ is given by

\[
f(R) = 1 - \frac{R}{L} + 2\Phi. \tag{15}
\]

Here $\Phi$ plays the role of the Newtonian potential and parametrizes the deviations in the geometry due to vacuum fluctuations in the energy conjugate to the time $T$.

Without any quantum gravity effects, the horizon is located at $R = L$. In general, its location is determined by $f(R) = 0$. This leads to the following relationship between the product of the lightcone time variations $\delta u(L)$ and $\delta v(L)$ and the value of Newton’s potential

\[
\frac{\delta v(L)\delta u(L)}{L^2} = 2\Phi(L). \tag{16}
\]

This equation should be regarded as an operator identity. Since $\langle \Phi \rangle = 0$ in vacuum, the right-hand-side of this equation actually
represents a fluctuation around the vacuum, whose amplitude is given by squaring the operators and taking its expectation value:

\[
\left( \frac{\delta v(L)\delta u(L)}{L^2} \right)^2 = \left\langle 4\Phi(L)^2 \right\rangle.
\] (17)

For a more detailed and formal discussion of this point in the context of AdS/CFT, we encourage the reader to consult Sec IV of our companion paper Ref. [18].

The goal of the next section is to determine the root-mean-square value of the fluctuations in \( \Phi \) in an ensemble averaged over many interferometer light crossings.

6. Holographic model for spacetime fluctuations

We are now ready to employ all of our postulates together to compute the deviations in the Newtonian potential, Eq. (16). The fluctuations in \( \Phi(L) \) will be induced by vacuum fluctuations in the energy conjugate to the time coordinate \( T \), Eq. (16). In the following analysis we follow closely the reasoning of Marolf in Ref. [16] for the quantum thickness of black hole horizons. Directly applying the holographic principle to the horizon of the causal diamond gives

\[
S_{\text{hor}} = \frac{A}{4G_N} = \frac{8\pi^2L^2}{\ell_p^2}.
\] (18)

Now the fluctuations in the Newtonian potential on the horizon obey

\[
2\Phi(L) = -\frac{\ell_p^2}{4\pi L} \Delta M,
\] (19)

where \( \Delta M \) represents the energy fluctuations in the holographic degrees of freedom. Heuristically, one expects the RMS value of \( \Delta M \) to scale as the square root of the number of pixels on the horizon, times the typical energy of the fluctuation, which is given by the Hawking temperature.

One of the standard methods to determine the Hawking temperature is to go to Euclidean time and impose that the resulting metric is free from conical singularities. In this way one finds

\[
T_{\text{hor}} = \frac{|f'(L)|}{4\pi L} = \frac{1}{4\pi L}.
\] (20)

In the present situation the temperature \( T_{\text{hor}} \) is measured by an accelerated observer whose event horizon coincides with the photon trajectory and whose own trajectory passes through the origin at \( T = 0 \). This observer stays at \( R = 0 \) and has \( T \) as proper time coordinate.

We now calculate the RMS value of the fluctuations, by assuming that the vacuum energy \( E \) vanishes. This implies that the free energy \( F(\beta) \) equals

\[
F(\beta) = -T_{\text{hor}}S_{\text{hor}} = -\frac{\beta}{2\ell_p^2},
\] (21)

where in the last step we eliminated the length \( L \) in favor of the inverse temperature \( \beta = 1/T_{\text{hor}} = 4\pi L \). In the canonical ensemble the mass fluctuations \( \Delta M \) are obtained by taking the second derivative of the free energy. One thus obtains

\[
\langle \Delta M^2 \rangle = -\frac{\partial^2}{\partial \beta^2} \left( \frac{\beta}{2\ell_p^2} \right) = \frac{1}{\ell_p^2}.
\] (22)

Note that \( \Delta M \sim T_{\text{hor}}\sqrt{S_{\text{hor}}} \), as expected from the heuristic argument. We now assume that at a coincident point, \( \delta v(L) \) and \( \delta u(L) \) take the same value \( \delta L \). In this situation, combining Eqs. (16)-(22), we learn that the amplitude of the length fluctuation is

\[
\left\langle \frac{\delta L^2}{L^2} \right\rangle = \frac{\ell_p^2\Delta M}{4\pi L} = \frac{\ell_p^2}{4\pi L}.
\] (23)

where here \( \Delta M = \sqrt{\langle \Delta M^2 \rangle} \) is interpreted as the root-mean-square of the mass fluctuation. Note this has precisely the behavior shown in Eq. (5) needed to be observable, where now we can fix the constant \( C \) via the holographic principle. We will propose in the next section that angular correlations between the interferometer arms respect the spherical symmetry of the measuring apparatus, and would give rise to a distinctive experimental signature.

7. Angular correlations and ’t Hooft’s S-matrix

We have considered so far the amplitude of the fluctuations only as a function of the longitudinal coordinates. Physically it is clear that the fluctuations will also have an angular dependence, which can be straightforwardly determined for an interferometer with two arms of equal length \( L \). In this case, a spherical coordinate system, with origin at the beamsplitter, is appropriate, with the far mirrors located at two positions \( \mathbf{r}_1 \), \( \mathbf{r}_2 \) on the sphere. In this experimental configuration, the angular information can be determined with the help of the Newtonian potential \( \Phi \) decomposed in terms of spherical harmonics, thus respecting the spherical symmetry of the measuring apparatus.

Accordingly, we propose the following natural Ansatz as a generalization of Eq. (23):

\[
\left\langle \delta L(\mathbf{r}_1)\delta L(\mathbf{r}_2) \right\rangle = \frac{\ell_p^2 L^2}{4\pi^2} G(\mathbf{r}_1, \mathbf{r}_2).
\] (24)

This equation can be further justified from a generalization of Eq. (17) to give the four-point correlation at separated points:

\[
\left\langle \delta u(\mathbf{r}_1)\delta v(\mathbf{r}_1)\delta u(\mathbf{r}_2)\delta v(\mathbf{r}_2) \right\rangle = \frac{\ell_p^2 L^2}{16\pi^2} \Theta^2(\mathbf{r}_1, \mathbf{r}_2).
\] (25)

If we again assume as above that at a coincident point \( \delta u(\mathbf{r}) = \delta v(\mathbf{r}) = \delta L(\mathbf{r}) \), the two-point Eq. (24) is seen to be the factorization of Eq. (25) into the product of two two-point functions.

Our suggestion, based on the symmetries of the system, is to identify the function \( G(\mathbf{r}_1, \mathbf{r}_2) \) with the Green function of a modified Laplacian on the sphere. It obeys

\[
\left( -\nabla^2_{\mathbf{r}_1} + \frac{1}{L^2} \right) G(\mathbf{r}_1, \mathbf{r}_2) = \delta^2(\mathbf{r}_1, \mathbf{r}_2),
\] (26)

and can be obtained by integrating the 3D Green function along the radial direction corresponding to the beam. At short distances it behaves as the normal Green function on the 2D-plane

\[
G(\mathbf{r}_1, \mathbf{r}_2) \sim \frac{1}{2\pi} \log \left( \frac{L}{|\mathbf{r}_1 - \mathbf{r}_2|} \right) \quad \text{for } |\mathbf{r}_1 - \mathbf{r}_2| \ll L.
\] (27)

In terms of spherical harmonics it has the expansion

\[
G(\mathbf{r}_1, \mathbf{r}_2) = \sum_{\ell,m} Y_{\ell,m}(\mathbf{r}_1)Y_{\ell,m}^*(\mathbf{r}_2).
\] (28)

We can write this result alternatively in terms of the coefficients, \( Y_{\ell,m} \) and \( \delta Y_{\ell,m} \), of the decomposition of \( \delta L(\mathbf{r}) \) and \( \delta \Phi(\mathbf{r}) \) in to spherical harmonics as

\[
\left\langle \delta L_{\ell,m} \delta L_{\ell',m'} \right\rangle = \frac{\ell_p^2 L}{4\pi} \frac{1}{\ell^2 + \ell' + 1} \delta_{\ell\ell'}\delta_{mm'}.
\] (29)

This relation tells us that much of the power in the fluctuations is contained in the low \( \ell \) modes, and thus appears on the largest
scales, contrary to one’s intuition about Planckian effects. While we think that our Ansatz of fluctuations obeying the Green function on the 2-d sphere is natural given the symmetries of the system, we have not rigorously derived this result here; we leave more detailed work in this direction for the future.

Our result implies a fundamental uncertainty relation between the longitudinal spacetime components. As ‘t Hooft showed, the in-going and out-going radiation at the horizon causes a spacetime shift due to gravitational shock waves. He then went on to postulate that there is an inherent uncertainty in the values of the position of the horizon. In fact, ‘t Hooft’s uncertainty relations described in e.g. [6,7], when translated into a correlation function, have an identical angular dependence as Eq. (29), though a different normalization. Due to our assumption of statistical independence and the resulting accumulation of the spacetime fluctuations, we find an extra factor \( l/\ell_p \) compared to ‘t Hooft.

8. Conclusion and discussion

In this Letter we have constructed a concrete holographic model of transversally correlated longitudinal distance fluctuations, due to vacuum energy fluctuations of the (holographic) degrees of freedom associated with a causally connected volume of spacetime. By assuming that the energy of these fluctuations is of the order \( 1/L \), where \( L \) is the length of the interferometer arm, and that number of fluctuating degrees of freedom is \( L^2/\ell_p \), we have derived length fluctuations of size \( \delta L^2 \sim \ell_p L \). The strong transverse correlation implies that in interferometer experiments the length fluctuations are sufficiently coherent across the light beam, so that they are in principle observable.

If a signal with the characteristic features of our model is observed it would be a confirmation that our postulates regarding the theory of quantum gravity in flat spacetime are realized. Conversely, if no signal is observed in an appropriately sensitive interferometer, it would tell us that one our proposed postulates does not hold. In either case, we will have obtained concrete experimental information about the underlying theory of quantum gravity.

Our results were derived for a simple toy Michelson interferometer, so the next step is to concretely connect the result in Eq. (29) to a power spectral density and to realistic interferometers. The closest experimental set-up to our toy is the “Holometer” [19], but to make a concrete comparison to experimental results requires at minimum a computer extension Extending this to the case of a holographic wavefront as discussed in Ref. [12]. To fully determine the observational implications of our results one needs to incorporate these and other experimental aspects in to our model.

The transversal correlation of our model also has other important phenomenological implications. Previous attempts to consider phenomenological effects from Planckian Brownian noise (see Refs. [20,13], and as suggested by Eq. (3)-(5)) were stymied by the blurring of images from distant astrophysical sources. If the fluctuations are uncorrelated in the transverse direction, this implies large deviations in the phase of the light [1-6]. In the model under consideration here, however, most of the power in the length fluctuations is contained in low \( \ell \)-modes, as shown in Eq. (29), that are coherent across the aperture diameter \( D \) of an optical device, namely those with \( \ell \leq 2\pi D/D \). These modes do not affect the quality of the image of astronomical objects. In our model the image quality is therefore improved compared to previously considered situations. To fully establish that the transversal corre-

lations are sufficient to evade the astrophysical constraints needs further study.

On the theoretical side the most important open question is the precise nature of the holographic degrees of freedom that are responsible for the spacetime fluctuations. A possible route to gain more control over the microscopic theory is to consider an interferometer in Anti-de Sitter space, and to reformulate the problem in terms of observables of the dual conformal field theory. The question is then to isolate the microscopic degrees of freedom dual to a small causal diamond deep inside the AdS geometry. Another promising direction is to relate our description to the recent works on the BMS-group, soft gravitons and gravitational memory effects [21,22]. In this context one is dealing with coordinate shifts at spatial infinity, or at event horizons of black holes [23]. The theoretical challenge in this case is to generalize these studies to finite size causal diamonds, and determine the fluctuation spectrum of the coordinate shifts. We will leave these theoretical, as well as the phenomenological, analyses to future work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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