The macroeconomics of banking

van der Kwaak, C.G.F.

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This thesis studies the macroeconomic effectiveness of monetary and fiscal policy in an environment where commercial banks are undercapitalized after a financial crisis and have large holdings of (risky) government bonds on their balance sheets. An undercapitalized banking system cannot perfectly elastically expand the balance sheet, and therefore has to choose whether an additional euro of funding is used to provide new credit to the real economy or purchase additional government bonds.

The main result of this thesis is that the effectiveness of monetary and fiscal policy is reduced in such an environment. Two channels play a key role.

The first is one where capital losses arise on existing holdings of risky government bonds held by commercial banks when the fiscal authority engages in debt-financed fiscal policy. More debt issue increases interest rates and lowers bond prices because of increased sovereign default risk. The ensuing capital losses reduce bank capital (bank equity) and therefore force commercial banks to shrink the balance sheet, with negative effects on financing investment in the real economy. This channel is at work in chapters one and two.

The second channel is at work in chapters three and four, where a change in government policy increases the attractiveness for commercial banks to hold government bonds. Because of limited balance sheet capacity after a financial crisis, commercial banks increase government bond holdings at the expense of loan provision to the real economy, which negatively affects the macroeconomy through lower investment.

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THE MACROECONOMICS OF BANKING
THE MACROECONOMICS OF BANKING

ACADEMISCH PROEFSCHRIFT

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aan de Universiteit van Amsterdam,
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ten overstaan van een door het College voor Promoties ingestelde
commissie, in het openbaar te verdedigen in de Aula der Universiteit
op vrijdag 20 januari 2017, te 13:00 uur
door Christiaan Gradus Frederik van der Kwaak
egeboren te Groningen
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Authors

Financial Fragility, Sovereign Default Risk and the Limits to Commercial Bank Bail-outs

Authors: Christiaan van der Kwaak and Sweder van Wijnbergen

This chapter is based on joint work with Sweder van Wijnbergen. Christiaan has developed the research idea and the structure of the paper together with Sweder van Wijnbergen. Christiaan has written up and developed the model, solved the model using numerical simulations, and written up the results. In addition, he was involved in rewriting and editing the paper.

Financial Fragility and the Fiscal Multiplier

Authors: Christiaan van der Kwaak and Sweder van Wijnbergen

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The Macroeconomic Impact of Changing Capital Requirements

Authors: Christiaan van der Kwaak and Sweder van Wijnbergen
AUTHORS

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Financial Fragility and Unconventional Central Bank Lending Operations

Authors: Christiaan van der Kwaak
This chapter has completely been written by Christiaan van der Kwaak.
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Introduction

When I started my Ph.D. thesis in September 2012, the European sovereign debt crisis was at its zenith. The European Central Bank (ECB) had announced its largest refinancing operation in the form of two unconventional Longer-Term Refinancing Operations (LTROs) in December 2011, the government of Spain had received a bailout from fellow Eurozone countries in June 2012, and Mario Draghi had announced “to do whatever it takes” to save the Eurozone in July 2012.

A key element in the European sovereign debt crisis was the poisonous interaction between sovereigns at increased default risk and weakly capitalized banks that have significant amounts of this risky sovereign debt on their balance sheets (Acharya, Drechsler, and Schnabl, 2014; Laeven and Valencia, 2013; Haidar, 2012; De Bruyckere, Gerhardt, Schepens, and Vander Vennet, 2013; Alter and Schüler, 2012; Alter and Beyer, 2012): increased sovereign default risk led to lower prices for government bonds, which imposed capital losses at the commercial banks holding those government bonds. Capital losses, in turn, led to a reduction in bank equity, and a tightening of banks’ leverage constraints, which led to higher interest rates on private credit. Arbitrage between private credit and government bonds led to subsequent interest rate increases on government bonds. Higher interest rates then further increased sovereign default risk, leading to a second round of capital losses on government bonds etc.

In this thesis, I will investigate the macroeconomic effectiveness of government policies in such an environment, and I will show that the presence of (risky)
sovereign debt on the balance sheet of undercapitalized commercial banks can reduce the effectiveness of government policies, and requires new alternative policies. I focus on the case where commercial banks do most or all of the intermediation between savers and borrowers, thus only allowing a limited role for capital markets.

The reason for this choice is that commercial banks are very important for credit provision to the real economy in the Eurozone, as commercial banks intermediate 80% of debt-financing to non-financial corporations (European Central Bank, 2015) and are thus important for financing investment in the real economy. However, European commercial banks were undercapitalized after the Great Recession of 2007-2009 (International Monetary Fund, 2011; Hoshi and Kashyap, 2014), which makes it harder for commercial banks to expand the balance sheet to provide new credit to the real economy. Less credit for firms and businesses in the real economy leads to lower investment and a drag on economic growth.

A second element is the fact that commercial banks in (Southern) European countries carried significant amounts of domestic sovereign debt on their balance sheet. Stress-tests by the European Banking Authority in 2011 revealed that domestic sovereign debt holdings totalled 150% of Tier-1 capital (bank equity/net worth) for Spanish banks, 200% for Italian banks, and 250% for Greek banks (European Banking Authority, 2011). Troubles in sovereign debt markets lead to losses on sovereign debt holdings, and thereby reduce Tier-1 capital of commercial banks. With large exposures to the domestic sovereign, troubles in sovereign debt markets can have a potentially destabilizing effect on commercial banks, as it has the potential to wipe away the complete Tier-1 capital or net worth of commercial banks.

It is clear that in such an environment, with undercapitalized commercial banks and large amounts of (risky) sovereign debt on their balance sheet, policies that affect either the riskiness of government debt, or the relative attractive-
ness of government debt, can have a macroeconomic effect through the credit provision channel.

So the focus of my thesis will be on the effectiveness of fiscal and monetary policy in an environment where weakly capitalized banks have significant amounts of sovereign debt that is possibly subject to default risk. For that purpose, I extend the Gertler and Karadi (2011) framework to incorporate financial intermediaries that finance both private loans to the real economy and sovereign debt that is (possibly) subject to sovereign default risk. This captures the poisonous interaction between sovereigns at increased default risk and weakly capitalized banks with large holdings of this risky sovereign debt on their balance sheet.

This setup allows me to investigate the role of fiscal and monetary policy in such an environment. Is fiscal policy still as effective as when commercial banks do not have significant holdings of (risky) sovereign debt on their balance sheet? Can debt-financed fiscal policy backfire on the sovereign when sovereign debt becomes more risky and leads to capital losses at commercial banks? What is the role of (unconventional) monetary policy in such an environment if commercial banks have to pledge government bonds as collateral in exchange for central bank funding? What is the macroeconomic impact of the structure and the transition to higher capital requirements for commercial banks in an environment where regulators force banks to hold Tier-1 capital for credit to the real economy, while not requiring any Tier-1 capital for government debt? These are the questions I will focus on in this thesis.

In particular, is it necessary to recapitalize commercial banks directly, as was done in the U.S. in 2009, before traditional macroeconomic policies regain their effectiveness? This is of particular importance for the Eurozone, where commercial banks were not forced to increase capital/equity levels, but were allowed to continue operating with hidden losses on their balance sheet (International Monetary Fund, 2011; Hoshi and Kashyap, 2014).
INTRODUCTION

I will next preview the five chapters of my thesis. For each chapter I will introduce, motivate and briefly discuss the key results.

The effectiveness of bank recaps

Commercial banks in many European countries were undercapitalized after the financial crisis of 2007-2009 (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). To get credit flowing to the real economy, many countries decided to recapitalize their financial sector, i.e. provide new equity/net worth to commercial banks, which led to a significant increase in public debt. Commercial banks, however, held significant amounts of domestic sovereign debt at the time, which amounted to more than 100% of core Tier-1 capital in most Southern European countries (European Banking Authority, 2011).

Large interventions in the form of debt-financed recapitalizations, which averaged 40% of GDP in the E.U. (International Herald Tribune, 2013), negatively affect bond prices, leading to capital losses on existing sovereign debt holdings. When commercial banks are already undercapitalized, this mechanism can lead to a further erosion of the capital base, thereby (partially) offsetting the positive effects from the recapitalization.

Chapter 1, based on Van der Kwaak and Van Wijnbergen (2014), studies the effect of such debt-financed recapitalizations in an environment of financial fragility and sovereign default risk, where sovereign debt is (partially) financed by undercapitalized banks. Contrary to traditional bank interventions where the government issues debt to recapitalize the domestic financial sector, a debt-financed recapitalization of banks can backfire on the sovereign and substantially reduce the effectiveness of bank bailouts in such an environment.

The feedback mechanism, through which capital losses on sovereign debt undermine the recapitalization effort for which the debt issuance was initiated in the first place, brings out the limits to traditional bank intervention when
commercial banks have sovereign debt on their balance sheets. This feedback mechanism might be so strong that recapitalization programs fail completely, as happened in Spain in May 2012.

**The effectiveness of fiscal stimuli**

Chapter 2, based on Van der Kwaak and Van Wijnbergen (2015), shifts the focus from the effectiveness of bank recapitalizations to fiscal stimuli. The initial policy response of most governments to the Great Recession of 2007 - 2009 was to implement fiscal stimuli consisting of tax breaks and substantial amounts of government spending. After the initial danger of a complete meltdown of the global financial system had faded, a debate erupted, both in academia and among policymakers, on the effectiveness of fiscal stimuli as a tool to increase demand for products.

I investigate the effectiveness of deficit-financed fiscal stimuli in a similar environment as the first chapter where banks are undercapitalized and carry significant amounts of sovereign debt subject to default risk on their balance sheet. I find that the effectiveness of fiscal stimuli is significantly reduced in such an environment, to the point where the cumulative impact on output may become negative. The mechanism is the following: the anticipation of a future fiscal expansion immediately leads to lower bond prices, which reduces the net worth of commercial banks. Credit provision to the real economy is negatively affected, leading to a fall in output before the package is actually being implemented. However, when monetary policy is constrained by the Zero Lower Bound, fiscal stimulus packages become much more effective, as the feedback effect from increased demand on interest rates is absent. This improves the future profitability of banks, which relaxes banks’ leverage constraints and leads to an increase in credit provision to the real economy.
The results that I find are highly relevant in the context of the European sovereign debt crisis, where banks are undercapitalized (International Monetary Fund, 2011; Hoshi and Kashyap, 2014), carry significant amounts of domestic sovereign debt, and where debt-ridden European sovereigns face substantial default risks. Implementing fiscal stimulus packages in such an environment to prop up the economy, as suggested by for instance Paul Krugman (Krugman, 2012), can be counterproductive, although fiscal stimuli become more effective at the Zero Lower Bound.

The general conclusion from the first two chapters is that undercapitalized banks reduce the effectiveness of fiscal policy, which is why commercial banks should have more capital/equity, which can be achieved by increasing Capital-Adequacy-Ratios (CARs) for commercial banks. I take a look this topic at in the next chapter.

The Transition to Higher Capital Requirements

Governments around the world responded to the financial crisis of 2007 - 2009 by raising capital requirements on financial institutions. While this should increase the buffers of financial institutions, and thereby make the financial system more resilient and less prone to financial crises, a debate erupted about the macroeconomic consequences of the newly adopted requirements.

An additional debate erupted on the structure of the requirements. Current capital requirements are applied to risk-weighted assets. Under a system of risk-weighted assets, a (different) risk-weight is applied to each asset class, as different asset classes pose different risks to the balance sheet of financial institutions. Government debt, for example, is considered riskless and has a zero risk-weight, while more risky private loans carry a high risk-weight.

This is potentially important, as empirical evidence (Hoshi and Kashyap, 2014) suggests that European banks shifted the portfolio composition of their
balance sheet from private credit to government bonds in response to higher capital requirements, a phenomenon known as asset substitution. In addition, the European sovereign debt crisis showed that the zero risk-weight on government bonds might not properly reflect the risks to the balance sheet. Proposals have therefore emerged to apply capital requirements against unweighted assets.

Chapter 3 studies the macroeconomic effects from the structure and the transition to higher capital requirements through the credit provision channel. I find that raising capital requirements has a negative effect on credit provision to the real economy. The negative effects from higher capital requirements can be ameliorated through a recapitalization by the fiscal authority. In addition, I find that the structure of the requirements has a large macroeconomic impact, as the risk-weighted structure of CAR induces banks to shift from high risk-weight assets, such as private loans, to low risk-weight assets, such as government bonds, when banks are forced to reduce the size of the balance sheet in response to a financial crisis or higher capital requirements.

The results are relevant for the current debate on the macroeconomic impact of the structure and the transition to a higher level of capital requirements. My results suggest that it is important that higher capital requirements should be accompanied by higher capital levels to prevent commercial banks from raising CAR through asset substitution. If commercial banks are unwilling to raise new equity privately, they should be forced by the government to do so, or even be recapitalized directly by the government. In addition, raising risk-weights on sovereign debt might limit the shift from private credit to government bonds in response to higher capital requirements and hence limit the impact on credit provision to the real economy.
Interaction with Unconventional Central Bank Lending Operations

Chapter 4 shifts the focus to the interaction between financial fragility and unconventional monetary policy in the form of lending operations of the central bank to an undercapitalized banking system. This is relevant as the European Central Bank (ECB) engaged in a massive increase in liquidity provision to the European banking system at the end of 2011 and beginning of 2012 under the unconventional three-year Longer-Term Refinancing Operations (LTROs).

One of the goals stated by ECB President Draghi was to expand credit provision to the real economy. However, empirical evidence shows that there was a shift from private credit to government bonds at banks from countries that took out most LTRO funding. This chapter provides an explanation for this shift by proposing a framework in which commercial banks have a portfolio choice between private loans and government bonds. In addition, commercial banks can obtain funding from the central bank for which they must pledge collateral in the form of government bonds. The portfolio decision between private loans and government bonds is affected by the collateral constraint when commercial banks are undercapitalized and the interest rate on LTRO funding is below that on regular deposit funding.

I find that the cumulative impact of the LTROs on output is zero, irrespective of the haircut policy, i.e., the amount of central bank funding obtained for one euro of collateral. When the interest rate on LTRO funding is equal to that on regular deposit funding, there are no dynamic effects from the LTRO policy, as LTRO funding and regular market funding are perfect substitutes. When the interest rate on LTRO funding is below that on regular market funding, the LTRO policy can be interpreted as a subsidy for commercial banks that indirectly recapitalizes the banking sector. Although the cumulative impact is still zero, the LTRO policy affects the time-pattern of output, as the LTRO induces a shift from
private credit to government bonds (asset substitution), which negatively affects credit provision to the real economy and therefore output. However, the indirect recapitalization restores bank balance sheets faster, and allows a stronger medium-run recovery, thereby offsetting the contractionary short-run asset substitution effect.

However, as the LTRO comes with a short-run contractionary effect on output, an obvious question is to ask whether a direct recapitalization by the fiscal authority is more effective. I find that this is the case. Bank balance-sheet-constraints are relaxed, similar to the LTRO case, but the recap does not distort the portfolio decision away from private credit. Hence the short-run negative effect on output is absent under a direct recap.

The policy relevance of these results is clear. Instead of expanding credit to the real economy, my model explains why commercial banks shifted from private credit to government bonds. Instead of letting the ECB implement the LTROs, it would have been much better to recapitalize the financial sector directly, either by the domestic fiscal authority or by European funds such as the EFSF or the ESM.

Summary

Chapter 5 summarizes and draws general conclusions from the previous chapters of the thesis. I find in general that the macroeconomic effectiveness of monetary and fiscal policy is much reduced in an environment where commercial banks are undercapitalized after a financial crisis and have large holdings of sovereign debt on their balance sheets. I find that that there are two channels through which aggregate investment and output are reduced after a policy intervention.

The first channel is through capital losses on sovereign debt, for example when the risk of a sovereign default increases and pushes down bond prices.
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The resulting capital losses reduce commercial banks’ net worth, and force commercial banks to reduce credit provision to the real economy, and through that channel aggregate investment.

The second channel is through changes in policy or regulation that makes sovereign debt more attractive compared with credit provision to the real economy. An example is the provision of low-interest-rate funding by the central bank to commercial banks for which they need to pledge government bonds as collateral. Such a policy induces a shift from private credit to government bonds, and results in a reduction of credit provision to the real economy when commercial banks are undercapitalized.

In such an environment some of the traditional macroeconomic policy responses become less effective. I show that a recapitalization by the fiscal authority can ameliorate macroeconomic outcomes, as it directly tackles the problem of an undercapitalized commercial banking system, and (partially) restores the effectiveness of traditional macroeconomic policy responses. The effectiveness of a recapitalization, however, might be reduced when sovereigns are subject to substantial default risk, as I show in chapter 1. In such a situation, academics and policymakers will have to rethink the policy options available to them and come up with alternative policy measures. This, however, is part of a future research agenda.
Chapter 1

Financial Fragility, Sovereign Default Risk and the Limits to Commercial Bank Bail-outs

1.1 Introduction

“The decision to downgrade the Kingdom of Spain’s rating reflects the following key factors:”

1. The Spanish government intends to borrow up to EUR 100 billion from the European Financial Stability Facility (EFSF) or from its successor, the European Stability Mechanism (ESM), to recapitalise its banking system. This will further increase the country’s debt burden, which has risen dramatically since the onset of the financial crisis.....”; Moody’s downgrades Spanish Sovereign bonds, June 13th 2012. (Moody’s Investors Service, 2012a).

“Today’s actions reflect, to various degrees across these banks, two main drivers:

(i) Moody’s assessment of the reduced creditworthiness of the Spanish sovereign, which not only affects the government’s ability to support the banks, but also

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1This chapter is based on joint work with Sweder van Wijnbergen. This article was published in the Journal of Economic Dynamics and Control, see Van der Kwaak and Van Wijnbergen (2014).
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

weighs on banks’ standalone credit profiles.....”; Moody’s downgrades 28 Spanish banks by one to four notches 6 days later. (Moody’s Investors Service, 2012b).

The same day Moody’s Investors Service (2012a,b) downgraded 28 Spanish banks, the political leaders of the G20 declared that: “Against the backdrop of renewed market tensions, Euro area members of the G20 will take all necessary measures to [.........] break the feedbackloop between sovereigns and banks” (G20 Leaders, 2012). And this concern is more than political hype, as Figure 1.1 shows: sovereign debt exposure to the “own” sovereign is in the order of total bank equity. In all periphery countries except Cyprus, sovereign debt exposure exceeds the Tier-1 capital of the banks holding the debt, sometimes by a very substantial margin; in Spain banks’ sovereign debt holdings equal 150% of Tier-1 capital, in Italy almost 200% and in Greece almost 250%. These data should make clear that, with sovereign debt exposure so high among especially the Southern European banks, stress in the sovereign debt market will have a very destabilizing impact on the financial system.

The home bias in those sovereign debt holdings differs across the eurozone. It averages a high 60% in the periphery countries, with Greece as an outlier at almost 80% European Banking Authority (2011). The home bias is less in the Northern countries, where the ratio averages about 20%, although again with a possibly surprising outlier: in Germany almost 60% of sovereign debt holdings is domestic sovereign debt.

Moreover, bank interventions led to very substantial increases in public debt, thereby completing the circle of dependence between sovereigns and commercial banks. When the financial crisis hit in October 2008, governments across advanced economies had to recapitalise their financial system. The U.S. adopted the T.A.R.P. program of $700 billion that in the end was mostly used to recapitalise various financial institutions. And interventions in Europe were even
1.1. INTRODUCTION


larger as a proportion of the intervening countries’ GDP. Table 1.1 shows that the size of European interventions in financial institutions ranges from a relatively low 8.2% of GDP for Italy, to the mind boggling number for Ireland, 365.2%. The average for the E.U. is more than 40% of GDP, so the bank interventions have had a major impact on the aggregate stock of outstanding sovereign debt. It should be clear that interventions this large will have an impact on bond prices, and from there potentially feed back on bank’s balance sheets through increased risk premia, lower bond prices and further capital losses.

Of course capital losses will only occur if the debt is of a significant maturity. Figure 1.2 shows that the average maturity of the sovereign debt portfolios is between 4 and 6 years for the banks in the periphery of the eurozone, and somewhat longer in the core countries of the eurozone (6-8 years).
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

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<th>Country</th>
<th>Total billions euros</th>
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<tr>
<td>Britain</td>
<td>873</td>
<td>50.0%</td>
</tr>
<tr>
<td>Germany</td>
<td>646</td>
<td>25.1</td>
</tr>
<tr>
<td>Denmark</td>
<td>613</td>
<td>256.1</td>
</tr>
<tr>
<td>Spain</td>
<td>575</td>
<td>53.6</td>
</tr>
<tr>
<td>Ireland</td>
<td>571</td>
<td>365.2</td>
</tr>
<tr>
<td>France</td>
<td>371</td>
<td>18.6</td>
</tr>
<tr>
<td>Belgium</td>
<td>359</td>
<td>97.4</td>
</tr>
<tr>
<td>Netherlands</td>
<td>313</td>
<td>52.0</td>
</tr>
<tr>
<td>Sweden</td>
<td>162</td>
<td>41.8</td>
</tr>
<tr>
<td>Italy</td>
<td>130</td>
<td>8.2</td>
</tr>
<tr>
<td>Greece</td>
<td>129</td>
<td>59.9</td>
</tr>
<tr>
<td>Austria</td>
<td>94</td>
<td>31.3</td>
</tr>
<tr>
<td>Portugal</td>
<td>77</td>
<td>45.0</td>
</tr>
<tr>
<td>Poland</td>
<td>68</td>
<td>18.3</td>
</tr>
<tr>
<td>Finland</td>
<td>54</td>
<td>28.5</td>
</tr>
<tr>
<td>Slovenia</td>
<td>13</td>
<td>35.4</td>
</tr>
<tr>
<td>Hungary</td>
<td>10</td>
<td>10.3</td>
</tr>
<tr>
<td>Latvia</td>
<td>9</td>
<td>46.2</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>9</td>
<td>20.9</td>
</tr>
<tr>
<td>Cyprus</td>
<td>5</td>
<td>27.0</td>
</tr>
<tr>
<td>Total E.U.</td>
<td>5,086</td>
<td>40.3</td>
</tr>
</tbody>
</table>

Table 1.1. Total bank bailouts approved (2008 to September 2012). Source: European Commission, as reported in International Herald Tribune (2013).

This implies that sovereign debt problems that cause yields to rise and prices to fall, will inflict substantial capital losses on the financial intermediaries. These capital losses will reduce net worth of the banks, which may well start off a vicious circle as banks increase credit spreads and interest rates, thereby crowding out credit to the private sector, with potentially harmful consequences for investment, tax revenues and long term growth. Lower tax revenues and higher interest rates increase deficits further, leading to further rounds of crowding out and a larger stock of debt, again increasing sovereign discounts. This amplification mechanism and the restrictions it implies on the ability of governments to intervene in and rescue their national commercial banks form the topic of this paper. The key point of this paper is that the negative amplification cycle
1.1. INTRODUCTION

Average maturity banks’ domestic sovereign debt holdings


triggered by the feedback loops back and forth between weak banks and weak governments severely limits the ability of governments to support their financial sector in situations of distress. When sovereign risk premia rise and bond prices go down, governments might not even be able to intervene and support their financial sector economy, contrary to what is commonly assumed in contemporaneous macroeconomic models used to analyze financial crises, that the government always has pockets deep enough to finance any possible intervention.

That this concern is not just a theoretical artefact was clearly shown in the case of Spain. Before the financial crisis started, the Spanish economy experienced a housing boom. When the bubble burst, Spanish banks were left with big losses on their real estate portfolios, effectively wiping out their net worth. This, in turn, depressed the flow of credit to the private sector, and contributed to
the ensuing recession. At the same time government deficits soared, and within a couple of years Spanish debt rose from 35% of GDP to more than 80% of GDP (Spanish Ministry of Economic Affairs, 2012). The Spanish government decided to restructure the Spanish financial system in May 2012, and committed to debt-financed public recapitalizations in case banks would not be able to raise new capital privately. It was expected that the new flow of credit by a recapitalized banking system would restart the economy, and thereby improve the long term budget position of the Spanish government, which should be reflected in lower yields on current Spanish government bonds. Instead, yields soared, undermining any effect of the intended bank recapitalization, and the Spanish government had to apply for external funds from the ESM to be transmitted directly to the banks on June 25th, 2012. Similar problems have emerged across Southern Europe, extremely so in Ireland. There bank intervention was in fact the only source of the subsequent debt problems, prior to the recent crisis Irish debt was as low as 25% of GDP (Eurostat, 2014) while their government budget was in fact in surplus in 2007. The bank rescue in 2008 led to an explosion of domestic debt, a collapse in debt prices and an effective drying up of capital market access for Ireland.

In order to capture the above described dynamics, we build a dynamic stochastic general equilibrium model that incorporates balance sheet constrained financial intermediaries supplying loans both to firms and to the government (i.e. they hold sovereign debt on their balance sheet). We also explicitly introduce sovereign risk. The methodological innovation is the fact that we combine financial intermediaries in our macromodel that are balance sheet constrained while holding both corporate loans and government bonds subject to sovereign default risk. Through this channel we capture the interconnectedness between the financial system and the fiscal problems of the government.

We introduce long term government bonds in a way similar to Woodford (1998, 2001), through a variable maturity structure of government debt captured
by the parameter \( \rho \), through which we can obtain any duration between 1 period bonds (\( \rho = 0 \)) and perpetuums, or ‘consols’ (\( \rho = 1 \)).\(^2\) Introducing maturity structure allows us to capture the stylized facts from Figure 1.2. Introducing maturities longer than the one period bonds commonly used in macroeconomic models is important because of the link with capital losses for the already balance sheet constrained commercial banks in the model. The longer the maturity of the government bonds, the higher the capital losses for the financial intermediaries, and the more pronounced the adverse effects on the economy in case of a financial crisis.

Long term government debt is commonly thought of as stabilizing because of lower roll over risk; while that is doubtlessly true, we show there is another side to this whereby long term debt may in fact exacerbate a given financial crisis. We do not try to derive an optimal maturity structure balancing these two conflicting effects on financial fragility; instead, more modestly, we take the maturity structure as given, and show how lengthening the maturity structure strengthens a poisonous link between financial fragility and sovereign weakness in the debt market.

Sovereign default risk is captured by postulating a so called maximum level of (lump-sum) taxation that is politically feasible, which is imposed by assumption. We then map this maximum level of taxation into a maximum level of debt. We assume that the government follows a core tax policy that guarantees intertemporal solvency in the no default setup and compute the amount of new debt that needs to be issued in order to finance all government obligations, and compare this with the maximum level of debt that is still politically feasible. If the so-called level of no default debt is smaller than the maximum level of debt, the government honors its obligations and does not default; when the no-default

\(^2\)We are indebted to a referee for suggesting this approach.
level exceeds the maximum level, a (partial) default occurs bringing back the number of government bonds to the maximum number possible.

We first use the model to assess the effect of varying the maturity of the government bonds on the impact of a financial crisis. We then proceed to investigate the effect of a recapitalization of the financial sector that is announced at the onset of a financial crisis, but implemented 4 quarters later, reflecting realistic delays in implementing rescue programs. This will introduce anticipation effects coming in before the recapitalization itself due to the forward looking nature of the model. We finally introduce sovereign default risk, and compare the same recapitalization exercise but now in the presence of endogenous sovereign default risk. In particular, we want to investigate whether and how financial sector bailout programs affect sovereign default risk, and whether sovereign default risk can feed back to the financial sector, thereby undermining the rescue action and creating an amplification mechanism exacerbating the initial impact of a financial shock.

Since the start of the credit crisis, the theoretical literature with general equilibrium models containing financial frictions is growing, although Bernanke, Gertler, and Gilchrist (1999) preceded the crisis. Gertler and Karadi (2011) introduce financial intermediaries that are balance sheet constrained by an agency problem between the deposit holders and the bank owners. This gives rise to an endogeneous leverage constraint, which becomes more binding when net worth is reduced by for example a negative shock to the quality of the loans. Several others have a similar mechanism, for example Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), and Kirchner and van Wijnbergen (2012), who include financial intermediaries holding short term government debt besides loans to the private sector. The current paper extends that model by introducing long term government bonds and sovereign default risk. Woodford (1998, 2001) introduces long term government debt by assuming that the government is financed through a bond with infinite maturity. The stream of payments that the
holder receives, though, decreases each period by a factor $\rho \leq 1$, thereby creating a bond with an effective duration that depends on the factor $\rho$. We follow this approach to modeling maturity. Gertler and Karadi (2013) also extend the number of assets held by financial intermediaries by letting them hold a long term government bond in the form of a perpetuity, a case that is encompassed as a special case in the setup used in this paper (for $\rho = 1$). The introduction of government bonds financed by financial intermediaries creates a second amplification mechanism, whereby increased government bond issuance, in order to stimulate the economy, can crowd out financing of the private sector. These papers, however, do not take into account the possibility of a government default. Acharya, Drechsler, and Schnabl (2014) have a setup containing both financial sector bailouts and sovereign default risk, but their analysis occurs within a partial equilibrium setup. Acharya and Steffen (2012), in their empirical research on systemic risk of the European banking sector, find that European banks have been at the center of the two major systemic crises that have faced the financial system since 2007, and specifically that markets have demanded more capital from banks with high sovereign debt exposures to peripheral countries, thereby indicating that sovereign debt holdings from those countries are a major contributor to systemic risk.

Designing the optimal maturity structure of public debt is not the ambition of this paper (cf Cole and Kehoe (2000), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012) for a discussion of the optimal maturity structure). Our focus is exclusively on the possibility of capital losses due to changes in sovereign risk. Sovereign default risk is captured in Arellano (2008), which contains an endogeneous default mechanism somewhat similar in outcome to our approach (see Davig, Leeper, and Walker (2011) for a similar approach). Our set up is close to Schabert and van Wijnbergen (2014) who introduce sovereign default risk by assuming that there exists a (stochastic) maximum
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

level of taxation that is politically feasible and derive from there a default risk
discount that is increasing with government debt.

Section 1.2 describes the version of the model without sovereign default risk. Section 1.3 introduces sovereign default risk into the model. Section 1.4 describes the calibration of the model. Section 1.5 discusses the results from the simulations, and section 1.6 concludes.

1.2 Model description

Financial frictions are introduced in a manner similar to the approach pioneered by Gertler and Karadi (2011), but in our set up banks extend credit to firms but also hold public sector debt on their balance sheet, like in Kirchner and van Wijnbergen (2012). Furthermore we introduce long term government debt and the possibility of a (partial) sovereign default. The government issues debt to financial intermediaries and raises taxes in a lump sum fashion from households to finance its expenditures and meet debt service obligations of its existing debt. The default probability is increasing in the real debt burden in a manner specified more fully below. The other part of the public sector is a central bank that is in charge of monetary policy. It sets the nominal interest rate on the deposits that the households bring to the financial intermediaries. The private sector consists of financial intermediaries and a non-financial sector that includes households and firms. The non-financial sector consists of capital producing firms that buy investment goods and used capital, and convert these into capital that is sold to the intermediate goods producers. The intermediate goods producers use the capital as an input, together with labor, to produce intermediate goods for the retail firms. Future gross profits are pledged to the financial intermediaries in order to obtain funding, hence the profits of the intermediate goods producers are zero in equilibrium. Each intermediate goods producer produces a differentiated product. The retail firms repackage and sell
the retail products to the final goods producer. Every retail firm is a monopo-
list and charges a markup for his product. The final goods producers buy these
goods and combine them into a single output good. The final good is purchased
by the households for consumption, by the capital producers to convert it into
capital, and by the government. The household maximizes life-time utility sub-
ject to a budget constraint, which contains income from deposits, profits from
the firms, both financial and non-financial, and from labor. The income is used
for consumption, lump sum taxes and investments in deposits.

1.2.1 Household

The household sector consists of a continuum of infinitely lived households
that exhibit identical preferences and asset endowments. A typical household
consists of bankers and workers. Every period, a fraction $f$ of the household
members is a banker running a financial intermediary. A fraction $1 - f$ of the
household members is a worker. At the end of every period, all members of
the household pool their resources, and every member of the household has the
same consumption pattern. Hence there is perfect insurance within the house-
hold, and the representative agent representation is preserved. Every period, the
household earns income from the labor of the working members and the profits
of the firms, which are owned by the household. And deposits are paid back
with interest. The household uses these funds to buy goods for consumption or
deposits them in financial intermediaries (but not the ones owned by the family,
in order to prevent self-financing). The household members derive utility from
consumption and leisure, with habit formation in consumption, in order to more
realistically capture consumption dynamics, as in Christiano, Eichenbaum, and Evans (2005). Households maximize expected discounted utility

$$\max_{\{c_{t+s}, h_{t+s}, d_{t+s}\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \log (c_{t+s} - \nu c_{t-1+s}) - \Psi \frac{h_{t+s}^{1+\varphi}}{1+\varphi} \right) \right],$$

$$\beta \in (0, 1), \quad \nu \in [0, 1), \quad \varphi \geq 0,$$

where $c_t$ is consumption per household, and $h_t$ are hours worked, subject to the following budget constraint:

$$c_t + d_t + \tau_t = w_t h_t + (1 + r_{t}^{d}) d_{t-1} + \Pi_t.$$  

The household optimizes with respect to the budget constraint. Intermediary deposits $d_{t-1}$ are deposited at $t - 1$; they receive interest $r_t^d$ and repayment of principal at time $t$. $w_t$ is the real wage rate, $\tau_t$ are the lump sum tax payments the household has to pay to the government, and $\Pi_t$ are the profits from the firms that are owned by the households. The profits of the financial intermediary are net of the startup capital for new bankers, as will be explained below. The first order conditions are now given by:

$$c_t : \quad \lambda_t = (c_t - \nu c_{t-1})^{-1} - \nu \beta E_t \left[ (c_{t+1} - \nu c_t)^{-1} \right],$$  

(1.1)

$$h_t : \quad \Psi h_t^{\varphi} = \lambda_t w_t,$$  

(1.2)

$$d_t : \quad 1 = \beta E_t \left[ \Lambda_{t,t+1} (1 + r_{t+1}^d) \right],$$  

(1.3)

where $\lambda_t$ is the Lagrange multiplier of the budget constraint, and $\Lambda_{t,t+i} = \lambda_{t+i}/\lambda_t$ the stochastic discount factor for $i \geq 0$.  

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1.2. MODEL DESCRIPTION

1.2.2 Financial intermediaries

Financial intermediaries lend funds obtained from households to intermediate goods producers and the government. The banker’s balance sheet is given by:

\[ p_{jt} = n_{jt} + d_{jt}, \]

where \( p_{jt} \) are the assets of bank \( j \) in period \( t \), \( n_{jt} \) and \( d_{jt} \) denote the net worth and deposits of bank \( j \). The financial intermediary invests its funds in claims issued by the intermediate goods producer, and in government bonds. Hence the asset side of the bank’s balance sheet has the following structure:

\[ p_{jt} = q_k^{s_k} + q_b^{s_b}, \]

where \( s_k \) are the number of claims on the intermediate goods producers with price \( q_k \), and \( s_b \) the number of government bonds acquired by intermediary \( j \), at a price \( q_b \). The claims on the producers pay a net real return \( r_k \) at the beginning of period \( t+1 \). Government bonds pay a net real return \( r_b \) at the beginning of period \( t+1 \). Financial intermediaries earn those returns on their assets, and pay a return on the deposits. The difference between the two is equal to the increase in the net worth from one period to the next. The balance sheet of intermediary \( j \) then evolves as follows:

\[
\begin{align*}
    n_{jt+1} &= (1 + r_k^{t+1})q_k^{s_k} + (1 + r_b^{t+1})q_b^{s_b} - (1 + r_d^{t+1})d_{jt} + n_g^{t+1} - \tilde{n}_g^{t+1} \\
    &= (r_k^{t+1} - r_d^{t+1})q_k^{s_k} + (r_b^{t+1} - r_d^{t+1})q_b^{s_b} + (1 + r_d^{t+1})n_{jt} \\
    &\quad + \tau_n^{t+1}n_{jt} - \tilde{\tau}_n^{t+1}n_{jt},
\end{align*}
\]

where \( n_g^{t+1} = \tau_n^{t+1}n_{jt} \) denotes net worth provided by the government to the financial intermediary \( j \) (for example a capital injection). \( \tilde{n}_g^{t+1} = \tilde{\tau}_n^{t+1}n_{jt} \) denotes the repayment of government support received in previous periods.
The financial intermediary maximizes expected profits. The probability that the banker has to exit the industry next period equals $1 - \theta$, in which case he will bring his net worth $n_{j,t+1}$ to the household. So $\theta$ is the probability that he will be allowed to continue operating. The banker discounts these outcomes by the stochastic discount factor $\beta \Lambda_{t,t+1}$, since financial intermediaries are owned by households. The banker’s objective is then given by the following recursively defined maximand:

$$V_{j,t} = \max E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) n_{j,t+1} + \theta V_{j,t+1} \right\} \right],$$

where $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$. We conjecture the solution to be of the following form, and later check whether this is the case:

$$V_{j,t} = \nu^k_t q^k_{t} s^k_{j,t} + \nu^b_t q^b_{t} s^b_{j,t} + \eta_t n_{j,t}. \quad (1.4)$$

Like in Gertler and Karadi (2011), bankers can divert a fraction $\lambda$ of the assets at the beginning of the period, and transfer these assets costlessly back to the household. If that happens, the depositors will force the intermediary into bankruptcy, but will only be able to recover the remaining fraction $1 - \lambda$ of the assets of the financial intermediary. Hence lenders will only supply funds if the gains from stealing are lower than the continuation value of the financial intermediary. This gives rise to the following constraint:

$$V_{j,t} \geq \lambda (q^k_{t} s^k_{j,t} + q^b_{t} s^b_{j,t}) \Rightarrow
\nu^k_t q^k_{t} s^k_{j,t} + \nu^b_t q^b_{t} s^b_{j,t} + \eta_t n_{j,t} \geq \lambda (q^k_{t} s^k_{j,t} + q^b_{t} s^b_{j,t}). \quad (1.5)$$

The optimization problem can now be formulated in the following way:

$$\max_{\{q^k_{t} s^k_{j,t}, q^b_{t} s^b_{j,t}\}} V_{j,t}, \quad \text{s.t.} \quad V_{j,t} \geq \lambda (q^k_{t} s^k_{j,t} + q^b_{t} s^b_{j,t}).$$
1.2. MODEL DESCRIPTION

From the first order conditions we find that $\nu^b_t = \nu^k_t$. Hence the leverage constraint (1.5) can be rewritten in the following way:

$$\nu^k_t(q^k_t s^k_{j,t} + q^b_t s^b_{j,t}) + \eta_t n_{j,t} \geq \lambda(q^k_t s^k_{j,t} + q^b_t s^b_{j,t}) \Rightarrow q^k_t s^k_{j,t} + q^b_t s^b_{j,t} \leq \phi_t n_{j,t},$$

$$\phi_t = \frac{\eta_t}{\lambda - \nu^k_t}, \quad (1.6)$$

where $\phi_t$ denotes the ratio of assets to net worth, which can be seen as the leverage constraint of the financial intermediary. The intuition for the leverage constraint is straightforward: a higher shadow value of assets $\nu^k_t$ implies a higher value from an additional unit of assets, which raises the continuation value of the financial intermediary, thereby making it less likely that the banker will steal. A higher shadow value of net worth $\eta_t$ implies a higher expected profit from an additional unit of net worth, while a higher fraction $\lambda$ implies that the banker can steal a larger fraction of assets, which induces the household to provide less funds to the banker, resulting in a lower leverage ratio everything else equal.

Substitution of the conjectured solution into the right hand side of the Bellman equation gives the following expression for the continuation value of the financial intermediary:

$$V_{j,t} = E_t \left[ \Omega_{t+1} n_{j,t+1} \right],$$

$$\Omega_{t+1} = \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta [\eta_{t+1} + \nu^k_{t+1} \phi_{t+1}] \right\}.$$
\( \Omega_{t+1} \) can be thought of as a stochastic discount factor that incorporates the financial friction. Now substitute the expression for next period’s net worth into the expression above:

\[
V_{j,t} = E_t \left[ \Omega_{t+1} n_{j,t+1} \right] \\
= E_t \left[ \Omega_{t+1} \right. \left\{ (1 + r_{t+1}^k)q_{t}^k s_{j,t}^k + (1 + r_{t+1}^b)q_{t}^b s_{j,t}^b \\
- (1 + r_{t+1}^d)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \right\} \\
= E_t \left[ \Omega_{t+1} \right. \left\{ (r_{t+1}^k - r_{t+1}^d)q_{t}^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d)q_{t}^b s_{j,t}^b \\
+ (1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n) n_{j,t} \right\} \right]. \tag{1.7}
\]

After combining the conjectured solution with (1.4), we find the following first order conditions:

\[
\eta_t = E_t \left[ \Omega_{t+1} \right. \left\{ (1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n) \right\}, \tag{1.8}
\nu_t^k = E_t \left[ \Omega_{t+1} \right. \left\{ (r_{t+1}^k - r_{t+1}^d) \right\}, \tag{1.9}
\nu_t^b = \nu_t^k = E_t \left[ \Omega_{t+1} \right. \left\{ (r_{t+1}^b - r_{t+1}^d) \right\}, \tag{1.10}
\Omega_{t+1} = \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta [\eta_{t+1} + \nu_t^k \phi_{t+1}] \right\}.
\]

**Financial sector support**

We assume that support provided to an individual intermediary, if provided, will be proportional to the intermediary’s net worth in the previous period. Hence individual financial support is given by:

\[
n_{j,t}^g = \tau_t^g n_{j,t-1}, \quad \zeta \leq 0, \quad l \geq 0, \\
\tau_t^n = \zeta (\xi_t - \xi).
\]
1.2. MODEL DESCRIPTION

Repayment of the support is parametrized proportionally to the sector’s net worth in the period preceding the pay back period:

\[ \tilde{n}_{j,t}^g = \tilde{\tau}_t n_{j,t-1}, \]

where \( \tilde{\tau}_t \) is a scaling factor that is obviously time dependent and incorporates the return paid by the sector to the government over the support funds.

**Aggregation of financial variables**

Integrating the individual balance sheets of the financial intermediaries yields the aggregate balance sheet of the financial sector:

\[ p_t = n_t + d_t. \] \hspace{1cm} (1.11)

Aggregation over the asset side of the balance sheet gives the composition of the aggregated financial system:

\[ p_t = q^k_t s^k_t + q^b_t s^b_t. \] \hspace{1cm} (1.12)

\( \phi_t \) does not depend on firm specific factors, so we can aggregate the leverage constraint (1.6) across financial intermediaries to link sector wide assets and net worth:

\[ p_t = q^k_t s^k_t + q^b_t s^b_t = \phi_t n_t. \] \hspace{1cm} (1.13)

The share of assets invested in private loans is given by:

\[ \omega_t = q^k_t s^k_t / p_t. \] \hspace{1cm} (1.14)

At the end of the period, only a fraction \( \theta \) of the current bankers will remain a banker, while the remaining fraction \( 1 - \theta \) will become a worker. Bankers only pay out dividends at the moment they quit the banking business. If they do not
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

quit, they retain their net worth and carry it into the next period. So the aggregate net worth of the continuing bankers at the end of the period equals:

\[ n_{e,t} = \theta \left[ (r^k_t - r^d_t)q^{k}_{t-1}s^{k}_{t-1} + (r^b_t - r^d_t)q^{b}_{t-1}s^{b}_{t-1} + (1 + r^d_t)n_{t-1} \right]. \]

Exiting bankers bring their net worth into the household’s income. A fraction \( 1 - \theta \) of the \( f \) bankers leaves the financial industry each period, equal to a fraction \( (1 - \theta)f \) of the household. The same fraction of the household will enter the financial industry next period. We assume that the household will provide a starting net worth to the new bankers proportional to the assets of the old bankers, equal to a fraction \( \chi/(1 - \theta) \) of the assets of the old bankers, as in Gertler Karadi (2011). Hence the aggregate net worth of the new bankers will be equal to:

\[ n_{n,t} = \chi p_{t-1}. \]

Then the total net worth at the end of the period, after the lottery has decided which bankers will leave the industry, is:

\[
\begin{align*}
    n_t &= n_{e,t} + n_{n,t} + n^g_t - \tilde{n}^g_t \\
    &= \theta \left[ (r^k_t - r^d_t)q^{k}_{t-1}s^{k}_{t-1} + (r^b_t - r^d_t)q^{b}_{t-1}s^{b}_{t-1} + (1 + r^d_t)n_{t-1} \right] \\
    &+ \chi p_{t-1} + n^g_t - \tilde{n}^g_t, \\
\end{align*}
\]

(1.15)

where \( n^g_t \) and \( \tilde{n}^g_t \) are aggregate financial sector support, respectively payback of (earlier) financial support. Since individual support is proportional to the individual intermediary’s net worth, it is straightforward to get aggregate financial sector support:

\[ n^g_t = \zeta(\xi_{t-1} - \bar{\xi})n_{t-1}. \]

(1.16)
Similarly, we can aggregate financial sector payback:

\[ \hat{n}_t^g = \hat{\tau}_t^n n_{t-1} \Rightarrow \hat{\tau}_t^n = \hat{n}_t^g / n_{t-1}. \]  (1.17)

We derive the expression for \( \hat{n}_t^g \) below, in section 1.2.4.

1.2.3 Production side

The production side of the economy is modeled in by now standard New-Keynesian fashion. We have a continuum of intermediate goods producers indexed by \( i \in [0,1] \) borrowing from the financial intermediary to purchase the capital necessary for production. With the proceeds from the sale of the output and the sale of the capital after it has been used, the firms pay workers and pay back the loans to the financial intermediary. The capital producers buy the capital that has been used, and transform the used capital, together with the goods purchased from the final goods producers, into new capital. This new capital is sold to the intermediate goods producers, who will use it for production next period. A continuum of retail firms, indexed by \( f \in [0,1] \), repackage the products bought from the intermediate goods producers to produce a unique differentiated retail product. The retail firms sell their products to a continuum of final goods producers. The products are differentiated, so each individual retail firm has “local” monopoly power, and charges a markup. A randomly selected fraction \( \psi \) of all retail firms can not change prices in a given period. The final goods producers convert the inputs from the retail firms into final goods. Due to perfect competition, profits are zero in equilibrium, and the final goods are sold to the households, the government, and the capital producers. We only derive the non-standard parts, and refer to appendix 1.A.2 for the rest of the production process.
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

Capital Producers

At the end of period \( t \), when the intermediate goods firms have produced, the capital producers buy the remaining stock of capital \((1 - \delta)\xi_t k_{t-1}\) from the intermediate goods producers at a price \(q^k_t\). They combine this capital with goods bought from the final goods producers (investment \(i_t\)) to produce next period’s beginning of period capital stock \(k_t\). This capital is being sold to the intermediate goods producers at a price \(q^k_t\). We assume that the capital producers face convex adjustment costs when transforming the final goods bought into capital goods, set up such that changing the level of gross investment is costly. Hence we get:

\[
k_t = (1 - \delta)\xi_t k_{t-1} + (1 - \Psi(\tau_t))i_t, \quad \Psi(x) = \frac{\gamma}{2}(x - 1)^2, \quad \tau_t = i_t / i_{t-1}. \quad (1.18)
\]

\(\xi_t\) represents a capital quality shock which will be discussed later. Profits are passed on to the households, who own the capital producers. The profit at the end of period \( t \) equals:

\[
\Pi^c_t = q^k_t k_t - q^k_t (1 - \delta)\xi_t k_{t-1} - i_t.
\]

The capital producers maximize expected current and (discounted) future profits (where we substitute in (1.18)):

\[
\max_{\{i_{t+i}\}_{i=0}^{\infty}} E_t \left[ \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left( q^k_t (1 - \Psi(\tau_{t+i}))i_{t+i} - i_{t+i} \right) \right].
\]

Differentiation with respect to investment gives the first order condition for the capital producers:

\[
q^k_t (1 - \Psi(\tau_t)) - 1 - q^k_t \tau_t \Psi'(\tau_t) + \beta E_t \Lambda_{t,t+1} q^k_{t+1} \tau_{t+1}^2 \Psi'(\tau_{t+1}) = 0,
\]
which gives the following expression for the price of capital:

\[
\frac{1}{q_t^k} = 1 - \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 - \frac{\gamma i_t}{i_{t-1}} \left( \frac{i_t}{i_{t-1}} - 1 \right) + \beta E_t \left[ \Lambda_{t+1} \frac{q_{t+1}^k}{q_t^k} \left( \frac{i_{t+1}}{i_t} \right)^2 \frac{\gamma}{i_t} \left( \frac{i_t}{i_{t-1}} - 1 \right) \right].
\] (1.19)

**Intermediate Goods Producers**

There exists a continuum of intermediate goods producers indexed by \( i \in [0, 1] \). Each of these firms produces a differentiated good. The intermediate goods producers obtain funds from the financial intermediaries by pledging next period’s profits, so banks are exposed to downside risk. We assume that there are no financial frictions between the financial intermediaries and the intermediate goods producers. The securities issued by the intermediate goods producers are best considered as state-contingent debt, like in Gertler and Kiyotaki (2010).\(^3\)

The price of the claims is equal to \( q_{t-1}^k \), and pay a state-contingent net real return \( r_t^k \) in period \( t \). The production technology of the intermediate goods producers is given by:

\[
y_{i,t} = a_t (\xi_t k_{i,t-1})^{\alpha h_{i,t}^{1-\alpha}},
\]

\[
\log(a_t) = \rho_a \log(a_{t-1}) + \epsilon_{a,t}, \quad \log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \epsilon_{\xi,t}.
\]

Both (log of) total factor productivity \( a_t \) and capital quality \( \xi_t \) are AR(1) processes driven by random shocks \( \epsilon_{a,t} \sim N(0, \sigma_a^2) \) and \( \epsilon_{\xi,t} \sim N(0, \sigma_\xi^2) \). The intermediate goods producer acquires the capital at the end of period \( t-1 \) and uses it for production in period \( t \). The capital quality shock \( \xi_t \) occurs at the beginning of period \( t \), so \( \xi_t k_{i,t-1} \) is the effective stock of capital used for production in period

---

\(^3\)It is therefore better to think of the claims of financial intermediaries as equity. Occhino and Pescatori (2015) explicitly model loans to producers with a fixed face value, where the goods producers have the possibility of defaulting on the loans. We refrain from explicitly modelling this default possibility, and note the equity characteristics of debt in the real world when firms are short of funds to pay off the loans.
A negative realization of $\xi_t$ lowers the quality of the capital stock, hence the return on the claims of the financial intermediary will be lower. The intermediate goods producer hires labor $h_{i,t}$ for a wage rate $w_t$ after the shock $\xi_t$ has been realized. When the firm has produced in period $t$, the output is sold for price $m_t$ to the retail firms. $m_t$ is the relative price of the intermediate goods with respect to the price level of the final goods, i.e. $m_t = P_t^m / P_t$. A fraction $\delta$ of the capital stock $\xi_{t-1}$ is used up in the production process. The intermediate goods producing firms sell back what is left of the effective capital stock to the capital producers for the end-of-period price of $q_{t-1}^k$ and thus receive $q_{t-1}^k(1-\delta)\xi_{t-1}$. Hence period $t$ profits are:

$$\Pi_{i,t} = m_t a_t(\xi_{t-1})^{1-\alpha} h_{i,t}^{-\alpha} + q_{t}^k (1-\delta)\xi_{t-1} - (1 + r_{t}^k) q_{t-1}^k k_{t-1} - w_t h_{i,t}.$$

The intermediate goods producing firms maximize expected current and future profits using the household’s stochastic discount factor $\beta^s \Lambda_{t,t+s}$ (since they are owned by the households), taking all prices as given:

$$\max_{\{k_{t+s}, h_{i+s}\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \Pi_{i,t+s} \right].$$

The resulting first order conditions are derived in a straightforward manner, and can be found in the appendix.

### 1.2.4 Government

The government issues $b_t$ bonds in period $t$, and raises $q_t^b b_t$ with $q_t^b$ the market price of bonds. We parametrize the maturity structure of government debt like Woodford (1998, 2001): maturity is introduced by assuming that one government bond issued in period $t$ pays out $r_c$ units (in real terms) in period $t+1$, $\rho r_c$ real units in period $t+2$, $\rho^2 r_c$ real units in period $t+3$ etc. This is equivalent to a payout of $r_c$ plus $\rho$ times one newly issued bond in period $t+1$, with a value of $r_c + \rho q_{t+1}^b$. So $\rho$ pins down the maturity of government debt, and gov-
1.2. MODEL DESCRIPTION

government debt service in period \( t \) is \((r_c + \rho q^b_t)b_{t-1}\). The duration\(^4\) of public debt is \(1/(1-\beta \rho)\). The government also raises revenue by levying lump sum taxes on the households. Government purchases are constant: \( g_t = G \). Furthermore the government may provide assistance to the financial intermediaries by injecting capital \( n_t^g \), and it receives repayment of support administered previously \((\tilde{n}_t^g)\). So the budget constraint becomes:

\[
q^b_t b_t + \tau_t + \tilde{n}^g_t = g_t + n_t^g + (r_c + \rho q^b_t)b_{t-1} = g_t + n_t^g + \left( \frac{r_c + \rho q^b_t}{q^b_{t-1}} \right) q^b_{t-1}b_{t-1},
\]

\[\implies q^b_t b_t + \tau_t + \tilde{n}^g_t = g_t + n_t^g + \left( 1 + r^b_t \right) q^b_{t-1}b_{t-1}. \quad (1.20)\]

\(r^b_t\) is the real return on government bonds:

\[
1 + r^b_t = \frac{r_c + \rho q^b_t}{q^b_{t-1}}. \quad (1.21)
\]

The tax rule of the government is given by a rule which Bohn (1998) has shown secures sustainability:

\[
\tau_t = \bar{\tau} + \kappa_b(b_{t-1} - \bar{b}) + \kappa_n n^g_t, \quad \kappa_b \in (0, 1], \quad \kappa_n \in [0, 1]. \quad (1.22)
\]

\(\bar{b}\) is the steady state level of debt. \(\kappa_n\) controls the way government transfers to the financial sector are financed. If \(\kappa_n = 0\), support is financed by new debt. \(\kappa_n = 1\) implies that the additional spending is completely financed by increasing lump sum taxes. We parametrize government support as follows:

\[
n^g_t = \tau^n_t n_{t-1}, \quad \zeta \leq 0, \quad l \geq 0, \quad \tau^n_l = \zeta(\xi_{l-1} - \xi). \quad (1.23)
\]

---

\(^4\)Duration is defined as: \(\sum_{j=0}^{\infty} \frac{B(\rho^{j-l} \xi)}{\sum_{j=0}^{\infty} \beta (\rho^{j-l} \xi)}\)
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

Thus the government provides funds to the financial sector if $\zeta < 0$ (a negative shock to the quality of capital). Depending on the value of $l$, the government can provide support instantaneously ($l = 0$), or with a lag ($l > 0$). Furthermore, $\vartheta$ indicates the extent to which the government needs to be repaid:

$$\tilde{n}^g_t = \vartheta n^g_{t-e}, \quad \vartheta \geq 0, \quad e \geq 1.$$  \hspace{1cm} (1.24)

$\vartheta = 0$ means the support is a gift from the government. In case $\vartheta = 1$, the government aid is a zero interest loan, while a $\vartheta > 1$ implies that the financial intermediaries have to pay interest over the support received earlier.\(^5\) The parameter $e$ denotes the amount of time after which the government aid has to be paid back.

1.2.5 Central Bank

The Central Bank sets the nominal interest rate on deposits $r^n_t$ according to a standard Taylor rule, in order to minimize output and inflation deviations:

$$r^n_t = (1 - \rho_r) \left( r^n + \kappa_\pi (\pi_t - \bar{\pi}) + \kappa_y \log(y_t/y_{t-1}) \right) + \rho_r r^n_{t-1} + \varv_{r,t}, \hspace{1cm} (1.25)$$

where $\varv_{r,t} \sim N(0, \sigma^2_r)$, and $\kappa_\pi > 0$ and $\kappa_y > 0$. The parameter $\bar{\pi}$ is the target inflation rate. We choose $\kappa_\pi > 1, \kappa_y > 0$, (leaning against the wind). The real interest rate on deposits then equals:

$$1 + r^d_t = \left(1 + r^n_{t-1}\right)/\pi_t.$$  \hspace{1cm} (1.26)

1.2.6 Market clearing

Equilibrium requires that the number of claims owned by the financial intermediaries ($s^k_t$) must be equal to aggregate capital ($k_t$), while the number of

\(^5\)The case where $\vartheta > 1$ happened in the Netherlands, where financial intermediaries received government aid with a penalty rate of 50 percent.
government bonds owned by the financial sector \( s^b_t \) must be equal to the number of bonds issued by the government \( b_t \):

\[
\begin{align*}
    s^k_t &= k_t, \\
    s^b_t &= b_t.
\end{align*}
\] (1.27) (1.28)

Goods market clearing requires that the aggregate demand equals aggregate supply:

\[
c_t + i_t + g_t = y_t.
\] (1.29)

1.3 Extension with government default

1.3.1 The default process

The government follows a simple tax rule consistent with the long term sustainability requirements outlined in Bohn (1998):

\[
\tau_{t+1} = \bar{\tau} + \kappa_b (b_t - \bar{b}) + \kappa_n n^g_{t+1}.
\]

But we furthermore assume that there is a maximum level beyond which taxes are not politically sustainable anymore, like in Schabert and van Wijnbergen (2014), or like the ‘fiscal limit’ in Davig, Leeper, and Walker (2011). This fiscal limit is not derived endogenously, but is instead introduced by assumption. This fiscal limit translates in a maximum level of debt that can be sustained, and introduces the possibility of (partial) default if shocks trigger higher levels of debt than the maximum level of debt implied by the ’fiscal limit’. Such a maximum level of taxes should probably be stochastic, as in Schabert and van Wijnbergen (2014), but for simplicity and without much loss of generality we follow Davig, Leeper, and Walker (2011) in assuming it to be fixed and known.
to be equal to $\tau^{\text{max}}$. The maximum level of debt $b_t^{\text{max}}$ implied by this fiscal limit then becomes:

$$b_t^{\text{max}} = \bar{b} + \frac{\tau^{\text{max}} - \bar{\tau}}{\kappa_b}.$$  \hfill (1.30)

As shown in section 2.4, the Woodford (1998, 2001) maturity structure leads to a government liability before financing of the primary deficit equal to $L_t^q = (r_c + \rho q_t^b)b_{t-1}$. Thus in the absence of government default, the end of period debt would become:

$$q_t^b \bar{b}_t = L_t^q + g_t + n_t^g - \tau_t - \bar{n}_t^g,$$

where $\bar{b}_t$ denotes the level of government debt if the government would not default on its obligations. The constraint can be rewritten in the following way:

$$q_t^b \bar{b}_t + \tau_t + \bar{n}_t^g = g_t + n_t^g + \left( r_c + \rho q_t^b \right) b_{t-1}.$$  \hfill (1.31)

As long as the level of debt that the government needs to issue in order not to default ($\bar{b}_t$) is smaller than the maximum level of debt $b_t^{\text{max}}$, the actual government debt $b_t$ will be equal to the no default level of government debt $\bar{b}_t$. But when $\bar{b}_t > b_t^{\text{max}}$, we assume that the government defaults on a large enough fraction of its outstanding debt and debtservice obligations to bring the actual end-of-period debt down to $b_t^{\text{max}}$:

$$b_t = \begin{cases} 
\bar{b}_t & \text{if } \bar{b}_t \leq b_t^{\text{max}}; \\
b_t^{\text{max}} & \text{if } \bar{b}_t > b_t^{\text{max}}.
\end{cases}$$  \hfill (1.32)

We assume that the government achieves this outcome through orderly renegotiation with its creditors. Since creditors have rational expectations, they know that they will not be able to get more from the government than what the government can raise through the maximum level of taxes $\tau^{\text{max}}$. The debt/tax limit may nevertheless become binding because of random shocks to the system affecting debt both directly and through the government’s tax and expenditure rules. We abstain from free-rider problems among creditors and assume all creditors par-
1.3. EXTENSION WITH GOVERNMENT DEFAULT

participate in the renegotiation. The government partially reneges on its debtservice obligations, applying the same discount $\Delta_t$ as used in the debt restructuring.\(^6\) We can see the debt structure $b_t$ as the blue solid line in Figure 1.8 (see appendix). It is informative to write the debt level structure (1.32) in the following way:

$$b_t = \min (\tilde{b}_t, b_{t}^{\text{max}}) = b_{t}^{\text{max}} - \max (b_{t}^{\text{max}} - \tilde{b}_t, 0).$$  \hspace{1cm} (1.33)

The second term is like the payout of a put option at maturity with underlying process $\tilde{b}_t$ and strike price $b_{t}^{\text{max}}$, see Claessens and van Wijnbergen (1993) who apply such a model in their evaluation of the Mexican Brady plan debt restructuring using option pricing methodology for ex ante valuation. As an ex post default function, however, (1.33) is not differentiable at $b_t = b_{t}^{\text{max}}$ which creates severe problems in solving the model. We therefore approximate the ex post default rule by its ex ante option pricing based valuation formula.\(^7\) Since option prices close to maturity are a good approximation to option-payouts at maturity, our option based formula approximates (1.33) closely but without differentiability problems. This is described in detail in 1.A.3.

1.3.2 Default and the government budget constraint

Of course this default process has implications for the government budget constraint. When $\tilde{b}_t \leq b_{t}^{\text{max}}$, $\Delta_t = 0$ but when $\tilde{b}_t > b_{t}^{\text{max}}$, the old government debt $b_{t-1}$ is restructured by converting the old bonds into new bonds against a pro-rata discount high enough to avoid overshooting the maximum debt level. Both coupon payments and all existing bonds are reduced by a factor $(1 - \Delta_t)$

This implies that the government saves an amount equal to $\Delta_t (r_c + \rho_d^{b}) b_{t-1}$ on

---

\(^6\)For analytical simplicity and without loss of generality we choose an equal discount percentage for both current debtservice and for the existing stock of debt.

\(^7\)We acknowledge that we introduce an approximation error in this way. In our simulations, though, we never reach the maximum level of debt, and find that the government maximally defaults over less than 1.5\% of the outstanding debt stock. Hence the sovereign default risk operates mostly through an ex-ante anticipation effect. We therefore think that the approximation does not significantly affect our results, see also appendix 1.B.
new debt issuance. The flow budget constraint of the government in period $t$ thus becomes:

$$q^b_t b_t = L^g_t + g_t + n^g_t - \tau_t - \tilde{n}^g_t - \Delta_t (r_c + \rho q_t^b) b_{t-1},$$

with $\Delta_t = 0$ when there is no default. This can be rearranged to get:

$$q^b_t b_t + \tau_t + \tilde{n}^g_t = g_t + n^g_t + (1 - \Delta_t) (r_c + \rho q_t^b) b_{t-1}. \quad (1.34)$$

### 1.3.3 Financial intermediaries and default

The returns to the financial intermediaries holding the sovereign debt are of course affected. Call the default inclusive return $r_t^{b*}$, which is given by:

$$1 + r_t^{b*} = (1 - \Delta_t) \left( 1 + r_t^b \right) = (1 - \Delta_t) \left( \frac{r_c + \rho q_t^b}{q_{t-1}^b} \right). \quad (1.35)$$

This definition of the return on sovereign debt captures the complete direct impact of the possible default on financial intermediaries holding the debt, so we do not need to change anything after the introduction of possible sovereign defaults other than replacing $r_t^b$ by $r_t^{b*}$. This in turn implies that the expression for the leverage constraint remains unchanged, as well as the expressions for the shadow value of private loans and net worth, and that the equations for the shadow value of government bonds and the law of motion of net worth of financial intermediaries only need minor adjustment (replacement of $r_t^b$ by $r_t^{b*}$):

$$\nu^b_t = E_t \left[ \Omega_{t+1} (r_{t+1}^{b*} - r_{t+1}^d) \right], \quad (1.36)$$

$$n_t = \theta \left[ (r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^{b*} - r_t^d) q_{t-1}^b s_{t-1} + (1 + r_t^d) n_{t-1} \right]$$

$$+ \chi p_{t-1} + n^g_t - \tilde{n}^g_t. \quad (1.37)$$

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1.4 Calibration

1.4.1 No default version

We calibrate the model on a quarterly frequency. The parameter values can be found in Table 1.2. Most of the parameters are common in the literature on DSGE models, or frequently used in models containing financial frictions. We mostly follow the calibration of Gertler and Karadi (2011). This is the case for the subjective discount factor $\beta$, the degree of habit formation $\nu$, the Frisch elasticity of labor supply,$^8$ $\varphi^{-1}$, the elasticity of substitution among intermediate goods $\varepsilon$, the price rigidity parameter $\psi$, the effective capital share $\alpha$, and the investment adjustment cost parameter $\gamma$. The calibration of the financial variables is also taken from Gertler and Karadi (2011). The steady state leverage ratio is set to 4, while the credit spread $\Gamma$ is set to 100 basis points annually (which amounts to $\Gamma = 0.0025$), which coincides with the pre-2007 spreads in US financial data between BAA corporate and government bonds. The parameter $\theta$ is calibrated by taking the average survival period ($\Theta = 1/(1 - \theta)$) to be equal to 36 quarters, or $\theta = 0.9722$. The parameters in the Taylor rule are set to conventional values.

The feedback from government debt on taxes is set to a value such that both the model with and without default are stable. We calibrate the steady state ratios of investment and government spending over GDP, $\bar{i}/\bar{y}$ and $\bar{g}/\bar{y}$ to 20 percent (a reasonable value for OECD countries), by calibrating the depreciation parameter $\delta$. The fixed payment in real terms that the holder of government bonds receives each period is set to 0.04. Different values have been tried but do

---

$^8$ As a robustness check we have set $\varphi = 4$. The results carry over qualitatively, although labor supply becomes more persistent, and therefore moderates the financial crisis. We have also investigated an RBC version of the model, and found that the effect from the maturity parameter $\rho$ becomes less severe, while preserving the results qualitatively. See also appendix 1.B.
### CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

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<th>Value</th>
<th>Definition</th>
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<tr>
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<td>( \varphi )</td>
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<td>Fraction of assets that can be diverted</td>
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<td>( \chi )</td>
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<td>Proportional transfer to entering bankers</td>
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<td>( \theta )</td>
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<td>Survival rate of the bankers</td>
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<td>( \Gamma )</td>
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</table>

Table 1.2. Model parameters.
not significantly affect the results. In our base case we set the maturity parameter at $\rho = 0.96$, equivalent to an average duration of 5 years. The steady state bond price, though, changes as we vary $\rho$ in the first experiment, the parameter governing the average duration of government debt. In order for different maturities to be comparable, we must make sure that the fraction of government debt on the balance sheet of the financial intermediaries does not change. Hence we calibrate on the outstanding government liabilities as a fraction of GDP $\bar{q}_b \bar{b}/\bar{y}$ instead of $\bar{b}/\bar{y}$, and set it equal to 2.4, implying an annual debt-to-GDP ratio of 60 percent. Even though government financing by financial intermediaries accounts for only a small part in the US, most financial friction models have been calibrated on US data. We follow the conventional calibration. The purpose of this paper is not to perform quantitative exercises specifically focused on the debt-distressed European periphery economies; our aim is more generally to highlight the relevant mechanisms, leaving calibration on European data for the future. We perform robustness checks to make sure that the mechanism does not depend on a specific set of parameters. We assume more aggressive monetary policy in the face of a credit crisis, and hence set $\rho_r = 0$ in times of crisis. We think this captures the way central banks reacted when the financial crisis erupted. A credit crisis is represented by a negative shock to capital quality $\xi_t$ of 5 percent on impact, with an autocorrelation coefficient $\rho_\xi = 0.66$, as in Gertler and Karadi (2011).

1.4.2 Default calibration

In this section we describe the calibration when sovereign default risk is introduced. The calibration of the real economy is not affected by the introduction of sovereign default risk. For the financial sector, the steady state bond price $\bar{q}_b$ changes, and hence $\bar{b}$. We calibrate the maximum level of government liabilities $\bar{q}_b \bar{b}_{max}/\bar{y}$ to be at 90% of annual steady state GDP. Different values could have been chosen, but the main point of the paper is to show the mechanisms that
interplay when debt levels get close to the maximum level of debt. The steady state fraction of government liabilities $\bar{q}_b \bar{b} / \bar{y}$ on the balance sheet of financial intermediaries does not change, since it is still calibrated to hit the 60% annual steady state output target. The reason for this freedom is the fact that we have a new variable, the level of debt in case of no government debt $\tilde{b}_t$, and the steady state tax rate that we can adjust in order to still be able to hit our original targets. The steady state default probability is set at a rather conservative estimate of $\bar{\Delta} = 0.005$, which implies an annual default probability of 2%, which is small given the observed bond spreads in the European periphery.

There are 2 ways in which we calibrate the model, which are in detail described in the appendix. We apply calibration strategy 1 when far away from the debt limit. In this case we always apply $\bar{\Delta} = 0.005$. When the steady state level of government debt comes close to the maximum level of debt, though, this strategy can not be followed anymore, due to numerical problems. This is the case for the last exercise, in which we investigate a delayed recapitalization when the steady state government liabilities are at 80% of annual steady state GDP. First we find the parameters of the (option) approximation when $\bar{q}_b \bar{b} / \bar{y}$ is at 75% of annual steady state GDP. Calibration strategy 1 cannot go further than this when the maximum level of government liabilities is at 90% of annual steady state GDP. We therefore change to calibration strategy 2 which can still be applied, and calibrate the model at 80% of annual steady state GDP. This changes the steady state default probability to $\bar{\Delta} = 0.0068$.

### 1.5 Results

As a prelude to the main results about the interaction between financial fragility, sovereign debt and commercial bank rescues, we first investigate the effect of a financial crisis, initiated through a credit (capital quality) shock in the model, to set the stage for the interventions that form the main topic of this paper. A
special point of interest highlighted in this subsection is the crucial importance of the maturity of sovereign debt. We then analyze the consequences of a classic commercial bank recapitalization by the government, where we realistically assume that the recaps are implemented 4 quarters after the announcement. Then we show that introducing sovereign default risk exacerbates the poetically negative interactions between sovereign debt holdings of commercial banks and debt financed rescue attempts.

1.5.1 The macroeconomic impact of a financial crisis

As a prelude to the analysis of the interaction between sovereign debt and bank rescues, we first set the benchmark, a financial crisis without government intervention to support banks. Like Gertler and Karadi (2011), we model a financial crisis as a decrease in the capital quality $\xi_t$. A deterioration of capital quality induces losses at the financial intermediaries on the loans provided to the intermediate goods producers. As a consequence, the net worth of the financial intermediaries decreases, and hence the intermediaries become more balance sheet constrained, and the credit spread increases by almost 120 basis points in the short maturity case (which corresponds to an average maturity of 2 quarters). The lower quality of capital also decreases the expected productivity from the capital that is purchased with the loans, and because of the lower net worth, the financial intermediaries have to cut back on lending, which further reduces the price of capital. Because of classic Dornbusch style overshooting, the price collapse leads to a higher expected return on the loans once the shock has hit. In response, financial intermediaries sell government bonds, thereby pushing down the current bond price, which in turn inflicts further capital losses on the financial intermediaries. This process continues until the forward looking expected return on government bonds has increased sufficiently to make the intermediaries willingly hold the outstanding stock of bonds. In the process, financial net worth falls further; the intermediaries’ balance sheet further deteriorates, rais-
Financial crisis impact and maturity of sovereign debt-1 (A)

Figure 1.3. Impulse response functions for the model without default for \( \rho = 0.5 \) (blue solid line) and \( \rho = 0.96 \) (red solid line with dots). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state, and no additional government policy is implemented.

...ing borrowing costs and so on. What we see (in the plots collected in Figure 1.3 and 1.4) is a pro-cyclical amplification cycle whereby investment and eventually capital drop by more than 10 percent.

A third balance sheet effect that plays a role was highlighted by Kirchner and van Wijnbergen (2012) and is due to crowding out by government debt. Government spending \( g_t \) is fixed; then government borrowing is primarily affected by the bond price \( q_t^b \). Since the increase in the (expected) interest rates on private loans pushes up the (expected) interest rates on government debt, the bond price drops. Besides inflicting capital losses on the financial intermediaries, it also increases the number of bonds the government needs to issue for a given amount of expenditures. Issuing more bonds implies that there are more creditors to
1.5. RESULTS

Financial crisis impact and maturity of sovereign debt-1 (B)

![Figure 1.4](image)

Figure 1.4. Impulse response functions for the model without default for \( \rho = 0.5 \) (blue solid line) and \( \rho = 0.96 \) (red solid line with dots). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state, and no additional government policy is implemented.

which the government has to pay the fixed payment \( r_c \), implying higher borrowing needs in future periods. Hence a smaller proportion of the intermediaries’ balance sheets is available for financing the capital purchases of the intermediate goods producers. This effect is amplified because the size of the balance sheet is reduced as well, due to a tightening of the balance sheet constraint.

The lower capital quality reduces the productivity of the capital. Wages fall as a consequence, as do profits from the production sector and the financial intermediaries, and (except in the period of the shock) the real return on deposits. Since we assume a very aggressive monetary policy response (the smoothing parameter is set to zero after the crisis hits), nominal rates initially fall to such an extent that the Zero Lower Bound is actually violated for one period. For more
realistic values of the smoothing parameter the ZLB is not violated. Obviously the household’s budget constraint is tightened, and consumption falls: We see that output and consumption are reduced by more than 4 percent. The model also reproduces the Reinhart and Rogoff (2009) finding of long recessions after financial crises: after 40 quarters the economy has still not recovered completely from the initial shock.

Figures 1.3 and 1.4 compare the case where \( \rho = 0.5 \), which coincides with an average duration of 2 quarters (solid blue line), with the case \( \rho = 0.96 \), which corresponds to an average duration of 5 years (dashed red line). The impact of a longer maturity structure is very clear: the longer the maturity of the government bonds, the larger the drop in the bond price to even 7\%, thereby increasing the capital losses faced by financial intermediaries, and a further deterioration of the net worth of the financial intermediaries. Hence they become more balance sheet constrained, as can be seen from an increase in the credit spread to almost 150 basis points. The tightening of the balance sheet induces the financial intermediaries to charge higher expected returns on both bonds and private loans, thereby reducing demand for new private loans, which in turn decreases the price of capital and investment. The drop in asset prices further erodes net worth, further raising interest rates etc. The effects on the real economy are clear: output drops further, and a decrease in investment pushes down the capital stock. The reduction in capital further reduces the demand for labor, and lowers wages and profits from the firms in the economy, which tightens the household budget constraint, further pushing down consumption.

Figure 1.5 elaborates on the importance of the maturity of sovereign debt for the economy without sovereign default risk. The Figure shows the average deviation from the steady state in percentages (except the credit spread, which is in absolute deviation in basis points) for selected variables as a function of the average duration of government bonds. The average deviation is taken over the first 40 periods after the capital shock hits the economy in period 1. By
1.5. RESULTS

Financial crisis impact and maturity of sovereign debt-2

Figure 1.5. Average deviation from the steady state in percentages (except the credit spread, which is in absolute deviation in basis points) for selected variables vs. average duration of government bonds for the model without sovereign default risk. The average is taken over the first 40 periods, where the financial crisis is initiated when a negative capital quality shock of 5 percent relative to the steady state hits the economy in period 1. The government does not engage in additional policy. 1 quarter duration corresponds to $\rho = 0$, while an average duration of 100 quarters corresponds practically speaking to the case of perpetual bonds, or ‘consols’, $\rho = 1$. The maturity parameter $\rho$ is transformed into an average duration in quarters $q$ through the formula $q = 1/(1 - \beta \rho)$.

assumption, the government does not engage in additional policy. 1 quarter duration corresponds to $\rho = 0$. $\rho = 1$ corresponds to perpetual bonds (consols), which we list in the figure as an average duration of 100 quarters. The maturity parameter $\rho$ is transformed into an average duration in quarters $q$ through the formula $q = 1/(1 - \beta \rho)$.

Figure 1.5 shows that a longer maturity of sovereign bonds substantially increases the impact of a crisis. The mechanism should be clear: longer maturities introduce larger capital losses on the stock of bonds on bank balance sheets,
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

giving the negative feedback between bank holdings of sovereign debt and fi-
nancial fragility another perverse twist. The relation is strikingly nonlinear. The
average deviation of all variables drops substantially when increasing the aver-
age duration from 1 quarter to approximately 30 quarters after which the pace
of the decline is lower. The average output decline is higher by about a half
over the range considered; the capital stock decline increases by about a quar-
ter. This is triggered by an almost doubling of the decline in net worth and a
substantially higher increase in credit spread as maturities lengthen. Clearly, the
maturity structure of government bonds is an important channel for further capi-
tal losses on bank balance sheets during financial crises, with substantial adverse
macroeconomic consequences.

1.5.2 Financial crisis and government response: the effect of a (delayed)
recapitalization

Since low capitalization is at the root of the credit tightening and macroe-
conomic fall out after a financial crisis, a recapitalization is a logical response
and has been the mainstay of government intervention on both sides of the At-
lantic. We evaluate the impact of a recapitalization of the financial system of
1.25% of annual steady state GDP by an issuance of new government bonds.
We assume that the recap is announced immediately after the crisis hits, but that
implementing it takes time: in our policy experiment 4 quarters. The measure is
designed to improve financing conditions: the recap alleviates the balance sheet
constraint. On the other hand, the recap requires the government to issue more
bonds, which will cause a drop in the bondprice due to an increased supply,
which in turn affects the balance sheet of the banks which hold bonds in their
asset portfolio. Figure 1.6 compares the case of no additional policy with the
delayed recapitalization. The maturity parameter is set at $\rho = 0.96$, which cor-
responds to an average maturity of sovereign debt of about 5 years, about the
average for the Eurozone area.
1.5. RESULTS

Delayed recapitalization

Figure 1.6. Impulse response functions for the model without default with no additional government policy (blue solid line) and a delayed recapitalization of the financial sector (red solid line with dots) occurring 4 quarters after the financial crisis hits and equal to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

We see that the announcement of the recapitalization already has an impact effect prior to the actual policy implementation: the financial intermediaries anticipate the recapitalization, which raises the continuation value of the financial intermediaries and so immediately relaxes the balance sheet constraint. Due to the relaxation of the balance sheet constraint, the supply of loans (demand for private sector liabilities) and demand for government bonds increases, driving up the price of these assets, thereby reducing the losses on the private loans.
triggered by the initial capital shock, as well as leading to lower losses on government bonds. Net worth increases with respect to the no policy case, further reducing the losses of the financial intermediaries caused by the initial deterioration of the quality of capital, as well as reducing the credit spread by 30 basis points. When the actual recapitalization is implemented, a further drop in the credit spread of approximately 70 basis points occurs. Investment increases by almost 5 percentage points, due to a higher demand for capital.

But at the moment the recap is implemented, debt issue goes up and the bond price drops again, although marginally so. In fact it is more accurate to say that the gradual increase in bond prices is reversed, an actual discrete price drop is prevented by intertemporal arbitrage, since everything has been priced in at time zero, when the intervention was announced. We see that the positive effects from a capital injection dominate the effects of an increased debt issuance, as the asset prices remain above the no policy case. The positive effects on the real economy are clear as well: higher investment pushes up the capital stock. A higher capital stock increases the marginal product of labor, inducing firms to offer higher wages, which causes households to increase their labor supply. This leads to higher consumption with respect to the no policy case, also because of increased profits/reduced losses from the financial intermediaries. The recap has unambiguously positive macroeconomic effects.

1.5.3  **Sovereign risk and the limits of government intervention**

The analysis of the commercial bank recap has sofar ignored the issue of sovereign debt discounts even though a standard government induced recap involves substantial issues of new public debt. The Spanish and Irish experience with large scale commercial bank rescues through the issue of new public debt and/or public guarantees of private claims has indicated, however, that such debt financed bank rescues do undermine capital market confidence in the public sector and its debt. This, in turn, may jeopardise the impact of the initial bank res-
1.5. RESULTS

Default vs. no default and bank recapitalizations

Figure 1.7. Impulse response functions comparing the case with no default (blue solid line) and default (red solid line with dots) in case of a delayed recapitalization of the financial sector of 1.25 percent of annual steady state GDP that is announced at the start of the financial crisis but implemented 4 quarters later. Steady state government liabilities are at 80% of annual steady state GDP. The financial crisis is initiated through an initial negative capital quality shock of 5 percent relative to the steady state.

cue action when the same banks hold sovereign debt on their balance sheet. To analyze this conflict, we introduce sovereign risk into the model. The specifics of the calibration strategies can be found in the appendix. We once again analyze the impact of a recap equal to 1.25% of annual steady state output, and announced at the onset of the financial crisis but implemented 4 quarters later. Figure 1.7 compares the macroeconomic responses set against the same experiment but without sovereign risk.
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The graphs clearly show that the government default possibility does have a significant effect on the economy. The announcement of a recapitalization immediately causes the bond price to drop by an additional 5% with respect to the no default case. Investors subsequently anticipate the extra bond issuance necessary for financing the recapitalization, and the accompanying increase in the sovereign risk discount. This causes additional losses at the financial intermediaries and a further tightening of the balance sheet constraints of those intermediaries. As a consequence, the cost of capital (required return) shoots up as the price drops on impact. The effects on the real economy are clear: investment goes down further, pushing down the capital stock, wages go down, and thereby the supply of labor. Consumption eventually decreases more than in the no default case. Clearly sovereign default risk affects the economy substantially as increased sovereign risk premia are translated into lower bond prices that further inflict capital losses on the financial intermediaries.

The drop in the bond price clearly shows the limits of government intervention: the bonds are issued to increase bank capitalization, but their very issuance causes prices to fall, triggering subsequent losses on commercial banks’ sovereign asset portfolios that substantially offset the impact of the recap they are financing. The sovereign risk impact of the recap makes it more difficult to implement effective support measures. This is reminiscent of the failed recapitalization of the Spanish banking sector that the Spanish government (Spanish Ministry of Economic Affairs, 2012) tried to implement in May 2012, by committing to provide new capital to banks if they could not raise it privately. It was expected that the recapitalization of the Spanish financial sector would relax the debt overhang problems, and create additional room for financing the Spanish private sector. Contrary to these expectations, though, bond yields on Spanish government debt shot up and bond prices went down commensurately when the recap was announced, and the Spanish government had to apply for a direct financial sector bailout by the EFSF/ESM, whereby the bank risk was taken over
by those financial institutions. Our model highlights the importance of the poi-
sonous nexus between banks and sovereigns in times of financial fragility.
1.6 Conclusion

We have investigated the poisonous interaction between financial fragility, commercial bank holdings of public debt and sovereign risk. We do so by introducing sovereign default risk into a model with financial intermediaries that hold public debt in their asset portfolio and are subject to a leverage constraint. We show that the maturity structure of government debt matters a great deal. Issuing more debt as part of a bank recapitalization exercise causes interest rates to rise. But at longer maturities this leads to larger capital losses for the very banks the debt issue is supposed to recapitalise, and the more so the longer is the average maturity of the sovereign debt outstanding.

We also introduce sovereign default risk and show that the real effects from sovereign default risk are again substantial under plausible parametrizations of the default process, leading to further difficulties when trying to recapitalise banks after a financial crisis. Higher debt issuance undermines sovereign credibility and thus leads to larger sovereign debt discounts, further increasing the capital losses on sovereign debt holdings of the banks being intervened. The additional capital losses cause the leverage constraint to tighten more, thereby exacerbating the link between sovereign debt problems and financial fragility, through which sovereign debt problems have a negative impact on the corporate sector: (required) returns increase, making finance more expensive, asset prices go down, balance sheet constraints are tightened, with substantial subsequent negative impact on investment, consumption and output. Obviously, the bond price is most significantly affected: the price drop is doubled compared with the case of no sovereign default. This also triggers dynamic problems: the government has to issue more bonds to finance the same intervention, which increases the payments that the government has to make in the future, thereby increasing future debt issuance and default probabilities once again, causing a further am-
plification channel through which bond prices fall even more and losses on bank
balance sheets rise again.

The mechanisms we highlighted in this paper bring out the limits to traditional bank intervention when commercial banks have sovereign bonds on their balance sheets. Capital losses on longer maturity bonds in commercial bank portfolios, through higher interest rates and/or through increased sovereign risk discounts triggered by increased fears of default, undermine the recapitalization for which the bonds are issued to begin with. The link between the sovereign debt-financial fragility nexus and the European debt crisis, and especially the failed Spanish recapitalization attempt in 2012 is clear: our model predicts the increase in sovereign risk discounts and drop in bond prices when announcing a debt financed bank recapitalization that caused the Spanish government’s recapitalization program announced in May 2012 to fail before even being implemented. We explored in this paper a mechanism the literature has abstracted from, the channel is clearly quantitatively relevant under a fairly standard calibration, suggesting the need and usefulness of further empirical assessment of macromodels incorporating financial frictions.

1.A Derivations

1.A.1 Derivation of structural equations financial intermediaries in presence of sovereign default risk

The introduction of sovereign default risk changes the equations for the financial intermediaries. In this section we show that the net worth of an individual intermediary is given by the same expression as the case with no default, except for the fact that we replace $r^b_t$ by $r^{b*}_t$. Hence we only have to replace $r^b_t$ in the equations governing the financial intermediaries by $r^{b*}_t$, and include the expression for $r^{b*}_t$ in the first order conditions. We start by observing that the funds obtained from selling the bonds in period $t + 1$, that were purchased in period $t$,
are reduced by $1 - \Delta_{t+1}$, just as the fixed real payment $r_c$ per bond is reduced to $(1 - \Delta_{t+1})r_c$. Hence the law of motion for the net worth of an individual intermediary changes into the following equation:

$$n_{j,t+1} = \left(1 + r_{t+1}^d\right)q_t^d s_{j,t} + (1 - \Delta_{t+1})r_c s_{j,t} + (1 - \Delta_{t+1})p q_{t+1}^b s_{j,t} - \left(1 + r_{t+1}^d\right)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \right) q_t^d s_{j,t} \right)$$

$$= \left(1 + r_{t+1}^k\right)q_t^k s_{j,t} + (1 - \Delta_{t+1})\left(1 + r_{t+1}^b\right) q_t^b s_{j,t} - \left(1 + r_{t+1}^d\right)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \right) q_t^b s_{j,t} \right)$$

$$= \left(1 + r_{t+1}^b\right) q_t^b s_{j,t} + \left(1 + r_{t+1}^d\right) q_t^d s_{j,t} + \left(1 - \Delta_{t+1}\right) \left(1 + r_{t+1}^b\right) q_t^b s_{j,t} - \left(1 + r_{t+1}^d\right)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \right) q_t^b s_{j,t} \right)$$

where $r_{t+1}^b$ is given by:

$$1 + r_{t+1}^b = (1 - \Delta_{t+1}) \left(1 + r_{t+1}^b\right) \left(1 - \Delta_{t+1}\right) \left(1 + r_{t+1}^b\right) q_t^b s_{j,t} \right)$$

We replace $r_t^b$ by $r_{t+1}^b$ in the equation for the shadow value of government bonds, and the law of motion of net worth:

$$v_t^b = E_t \left[ \Omega_{t+1} (r_{t+1}^b - r_{t+1}^d) \right],$$

$$n_t = \Theta \left[ (r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) q_{t-1}^b s_{t-1}^b + \chi p_{t-1} + n_t^g - \tilde{n}_t^g. \right]$$

The other equations for the financial intermediaries remain the same.
1.A.2 Production Process

Intermediate Goods Producers

We remember that period $t$ profits are given by:

\[
\Pi_{i,t} = m_t a_t (\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha} + q_t^k (1 - \delta) \xi_t k_{i,t-1} - (1 + r_t^k) q_{t-1}^k k_{i,t-1} - w_t h_{i,t}.
\]

The intermediate goods producing firms maximize expected current and future profits using the household’s stochastic discount factor $\beta^s \Lambda_{t,t+s}$ (since they are owned by the households), taking all prices as given:

\[
\max_{\{k_{i,t+s}, h_{i,t+s}\}_{s=0}^\infty} E_t \left[ \sum_{s=0}^\infty \beta^s \Lambda_{t,t+s} \Pi_{i,t+s} \right].
\]

The first order conditions belonging to this problem are given by:

\[
k_{i,t} : E_t \left[ \beta \Lambda_{t,t+1} q_t^k (1 + r_{t+1}^k) \right] = E_t \left[ \beta \Lambda_{t,t+1} (\alpha m_t y_{i,t+1} / k_{i,t} + q_{t+1}^k (1 - \delta) \xi_{t+1}) \right],
\]

\[
h_{i,t} : w_t = (1 - \alpha) m_t y_{i,t} / h_{i,t}.
\]

In equilibrium profits will be zero. By substituting the first order condition for the wage rate into the zero-profit condition $\Pi_{i,t} = 0$, we can find an expression for the ex-post return on capital:

\[
r_t^k = (q_{t-1}^k)^{-1} (\alpha m_t y_{i,t} / k_{i,t-1} + q_t^k (1 - \delta) \xi_t) - 1.
\]

Now we rewrite the first order condition for labor and the expression for the ex-post return on capital to find the factor demands:

\[
k_{i,t-1} = \alpha m_t y_{i,t} / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t],
\]

\[
h_{i,t} = (1 - \alpha) m_t y_{i,t} / w_t.
\]
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By substituting the factor demands into the production technology function, we get for the relative intermediate output price \( m_t \):

\[
m_t = \alpha^{-\alpha}(1 - \alpha)^{\alpha - 1} \alpha^{-1} \left( w_t^{1 - \alpha} [q_{t-1}^k (1 + r_t^k) \xi_t^{-1} - q_t^k (1 - \delta)] \right)^\alpha. \tag{1.38}
\]

Retail firms

Retail firms purchase goods \((y_{i,t})\) from the intermediate goods producing firms for a nominal price \(P_{m_t}^m\), and convert these into retail goods \((y_{f,t})\). These goods are sold for a nominal price \(P_{f,t}\) to the final goods producer. It takes one intermediate goods unit to produce one retail good \((y_{i,t} = y_{f,t})\). All the retail firms produce a differentiated retail good by assumption, therefore operate in a monopolistically competitive market, and charge a markup over the input price earning them profits \((P_{f,t} - P_{m_t}^m)y_{f,t}\).

Each period, only a fraction \(1 - \psi\) of retail firms is allowed to reset their price, while the \(\psi\) remaining firms are not allowed to do so, like in Calvo (1983) and Yun (1996). The firms allowed to adjust prices are randomly selected each period. Once selected, they set prices so as to maximize expected current and future profits, using the stochastic discount factor \(\beta^s \Lambda_{t,t+s}(P_t/P_{t+s})\):

\[
\max_{P_{f,t}} E_t \left[ \sum_{s=0}^\infty (\beta \psi)^s \Lambda_{t,t+s}(P_t/P_{t+s}) [P_{f,t} - P_{m_t}^m] \right] y_{f,t+s},
\]

where \(y_{f,t} = (P_{f,t}/P_t)^{-\epsilon} y_t\) is the demand function. \(y_t\) is the output of the final goods producing firms, and \(P_t\) the general price level. Symmetry implies that all firms allowed to reset their prices choose the same new price \((P_t^*)\). Differentiation with respect to \(P_{f,t}\) and using symmetry then yields:

\[
\frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^\infty (\beta \psi)^s \Lambda_{t,t+s} P_{t+s}^e P_t^{-\epsilon} m_{t+s} y_{t+s}}{E_t \sum_{s=0}^\infty (\beta \psi)^s \Lambda_{t,t+s} P_{t+s}^e P_t^{-1-\epsilon} y_{t+s}}.
\]
Defining the relative price of the firms that are allowed to reset their prices as \( \pi_t^* = P_t^* / P_t \) and gross inflation as \( \pi_t = P_t / P_{t-1} \), we can rewrite this as:

\[
\pi_t^* = \frac{\varepsilon}{\varepsilon - 1} \Xi_{1,t}, \\
\Xi_{1,t} = \lambda_t m_t y_t + \beta \psi E_t \pi_{t+1}^{\pi_{t+1}^*} \Xi_{1,t+1}, \\
\Xi_{2,t} = \lambda_t y_t + \beta \psi E_t \pi_{t+1}^{\pi_{t+1}^*} \Xi_{2,t+1}.
\]

The aggregate price level equals:

\[
P_t^{1-\varepsilon} = (1 - \psi)(P_t^*)^{1-\varepsilon} + \psi P_{t-1}^{1-\varepsilon}.
\]

Dividing by \( P_t^{1-\varepsilon} \) yields the following law of motion:

\[
(1 - \psi)(\pi_t^*)^{1-\varepsilon} + \psi \pi_{t+1}^{\pi_{t+1}^*} = 1.
\]

**Final Goods Producers**

Final goods firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by \( y_t^{(\varepsilon-1)/\varepsilon} = \int_0^1 y_{f,t}^{(\varepsilon-1)/\varepsilon} df \), where \( y_{f,t} \) is the output of the retail firm indexed by \( f \). \( \varepsilon \) is the elasticity of substitution between the intermediate goods purchased from the different retail firms. The final goods firms face perfect competition, and therefore take prices as given. Thus they maximize profits by choosing \( y_{f,t} \) such that \( P_t y_t - \int_0^1 P_{f,t} y_{f,t} df \) is maximized. Taking the first order conditions with respect to \( y_{f,t} \), gives the demand function of the final goods producers for the retail goods. Substitution of the demand function into the technology constraint gives the relation between the price level of the final goods and the price level of the individual retail firms:

\[
y_{f,t} = (P_{f,t}/P_t)^{-\varepsilon} y_t, \\
P_t^{1-\varepsilon} = \int_0^1 P_{f,t}^{1-\varepsilon} df.
\]
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Aggregation

Substituting \( y_{f,t} = y_{i,t} = y_t \left( \frac{P_{f,t}}{P_t} \right)^{-\varepsilon} \) into the factor demands derived earlier yields:

\[
h_{i,t} = (1 - \alpha) m_t y_{f,t} / w_t, \quad k_{i,t-1} = \alpha m_t y_{f,t} / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t].
\]

Aggregation over all firms \( i \) gives us aggregate labor and capital:

\[
h_t = (1 - \alpha) m_t y_t D_t / w_t, \quad k_{t-1} = \alpha m_t y_t D_t / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t],
\]

where \( D_t = \int_0^1 \left( \frac{P_{f,t}}{P_t} \right)^{-\varepsilon} df \) denotes the price dispersion. It is given by the following recursive form:

\[
D_t = (1 - \psi)(\pi_t^*)^{-\varepsilon} + \psi \pi_t^\varepsilon D_{t-1}. \tag{1.43}
\]

The aggregate capital-labor ratio is equal to the individual capital-labor ratio:

\[
k_{t-1}/h_t = \alpha (1 - \alpha)^{-1} w_t / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t] = k_{i,t-1}/h_{i,t}. \tag{1.44}
\]

Now calculate aggregate supply by aggregating \( y_{i,t} = a_t(\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha} \):

\[
\int_0^1 a_t(\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha} \, di = a_t \xi_t^\alpha \left( \frac{k_{i,t-1}}{h_t} \right)^\alpha \int_0^1 h_{i,t} \, di = a_t(\xi_t k_{i,t-1})^\alpha h_t^{1-\alpha},
\]

while aggregation over \( y_{i,t} \) gives:

\[
\int_0^1 y_{i,t} \, df = y_t \int_0^1 \left( \frac{P_{f,t}}{P_t} \right)^{-\varepsilon} df = y_t D_t.
\]

So we get the following relation for aggregate supply \( y_t \):

\[
y_t D_t = a_t(\xi_t k_{t-1})^\alpha h_t^{1-\alpha}. \tag{1.45}
\]
1.A.3 Approximation of the default function

We can also write the debt level structure (1.32) in the following way:

\[ b_t = \min (\bar{b}_t, b_t^{max}) = b_t^{max} - \max (b_t^{max} - \bar{b}_t, 0). \] (1.46)

We can interpret the second term of the new debt level as the payoff of a put option at maturity with underlying process \( \bar{b}_t \) and strike price \( b_t^{max} \). The formula for \( b_t \), however, does not have a defined derivative at \( \bar{b}_t = b_t^{max} \). Therefore we apply an approximation for the payoff structure of the put option, and use the option pricing formula, which gives the price of the put option when time to maturity is equal to \( T \), compounded risk-free interest rate \( r \), and volatility of the underlying process \( \sigma \). This is an approximation to the actual mapping from \( \bar{b}_t \) to \( b_t \), and has in this sense no economic interpretation in our model. The dashed line in Figure 1.8 is the approximation to the actual mapping of \( \bar{b}_t \) to \( b_t \). We then get the following approximation for \( b_t \), with \( \Phi(\cdot) \) denoting the standard normal CDF, which is indeed continuous:

\[
\begin{align*}
    b_t & = b_t^{max} - put_t, \\
    put_t & = X e^{-rT} \Phi (-d_{2,t}) - S_t \Phi (-d_{1,t}), \\
    d_{1,t} & = \frac{\log (S/X) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}}, \\
    d_{2,t} & = \frac{\log (S/X) + \left( r - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}}, \\
    X & = b_t^{max}, \\
    S_t & = \bar{b}_t.
\end{align*}
\] (1.47-1.52)
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

Figure 1.8. Plot showing the mapping from the no default level of debt $\tilde{b}_t$ to the actual debt level $b_t$. The solid blue line is the actual mapping, while the dashed red line is an approximation to it, where option pricing formulas have been used.

1.B Calibration Strategy and Robustness

1.B.1 Calibration strategies for the default function

In this section we will write down the 2 calibration strategies regarding the sovereign debt in the current paper, since other parts of the model are straightforward to calibrate. The steady state value of a variable $x_t$ is denoted by $\bar{x}$. We have the following 2 equations from the financial interemdiaries’ problem, from which we can derive the steady state return on bonds ex-post a possible default.

\[
\nu^k_t = E_t \left[ \Omega_{t+1} \left( r^k_{t+1} - r^d_{t+1} \right) \right], \\
\nu^b_t = E_t \left[ \Omega_{t+1} \left( r^{b*}_{t+1} - r^d_{t+1} \right) \right], \\
\Rightarrow E_t \left[ \Omega_{t+1} \left( r^k_{t+1} - r^{b*}_{t+1} \right) \right] = 0.
\]
From these equations it is clear that \( \bar{r}_k = \bar{r}_{b^*} \). Now we have the following equation for the maximum level of debt:

\[
b_t^{max} = \bar{b} + \frac{E_t[max]_t - \bar{\bar{r}}}{\kappa_b}.
\] (1.53)

The government budget constraint in case of no default by the government is equal to:

\[
q_t^b b_t + \tau_t + \bar{n}_t^g = g_t + n_t^g + (r_c + \rho q_t^b) b_{t-1},
\]

\[
= g_t + n_t^g + \frac{(r_c + \rho q_t^b)}{q_{t-1}^b} q_{t-1}^b b_{t-1} \implies
\]

\[
q_t^b b_t + \tau_t + \bar{n}_t^g = g_t + n_t^g + (1 + r_{t}) q_{t-1}^b b_{t-1}.
\] (1.54)

The mapping from the number of no default bonds to the actual number of bonds is given by:

\[
b_t = b_t^{max} - \max (b_t^{max} - \bar{b}_t, 0) \approx b_t^{max} - \text{put} (b_t^{max}, \bar{b}_t).
\] (1.55)

The actual number of government bonds is given by:

\[
q_t^g b_t + \tau_t + \bar{n}_t^g = g_t + n_t^g + (1 - \Delta_t) \left( r_c + \rho q_t^b \right) b_{t-1}
\]

\[
= g_t + n_t^g + (1 - \Delta_t) \frac{(r_c + \rho q_t^b)}{q_{t-1}^b} q_{t-1}^b b_{t-1} \implies
\]

\[
q_t^g b_t + \tau_t + \bar{n}_t^g = g_t + n_t^g + \left( 1 + r_{t}^{b*} \right) q_{t-1}^b b_{t-1},
\]

(1.56)

together with the ex-post default return on bonds:

\[
1 + r_{t}^{b*} = (1 - \Delta_t) \left( 1 + r_{t}^b \right),
\] (1.57)
and the return on bonds before default:

\[ 1 + r^b_f = \frac{(r_c + \rho q^b_f)}{q^b_{f-1}}. \] (1.58)

Throughout the paper we assume that the financial intermediaries do not receive any support from the government in the steady state, nor are they paying back support in the steady state, i.e. \( \bar{n}_g = \bar{n}_\bar{g} = 0 \). Finally, we have the option pricing formulas:

\[ \text{put}_t = b^\max_t e^{-rT} \Phi(-d_{2,t}) - \tilde{b}_t \Phi(-d_{1,t}), \] (1.59)

\[ d_{1,t} = \frac{\log(\tilde{b}_t/b^\max_t) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \] (1.60)

\[ d_{2,t} = \frac{\log(\tilde{b}_t/b^\max_t) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}. \] (1.61)

**Calibration strategy 1**

The first strategy targets \( \bar{q}_b \tilde{b}, \bar{q}_b b^{\max} \) and \( \tilde{A} \), for which we take 60\%, respectively 90\% of annual GDP and \( \tilde{A} = 0.005 \). Since we know \( \bar{q}_b \tilde{b} \), the steady state return on bonds after default \( \bar{r}_b^* \) and \( \bar{g} \) (which is calibrated to be 20\% of steady state output), we can find the steady state level of taxes from (1.56):

\[ \bar{q}_b \tilde{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_b^*) \bar{q}_b \tilde{b} \implies \bar{\tau} = \bar{g} + \bar{r}_b^* \bar{q}_b \tilde{b}. \]

Since we know the steady state default fraction \( \tilde{A} \) and the ex-post return on bonds, we can calculate \( \bar{r}_b \) through (1.57):

\[ 1 + \bar{r}_b^* = (1 - \tilde{A}) (1 + \bar{r}_b) \implies \bar{r}_b = \frac{1 + \bar{r}_b^*}{1 - \tilde{A}} - 1. \]
1.B. CALIBRATION STRATEGY AND ROBUSTNESS

Since we know $r_c$, we can find the steady state bond price through (1.58):

$$1 + \bar{r}_b = \frac{r_c + \rho \bar{q}_b}{\bar{q}_b} \implies \bar{q}_b (1 + \bar{r}_b) = r_c + \rho \bar{q}_b \implies$$

$$\bar{q}_b = \frac{r_c}{1 + \bar{r}_b - \rho}.$$ 

Now that the steady state bond price is known, we can find the steady state number of bonds and the maximum number of bonds $\bar{b}$ and $\bar{b}_{max}$. Since we know the return on bonds $\bar{r}_b$, we can find the number of bonds if the government does not default:

$$\bar{q}_b \bar{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_b) \bar{q}_b \bar{b}.$$ 

Now that we have found steady state number of bonds $\bar{b}$, the maximum number of bonds in the steady state $\bar{b}_{max}$, and the steady state number of bonds in case the government does not default $\bar{b}$. With these 3 numbers, and the requirement that the derivative of the put option $-\Phi (-d_{1,t})$ is equal to $-0.99$ (otherwise the default probability goes down when debt increases, which is counterintuitive), we can find the variables $r, \sigma$ and $T$ from the option pricing formulas.

**Calibration strategy 2**

The second strategy targets $\bar{q}_b \bar{b}$, and $\bar{q}_b \bar{b}_{max}$, and takes the option pricing parameters $r, \sigma$ and $T$ as given. We calibrate $\bar{q}_b \bar{b}$ to be equal to 80% of annual GDP, while $\bar{q}_b \bar{b}_{max}$ is equal to 90% of annual GDP. Since $\bar{g}$ is also known, we can find the steady state level of taxes from (1.56):

$$\bar{q}_b \bar{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_{b*}) \bar{q}_b \bar{b} \implies$$

$$\bar{\tau} = \bar{g} + \bar{r}_{b*} \bar{q}_b \bar{b}.$$  

55
CHAPTER 1. LIMITS TO COMMERCIAL BANK BAIL-OUTS

Since we know \( \bar{q}_b \bar{b} \) and \( \bar{q}_b \bar{b}_{max} \), we can divide the two to find the ratio \( \bar{b}_{max}/\bar{b} \).

Now we look at the option pricing formulas, and remember that the parameters \( r, \sigma \) and \( T \) are given. We rewrite the put option in the following way:

\[
b_t = b_t^{\text{max}} - \text{put}_t = b_t^{\text{max}} - \left\{ b_t^{\text{max}} e^{-rT} \Phi(-d_{2,t}) - \bar{b}_t \Phi(-d_{1,t}) \right\}
\]

Division by \( b_t \) gives the following expression:

\[
1 = \left( \frac{b_t^{\text{max}}}{b_t} \right) - \left( \frac{b_t^{\text{max}}}{b_t} \right) e^{-rT} \Phi(-d_{2,t}) + \bar{b}_t \Phi(-d_{1,t}).
\] (1.62)

Now we look at the formula for \( d_{1,t} \):

\[
d_{1,t} = \frac{\log(\bar{b}_t/b_t^{\text{max}}) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}},
\]

\[
= \frac{\log(\bar{b}_t/b_t^{\text{max}}) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} = f \left( \frac{\bar{b}_t}{b_t^{\text{max}}} \right).
\] (1.63)

Similarly we find for \( d_{2,t} \):

\[
d_{2,t} = \frac{\log(\bar{b}_t/b_t^{\text{max}}) + \left( r - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}},
\]

\[
= \frac{\log(\bar{b}_t/b_t^{\text{max}}) + \left( r - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} = g \left( \frac{\bar{b}_t}{b_t^{\text{max}}} \right).
\] (1.64)

Hence we see that equations (1.62), (1.63) and (1.64) only depend on the ratios \( b_t^{\text{max}}/b_t \) and \( \bar{b}_t/b_t \). We know the first ratio in steady state, so we can solve for the steady state ratio \( \bar{\bar{b}}/\bar{b} \). We also see that regarding the default function, it does not matter whether we calibrate on the number of bonds \( b_t \) or the value of government liabilities \( q_t^b b_t \), since the bond ratios are the variables that matter.
1.B. CALIBRATION STRATEGY AND ROBUSTNESS

Now we can find $\bar{b}/\tilde{b}$, and hence we find $\bar{q}_b\bar{b} = \bar{q}_b\bar{b}\left(\bar{b}/\tilde{b}\right)$. Since we know $\bar{q}_b\tilde{b}$, we can find $\tilde{r}_b$ from (1.54):

$$
\bar{q}_b\tilde{b} + \tau = \bar{g} + (1 + \tilde{r}_b) \bar{q}_b\bar{b} \implies \\
\tilde{r}_b = \frac{\bar{q}_b\tilde{b} + \tau - \bar{g}}{\bar{q}_b\bar{b}} - 1.
$$

Now we can find the steady state bond price from (1.58):

$$
1 + \tilde{r}_b = \frac{r_c + \rho \bar{q}_b}{\bar{q}_b} \implies \bar{q}_b (1 + \tilde{r}_b) = r_c + \rho \bar{q}_b \implies \\
\tilde{q}_b = \frac{r_c}{1 + \tilde{r}_b - \rho}.
$$

after which we know $\bar{b}, \bar{b}_{\text{max}}$ and $\tilde{b}$. From (1.57) we can find the steady state default probability:

$$
1 + \tilde{r}_{b*} = \left(1 - \tilde{\Delta}\right) \left(1 + \tilde{r}_b\right) \implies \\
\tilde{\Delta} = 1 - \frac{1 + \tilde{r}_{b*}}{1 + \tilde{r}_b}.
$$

1.B.2 Robustness

RBC version

First we investigate the effect of price-stickiness by performing the same experiments as in the main text, but now the model does not contain price-stickiness and monetary policy anymore, i.e. an RBC (real business cycle) model. The results are reported in Figure 1.9, 1.10, and 1.11. It is clear that the main mechanisms work in the same way as the model that contains price-stickiness, although the effects of increasing the maturity parameter $\rho$ are small. A delayed recapitalization still has positive effects in the no default case. Introducing sovereign default risk has an effect on the real economy when close to the debt limit, although the effects are again smaller than in the model with
price-stickiness and monetary policy. So price-stickiness and monetary policy amplify the negative consequences for the economy. In case of a financial crisis, financial intermediaries need the real rate of return on deposits to be as low as possible, because the lower the return, the more net worth is left in the financial intermediaries, and the less balance sheet constrained they are. The central bank, though, is primarily concerned with price stability and to a lesser extent (under the current calibration) with the output gap. This causes deflation in the period of the shock, and since the nominal interest rate was set in the period before the shock, the real return on deposits increases by 100 basispoints. This, however, harms financial intermediaries, because their funding costs increase significantly, thereby causing more balance sheet tightening in a financial crisis, with all the adverse consequences for the real economy that result.

**Errors due to approximation default function**

But to what extent influences the approximation method for the default function the results? On the one hand, the effects of the sovereign default risk will be stronger under the approximation method, due to the fact that the government already starts to default on a small part of the debt before the maximum debt level is reached, as can be seen from figure 1.8. This implies that capital losses will arise before the maximum debt level is reached, which tightens the leverage constraint. At the same time, we see that the possibility of a sovereign default gives the same results as in the case of no default when far away from the debt limit, while the effects only show up when we get close to the maximum level of debt, which is to be expected. The approximation is worst at the kink, but in our simulations we never reach this point. Besides that, it is reasonable to expect a sovereign risk premium to arise before the maximum debt level is reached, because it is always possible that a big shock arrives that pushes the no default level of debt $\tilde{b}_t$ over the maximum level of debt $b_t^{max}$. The increased interest rate on sovereign bonds induces an additional capital loss through a lower market
1.B. CALIBRATION STRATEGY AND ROBUSTNESS

Financial crisis impact and maturity of sovereign debt (RBC)

Figure 1.9. Impulse response functions for the model (RBC version) without default for \( \rho = 0.5 \) (blue solid line) and \( \rho = 0.96 \) (red solid line with dots). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state, and no additional government policy is implemented.

price for bonds, while the fraction of debt over which the government defaults is small. Hence the current approximation method will probably exacerbate the effects from sovereign default risk slightly, but we do not think that the error in doing so is significant, since the fraction over which the government defaults remains below 0.012 of outstanding government debt.
Delayed recapitalization (RBC)

Figure 1.10. Impulse response functions for the model (RBC version) without default with no additional government policy (blue solid line) and a delayed recapitalization of the financial sector (red solid line with dots) occurring 4 quarters after the financial crisis hits and equal to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

Inverse labor supply elasticity

As a last check we assume a low Frisch elasticity in Figure 1.12 and 1.13. The results do not change qualitatively, except for more persistence. The effects of the shock are less severe, because labor does not fall as much, despite lower wages. This improves the ex-post return on capital with respect to the case with a high Frisch elasticity, and hence investment and output do not fall by as much. The same persistence, though, prevents the workers from increasing the labor
1.B. CALIBRATION STRATEGY AND ROBUSTNESS

Default vs. no default and bank recapitalizations (RBC)

Figure 1.11. Impulse response functions (RBC version) comparing the case with no default (blue solid line) and default (red solid line with dots) in case of a delayed recapitalization of the financial sector of 1.25 percent of annual steady state GDP that is announced at the start of the financial crisis but implemented 4 quarters later. The financial crisis is initiated through an initial negative capital quality shock of 5 percent relative to the steady state. Contrary to the previous cases, the value of the government debt is now 80% of annual GDP.
Variation Frisch elasticity (A)

Figure 1.12. Impulse response functions for the model without default ($\rho = 0.96$) with the Frisch elasticity at $\varphi = 0.276$ (blue solid line), and for $\varphi = 4$ (red solid line with dots). There is no additional government policy, and the crisis is initiated through a negative capital quality shock of 5% of steady state.

...supply when the capital quality is improving, and hence the negative effects of the capital quality shock are more persistent.
Variation Frisch elasticity (B)

Figure 1.13. Impulse response functions for the model without default (ρ = 0.96) with the Frisch elasticity at ϕ = 0.276 (blue solid line), and for ϕ = 4 (red solid line with dots). There is no additional government policy, and the crisis is initiated through a negative capital quality shock of 5% of steady state.
Chapter 2

Financial Fragility and the Fiscal Multiplier

2.1 Introduction

The post-credit-crisis recession of 2008 has sparked a renewed debate on the merits of fiscal stimulus. Driven by crumbling financial markets and actual and/or near-bankruptcies of major financial institutions and the global recession that followed, policymakers around the world responded by implementing fiscal stimulus packages consisting of tax breaks and substantial amounts of government spending, as can be seen from Table 2.1. The ensuing debate, both in academia and among policymakers, centered around the question of the effectiveness of fiscal policy: faced by a large fallout of demand, should the government step in and make up for the lost demand? We analyze this question for a particular set of circumstances that has become very relevant after the credit crisis, particularly in Europe: an environment characterized by financial fragility, weakly capitalized banks and sovereign debt discounts in the face of widening public sector deficits. We argue that the effectiveness of fiscal stimuli is much reduced in such circumstances to the point of possibly becoming counterpro-

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1 This chapter is based on joint work with Sweder van Wijnbergen.
DUCTIVE AS BANK BALANCE SHEETS COME UNDER STRESS AND SOVEREIGN DEBT DISCOUNTS RISE IN THE FACE OF DEFICIT FINANCED FISCAL STIMULI.

<table>
<thead>
<tr>
<th>% of GDP</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1.98</td>
<td>1.77</td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.83</td>
<td>0.73</td>
</tr>
<tr>
<td>Japan</td>
<td>2.42</td>
<td>1.79</td>
</tr>
<tr>
<td>Emerging Asia</td>
<td>2.16</td>
<td>2.01</td>
</tr>
<tr>
<td>Other Countries</td>
<td>0.85</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 2.1. Table displaying total fiscal stimulus packages as percentage of GDP in early crisis years. Source: Coenen, Erceg, Freedman, Furceri, Kumhof, Lalonde, Laxton, Lind, Mourougane, Muir, Mursula, and d (2012).

Specifically, the focus of this paper is the negative amplification cycle that may arise from the feedback loops between weak banks and weak governments, and the way in which the positive effects arising from additional demand from increased government purchases, are possibly reduced or even reversed. We identify a variety of channels through which this feedback loop interferes with the effectiveness of fiscal policy. As a consequence, the room for governments to engage in additional policy is severely limited in the circumstances we highlight, a conclusion that contrasts with results from the many contemporaneous macromodels that assume the government has pockets deep enough to finance any possible intervention.

**Empirical observations regarding the connection between weak banks and weak sovereigns**

For several reasons, the poisonous interaction between weak banks and weak governments is more of an issue in current day Europe than in the U.S. First, banks matter much more both for the financing of corporate debt and for the financing of public debt in Europe than they do in the U.S. Banks account for approximately 40% of debt financing for U.S. non-financial corporations before the sub-prime crisis, with more than 60% of corporate debt being raised through
bond issuance in capital markets (Federal Reserve, 2015). This ratio is more than reversed in the Euro Area: bank loans account for more than 80% of debt finance to non-financial corporations (European Central Bank, 2015). European non-financial corporates will therefore be more affected by financial fragility in the banking sector.

Besides this difference in financial architecture, the response to problems in the financial sector has been very different in the U.S. and the Euro Area. The U.S. government started its Troubled Asset Relief Program (TARP) in October 2008, and subsequently announced its Supervisory Capital Assessment Program in March 2009 under which it stress-tested the big financial institutions, with mandatory recapitalization in case of a capital shortfall. Losses were revealed and written down, and several banks raised substantial amounts of new capital through equity issues in spite of distressed circumstances, most (97%) from private sources, as can be seen in Figure 2.1. In Europe, on the other hand, stress-tests were not as severe as in the U.S., which left many financial institutions with hidden losses on their balance sheets (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). As a consequence, financial fragility has persisted for much longer in Europe than in the US.

The sovereign debt problems of several countries in the European periphery have added to the problems caused by financial fragility. Not only are European banks carrying unrecognized losses, they are also heavily exposed to domestic sovereign bonds, as can be seen from Figure 2.2, where we show the interconnectedness between sovereign default risk and the financial system. The figure presents the exposure of financial intermediaries to own-country sovereign debt as a percentage of their Tier-1 capital. The exposure to the domestic sovereign creates a strong link between the domestic sovereign and its banks. In all periphery countries except Cyprus, domestic sovereign debt exposure exceeds Tier-1 capital of the banks holding the debt, often by a substantial margin: in Spain banks’ sovereign debt holdings equal 150% of Tier-1 capital in 2011, in Italy
Figure 2.1. Figure displaying the volume of new share emissions of U.S. and Euro Area banks. The volumes are denoted as percentage of the consolidated balance sheet of the banking system. Figure for 2013 is to 24-09-2013. Source: Thomson One Banker, BIS, ECB Consolidated Banking Data, as reported in De Nederlandsche Bank (2013).

almost 200% and in Greece almost 250%. Among the Euro-core countries, Germany is a surprising outlier, with banks holding almost three times their Tier-1 capital in German sovereign debt. With domestic sovereign debt exposure so high among especially the Southern European banks, stress in the sovereign debt market can potentially have a destabilizing impact on the financial system.

Figure 2.3 suggests that crowding out of private sector loans by public sector debt holdings is not just a theoretical concern. This table shows the change in aggregate risk exposure2 (EAD for Exposure At Default) for 16 European global

---

2Aggregate risk exposure refers to the part of the asset that exposes risk to the financial intermediary. Fitch Ratings (2013) reports for example, that the 16 G-SIBs represent end-2012 EUR 21 trillon in assets, which translates into to an aggregate risk exposure (EAD) of EUR 13.5 trillion.
systemically important banks (G-SIBs). The exposure to sovereign debt of the 16 major European banks considered has increased by EUR 550 billion, a 26% increase in sovereign EAD between end-of-2010 and end-of-2012. Over the same period, bank risk exposure to corporates has fallen by EUR 440 billion.
Risk Exposure (EAD): Modest Reduction, Major Reallocation

![Risk Exposure Graph]

Figure 2.3. Bank pillar 3 disclosures from a sample of 16 European global systemically important banks (G-SIBs). EAD denotes Aggregate Exposure at Default, a measure for aggregate risk exposure. Source: Fitch Ratings (2013).

Key features of our setup

To analyze the effectiveness of fiscal stimulus policies in the context of financial fragility and sovereign risk, we construct a DSGE model with financial frictions like in Gertler and Karadi (2011), but extend their framework in several ways to accommodate the problems we want to highlight. In particular we allow banks to allocate their funds over corporate loans and long term government debt, and endogenize sovereign risk to the latter. Like in Gertler and Karadi (2011), banks face an endogenously determined leverage constraint. Through this setup we capture the interconnectedness between the financial system and (potential) fiscal/debt problems of the government. Long term government bonds are introduced in a way similar to Woodford (1998, 2001), with a variable maturity structure captured by a single parameter $\rho$, through which any duration between 1 period bonds ($\rho = 0$) and perpetuals, or ‘consols’ ($\rho = 1$) can be obtained. The average maturity of domestic sovereign debt held by Southern European banks is approximately 5 years. Introducing longer maturities is
important because of the link with capital losses for the already balance-sheet-constrained commercial banks in the model: the longer the maturity of the government bonds, the higher the potential capital losses for the financial intermediaries, and the more pronounced the adverse effects on the economy in case of a financial crisis. We do not try to derive an optimal maturity structure, one would need a very different model for this; instead we more modestly show that exogenously lengthening the maturity structure exacerbates the poisonous link between financial fragility and sovereign weakness in the debt market.

Sovereign default risk is captured by postulating a maximum politically feasible level of taxation. We then map this maximum level of taxation into a maximum level of debt. We assume that the government follows a core tax policy that aims to maintain intertemporal solvency, but shocks to the system may necessitate issuing debt beyond or too close to the maximum level, in which case partial default follows to maintain debt below its maximum limit.

We use the model to explain the potentially damaging amplification mechanism that can arise through interactions between capital losses on sovereign bonds, increasing sovereign risk, and crowding out mechanisms within the balance sheets of leverage constrained financial intermediaries when governments resort to deficit financed Keynesian demand management in such circumstances. We calibrate the model to the Spanish economy, since Spain is one of the countries where the above mentioned mechanisms were visibly at play: the Spanish banking sector has been undercapitalized after the housing boom of the early 2000’s burst, while the commercial banks have approximately 30% of outstanding Spanish sovereign debt on their balance sheet.
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Relation to the literature

Empirical evidence on the effectiveness of fiscal stimulus is mixed. Barro and Redlick (2011) find a multiplier of 0.7, which increases to unity when they allow for interactions with the unemployment rate. Auerbach and Gorodnichenko (2012a,b) and Bachmann and Sims (2012) show that the multiplier heavily depends on the state of the economy: it is moderate, or even negative in expansions, while it is larger than 2 in recessions. Blanchard and Perotti (2002), using the SVAR (Structural Vector Autoregression) approach, find a multiplier of 1 in the U.S. for government purchases. Corsetti, Meier, and Müller (2012) show that the multiplier is lower when debt is high (60% of GDP), but larger during financial crises. Ilzetzki, Mendoza, and Végh (2013), on the other hand, find that for countries with debt levels exceeding 60% of GDP, the impact multiplier is close to zero, and the long run multiplier -3, suggesting that debt sustainability is an important determinant for the output effect of fiscal stimulus. Theoretically results have been mixed as well: standard flexible-price neoclassical models always have multipliers smaller than unity, while New-Keynesian models usually have a larger multiplier than their neoclassical equivalents, but dependent on the stance of monetary policy, and most of the time below unity as well. Christiano, Eichenbaum, and Rebelo (2011), though, also investigate fiscal stimulus in a relatively standard New-Keynesian model and find that fiscal multipliers are significantly above 1 when the zero lower bound (ZLB) binds. Fernández-Villaverde (2010) investigates the effects of fiscal policy in a model with financial frictions in the spirit of Bernanke, Gertler, and Gilchrist (1999), and finds that increased government spending is a more powerful tool to increase output in the short term than cutting distortionary taxes. Our model goes beyond this literature by looking specifically at debt financed fiscal stimuli in an environment with weakly capitalized banks and doubtful government finances (sovereign debt discounts),
highlighting the negative feedbacks between these two features when banks are loaded up on government debt.

Since the start of the credit crisis, the theoretical literature with general equilibrium models containing financial frictions is booming. Possibly the first paper to incorporate financial frictions in a general equilibrium setup is Bernanke, Gertler, and Gilchrist (1999). We build on Gertler and Karadi (2011) who introduce financial intermediaries that are balance-sheet-constrained by an agency problem between the deposit holders and the bank owners. This gives rise to an endogeneous leverage constraint, which becomes more binding when net worth is reduced by for example a negative shock to the quality of the loans. Several others have a similar mechanism, for example Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), and Kirchner and van Wijnbergen (2012), who include financial intermediaries holding short term government debt besides loans to the private sector. The current paper extends the Kirchner and van Wijnbergen (2012) model by introducing long term government bonds and sovereign default risk, as well as differential frictions related to different asset classes, like in Gertler and Karadi (2013). Woodford (1998, 2001) introduces long term government debt by assuming that the government is financed through a bond with infinite maturity but declining coupon size, thereby shortening the effective maturity. We follow his approach to modeling maturity. These papers, however, do not take into account the possibility of a government default. Acharya, Drechsler, and Schnabl (2014) have a setup containing both financial sector bailouts and sovereign default risk, but their analysis occurs within a partial equilibrium setup. Acharya and Steffen (2012), in their empirical research on systemic risk of the European banking sector, find that European banks have been at the center of the two major systemic crises that have faced the financial system since 2007, and specifically that markets have demanded more capital from banks with high sovereign debt exposures to peripheral countries, thereby indicating that sovereign debt holdings from those countries are a major contributor to
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systemic risk. A full general equilibrium model with banks financing government bonds subject to sovereign default risk is investigated by Kollmann, Ratto, Roeger, and int Veld (2013). They have 1 period government bonds and exogenous sovereign default risk, without feedback from government debt onto default risk.

Questions concerning the costs and benefits of long term government debt and the optimal maturity structure are discussed in Cole and Kehoe (2000), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012). Designing the optimal maturity structure is not the ambition of this paper, so we take maturity structure as exogenous. Staggered price setting and price stickiness go back to Calvo (1983) and Yun (1996). Sovereign default risk is captured in Arellano (2008), which contains an endogeneous “strategic” default mechanism somewhat similar in outcome to our approach. She finds a maximum level of debt conditional on the income shock. Uribe (2006) contains a government that defaults on part of its government debt each period in such a way that the expected future tax receipts and liabilities match afterwards. Schabert and van Wijnbergen (2014) introduce sovereign default risk by assuming that there exists a maximum level of taxation that is politically feasible and stochastic. Investors do not know this maximum level ex-ante, but they do know the distribution, and can deduce that the risk of a default is increasing with government debt. Davig, Leeper, and Walker (2011) do not model an explicit default, but they do assume a maximum level of taxation, or ‘fiscal limit’, exists. Our approach is most related to the latter three contributions, and follows Van der Kwaak and Van Wijnbergen (2014).

2.2 Model description

The financial sector faces endogenous leverage constraints like in Gertler and Karadi (2011), because of the intuitive way it allows for introducing and quan-
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tifying financial fragility. But in our set up banks do not only lend to corporates but also to governments and we introduce long term government debt and endogenous sovereign default risk. The government issues debt to financial intermediaries and raises taxes in a lump sum fashion from the households to finance its expenditures and repay existing debt. Like in Gertler and Karadi (2011), the central bank sets the nominal interest rate on the deposits that the households bring to the financial intermediaries. The private sector consists of financial intermediaries and a non-financial sector (households and firms). The structure of the non financial private sector is relatively standard: capital producing firms buy investment goods and used capital, and convert these into new capital that is sold to the intermediate goods producers. The intermediate goods producers use the capital together with labor to produce intermediate goods for the retail firms. There is perfect competition in the intermediate goods market, and hence the ex-ante profits of the intermediate goods producers are zero in equilibrium. The retail firm repackages and sells his unique retail product to the final good producers, while exploiting his (local) monopoly power to charge a mark-up for his product. The final good producers buy these goods and combine them into a single output good. The final good is purchased by the households for consumption, by the capital producers to convert it into capital, and by the government. The household maximizes life-time utility subject to a budget constraint, which contains income from deposits, profits from the firms, both financial and non-financial, and from labor. The income is used for consumption, lump sum taxes and investments in deposits. The government can intervene and provide the financial sector with new capital (net worth). We describe the model below, but leave some of the more elaborate derivations for the appendix.

2.2.1 The household sector

There is a continuum of infinitely lived households with identical preferences and asset endowments. A typical household consists of bankers and workers.
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Every period, a fraction $f$ of the household members is a banker running a financial intermediary. A fraction $1 - f$ of the household members is a worker. At the end of every period, all members of the household pool their resources, and every member of the household has the same consumption pattern. Hence there is perfect insurance within the household, and the representative agent representation is preserved. Every period, the household earns income from the labor of the working members and the profits of the firms that are owned by the household. In addition, households keep short term deposits in commercial banks, which are paid back with interest. The household uses these incoming cashflows to buy consumption goods which are consumed immediately upon purchase, and make new deposits into financial intermediaries. The household members derive utility from consumption and leisure, with habit formation in consumption, in order to capture realistic consumption dynamics (Christiano, Eichenbaum, and Evans, 2005). Households optimize expected discounted utility:

$$\max_{\{c_{t+s}, h_{t+s}, d_{t+s}\}_{s=0}^\infty} E_t \left[ \sum_{s=0}^\infty \beta^s \left( \log (c_{t+s} - \nu c_{t-1+s}) - \Psi h_{t+s}^{1+\varphi} \right) \right],$$

with $\beta \in (0, 1)$, $\nu \in [0, 1)$, $\varphi \geq 0$

where $c_t$ is consumption per household, and $h_t$ are hours worked by the members of the household that are workers. The utility function is maximized subject to the following budget constraint:

$$c_t + d_t + \tau_t = w_t h_t + (1 + r_t^d) d_{t-1} + \Pi_t$$

Deposits $d_{t-1}$ are posted at the financial intermediary in period $t - 1$, and pay real interest $r_t^d$ and principal at time $t$. $w_t$ is the real wage rate, $\tau_t$ are lump sum taxes the household has to pay to the government, and $\Pi_t$ are the profits from

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3 but not in the ones owned by the family, in order to prevent self-financing.
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the firms owned by the households. The profits of the financial intermediary are net of the startup capital for new bankers, as will be explained below. The first order conditions are then given by:

\[
\begin{align*}
    c_t & : \lambda_t = (c_t - \nu c_{t-1})^{-1} - \nu \beta E_t \left[ (c_{t+1} - \nu c_t)^{-1} \right] \quad (2.1) \\
    h_t & : \Psi^h_t = \lambda_t w_t \quad (2.2) \\
    d_t & : 1 = \beta E_t \left[ \Lambda_{t,t+1}(1 + \nu_{t+1}^d) \right] \quad (2.3)
\end{align*}
\]

where \( \lambda_t \) is the Lagrange multiplier of the budget constraint, and the stochastic discount factor \( \Lambda_{t,t+i} = \lambda_{t+i}/\lambda_t \) for \( i \geq 0 \).

2.2.2 The Fiscal Authority and the Central Bank

Fiscal Authority

The Fiscal Authority (the Government) levies lump sum taxes on households and issues bonds to finance its (exogeneous) expenditures \( g_t \) and to pay back the principal of bonds that come due this period. We also allow the government to support the financial sector by providing additional net worth \( n_t^g \) when the quality of the capital, and hence the value of the financial sector’s assets, deteriorates due to a negative capital quality shock. Repayment of previously administered support \( \tilde{n}_t^g \) by financial intermediaries provides the government with additional revenue. Government financing is modeled as in Woodford (1998, 2001) in order to have a flexible maturity structure for the government bonds. \( q_t^b \) is the real price of the outstanding bonds \( b_t \), while the maturity is controlled by the parameter \( \rho \), see appendix 2.A.1 for details. Government debt is subject to sovereign default risk because of the existence of a maximum level of taxation that is politically feasible, like in Schabert and van Wijnbergen (2014), Van der Kwaak and Van Wijnbergen (2014), and similar to the ‘fiscal limit’, as in Davig, Leeper, and
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Walker (2011). This maximum level of taxation can be mapped one to one to a corresponding maximum level of debt $b_{t}^{max}$. Government liabilities at the beginning of period $t$ are given by $(1 + r_{i}^{b}) q_{i-1}^{b} b_{t-1}$, where $r_{i}^{b}$ is the default-exclusive net return on government bonds. Denote by $\tilde{b}_{t}$ the level of government debt if the government would not default on its obligations in the current period. The government budget constraint is in that case given by:

$$q_{t}^{b} \tilde{b}_{t} + \tau_{t} + \tilde{n}_{t}^{g} = g_{t} + n_{t}^{g} + \left(1 + r_{i}^{b}\right) q_{i-1}^{b} b_{t-1}$$

(2.4)

We assume that the government follows a simple fiscal rule for its core tax policy $\tau_{t}$, as in Bohn (1998):

$$\tau_{t} = \bar{\tau} + \kappa_{b}(b_{t-1} - \tilde{b}) + \kappa_{g}(g_{t} - \tilde{g}) + \kappa_{n} n_{t}^{g}, \quad \kappa_{b} \in (0, 1], \quad \kappa_{g}, \kappa_{n} \in [0, 1]$$

(2.6)

where $\tilde{b}$ and $\tilde{g}$ are the steady state levels of debt, respectively government spending. The Bohn (1998) policy rule formulation guarantees that the real value of public debt eventually grows at a rate smaller than the gross real rate of interest. Bohn (1998) proves that following this rule is a sufficient condition for government solvency. If we set $\kappa_{n} = 0$, the additional government transfers to the financial sector are completely financed by issuing new debt. $\kappa_{n} = 1$ implies that the additional spending is completely financed by increasing lump sum taxes. Similarly $\kappa_{g} = 0$ implies that all government spending above its steady state value is completely deficit financed, whereas $\kappa_{g} = 1$ implies a completely tax-financed government spending stimulus.

Government purchases are driven by an exogenous stochastic process, and in addition can possibly respond to a recession caused by an adverse quality of
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capital shock. The second component would correspond to a Keynesian stimulus package, in that case the government attempts to stimulate the economy by increasing government purchases. Combining these two components yields the actual time path for government expenditures/purchases of final goods in period $t$:

$$g_t = \tilde{g}_t + \varsigma(\xi_{t-l} - \tilde{\xi}), \quad \varsigma < 0, \quad l \geq 0$$  (2.7)

$$\log(\tilde{g}_t/\bar{g}) = \rho_g \log(\tilde{g}_{t-1}/\bar{g}) + \varepsilon_{g,t}^{u} + \varepsilon_{g,t-4}^{a}$$  (2.8)

where $\varepsilon_x \sim N(0, \sigma_x^2)$ with $x = (u, a)$. We assume that the autocorrelation coefficient $\rho_g \in [0, 1]$, and the steady state value of government purchases to be larger than zero ($\bar{g} > 0$). This way we can study the effects of surprise shocks to government spending ($\varepsilon_{g,t}^{u}$), but also the effects of shocks that are announced one year in advance ($\varepsilon_{g,t}^{a}$). The parameter $\varsigma$ determines the size of the response to a deterioration in the quality of capital. If $\varsigma = 0$, the government does not respond to a deterioration in the quality of capital. $\varsigma < 0$ implies that the government reacts to a deterioration in the quality of capital by increasing government spending above the steady state value. The case where $l = 0$ implies that the government reacts instantaneously to a capital quality shock, while $l > 0$ implies that the government reacts with some lag. Whereas it might be preferable in general to model the government response as an endogeneous optimizing feedback from output, we choose to model government intervention as an exogenous process because of our focus on the size of fiscal multipliers. This allows us to make policy impact comparisons that are not “polluted” by second round effects triggered by the macroeconomic response to government expenditure shocks leading to subsequent rounds of government interventions.

Default rule

Following Schabert and van Wijnbergen (2014), Van der Kwaak and Van Wijnbergen (2014), and Davig, Leeper, and Walker (2011), we introduce a maxi-
mum level of taxation that is politically feasible. This maximum level of taxation can then be mapped into a maximum level of debt $b_{t}^{\text{max}}$. When the level of debt in the absence of default ($\tilde{b}_{t}$) is smaller than the maximum level of debt $b_{t}^{\text{max}}$, the government will honor all of its outstanding liabilities, and the actual level of government debt $b_{t}$ will be equal to $\tilde{b}_{t}$. But when $\tilde{b}_{t} > b_{t}^{\text{max}}$, the government defaults over a fraction $\Delta_{t}$ of the debt such that the new actual debt $b_{t}$ is equal to the maximum level of debt $b_{t}^{\text{max}}$:

$$b_{t} = \begin{cases} \tilde{b}_{t} & \text{if } \tilde{b}_{t} \leq b_{t}^{\text{max}}, \\ b_{t}^{\text{max}} & \text{if } \tilde{b}_{t} > b_{t}^{\text{max}}. \end{cases} \quad (2.9)$$

The government achieves this outcome through renegotiation with its creditors once the unfavorable debt shocks have occurred. Creditors know that they will not be able to get more from the government than what the government can raise sustainably through the maximum level of taxes $\tau_{t}^{\text{max}}$, hence they are willing to accept the haircut $\Delta_{t}$ on their bond portfolio. We assume all creditors participate in the renegotiation, and abstract from free-rider problems among creditors. So the default exclusive return $r_{t}^{b}$ and the default inclusive return $r_{t}^{b*}$ are linked through the following relation in period $t$:

$$1 + r_{t}^{b*} = (1 - \Delta_{t}) \left(1 + r_{t}^{b}\right) \quad (2.10)$$

We show the debt structure $b_{t}$ as the solid blue line in Figure 2.14 in appendix 2.A.5. We can also write the debt level structure (2.9) in the following way:

$$b_{t} = \min (\tilde{b}_{t}, b_{t}^{\text{max}}) = b_{t}^{\text{max}} - \max (b_{t}^{\text{max}} - \tilde{b}_{t}, 0). \quad (2.11)$$

We can interpret the second term of the new debt level as the payoff of a put option at maturity with underlying process $\tilde{b}_{t}$ and strike price $b_{t}^{\text{max}}$, similar to the debt pricing model presented in Claessens and van Wijnbergen (1993). The debt-level, or default function, however, is not differentiable at $b_{t} = b_{t}^{\text{max}}$. In
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order to be able to solve the model using perturbation techniques, which require

differentiability, we convexify (2.11) by replacing it by the corresponding put

option valuation formula (see Figure 2.14, and for more detail see appendix

2.A.5).

*The Central Bank*

The Central Bank sets the nominal interest rate on deposits $r^n_t$ according to

the following Taylor rule, in order to minimize output and inflation deviations:

$$ r^n_t = (1 - \rho_r)(r^n + \kappa_\pi(\pi_t - \bar{\pi}) + \kappa_y \log(y_t/y_{t-1})) + \rho_r r^n_{t-1} + \varepsilon_{r,t} \tag{2.12} $$

where $\varepsilon_{r,t} \sim N(0, \sigma^2_r)$, and $\kappa_\pi > 0$ and $\kappa_y > 0$. $\rho_r$ is a smoothing parameter. The parameter $\bar{\pi}$ is the target inflation rate or the natural inflation rate. In order to satisfy the Taylor principle, we choose $\kappa_\pi > 1$ (leaning against the wind). The values of $\kappa_\pi$ and $\kappa_y$ determine the strength with which the authorities react to deviations from the natural rate of inflation and output. The nominal and the real interest rate on deposits are linked through the following Fisher relation:

$$ 1 + r^d_t = (1 + r^n_{t-1})/\pi_t \tag{2.13} $$

Hence monetary policy is executed through the control of interest rates on de-

posits rather than the interest rates on the government bonds: the latter are en-
dogeneously determined in equilibrium.

2.2.3 Financial intermediaries

The financial sector setup is similar to Gertler and Kiyotaki (2010) and Gertler

and Karadi (2011), except that we allow the financial intermediary to invest both

in private loans and government bonds. Thus financial intermediaries lend funds

obtained from households to the intermediate goods producers and to the gov-
government. We assume that households do not deposit with the bankers belonging
to the same household, in order to prevent self-financing, bypassing financial
frictions. The banker’s balance sheet is given by:

\[ p_{j,t} = n_{j,t} + d_{j,t} \]

where \( p_{j,t} \) are the assets on the balance sheet of bank \( j \) in period \( t \), \( n_{j,t} \) denotes
the net worth of the bank, while \( d_{j,t} \) denotes the deposits of the bank. The
financial intermediary invests the funds obtained from the household in claims
issued by the intermediate goods producer and in government bonds. Hence the
asset side of the bank’s balance sheet has the following structure:

\[ p_{j,t} = q_k^k s_{j,t}^k + q_b^b s_{j,t}^b \]

where \( s_{j,t}^k \) are the number of claims financial intermediary \( j \) acquired for a price
\( q_t^k \), paying out a net real return \( r_{t+1}^k \) at the beginning of period \( t + 1 \). The number
of government bonds \( s_{j,t}^b \) are acquired by intermediary \( j \), for a price \( q_t^b \). At
the beginning of period \( t + 1 \) a net real return \( r_{t+1}^b \) is paid out, which includes the
impact of a possible sovereign default. Financial intermediaries earn a return on
their assets, and pay a return on the deposits. The difference between the two
adds to the increase in net worth from one period to the next, possibly supple-
mented by government support measures. The balance sheet of intermediary \( j \)
then evolves as follows:

\[
\begin{align*}
n_{j,t+1} &= (1 + r_{t+1}^k)q_t^k s_{j,t}^k + (1 + r_{t+1}^b)q_t^b s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\
&= (r_{t+1}^k - r_{t+1}^d)q_t^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d)q_t^b s_{j,t}^b + (1 + r_{t+1}^d) n_{j,t} \\
&+ \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t}
\end{align*}
\]

where \( n_{j,t+1}^g = \tau_{t+1}^n n_{j,t} \) denotes net worth provided by the government to finan-
cial intermediary \( j \) (for example a capital injection). \( \tilde{n}_{j,t+1}^g = \tilde{\tau}_{t+1}^n n_{j,t} \) denotes
the repayment of government support received in previous periods.
The financial intermediary is interested in maximizing expected profits. There is a probability of \(1 - \theta\) that the banker has to exit the industry next period, in which case he will bring the net worth \(n_{j,t+1}\) to the household, while he is allowed to continue operating with a probability \(\theta\). The banker discounts these outcomes by the household’s stochastic discount factor \(\beta \Lambda_{t,t+1}\), since the banker is part of the household, the ultimate owner of the financial intermediary. The banker’s objective is then given by the following recursive optimization problem:

\[
V_{j,t} = \max_{\bar{E}_t} \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) n_{j,t+1} + \theta V_{j,t+1} \right\} \right]
\]

where \(\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t\). We conjecture the solution to be of the following form, and later check whether this is the case:

\[
V_{j,t} = \nu_k q_{t}^k s_{j,t}^k + \nu_b q_{t}^b s_{j,t}^b + \eta_t n_{j,t}
\]

We follow Gertler and Karadi (2011) and proxy governance problems in banks by assuming that in any period, bankers running the financial intermediaries can divert the assets on the bank’s balance sheet. If that would happen, depositors will force the intermediary into bankruptcy, but in that case they can only recoup a fraction \(1 - \lambda_a\) of asset class \(a = \{k, b\}\) that was diverted by the banker. We apply different effective diversion rates \(\lambda_a\) for different asset classes to capture the fact that government bonds are easier to monitor for depositors than a private loan portfolio. \(\Delta = \frac{\lambda_b}{\lambda_k}\) is the relative diversion rate between bonds and private loans. Because of the asset diversion opportunity, households will in equilibrium only provide deposits up until the level where the continuation value of the intermediaries is equal to the value of the assets that can be diverted by the bankers net of the part that can be recovered by the depositors. Imposing this incentive compatibility constraint ensures that the bankers will actually choose to continue as banker: depositors will only supply funds to the bank if the
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Gains from diverting assets are lower than the continuation value of the financial intermediary. This gives rise to the following leverage constraint:

\[ V_{j,t} \geq \lambda_k q^k_t s^k_{j,t} + \lambda_h q^h_t s^h_{j,t} \]  

(2.14)

The optimization problem can now be formulated in the following way:

\[
\max_{\{s^k_{j,t}, s^h_{j,t}\}} V_{j,t}, \quad \text{s.t.} \quad V_{j,t} \geq \lambda_k q^k_t s^k_{j,t} + \lambda_h q^h_t s^h_{j,t}
\]

From the first order conditions we find that \( \nu_t^b = \frac{\lambda_k}{\lambda_k} \nu_t^k \). Hence the leverage constraint (2.14) can be rewritten in the following way:

\[ q^k_t s^k_{j,t} + \frac{\lambda_h}{\lambda_k} q^h_t s^h_{j,t} \leq \phi_t n_{j,t}, \quad \phi_t = \frac{\eta_t}{\lambda_k - \nu_t^k} \]  

(2.15)

where \( \phi_t \) is the ratio of assets (weighted by the relative diversion rates) to net worth, or the leverage constraint of the financial intermediary. The intuition for the leverage constraint is straightforward: a higher shadow value of assets \( \nu_t^k \) indicates a higher value from an additional unit of assets, increasing expected profits everything else equal, thereby reducing the incentive for bankers to divert the assets. A higher value of \( \eta_t \) implies higher expected profits from an additional unit of net worth, therefore allowing a higher leverage ratio. A higher fraction \( \lambda_k \) implies bankers can divert more, inducing households to provide less deposits everything else equal. After substitution of the conjectured solution into the right hand side of the Bellman equation, application of the first order conditions, and comparing the left and right hand side of the Bellman equation,
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we obtain the following expressions for the shadow values of net worth, private loans, and government bonds:

\[ \eta_t = E_t \left[ \Omega_{t+1} \left( 1 + r^d_{t+1} + \tau^n_{t+1} - \tau^{\lambda}_{t+1} \right) \right] \]  
(2.16)

\[ \nu^k_t = E_t \left[ \Omega_{t+1} (r^k_{t+1} - r^d_{t+1}) \right] \]  
(2.17)

\[ \nu^b_t = \frac{\lambda_b}{\lambda_k} \nu^k_t = E_t \left[ \Omega_{t+1} (r^{bs}_{t+1} - r^d_{t+1}) \right] \]  
(2.18)

\[ \Omega_{t+1} = \beta \Lambda_{t,t+1} \{ (1 - \theta) + \theta [\eta_{t+1} + \nu^k_{t+1} \phi_{t+1}] \} \]

where \( \Omega_{t+1} \) can be thought of as a stochastic discount factor that incorporates the financial friction. A detailed derivation is provided in the appendix.

Aggregation of financial variables

Aggregating the balance sheet identities is straightforward:

\[ p_t = q^k_t s^k_t + q^b_t s^b_t, \]  
(2.19)

where \( p_t \) denotes the aggregate quantity of assets that are on the balance sheets of the financial intermediaries. Since \( \phi_t \) does not depend on firm specific factors, we can aggregate the leverage constraint (2.15) across financial intermediaries:

\[ q^k_t s^k_t + \frac{\lambda_b}{\lambda_k} q^b_t s^b_t = \phi_t n_t, \]  
(2.20)

where \( n_t \) denotes the aggregate intermediary net worth. The share of assets invested in private loans, to which we will refer in our simulations as “Portfolio weight claims” is given by:

\[ \omega_t = q^k_t s^k_t / p_t \]  
(2.21)

At the same time, we know that at the end of the period, only a fraction \( \theta \) of the current bankers will remain a banker, while the remaining fraction \( 1 - \theta \)
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will (again) become a worker. We assume that current bankers only pay out dividends at the moment they quit the banking business. If they do not quit but continue as a banker, they retain their earnings to expand their net worth and expand their balance sheet. Thus the net worth of existing bankers at the end of the period is equal to:

\[ n_{e,t} = \theta \left( (r^k_t - r^d_t) q^k_{t-1} s^k_{t-1} + (r^{b*}_t - r^d_t) q^b_{t-1} s^b_{t-1} + (1 + r^d_t) n_{t-1} \right) \]

The exiting bankers bring back the net worth to the household income. A fraction \(1 - \theta\) of the bankers has left the financial industry, which is equal to a fraction \((1 - \theta)f\) of the households. The same fraction of the households will enter the financial industry next period by leaving their working job. We assume that the household will provide a starting net worth to the new bankers proportional to the assets of the old bankers, as in Gertler and Karadi (2011). We assume that the household transfers a fraction of \(\chi/(1 - \theta)\) of the assets of the old bankers to the new bankers. Then aggregate net worth of new bankers equals:

\[ n_{n,t} = \chi p_{t-1} \]

At the beginning of the new period, random draws determine which bankers will leave the industry, taking their accumulated net worth with them. Total net worth of the bankers active in the new period (old bankers that continue plus new bankers and possibly government support) equals:

\[
\begin{align*}
n_t &= n_{e,t} + n_{n,t} + n^g_t - \bar{n}^g_t \\
&= \theta \left( (r^k_t - r^d_t) q^k_{t-1} s^k_{t-1} + (r^{b*}_t - r^d_t) q^b_{t-1} s^b_{t-1} + (1 + r^d_t) n_{t-1} \right) \\
&\quad + \chi p_{t-1} + n^g_t - \bar{n}^g_t \tag{2.22}
\end{align*}
\]
where $n^g_t$ and $\tilde{n}^g_t$ are aggregate financial sector support, respectively financial support payback.\footnote{We introduce this policy measure because we will later in this paper explore the link between the fiscal multiplier and bank capitalization.}

### 2.2.4 Production side

#### Intermediate Goods Producers

There exists a continuum of intermediate goods producers indexed by $i \in [0, 1]$. Each of these firms produce a differentiated good. The intermediate goods producers borrow from the financial intermediaries against future profits. We assume that there are no financial frictions between the financial intermediaries and the intermediate goods producers. Hence there are no monitoring costs for the financial intermediaries, and the intermediate goods producers can commit next period’s profits to the financial intermediaries. The securities issued by the intermediate goods producers are therefore really state-contingent debt, like in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). The production technology of the intermediate goods producers is given by:

$$y_{i,t} = a_t(\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha},$$

where $a_t$ equals total factor productivity and $\xi_t$ capital quality, which follow a lognormal AR(1) process:

$$\log(x_t) = \rho_x \log(x_{t-1}) + \varepsilon_{x,t},$$

where $x = \{a, \xi\}$. The innovations $\varepsilon_{x,t}$ are distributed as $\varepsilon_{x,t} \sim N(0, \sigma_x^2)$ for $x = a, \xi$. The intermediate goods producer acquires the capital at the end of period $t-1$, while the production only occurs after the capital quality shock $\xi_t$ has hit at the beginning of period $t$. Hence $\xi_t k_{i,t-1}$ denotes the effective capital
in our model. We can think of the shock $\xi_t$ as accelerated economic depreciation or obsolescence of capital. The intermediate goods producer decides at the end of period $t - 1$ how much capital to purchase. At the moment the intermediate goods producer purchases the capital, he does not know the realization of $\xi_t$ in period $t$ yet. To finance his purchase at the end of period $t - 1$, he needs to issue claims $s_{i,t-1}^k$, with the number of claims $s_{i,t-1}^k$ equal to the number of capital units ($k_{i,t-1}$) acquired. The price at which the claims are sold equals $q_{t-1}^k$, and they pay a state-contingent net real return $r_t^k$ in period $t$. Hence a negative capital quality shock will lower the return on the claims of the financial intermediary. The intermediate goods producer also hires labor $h_{i,t}$ for a wage rate $w_t$ after the shock ($\xi_t$) has been realized. When the firm has produced in period $t$, the output is sold for a relative price $m_t$ to the retail firms. $m_t$ is the relative price of the intermediate goods with respect to the price level of the final goods, i.e. $m_t = P_{i,t}^{mn}/P_t$. After production, the intermediate goods producing firms sell back what remains of the effective capital to the capital producers at a price of $q_t^k$. The effective capital stock is also subject to regular depreciation $\delta$ during production. So the intermediate goods producer receives $q_t^k(1 - \delta)\xi_t k_{i,t-1}$ for his end of period capital stock.

Capital Producers

At the end of period $t$, when the intermediate goods firms have produced, they sell what remains of the capital stock after depreciation $\delta$ to the capital producers at a price $q_t^k$. The capital producers also buy $i_t$ final goods from the final good producers; these purchases (investment) are an input in the capital production process: they are used to produce additional capital. Capital producers combine this additional capital with the old, partially depreciated stock bought earlier from the intermediate goods producers and so produce the new capital stock. This new capital is being sold to the intermediate goods producers at a price $q_t^k$. We assume that the capital producers face convex adjustment costs whenever
2.2. MODEL DESCRIPTION

investment \(i_t\) deviates from previous period investment \(i_{t-1}\). Hence we have the following capital production technology:

\[
k_t = (1 - \delta)\xi_t k_{t-1} + (1 - \Psi(t))i_t, \quad \Psi(x) = \frac{\gamma}{2}(x - 1)^2, \quad \upsilon_t = i_t/i_{t-1} \quad (2.23)
\]

Retail firms

Retail firms purchase goods \((y_{i,t})\) from the intermediate goods producing firms for a nominal price \(P_{m,t}\), and convert these into retail goods \((y_{f,t})\). These goods are sold for a nominal price \(P_{f,t}\) to the final goods producer. We assume that it takes one intermediate goods unit to produce one retail good \((y_{i,t} = y_{f,t})\). All the retail firms produce a differentiated retail good. The individual retail firm therefore operates in a market characterized by monopolistic competition, and can therefore charge a markup over the input price \(P_{m,t}\), so the nominal profit of the retail firm is given by \((P_{f,t} - P_{m,t})y_{f,t}\).

We assume that in each period a fraction \(1 - \psi\) of retail firms is allowed to reset their prices optimally, while the \(\psi\) remaining firms are not allowed to do so, like in Calvo (1983) and Yun (1996), where the probability \(\psi\) is assumed to be i.i.d. When a firm is allowed to reset its prices, it will do it in such a way that the expected sum of discounted profits is maximized. The resulting first order conditions can be found in appendix 2.A.3.

Final Good Producers

The final good firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by \(y_t^{(\varepsilon-1)/\varepsilon} = \int_0^1 y_{f,t}^{(\varepsilon-1)/\varepsilon} df\), where \(y_{f,t}\) is the output of the retail firm indexed by \(f\). \(\varepsilon\) is the elasticity of substitution between the intermediate goods purchased from the different retail firms. We assume that the final good firms operate in an environment of perfect competition,
and hence they maximize profits by choosing $y_{f,t}$ such that $P_t y_t - \int_0^1 P_{f,t} y_{f,t} \, df$ is maximized. The final good producer takes $P_t$ and $P_{f,t}$ as given.

**Aggregation**

Price dispersion, which is given by $D_t = \int_0^1 (P_{f,t} / P_t)^{-\varepsilon} \, df$ is given by the following recursive form:

$$D_t = (1 - \psi)(\pi_t^*)^{-\varepsilon} + \psi \pi_t^\varepsilon D_{t-1},$$

where $\pi_t^*$ is the price set by retail firms that were allowed to reset its prices, expressed in terms of the general price level $P_t$. Aggregation over the intermediate goods production technology gives the following relation for aggregate output:

$$y_t D_t = a_t (\xi_t k_{t-1})^{\alpha} h_t^{1-\alpha}$$

### 2.2.5 Market clearing

Equilibrium requires that the number of claims owned by the financial intermediaries ($s_t^k$) is equal to aggregate capital ($k_t$), while the number of government bonds owned by the financial sector ($s_t^b$) must equal the number of bonds issued by the government ($b_t$):

$$s_t^k = k_t \quad (2.26)$$
$$s_t^b = b_t \quad (2.27)$$

Goods market clearing requires that the aggregate demand equals aggregate supply:

$$c_t + i_t + g_t = y_t \quad (2.28)$$
2.3 Calibration

Before using the model for simulation analysis, we need to assign numerical values to the parameters. We calibrate the model on the Spanish economy with a quarterly frequency. Several parameter values are taken from Burriel, Fernández-Villaverde, and Rubio-Ramrez (2010), who develop a model for the Spanish economy, and perform a Bayesian estimation for several parameters. The calibration of the default process is explained in more detail in appendix 2.A.6. The parameter values can be found there in Table 2.6. The steady state leverage ratio is set to 5.1, which is derived from taking the average ratio of consolidated equity to consolidated financial assets from the OECD database. The credit spread \( \Gamma \) between private loans and deposits is set to 216 basis points annually (which amounts to \( \Gamma = 0.0054 \)). This number is derived from the Worldbank database and is equal to the 9-year average interest rate spread (lending minus deposit rate) between 1994 and 2002. The parameter \( \theta \) is calibrated by taking the average survival period \( \Theta = 1/(1 - \theta) \) to be equal to 24 quarters, or \( \theta = 0.9583 \). The parameters in the Taylor rule are set to values in line with the Taylor principle. The feedback from government debt on taxes is set to a value such that both the model with and without default are stable. We calibrate the steady state ratios of investment and government spending over GDP, \( \tilde{i}/\tilde{y} \) and \( \tilde{g}/\tilde{y} \) to 22.6 percent, respectively 17.8 percent, by calibrating the depreciation parameter \( \delta \) in order to match the 15-year average investment and government spending over GDP ratios from 1994-2008. The fixed payment in real terms that the holder of government bonds receives each period is set to 0.041, the average interest rate on 10-year Spanish bonds from 1998-2008 (European Central Bank, 2015). We set the maturity parameter at \( \rho = 0.97 \), equivalent to an average duration of 25 quarters. As we vary \( \rho \) in one of the experiments, the parameter governing the average duration of government debt, the steady state bond price changes.
as well. In order for different maturities to be comparable, we must make sure that the fraction of government debt on the balance sheet of the financial intermediaries does not change. Hence we calibrate on the outstanding government liabilities as a fraction of GDP $\bar{q}_b \bar{b}/\bar{y}$ instead of $\bar{b}/\bar{y}$, and set this equal to 2.128 implying an annual debt-to-GDP ratio of 53.2 percent, the average ratio over the period 1995-2008. We calibrate the diversion rate of government bonds $\lambda_b$ to be equal to half the diversion rate on private loans $\lambda_k$, i.e. $\lambda_b/\lambda_k = 0.5$, as in Gertler and Karadi (2013). Subsequently we will look at the effect of varying this ratio. We assume more aggressive monetary policy in the face of a credit crisis, and hence set $\rho_r = 0$ in times of crisis. We think this captures the way central banks reacted when the financial crisis erupted. A credit crisis is represented by a negative shock to capital quality $\xi_t$ of 5 percent on impact, with an autocorrelation coefficient $\rho_{\xi} = 0.66$, as in Gertler and Karadi (2011).

The calibration of the real economy is not affected by the introduction of sovereign default risk. For the financial sector, the steady state bond price $\bar{q}_b$ changes, and hence $\bar{b}$. We calibrate the maximum level of government liabilities $\bar{q}_b\bar{b}_{max}/\bar{y}$ to be at 60% of annual steady state GDP. The steady state fraction of government liabilities $\bar{q}_b\bar{b}/\bar{y}$ on the balance sheet of financial intermediaries does not change, since it is still calibrated to equal 53.2% of annualized steady state output. The steady state default probability is set at a rather conservative estimate of $\bar{\Delta} = 0.0077$, which implies an annual default probability of 3.2%, which is small given the observed bond spreads in the European periphery. A more elaborate exposition of the calibration of the variables related to the sovereign default risk can be found in appendix 2.A.6.

### 2.4 Results

In this section we trace out dynamic multipliers after Keynesian demand stimuli. We start off with an economy that has been shocked by a financial
2.4. RESULTS

crisis, modeled as a sudden unanticipated downward shock to the “quality” of capital, following Gertler and Karadi (2011). We then analyze the output response to an expansionary shift in government expenditure in response to the shock, mimicking in this way the impact of a weakly capitalized banking system on the effectiveness of such a stimulus program. In the first set of policy experiments, we use the full model, with long term debt held on banks’ balance sheets and endogenously generated sovereign risk built in. The stimulus package is announced at the onset of the financial crisis, but implemented only four quarters (one budget year) after its announcement, in line with standard budget procedures. After presenting our core results and the associated dynamic multiplier patterns, we dissect these results by running similar simulations but trimming the model down step by step, so as to find out which new feature is the most significant driver of the strong results we obtain on the size of multipliers. The fiscal policy response, which we feed into the model as a second, anticipated shock, is graphically represented in Figure 2.4.

2.4.1 The effects of a stimulus package in the presence of long term debt and sovereign default risk

To set the stage, Figure 2.5 and 2.6 compare the stimulus package (red, slotted) with the case where sovereign default risk is present, but with no additional government policy being implemented (blue solid line in the figure). Looking at the no policy response case, we see that the deterioration in the capital quality $\xi_t$ induces losses at the financial intermediaries on the private loans; these lower than expected returns on the private loans reduce the value of the loans and therefore the banks’ net worth. This leads to a rising credit spread and thus higher interest rates on loans, which reduces demand for loans from intermediate goods producers. Higher interest rates on loans and arbitrage between private loans and government bonds causes the returns on bonds to go up as well, with a subsequent fall in bond prices as a results. This price fall triggers further capital
losses at the financial intermediaries. Because of these additional declines in net worth, a second round of interest rate increases follows, further reducing the demand for capital. The subsequent rounds of balance sheet deterioration cause the credit spread to increase by almost 200 basispoints, investment to drop by more than 20%, and eventually the capital stock to fall by almost 15%. Even after 20 quarters the economy still has not recovered completely from the initial shock. This model therefore highlights the point made by Reinhart and Rogoff (2009): financial crises lead to long lasting declines in output.

Figure 2.4. Figure displaying the stimulus spending in case the government engages in additional policy. The size of the stimulus is equal to 5% of quarterly steady state output, or equivalently, 1.25% of annual steady state output.
Figure 2.5. Plot of the impulse response functions comparing no additional policy (blue, solid) and fiscal stimulus (red, slotted). The stimulus is announced as the crisis hits, and implemented 4 quarters later through additional debt issuance, and equal to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

The effects of a fiscal stimulus

Now consider the impact of a government response through a Keynesian deficit financed stimulus package that is announced immediately but implemented 4 quarters later. Due to the forward looking nature of the agents, they immediately anticipate the future increase in debt, which increases the probability of default. The effects of sovereign default risk are propagated through a
Figure 2.6. Plot of the impulse response functions comparing no additional policy (blue, solid) and fiscal stimulus (red, slotted). The stimulus is announced as the crisis hits, and implemented 4 quarters later through additional debt issuance, and equal to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

The bond price that is already lowered ex-ante. This in turn leads to capital losses on the banks’ holdings of sovereign debt, already before or possibly even without an actual sovereign default. The anticipation of having to finance riskier debt in the future makes the financial intermediaries immediately more balance-sheet-constrained, which pushes up the credit spread, and triggers higher interest rates for commercial borrowers and on government bonds. A higher interest rate implies again a lower bond price, which falls by another 5% with respect to the
2.4. RESULTS

In the absence of any additional government policy, capital losses on private loans and government bonds, investment (and hence capital formation) are also negatively affected because of direct crowding out by government bonds in the portfolios of the banks, a crowding out channel highlighted in Kirchner and van Wijnbergen (2012): the government and private enterprise compete for funds that are not perfectly elastically supplied because of the leverage constraint under which the banks have to operate. This leads again to higher interest rates and thus falling bond prices, another amplification channel. Obviously, all these effects are stronger the larger the additional government debt issue.

Intermediate goods producers also anticipate future crowding out of private loans by government debt once the stimulus will be implemented, and demand fewer loans, as investment falls due to higher capital costs. Investment falls by more than 5% of steady state investment with respect to the no policy case, with a bigger decline in the capital stock as a consequence. This, in turn leads to a fall in potential output. More government debt causes households to anticipate higher future taxes, hence the anticipatory fall in consumption. Of course, there are positive direct effects of the additional government expenditures once they occur, but despite the positive direct effects from higher anticipated government demand in the future, which leads firms to demand more loans everything else remaining equal, the current and anticipated future deterioration in financial conditions trigger a net fall in demand for private loans and a fall in investment, already before implementation of the stimulus. The direct positive impact effects from anticipated government stimulus 4 quarters from now are apparently not enough to offset the fall in consumption, investment, and output with respect to the no policy case.

Figure 2.7 shows the net impact over time of all these occasionally conflicting impact effects. The anticipation of future tightening leads to tightening conditions today and thus a negative output effect in the months preceding the actual
Figure 2.7. The solid (blue) line represents the fiscal stimulus itself, expressed as a percentage of quarterly steady state output. The slotted and starred lines are fiscal multipliers: the difference between output-with-intervention and output-without-stimulus. The starred (black) line is our base case, with sovereign risk, long term (duration 25 quarters) debt and deficit financed stimulus package. The slotted (red) line is the fiscal multiplier in the benchmark case without sovereign risk, with short term bonds only and no financial frictions.

start of the stimulus program. This also happens in the no frictions case, but to a much smaller extent. The net impact turns positive at the start of the implementation, although less than in the case without financial frictions. Without financial frictions and without sovereign risk (red slotted line), the output effects fade away to zero in about 20 quarters. But with financial frictions, leverage constrained banks and sovereign risk, the negative channels dominate again within
a year after the start of the stimulus for a prolonged net negative impact, before also reaching zero a few quarters later than in the no-frictions case.

The mixture of negative and positive effects raise the question of whether the cumulative policy impact can actually turn negative. To answer that question, we calculate a cumulative discounted multiplier. Denoting a variable from the stimulus scenario $x_{st}$ and from the no-policy-response case $x_{np}$, the cumulative discounted multiplier is defined as:

$$
\mu_D = \frac{\sum_j \beta^j (y_{st,t+j} - y_{np,t+j})}{\sum_j \beta^j (g_{st,t+j} - g_{np,t+j})}
$$

The numerator of $\mu_D$ is equal to the cumulative area between the black starred line and the zero line in Figure 2.7, with areas below zero having a minus sign, while the denominator is the difference between the solid blue line and the zero line. The solid blue line represents the additional government spending. For the base case, with sovereign risk, long term (duration 25 quarters) debt, leverage constrained and weakly capitalized banks, and deficit financed stimulus package, the answer is yes, the overall effect is in fact negative, for that case $\mu_D = -0.25$. Without any frictions or sovereign risk or crowding out problems, $\mu_D$ equals 0.88, see Table 2.2. Introducing leverage constrained banks and the associated financial frictions (FF) reduces the cumulative discounted multiplier substantially, from 0.88 down to 0.49, but introducing all the crowding out channels, sovereign risk problems and potential for capital losses on commercial bank holdings of sovereign debt reduces $\mu_D$ much more, from 0.49 all the way down to an overall negative value of -0.25.

2.4.2 Dissecting & quantifying the various amplification mechanisms at play

Table 2.2 indicates that the interaction between capital losses on debt (whether they are due to shifts in the term structure or sovereign risk concerns) and weak


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<table>
<thead>
<tr>
<th>Stimulus policy</th>
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<tbody>
<tr>
<td>1: no financial frictions, no sovereign risk</td>
<td>0.88</td>
</tr>
<tr>
<td>2: FF, short term debt, tax-financed, no sovereign risk</td>
<td>0.49</td>
</tr>
<tr>
<td>3: FF, long term debt, sovereign risk</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

Table 2.2. Table displaying the discounted cumulative dynamic multiplier for the case of no financial frictions and no sovereign default risk (scenario 1), Financial frictions but no debt problems (scenario 2), and scenario 3 with financial frictions, long term government debt and sovereign default risk. Government liabilities represent 53.2% of annual steady state output.

capitalization in balance-sheet-constrained banks have a major negative impact on the fiscal multiplier. In this section we look at the different mechanisms that are causing the discounted cumulative multiplier to actually turn negative in the presence of financial friction. The first channel considered is sovereign default risk, and the second channel runs through the maturity of the government debt and the impact of capital losses on the balance sheet of the intermediaries.

The negative impact of sovereign default risk on the effectiveness of fiscal stimuli.

Consider first the effect of introducing sovereign default risk into the model. The simulations on which we report below show the response to the same financial shock-cum-stimulus package as analyzed before, but now comparing the results from one set of runs with debt levels in a region where sovereign risk is an issue, with the model response from another set of runs where all assumptions are the same except that the distance to default for the sovereign is much larger so that sovereign risk does not play a role. The stimulus package is the same as in the previous experiments. The (red) slotted line gives our base case with all the amplification channels in, and the (blue) solid line refers to the case where sovereign risk has been made ineffective by increasing the sovereign’s distance to default: when far away from the maximum level of government debt, the additional sovereign default channel does not generate any extra dynamics. All other assumptions and parameters are the same in the two sets. Figure 2.8 compares a subset of the results for the two cases.
Figure 2.8. Plot of the impulse response functions comparing the economy with no sovereign default risk (blue, solid), and with sovereign default risk present (red, slotted). In both cases the government engages in preannounced stimulus spending implemented 4 quarters after announcement (at the onset of the crisis) through additional debt issuance. The size of the package amounts to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

Incorporating sovereign default risk in the analysis of a deficit financed fiscal stimulus leads to a significant deterioration of outcomes compared with the no default risk outcome. The initial shock works out in a similar pattern in both set of runs: due to the fact that the financial intermediaries have to take losses on the private loans because of the initial capital quality shock (the crisis), they become more balance-sheet-constrained. The leverage constraint becomes more binding, which pushes up (expected) interest rates, reducing the demand for new capital. Due to arbitrage between private loans and bonds, the (expected) return on bonds is driven up as well, driving down the price of bonds. This in turn increases the nominal amount of debt the government needs to issue in order to finance the additional government spending. But then the differences start.
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As a consequence of the higher debt levels, the risk of a sovereign default increases, with associated additional impact on bond pricing. This in turn makes future deficit finance even more expensive. The credit spread increases by almost 50 basis points more in the sovereign risk case, as well as the leverage ratio, since the net worth of the financial intermediaries decreases by an additional 10 percent. The financial intermediaries financing the government anticipate the higher default fraction and higher interest rates, which explains the higher credit spread. The additional fall in the bond price almost doubles the overall bond price impact, with therefore higher losses on bank holdings of sovereign debt, which in turn sets off an additional negative amplification round.

The impact on the real economy of sovereign risk is thus substantial. The trough of the recession is almost a percentage point deeper, and output remains below the no default case over the entire time path. Due to substantial additional capital losses on government bonds, net worth goes down more, which in turn pushes up (expected) interest rates causing the price of capital to fall further. This has a negative effect on investment, pushing down the capital stock, and thereby pushing down output in the long run. Households become more constrained through lower wages, and lower profits from both the financial and non-financial firms they own, so consumption goes down.

<table>
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<td>2: FF, short term, tax-financed, no sovereign risk</td>
<td>0.49</td>
</tr>
<tr>
<td>3: FF, long term debt debt, no sovereign risk</td>
<td>0.30</td>
</tr>
<tr>
<td>4: FF, long term debt with sovereign risk</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

Table 2.3. Table displaying the discounted cumulative multiplier as defined in the text for short term debt and long term financing with and without default, where steady state government liabilities represent 53.2% of annual steady state output. Short term debt has an average duration of 2 quarters, whereas average duration of long term debt is equal to 5 years.

Table 2.3 shows the impact on the cumulative multiplier $\mu_D$. Clearly the emergence of sovereign risk and its interaction with financial frictions through
capital losses on bank holdings of sovereign debt have a major impact on the multiplier: of the total decline in $\mu_D$ from 0.88 down to -0.25, a full 55 percentage points (from 0.30 down to -0.25, or about half of the total decline of 113 basis points) is explained by the capital losses associated with the higher sovereign risk triggered by the debt financed stimulus package.

Fiscal stimuli and the maturity of government bonds

Consider next the influence of debt maturity. To bring that out we compare the same base run as in the previous section (without sovereign risk but with long term debt) with a set of runs where everything is the same except that the maturity of the debt has been shortened to short term debt (two quarters) only (Figure 2.9). The case where all government debt, both old and new, is short term is represented by the solid (blue) line, and the case where 25 quarters debt (LT) is being used by the slotted (red) line. In both cases, the sovereign default channel has been shut off. Steady state government liabilities are at 53.2% of annual steady state output as before. And also as before, the fiscal stimulus is announced as the crisis hits, implemented 4 quarters later, and has a size of 1.25% of annual steady state output. The results of the simulations clearly show that lengthening the maturity of the government debt has a strong impact on the bond price. Capital losses on government bonds due to falling bond prices are substantially larger in the case of long term debt, by some 5% of their steady state value, so the initial drop in banks’ net worth is correspondingly larger. Similar to the previous cases, the capital losses further deteriorate credit conditions, and ultimately reduce the effectiveness of a fiscal stimulus package. Table 2.5 spells out the impact of lengthening maturity on the cumulative multiplier $\mu_D$.

Comparing the ST debt case 3 with the LT (25 quarters) debt case 4 shows that the dynamic cumulative multiplier is higher in the ST debt case, it increases from 0.30 to 0.43. So of the total difference of 1.13 (113 percentage points) between the benchmark and the base case, 55 points (or about half, 49%) can
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<table>
<thead>
<tr>
<th>Stimulus policy</th>
<th>Discounted cumulative multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: no financial frictions (FF), no sovereign risk</td>
<td>0.88</td>
</tr>
<tr>
<td>2: FF, short term debt, stimulus is tax-financed, no sovereign risk</td>
<td>0.49</td>
</tr>
<tr>
<td>3: FF, short term debt, stimulus is debt-financed, no sovereign risk</td>
<td>0.43</td>
</tr>
<tr>
<td>4: FF, long term debt, stimulus is debt-financed, no sovereign risk</td>
<td>0.30</td>
</tr>
<tr>
<td>5: FF, long term debt, stimulus is debt-financed, sovereign risk</td>
<td>-0.25</td>
</tr>
<tr>
<td>6: FF, long term debt (10 yrs.), stimulus is debt-financed, sovereign risk</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

Table 2.4. Table displaying the discounted cumulative dynamic multiplier for listed scenarios. In all cases steady state government liabilities represent 53.2% of annual steady state output. Short term debt has an average maturity of 2 quarters (scenario 2 and 3), whereas duration of long term debt is equal to 25 quarters in scenario 4 and 5, and 10 years in scenario 6.

be attributed to the sovereign risk channel, as seen in the previous subsection (compare cases 4 and 5), 11 percentage points to the impact of longer debt maturity in the base case compared to the benchmark (compare case 3 and 4). And 6 points of the difference (going from 0.43 in case 3 to 0.49 in case 2, or 5.3% of the total 113 basis points difference) can be attributed to the balance sheet crowding out channel: this refers to the fact that government debt and corporate loans compete for limited lending capacity of the leverage constrained banks (cf Kirchner and van Wijnbergen (2012)). The last 39 percentage points can be attributed to the presence of financial frictions (compare case 1 and 2, going from 0.88 down to 0.49, or 34.5% of the total decline).

The impact on bond prices over time is given in Figure 2.9 for a maturity of 2 quarters versus a maturity of 25 quarters, while Figure 2.10 shows the decline in the cumulative dynamic multiplier $\mu_D$ as a function of average debt maturity of existing and new debt (we recalculate $\rho$ into the more intuitive but equivalent metric of average maturity (duration), measured in quarters). The figure shows a gradual decline of the multiplier as the maturity increases.
2.4. RESULTS

Figure 2.9. The impulse response functions for the no default economy comparing the bond price reaction for government debt with an average duration of 2 quarters (blue, solid) versus a maturity of 25 quarters (red, slotted).

2.4.3 The multiplier when government bonds are not or to a lesser extent subject to a leverage constraint

In this section we look at the effect of varying the ratio of the diversion rate of government bonds over the diversion rate of private loans, i.e. $\Delta = \frac{\lambda_b}{\lambda_k}$ on the dynamic multiplier $\mu_D$. The results can be found in Figure 2.11. We clearly see that as the relative diversion rate of government bonds is higher (higher $\Delta$), the dynamic multiplier $\mu_D$ increases, from approximately 0.2 for $\lambda_b = 0$ ($\Delta = 0$), to almost 0.4 for $\lambda_b = \lambda_k$ ($\Delta = 1$). A higher $\Delta$ makes it less attractive to shift out of corporate lending in response to capital losses, which explains the higher multiplier for higher $\Delta$.

An interesting interpretation is with the double edged capital requirement regime used under the current Basel-II and Basel-III approaches. Our $\Delta = 0$ regime is very similar to the risk weighted BIS Capital-Asset Ratio (RWCAR) case, because shifting from risky assets to government debt eases the leverage
constraint both in our set up with $\Delta = 0$ and under the Basel RWCAR approach. But with $\Delta = 1$ our endogenous leverage constraint case is closer to the unweighted leverage ratio in Basel-III calculated over all assets. In the latter case ($\Delta = 1$ and/or the Basel-III unweighted leverage ratio), shifting from one asset category to the other has no impact on the tightness of the leverage constraint. As a consequence the overall multiplier is much higher, banks have no incentive to substitute out of corporate loans into sovereign debt anymore when they are subjected to an unweighted leverage ratio. Whatever the microprudential merits of the Risk Weighted Approach may be, from a macro perspective the case for an unweighted leverage ratio is strong.
2.4. RESULTS

Figure 2.11. Relative diversion rate $\Delta$ vs. discounted multiplier with steady state government liabilities at 53.2% of annualized steady state GDP in the no default economy.

2.4.4 Bank capitalization and the effects of a stimulus package

In previous sections we have shown that weakly capitalized banks are an impediment to the effectiveness of fiscal stimulus measures. To stress that point, we rerun the model while assuming a better capitalized system and again assess the effectiveness of fiscal policy. We compare the base case experiment of a fiscal stimulus when banks finance long term government debt subject to sovereign default risk, and compare this with the impact of the same stimulus, but accompanied by an immediate tax-financed bank recapitalization equal to 5% of annual steady state GDP, as in Figure 2.12.
CHAPTER 2. FISCAL MULTIPLIER

Figure 2.12. Plot of the impulse response functions comparing a fiscal stimulus for a weakly capitalized banking system (blue, solid), and a better capitalized banking system (red, slotted), in an economy with sovereign default risk. The stimulus is announced as the crisis hits, and implemented 4 quarters later through additional debt issuance, and equal to 1.25% of annual steady state GDP. A better capitalized banking system is captured by immediately recapitalizing the financial sector as the crisis hits through a tax financed recap of 5% of annual GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

It is clear from the output response that a better capitalized financial sector significantly improves the response to the initial capital quality shock. The recap alleviates the banks’ balance sheet constraint, which explains the decrease in the credit spread by almost 100 basis points. So interest rates come down, and demand for private loans increases. Investment falls less by almost 10% of the steady state value, which ultimately leads to output increasing across the entire time path, with the troughs up by almost 2 percentage points of steady state output. This may well explain why the recovery in the US seems to have come much faster: financial intermediaries were forced to clean up their balance sheets and raise new capital early on. In Europe commercial banks were not forced to clean
up their balance sheets to a comparable extent, with hidden losses still on their balance sheet (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). Our simulations suggest that failure to do so has impeded the recovery. Intriguingly, the effectiveness of fiscal policy is still low, the cumulative multiplier only increases from -0.25 to -0.20; clearly the recap is large enough to lead to a more robust response to the initial capital quality shock, but the remaining financial frictions cause the multiplier to stay well below its frictionless and sovereign risk-free value of 0.88. To completely offset the financial frictions, if possible at all, substantially higher capital asset ratios would be needed for banks than are currently under discussion and in fact higher than within the feasible calibration range of our current model parameters.

2.4.5 The Zero Lower Bound and the Effectiveness of a Fiscal Stimulus

Until now we have analyzed the impact of fiscal policy assuming a Taylor-rule type “Leaning against the wind” monetary policy, which in old fashioned language is similar to an upward sloping LM curve. But what if monetary policy becomes extremely accommodating? The question takes on additional relevance because strongly expansionary CB policies in both the US and the Eurozone have landed both regions at the ZLB (Zero Lower Bound), the region where traditional monetary policy cannot be used anymore because interest rates have hit their lower bound of zero. In classical IS-LM language, we are in a flat LM curve region. With fiscal policy not running into an upward sloping LM curve anymore (to continue using IS-LM language...), one should expect the effectiveness of fiscal policy to increase. To see to what extent that affects our key results about how the interaction between fragile banks and weak sovereigns undermines the effectiveness of fiscal policy, we repeat our simulations in a ZLB environment.

To that end, we compare in Figure 2.13 the base case experiment of a fiscal stimulus when banks finance long term government debt subject to sovereign
default risk with the case where the interest rate smoothing parameter has been set at $\rho_r = 0.999$ (red, slotted). This effectively reproduces the “flat LM curve” we are looking for: by slowing down the interest rate response to practically zero, we effectively mimick a ZLB environment. Although setting $\rho_r = 0.999$ is strictly speaking not the same as being stuck at the ZLB, it does capture the key characteristic of the ZLB, namely that the interest rate does not respond to endogenous changes in output or inflation.

Figure 2.13 clearly shows that the absence of an endogenous response in the interest rate strengthens the stimulative impact on the economy of a fiscal push; the initial negative output response disappears altogether. Moreover, the anticipation that there will be no interest rate increases once the economy picks up after the financial crisis increases financial intermediaries’ future profits, which
relaxes the intermediaries’ balance sheet constraint immediately. This in turn leads to lower (expected) interest rates for corporate borrowers. Lower interest rates increase the demand for private loans, which leads to a higher price for capital. Losses due to the capital quality shock are thus reduced. Net worth increases with respect to the case where there is an endogenous interest rate response to inflation and output. This further decreases credit spreads and (expected) interest rates. Lower (expected) interest rates on private loans and a relaxed balance sheet constraint induce financial intermediaries to demand more government bonds. The price of government bonds increases as a result, and reduces the losses on existing holdings of sovereign debt, thereby further increasing net worth. As a consequence, the credit spread comes down from almost 300 basis points to 50 basis points, net worth increases with 60% of steady state net worth, and investment increases with respect to the steady state instead of falling by almost 30% of steady state investment. Higher investment, together with higher consumption (not shown) leads to output increasing instead of falling by 5% on impact.

Table 2.5 shows the DCMs of the various cases, now with the ZLB variant included. The table makes clear that without an upward sloping LM curve (to use the classical language for the ZLB once again...), the negative impact of financial frictions are much reduced, but do not disappear entirely. We find a value of 0.51 for the ZLB case with financial frictions and sovereign debt problems, while the discounted cumulative multiplier for the regular Taylor rule but without financial fragility and without sovereign debt problems (our regular benchmark case) still is 0.88. Clearly, fiscal stimulus packages become much more effective when the economy is stuck at the ZLB, so an accommodating monetary policy is therefore capable of offsetting to a substantial extent the negative effects of a debt-financed fiscal stimulus package in a weak banks-weak sovereign environment. But we still observe that the financial frictions cannot be offset entirely when the ZLB is hit. Although the right monetary policy is
able to substantially mitigate the negative effects from a financial crisis, it is not capable of fully offsetting them.

<table>
<thead>
<tr>
<th>Stimulus policy</th>
<th>Discounted cumulative multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: no financial frictions, no sovereign risk</td>
<td>0.88</td>
</tr>
<tr>
<td>2: FF, long term debt, stimulus is debt-financed, sovereign risk, ZLB</td>
<td>0.51</td>
</tr>
<tr>
<td>3: FF, short term debt, stimulus is tax-financed, no sovereign risk</td>
<td>0.49</td>
</tr>
<tr>
<td>4: FF, short term debt, stimulus is debt-financed, no sovereign risk</td>
<td>0.43</td>
</tr>
<tr>
<td>5: FF, long term debt, stimulus is debt-financed, no sovereign risk</td>
<td>0.30</td>
</tr>
<tr>
<td>6: FF, long term debt, stimulus is debt-financed, sovereign risk</td>
<td>-0.25</td>
</tr>
<tr>
<td>7: FF, very long term debt (10 yrs.), stimulus is debt-financed, sovereign risk</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

Table 2.5. Table displaying the discounted cumulative dynamic multiplier for different scenarios. Short term debt has an average maturity of 2 quarters (scenario 3 and 4), whereas duration of long term debt is equal to 25 quarters in scenario 2, 5 and 6, and 10 years in scenario 7.
2.5 Conclusion

The financial crisis of 2007-2009 ignited a renewed debate on the effectiveness of government spending packages to stimulate the economy. Both in the US and in parts of the Eurozone, governments adopted expansionary fiscal measures, with the notable exception of Germany. The subsequent sovereign debt crisis in the countries on the Eurozone’s Southern rim clearly limited the room for more decisive Keynesian measures. Whatever the reasons, Keynesian policies were tried in both the US and the Eurozone, but much more haphazardly in the latter, while a fierce debate erupted about their effectiveness. In this paper we argue that there is more than dogma to the doubts about the fiscal expansion tool in the European context. Across the Eurozone banks are not only more important vehicles for the provision of corporate credit than they are in the US, but the banking system was generally in worse shape for much longer as European governments were hesitant to recognize balance sheet weaknesses in their banking system and acted much less decisively in dealing with capital shortage in the financial system than the US did. This may well have been due to widespread awareness of sovereign risk in much of the Eurozone, an issue brought to the fore by the eruption of a sovereign debt crisis in Greece in May 2010 and its subsequent rapid contagious spread to other Eurozone periphery countries.

In this paper we argue that the feedback loop between weak banks and weak sovereigns has not only exacerbated the European sovereign debt crisis but through its impact on financial fragility has undermined the effectiveness of traditional Keynesian expansionary macroeconomic policy, or would have if tried. The impact of weak balance sheets in the financial sector and its interaction with sovereign debt discounts is key to our argument. The interaction between undercapitalized banks with extensive sovereign debt holdings on their balance sheet on the one hand and fiscal authorities whose repayment capacity is questioned as debt levels rise, not only limits the possibility of intervening in
weak banks, as we have argued elsewhere (Van der Kwaak and Van Wijnbergen, 2014), but also undermines the effectiveness of traditional Keynesian demand management policies. In such circumstances banks act as a break on expansion, as we demonstrate in this paper, possibly to such an extent that Keynesian multipliers actually become negative.

To make the point on the effectiveness of deficit-financed fiscal stimulus packages, we embed endogenous sovereign debt discounts and a possibly undercapitalized financial sector that is subject to financial frictions within an otherwise mostly standard New-Keynesian DSGE model. Financial intermediaries provide loans to the non-financial sector, and (partially) finance government deficits by buying long-term sovereign debt that is subject to default risk. Within this framework, we first investigate a deficit-financed fiscal expansion that is announced at the onset of a financial crisis, but implemented four quarters (one budget cycle) later. We find that the effectiveness of fiscal stimulus packages is severely diminished in such an environment: the announcement of the package with its associated future debt issues and tightening leverage constraints in the financial sector already leads to lower output before the package has been implemented. Although the actual implementation briefly reverses that negative output effect, negative output responses re-emerge in the worst cases we consider, with undercapitalized banks with substantial long term debt holdings and weak sovereigns. We identify several mechanisms through which such perverse crowding out effects are triggered. To measure the cumulative impact, we introduce the concept of a discounted cumulative multiplier to measure the effectiveness of government spending packages, and find that the cumulative effect becomes negative in an environment of weakly capitalized banks that carry long term government debt subject to sovereign default risk on their balance sheets.

We dissect the cumulative results and find three main channels: capital losses on sovereign debt because of increasing sovereign default risk (channel 1),
creasing with the maturity of that debt (channel 2), and crowding out of private loan provision by government bonds (channel 3) since financial intermediaries are balance-sheet-constrained and become more so if they have to absorb additional government debt issues. Hence our results suggest that fiscal stimulus packages in the Eurozone where banks were severely undercapitalized (International Monetary Fund, 2011; Hoshi and Kashyap, 2014), have been and/or would have been less effective than they have been in the US. And this would have been an even bigger problem in the Eurozone’s Southern periphery countries as those had a large exposure to risky domestic sovereign debt.

We also present further results suggesting that extremely accommodating monetary policy (extreme in the sense of hitting the Zero Lower Bound, or in old fashioned language, flattening the LM curve), would substantially although definitely not completely have restored the efficacy of fiscal policy. In our setup, about two-thirds of the negative impact of the weak-banks/weak-sovereigns nexus can be offset by a monetary policy pushing the economy down to the ZLB. Finally we also analyze the case where government debt held on bank balance sheets is not subject to leverage constraints. This is an interesting comparison because this variant is closely related to the BIS Risk-Weighted Capital-Asset Ratio (RWCAR) requirements while the other extreme, government debt held by banks equally subject to the leverage constraint as corporate loans is close to the BIS unweighted leverage ratio requirement. Our results suggest that shifting from Risk Weighted Assets as the basis for a leverage constraint to a leverage constraint based on unweighted definitions of the asset base will about double (!!!) the fiscal multiplier. This confirms the often expressed fear that the risk-weighted-asset approach, by encouraging firms to shift their assets out of corporate loans into sovereign debt when under pressure to raise their capital asset ratios, has actually worsened the European post-credit-crisis recession.
2.A Derivations

2.A.1 Maturity government bonds

The maturity of government bonds is modeled in a similar way as in Woodford (1998, 2001). Bonds $b_t$ issued in period $t$ provide the government with funds $q_t^b b_t$ in real terms, where $q_t^b$ is the real price of the outstanding bonds, and $b_t$ the number of bonds. One government bond entitles the investor to a payment $r_c$ fixed in real terms in period $t+1$, $\rho r_c$ real units in period $t+2$, $\rho^2 r_c$ real units in period $t+3$ etc. This pay off structure is equivalent to entitling a bond holder to a full payment of $r_c$ in the first period only, plus at the end of that period a new similar bond equal to $\rho$ and so on. The case of one period bonds is captured by $\rho = 0$, while the case of perpetual bonds, or ‘consols’ is obtained by setting $\rho = 1$. Any value of $\rho \in [0, 1]$ gives a maturity structure in between these 2 extremes. The average maturity is $1/(1 - \beta \rho)$.

The default exclusive real return on government bonds is $r_t^b$:

$$1 + r_t^b = \frac{r_c + \rho q_t^b}{q_{t-1}^b}$$

(2.29)

2.A.2 Financial intermediaries

The banker’s intermediate period balance sheet is given by:

$$p_{j,t} = n_{j,t} + d_{j,t}$$

The asset side of the bank’s balance sheet has the following structure:

$$p_{j,t} = q_t^k s_{j,t}^k + q_t^b s_{j,t}^b$$

---

5The duration (average maturity) equals $\frac{\sum_{j=1}^\infty \beta^j (\rho^{j-1} r_c)}{\sum_{j=1}^\infty \beta^j (\rho^{j-1} r_c)} = 1/(1 - \beta \rho)$. 

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The balance sheet of intermediary \( j \) evolves according to the following law of motion:

\[
\begin{align*}
\bar{n}_{j,t+1} &= (1 + r_{t+1}^k)q_t^k s_{j,t}^k + (1 + r_{t+1}^b)q_t^b s_{j,t}^b - (1 + r_{t+1}^d)d_{j,t} + n_{j,t+1}^g - \bar{n}_{j,t+1} \\
&= (r_{t+1}^k - r_{t+1}^d)q_t^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d)q_t^b s_{j,t}^b \\
&+ (1 + r_{t+1}^d)n_{j,t} + \tau_{t+1}^g n_{j,t} - \bar{\tau}_{t+1}^g n_{j,t}
\end{align*}
\]

The financial intermediary is interested in maximizing expected profits. The banker’s objective is then given by the following recursive optimization problem:

\[
V_{j,t} = \max E_t [\beta \Lambda_{t+1} \{(1 - \theta)n_{j,t+1} + \theta V_{j,t+1}\}]
\]

where \( \Lambda_{t+1} = \lambda_{t+1}/\lambda_t \). We conjecture the solution to be of the following form, and later check whether this is the case:

\[
V_{j,t} = \nu_t q_t^k s_{j,t}^k + \nu_t q_t^b s_{j,t}^b + \eta_t n_{j,t}
\]

We follow Gertler and Karadi (2011) and assume the banker can divert a fraction \( \lambda_a \) of asset class \( a = \{k, b\} \) at the beginning of the period, and transfer these assets costlessly back to the household. This gives rise to the following constraint:

\[
\begin{align*}
V_{j,t} &\geq \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b \\
\nu_t q_t^k s_{j,t}^k + \nu_t q_t^b s_{j,t}^b + \eta_t n_{j,t} &\geq \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b 
\end{align*}
\]

The optimization problem can now be formulated in the following way:

\[
\max_{\{s_{j,t}^k, s_{j,t}^b\}} V_{j,t}, \quad \text{s.t.} \quad V_{j,t} \geq \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b
\]

The Lagrangian for this problem is now given by:

\[
L = (1 + \mu_t)(\nu_t q_t^k s_{j,t}^k + \nu_t q_t^b s_{j,t}^b + \eta_t n_{j,t}) - \mu_t (\lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b)
\]
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where $\mu_t$ is the Lagrangian multiplier on the constraint. Hence we get the following first order conditions:

$$s^k_{j,t} : (1 + \mu_t)v^k_t - \lambda_k \mu_t = 0 \implies v^k_t = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right)$$

$$s^b_{j,t} : (1 + \mu_t)v^b_t - \lambda_b \mu_t = 0 \implies v^b_t = \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right)$$

$$\mu_t : \left\{ v^k_t q^k_t s^k_{j,t} + v^b_t q^b_t s^b_{j,t} + \eta_t n_{j,t} - \lambda_k q^k_t s^k_{j,t} - \lambda_b q^b_t s^b_{j,t} \right\} \mu_t = 0$$

From the first order conditions we find that $v^b_t = \frac{\lambda_k}{\lambda_b} v^k_t$. Hence the leverage constraint (2.30) can be rewritten in the following way:

$$v^k_t \left( q^k_t s^k_{j,t} + \frac{\lambda_b}{\lambda_k} q^b_t s^b_{j,t} \right) + \eta_t n_{j,t} \geq \lambda_k \left( q^k_t s^k_{j,t} + \frac{\lambda_b}{\lambda_k} q^b_t s^b_{j,t} \right)$$

$$\Rightarrow q^k_t s^k_{j,t} + \frac{\lambda_b}{\lambda_k} q^b_t s^b_{j,t} \leq \phi_t n_{j,t}, \quad \phi_t = \frac{\eta_t}{\lambda_k - v^k_t}$$ \hspace{1cm} (2.31)

where $\phi_t$ can be interpreted as the ratio of “risk-weighted” assets to net worth, or the leverage constraint of the financial intermediary. Substitution of the conjectured solution into the right hand side of the Bellman equation gives the following expression for the continuation value of the financial intermediary:

$$V_{j,t} = E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta)n_{j,t+1} + \theta V_{j,t+1} \right\} \right] = E_t \left[ \Omega_{t+1} n_{j,t+1} \right],$$

$$\Omega_{t+1} = \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta [\eta_{t+1} + v^k_t \phi_{t+1}] \right\}$$
Ω_{t+1} can be thought of as a stochastic discount factor that incorporates the financial friction. Now we can substitute the expression for next period’s net worth into the expression above:

\[
V_{j,t} = E_t\left[\Omega_{t+1} n_{j,t+1}\right] \\
= E_t\left[\Omega_{t+1}\left\{ (1 + r_{t+1}^k) q_{k,t} s_{j,t}^k + (1 + r_{t+1}^b) q_{b,t} s_{j,t}^b \right. \right. \\
- \left. \left. (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \right\} \right] \\
= E_t\left[\Omega_{t+1}\left\{ (r_{t+1}^k - r_{t+1}^d) q_{k,t} s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d) q_{b,t} s_{j,t}^b \right. \right. \\
+ \left. \left. (1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n) n_{j,t} \right\} \right] \\
(2.32)
\]

After combining the conjectured solution with (2.32), we find the following first order conditions:

\[
\eta_t = E_t\left[\Omega_{t+1}\left(1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n\right)\right], \\
(2.33)
\]

\[
\nu_t^k = E_t\left[\Omega_{t+1}\left(r_{t+1}^k - r_{t+1}^d\right)\right], \\
(2.34)
\]

\[
\nu_t^b = \frac{\lambda^b}{\lambda^k} \nu_t^k = E_t\left[\Omega_{t+1}\left(r_{t+1}^b - r_{t+1}^d\right)\right], \\
(2.35)
\]

\[
\Omega_{t+1} = \beta \Lambda_{t+1}\left\{ (1 - \theta) + \theta \left[\eta_{t+1} + \nu_{t+1}^k \phi_{t+1}\right]\right\}.
\]

2.A.3 Production side

Intermediate Goods Producers

There exists a continuum of intermediate goods producers indexed by \( i \in [0, 1] \). Each of these firms produce a differentiated good. The intermediate goods producers borrow from the financial intermediaries against future profits. We assume that there are no financial frictions between the financial intermediaries and the intermediate goods producers. Hence there are no monitoring costs for the financial intermediaries, and the intermediate goods producers can commit next period’s profits to the financial intermediaries. The securities issued by the
intermediate goods producers are therefore really state-contingent debt, like in Gertler and Kiyotaki (2010). The production technology of the intermediate goods producers is given by:

\[ y_{i,t} = a_t (\xi_t k_{i,t-1})^{\alpha h_{i,t}^{1-\alpha}}, \]

where \( a_t \) equals total factor productivity and \( \xi_t \) capital quality, which follow a lognormal AR(1) process:

\[ \log(x_t) = \rho_x \log(x_{t-1}) + \epsilon_{x,t}, \]

where \( x = \{a, \xi\} \). The innovations \( \epsilon_{x,t} \) are distributed as \( \epsilon_{x,t} \sim N(0, \sigma^2_x) \) for \( x = a, \xi \). The intermediate goods producer acquires the capital at the end of period \( t - 1 \), while the production only occurs after the capital quality shock \( \xi_t \) has hit at the beginning of period \( t \). Hence \( \xi_t k_{i,t-1} \) denotes the effective capital in our model. We see that if a negative realization of \( \epsilon_{\xi,t} \) occurs, the quality of the capital deteriorates immediately. Hence the firm will not be able to produce as much as when the shock does not occur. Remember that the number of claims \( s_{k,i,t}^k \) is equal to the number of units of capital purchased \( (k_{i,t}) \); hence the return on the claims of the financial intermediary will be lower. We can think of the shock \( \xi_t \) as accelerated economic depreciation or obsolescence of capital. The intermediate goods producer decides at the end of period \( t - 1 \) how much capital to purchase. At the moment the intermediate goods producer purchases the capital, he does not know the realization of \( \xi_t \) in period \( t \) yet. To finance his purchase at the end of period \( t - 1 \), he needs to issue claims \( s_{i,t-1}^k \), with the number of claims \( s_{i,t-1}^k \) equal to the number of capital units \( (k_{i,t-1}) \) acquired. The price at which the claims are sold equals \( q_{t-1}^k \), and they pay a state-contingent

---

\( ^6 \)The claims of financial intermediaries can therefore be better thought of as equity. Occhino and Pescatori (2015) explicitly model loans to producers with a fixed face value, and include the possibility of a default by the goods producers. We refrain from explicitly modelling the producers’ default, and note the equity characteristics of debt when firms have not enough funds to pay off the loan.
net real return $r_t^k$ in period $t$. The intermediate goods producer also hires labor $h_{i,t}$ for a wage rate $w_t$ after the shock ($\xi_t$) has been realized. When the firm has produced in period $t$, the output is sold for a relative price $m_t$ to the retail firms. $m_t$ is the relative price of the intermediate goods with respect to the price level of the final goods, i.e. $m_t = \frac{P_{m_t}}{P_t}$. After production, the intermediate goods producing firms sell back what remains of the effective capital to the capital producers at a price of $q_t^k$. The effective capital stock is also subject to regular depreciation $\delta$ during production. So the intermediate goods producer receives $q_t^k(1-\delta)\xi_t k_{i,t-1}$ for his end of period capital stock and real profits in period $t$ are given by:

$$\Pi_{i,t} = m_t a_t (\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha} + q_t^k (1-\delta)\xi_t k_{i,t-1} - (1+r_t^k)q_{t-1}^k k_{i,t-1} - w_t h_{i,t}$$

The objective of the intermediate goods producing firms is to maximize gross profits. They take the relative output price ($m_t$), and the input prices $q_t^k$, $r_t^k$ and $w_t$ as given when maximizing profits. The first order condition with respect to capital and labor are given by:

$$k_{i,t} : E_t \left[ \beta \Lambda_{t,t+1} q_t^k (1+r_{t+1}^k) \right] = E_t \left[ \beta \Lambda_{t,t+1} (\alpha m_{t+1} y_{i,t+1}/k_{i,t} + q_{t+1}^k (1-\delta)\xi_{t+1}) \right]$$

$$h_{i,t} : w_t = (1-\alpha) m_t y_{i,t}/h_{i,t}$$

Firms pay out residual revenues to the financial intermediaries. By substituting the first order condition for the wage rate into the zero-profit condition $\Pi_{i,t} = 0$, we can find an expression for the ex-post return on capital:

$$r_t^k = (q_{t-1}^k)^{-1} \left( \alpha m_t y_{i,t}/k_{i,t-1} + q_t^k (1-\delta)\xi_t \right) - 1$$
CHAPTER 2. FISCAL MULTIPLIER

The first order condition for labor and the expression for the ex-post return on capital can be rearranged to derive factor demands. These are given by:

\[
k_{i,t-1} = \alpha m_t y_{i,t} / [q_{k,t-1}(1+r_k^t) - q_k^t(1-\delta)]
\]

\[
h_{i,t} = (1-\alpha) m_t y_{i,t} / w_t
\]

Finally the relative intermediate output price \( m_t \) can be obtained by substituting the factor demands into the production function:

\[
m_t = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} \left[ w_t^{1-\alpha} \left[ q_{k,t-1}(1+r_k^t) - q_k^t(1-\delta) \right]^{-1} \right] \] (2.36)

**Capital Producers**

In this section we describe the capital producers. At the end of period \( t \), when the intermediate goods firms have produced, they sell what remains of the capital stock after depreciation \( \delta \) to the capital producers at a price \( q_k^t \). The capital producers also buy \( i_t \) final goods from the final good producers; these purchases (investment) are an input in the capital production process: they are used to produce additional capital. Capital producers combine this additional capital with the old, partially depreciated stock bought earlier from the intermediate goods producers and so produce the new capital stock. This new capital is being sold to the intermediate goods producers at a price \( q_k^t \). We assume that the capital producers face convex adjustment costs whenever investment \( i_t \) deviates from previous period investment \( i_{t-1} \). These adjustment costs are the reason that one unit of investment goods cannot be transformed into one unit of capital, unless \( i_t = i_{t-1} \). Hence we have the following capital production technology:

\[
k_t = (1-\delta)\xi_t k_{t-1} + (1-\Psi(t_t))i_t, \quad \Psi(x) = \frac{\gamma}{2}(x-1)^2, \quad t_t = i_t/i_{t-1}. \] (2.37)
The capital producers are profit maximizing, and profits are passed on to the households, who are the owners of the capital producers. The profit in period \( t \) is given by:

\[
\Pi^c_t = q^k_t k_t - q^k_t (1 - \delta) \xi_t k_{t-1} - i_t
\]

The capital producers’ optimization problem is then given by:

\[
\max_{\{i_{t+i}\}_{i=0}^{\infty}} E_t \left[ \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left( q^k_{t+i} (1 - \Psi(t_{t+i})) (i_t + i_{t+i}) \right) \right]
\]

Differentiation with respect to investment gives the first order condition for the capital producers:

\[
q^k_t (1 - \Psi(t_t)) - 1 - q^k_t \Psi'(t_t) + \beta E_t \Lambda_{t,t+1} q^k_{t+1} i_{t+1} \Psi'(t_{t+1}) = 0
\]

This equation can be rewritten to find the price of capital to be:

\[
\frac{1}{q^k_t} = 1 - \gamma \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 - \gamma \frac{i_t}{i_{t-1}} \left( \frac{i_t}{i_{t-1}} - 1 \right) + \beta E_t \left[ \Lambda_{t,t+1} q^k_{t+1} \left( \frac{i_{t+1}}{i_t} \right)^2 \frac{i_{t+1}}{i_t} \right]
\]

\[\tag{2.38}\]

Retail firms

The relevant part of the optimization problem of the typical retail firm is now given by:

\[
\max_{P_{f,t}} E_t \left[ \sum_{s=0}^{\infty} (\beta \psi)^s \Lambda_{t,t+s} (1/P_{t+s}) [P_{f,t} - P^m_{t+s}] y_{f,t+s} \right]
\]

where \( y_{f,t} = (P_{f,t}/P_t)^{-\epsilon} y_t \) is the demand function. \( y_t \) is the output of the final good producing firms, and \( P_t \) the general price level. The expression for the demand function for the retail firms products will be derived in the next section. Since all the retail firms have access to the same technology, all the firms that are
allowed to reset their prices will choose the same new price \( P^*_t \) for their goods. We remember that the relative price \( m_t \) is equal to \( m_t = \frac{P^*_t}{P_t} \). Differentiation with respect to \( P_{f,t} \) gives the first order condition for the price the retail firms will charge for their products:

\[
\frac{P^*_t}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta \psi)^s \lambda_{t+s} P_{f,t}^{\varepsilon} m_t y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta \psi)^s \lambda_{t+s} P_{f,t}^{\varepsilon-1} P_t^{1-\varepsilon} y_{t+s}}
\]

We define the relative price of the firms that are allowed to reset their prices to be equal to \( \pi^*_t = \frac{P^*_t}{P_t} \), while gross inflation is defined to be equal to \( \pi_t = \frac{P_t}{P_{t-1}} - 1 \). The above first order condition can now be rewritten in the following form:

\[
\pi^*_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\Xi_{1,t}}{\Xi_{2,t}} \tag{2.39}
\]

\[
\Xi_{1,t} = \lambda_t m_t y_t + \beta \psi E_t \pi_{t+1}^{\varepsilon-1} \Xi_{1,t+1} \tag{2.40}
\]

\[
\Xi_{2,t} = \lambda_t y_t + \beta \psi E_t \pi_{t+1}^{\varepsilon-1} \Xi_{2,t+1} \tag{2.41}
\]

The aggregate price level equals:

\[
P_t^{1-\varepsilon} = (1 - \psi)(P_t^*)^{1-\varepsilon} + \psi P_{t-1}^{1-\varepsilon}
\]

The aggregate price level will be given by the following law of motion:

\[
(1 - \psi)(\pi^*_t)^{1-\varepsilon} + \psi \pi_t^{\varepsilon-1} = 1 \tag{2.42}
\]

**Final Good Producers**

The final good firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by \( y_t^{(\varepsilon-1)/\varepsilon} = \int_0^1 y_{f,t}^{(\varepsilon-1)/\varepsilon} \, df \), where \( y_{f,t} \) is the output of the retail firm indexed by \( f \). \( \varepsilon \) is the elasticity of substitution between the intermediate goods purchased from the different retail firms. We assume that the final good firms operate in an environment of perfect competition,
and hence they maximize profits by choosing \( y_{f,t} \) such that \( P_t y_t - \int_0^1 P_{f,t} y_{f,t} df \) is maximized. The final good producer takes \( P_t \) and \( P_{f,t} \) as given. Taking the first order conditions with respect to \( y_{f,t} \) gives the demand function of the final good producers for the retail goods. Substitution of the demand function into the technology constraint gives the relation between the price level of the final good and the price level of the individual retail firms:

\[
\begin{align*}
y_{f,t} &= \left( \frac{P_{f,t}}{P_t} \right)^{-\varepsilon} y_t \\
P_t^{-\varepsilon} &= \int_0^1 P_{f,t}^{-\varepsilon} df
\end{align*}
\]

**Aggregation**

First recall that \( y_{f,t} = y_{i,t} = y_t \left( \frac{P_{f,t}}{P_t} \right)^{-\varepsilon} \), for all \( f \) and \( i \). Hence we can write the factor demands by firm \( i \) as:

\[
h_{i,t} = (1 - \alpha) m_t y_{f,t} / w_t, \quad k_{i,t-1} = \alpha m_t y_{f,t} / \left[ q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t \right]
\]

Aggregation over all firms \( i \) gives us aggregate labor and capital:

\[
h_t = (1 - \alpha) m_t y_t \mathcal{D}_t / w_t, \quad k_{t-1} = \alpha m_t y_t \mathcal{D}_t / \left[ q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t \right]
\]

where \( \mathcal{D}_t = \int_0^1 (P_{f,t}/P_t)^{-\varepsilon} df \) denotes the price dispersion. It is given by the following recursive form:

\[
\mathcal{D}_t = (1 - \psi) (\pi_t^{*})^{-\varepsilon} + \psi \pi_t^{\varepsilon} \mathcal{D}_{t-1}
\] (2.43)

Now we calculate the aggregate capital-labor ratio, and see that it is equal to the individual capital-labor ratio:

\[
k_{t-1}/h_t = \alpha (1 - \alpha)^{-1} w_t / \left[ q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t \right] = k_{i,t-1}/h_{i,t}
\] (2.44)
CHAPTER 2. FISCAL MULTIPLIER

Calculate aggregate supply by aggregating \( y_{i,t} = a_t(\xi_t k_{i,t-1})^{\alpha h_{i,t}^{1-\alpha}} \):

\[
\int_0^1 a_t(\xi_t k_{i,t-1})^{\alpha h_{i,t}^{1-\alpha}} di = a_t \xi_t^{\alpha} \int_0^1 h_{i,t}^{1-\alpha} di = a_t(\xi_t k_{i,t-1})^{\alpha h_{i,t}^{1-\alpha}}
\]

while aggregation over \( y_{i,t} \) gives:

\[
\int_0^1 y_{i,t} df = y_t \int_0^1 \left( \frac{P_{f,t}}{P_t} \right)^{-\epsilon} df = y_t D_t
\]

Hence we get the following relation for aggregate supply \( y_t \):

\[
y_t D_t = a_t(\xi_t k_{t-1})^{\alpha h_{t}^{1-\alpha}} \tag{2.45}
\]

2.A.4 Calibration

The numerical values of the parameters can be found in Table 2.6.

2.A.5 Approximation of the default function

We can also write the debt level structure (2.9) in the following way:

\[
b_t = \min (\tilde{b}_t, b_t^{max}) = b_t^{max} - \max (b_t^{max} - \tilde{b}_t, 0) \tag{2.46}
\]

We can interpret the second term of the new debt level as the payoff of a put option at maturity with underlying process \( \tilde{b}_t \) and strike price \( b_t^{max} \). The formula for \( b_t \), however, does not have a defined derivative at \( \tilde{b}_t = b_t^{max} \). Therefore we apply an approximation for the payoff structure of the put option, and use the option pricing formula, which gives the price of the put option when time to maturity is equal to \( T \), compounded risk-free interest rate \( r \), and volatility of the underlying process \( \sigma \). This is an approximation to the actual mapping from \( \tilde{b}_t \) to \( b_t \), and has in this sense no economic interpretation in our model. The red line in Figure 2.14 is the approximation to the actual mapping of \( \tilde{b}_t \) to \( b_t \). We then
## 2.A. DERIVATIONS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.990</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.847</td>
<td>Degree of habit formation</td>
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<tr>
<td>$\Psi$</td>
<td>3.342</td>
<td>Relative utility weight of labor</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.100</td>
<td>Inverse Frisch elasticity of labor supply</td>
</tr>
<tr>
<td><strong>Financial Intermediaries</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_k$</td>
<td>0.5401</td>
<td>Fraction of private loans that can be diverted</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.001</td>
<td>Proportional transfer to entering bankers</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.9583</td>
<td>Survival rate of the bankers</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.0054</td>
<td>Steady state credit spread $E[r^k - r^d]$</td>
</tr>
<tr>
<td><strong>Intermediate good firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>8.577</td>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>$\psi$</td>
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<td>Calvo probability of keeping prices fixed</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>Effective capital share</td>
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<td><strong>Capital good firms</strong></td>
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<tr>
<td>$\gamma$</td>
<td>1.728</td>
<td>Investment adjustment cost parameter</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0374</td>
<td>Steady state depreciation rate</td>
</tr>
<tr>
<td><strong>Autoregressive components</strong></td>
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</tr>
<tr>
<td>$\rho_z$</td>
<td>0.95</td>
<td>Autoregressive component of productivity</td>
</tr>
<tr>
<td>$\rho_\xi$</td>
<td>0.66</td>
<td>Autoregressive component of capital quality</td>
</tr>
<tr>
<td>$\rho_g$</td>
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<td>Persistence in government spending</td>
</tr>
<tr>
<td>$\rho_r$</td>
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<td>Interest rate smoothing parameter</td>
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<tr>
<td><strong>Policy</strong></td>
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</tr>
<tr>
<td>$r_c$</td>
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<td>Real payment to government bondholder</td>
</tr>
<tr>
<td>$\rho$</td>
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<td>Parameter government debt duration (5 yrs)</td>
</tr>
<tr>
<td>$\kappa_b$</td>
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<td>Tax feedback parameter from government debt</td>
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<tr>
<td>$\kappa_\pi$</td>
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<td>Inflation feedback on nominal interest rate</td>
</tr>
<tr>
<td>$\kappa_y$</td>
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<td>Output feedback on nominal interest rate</td>
</tr>
<tr>
<td><strong>Default parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$</td>
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<td>Steady state share of default indicator</td>
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<tr>
<td><strong>Shocks</strong></td>
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<td></td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.010</td>
<td>Standard deviation productivity shock</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>0.050</td>
<td>Standard deviation capital quality shock</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.050</td>
<td>Standard deviation unannounced government spending shock</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.050</td>
<td>Standard deviation pre-announced government spending shock</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.0025</td>
<td>Standard deviation interest rate surprise shock</td>
</tr>
<tr>
<td><strong>Option parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>-0.1148</td>
<td>Compounded risk-free interest rate</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.4546</td>
<td>Standard deviation underlying process</td>
</tr>
<tr>
<td>$T$</td>
<td>0.0342</td>
<td>Time to maturity</td>
</tr>
</tbody>
</table>

Table 2.6. Table with model parameters.
get the following approximation for $b_t$, with $\Phi(\cdot)$ denoting the standard normal CDF, which is indeed continuous:

$$b_t = b_{t}^{\max} - put_t$$  \hspace{1cm} (2.47)

$$put_t = Xe^{-rT} \Phi(-d_{2,t}) - S_t \Phi(-d_{1,t})$$  \hspace{1cm} (2.48)

$$d_{1,t} = \frac{\log(S/X) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma \sqrt{T}}$$  \hspace{1cm} (2.49)

$$d_{2,t} = \frac{\log(S/X) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma \sqrt{T}}$$  \hspace{1cm} (2.50)

$$X = b_{t}^{\max}$$  \hspace{1cm} (2.51)

$$S_t = \tilde{b}_t$$  \hspace{1cm} (2.52)

Figure 2.14. Plot showing the mapping from the no default level of debt $\tilde{b}_t$ to the actual debt level $b_t$. The blue line is the actual mapping, while the red line is an approximation to it, where option pricing formulas have been used.
2. A. DERIVATIONS

2.A.6 Calibration strategies

In this section we will write down the 2 calibration strategies regarding the sovereign debt in the current paper, since other parts of the model are straightforward to calibrate. We have the following 2 equations from the financial intermediaries’ problem, from which we can derive the steady state return on bonds ex-post a possible default.

\[
\nu^b_t = \frac{\lambda_b}{\lambda_k} \nu^k_t \\
\nu^k_t = E_t \left[ \Omega_{t+1} \left( r^{k}_{t+1} - r^d_{t+1} \right) \right] \\
\nu^b_t = E_t \left[ \Omega_{t+1} \left( r^{b*}_{t+1} - r^d_{t+1} \right) \right] \\
\Rightarrow E_t \left[ \Omega_{t+1} \left( r^{k}_{t+1} - r^{b*}_{t+1} \right) \right] = 0
\]

From these equations it is clear that \( \bar{r}^b = \frac{\lambda_b}{\lambda_k} \bar{r}_k \). Now we have the following equation for the maximum level of debt:

\[
b_{t}^{\text{max}} = \bar{b} + \frac{E_t \left[ \tau_0^{\text{max}} \right] - \bar{\tau}}{\kappa_b} \tag{2.53}
\]

The government budget constraint in case of no default by the government is equal to:

\[
q^b_t \bar{b}_t + \tau_t + \bar{n}^g_t = g_t + n^g_t + \left( r_c + \rho q^b_t \right) b_{t-1} \\
= g_t + n^g_t + \left( r_c + \rho q^b_t \right) q^b_{t-1} b_{t-1} \Rightarrow \\
q^b_t \bar{b}_t + \tau_t + \bar{n}^g_t = g_t + n^g_t + \left( 1 + r^b_t \right) q^b_{t-1} b_{t-1} \tag{2.54}
\]

The mapping from the number of no default bonds to the actual number of bonds is given by:

\[
b_t = b_t^{\text{max}} - \max \left( b_t^{\text{max}} - \bar{b}_t, 0 \right) \approx b_t^{\text{max}} - \text{put} \left( b_t^{\text{max}}, \bar{b}_t \right) \tag{2.55}
\]
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The actual number of government bonds is given by:

\[ q_t^b b_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + (1 - \Delta_t) \left( r_c + \rho q_t^b \right) b_{t-1} \]

\[ = g_t + n_t^g + (1 - \Delta_t) \frac{(r_c + \rho q_t^b)}{q_{t-1}^b} q_{t-1}^b b_{t-1} \]

\[ = g_t + n_t^g + (1 - \Delta_t) \left( 1 + r_t^b \right) q_{t-1}^b b_{t-1} \implies \]

\[ q_t^b b_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + \left( 1 + r_t^{bs} \right) q_{t-1}^b b_{t-1} \tag{2.56} \]

together with the ex-post default return on bonds:

\[ 1 + r_t^{bs} = (1 - \Delta_t) \left( 1 + r_t^b \right) \tag{2.57} \]

and the return on bonds before default:

\[ 1 + r_t^b = \frac{(r_c + \rho q_t^b)}{q_{t-1}^b} \tag{2.58} \]

Throughout the paper we assume that the financial intermediaries do not receive any support from the government in the steady state, nor are they paying back support in the steady state, i.e. \( \bar{n}_g = \bar{n}_g = 0 \). Finally, we have the option pricing formulas:

\[ put_t = b_t^{max} e^{-rT} \Phi(-d_{2,t}) - \tilde{b}_t \Phi(-d_{1,t}) \tag{2.59} \]

\[ d_{1,t} = \frac{\log \left( \tilde{b}_t / b_{t}^{max} \right) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \tag{2.60} \]

\[ d_{2,t} = \frac{\log \left( \tilde{b}_t / b_{t}^{max} \right) + \left( r - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \tag{2.61} \]

**Calibration strategy 1**

The first strategy targets \( \bar{q}_b \bar{b}, \bar{q}_b \bar{b}_{max} \) and \( \bar{\Delta} \), for which we take 53.2\%, respectively 65\% of annual GDP and \( \bar{\Delta} = 0.005 \). Since we know \( \bar{q}_b \bar{b} \), the steady state
return on bonds after default $\bar{r}_{bs}$ and $\bar{g}$, we can find the steady state level of taxes from (2.56):

$$\bar{q}_b \bar{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_{bs}) \bar{q}_b \bar{b} \implies \bar{\tau} = \bar{g} + \bar{r}_{bs} \bar{q}_b \bar{b}$$

Since we know the steady state default fraction $\bar{\Delta}$ and the ex-post return on bonds, we can calculate $\bar{r}_b$ through (2.57):

$$1 + \bar{r}_{bs} = (1 - \bar{\Delta}) (1 + \bar{r}_b) \implies \bar{r}_b = \frac{1 + \bar{r}_{bs}}{1 - \bar{\Delta}} - 1$$

Since we know $r_c$, we can find the steady state bond price through (2.58):

$$1 + \bar{r}_b = \frac{r_c + \rho \bar{q}_b}{\bar{q}_b} \implies \bar{q}_b (1 + \bar{r}_b) = r_c + \rho \bar{q}_b \implies \bar{q}_b = \frac{r_c}{1 + \bar{r}_b - \rho}$$

Now that the steady state bond price is known, we can find the steady state number of bonds and the maximum number of bonds $\bar{b}$ and $\bar{b}_{max}$. Since we know the return on bonds $\bar{r}_b$, we can find the number of bonds if the government does not default:

$$\bar{q}_b \bar{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_b) \bar{q}_b \bar{b}$$

Now that we have found steady state number of bonds $\bar{b}$, maximum number of bonds in the steady state $\bar{b}_{max}$, and the steady state number of bonds in case the government does not default $\bar{b}$. With these 3 numbers, and the requirement that the derivative of the put option $-\Phi (-d_{1,t})$ is equal to $-0.99$, we can find the variables $r, \sigma$ and $T$ from the option pricing formulas.
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Calibration strategy 2

The second strategy targets \( \bar{q}_b \bar{b} \) and \( \bar{q}_b \bar{b}_{\text{max}} \), and takes the option pricing parameters \( r, \sigma \) and \( T \) as given. We calibrate \( \bar{q}_b \bar{b} \) to be equal to 53.2% of annual GDP, while \( \bar{q}_b \bar{b}_{\text{max}} \) is equal to 60% of annual GDP. Since \( \bar{g} \) is also known, we can find the steady state level of taxes from (2.56):

\[
\bar{q}_b \bar{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_b) \bar{q}_b \bar{b} \implies \\
\bar{\tau} = \bar{g} + \bar{r}_b \bar{q}_b \bar{b}
\]

Since we know \( \bar{q}_b \bar{b} \) and \( \bar{q}_b \bar{b}_{\text{max}} \), we can divide the two to find the ratio \( \bar{b}_{\text{max}}/\bar{b} \).

Now we look at the formula for \( d_{1,t} \):

\[
d_{1,t} = \log \left( \frac{\bar{b}_t}{b_{t,\text{max}}} \right) + \left( r + \frac{\sigma^2}{2} \right) T \\
= \frac{\log \left( \frac{\bar{b}_t}{b_{t,\text{max}}} \right) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} = f \left( \frac{\bar{b}_t}{b_{t,\text{max}}} \right)
\]

Division by \( b_t \) gives the following expression:

\[
1 = \frac{b_{t,\text{max}}}{b_t} \frac{b_t}{b_{t,\text{max}}} e^{-rT} \Phi(-d_{2,t}) + \bar{b}_t \Phi(-d_{1,t})
\]

Now we look at the formula for \( d_{1,t} \):

\[
d_{1,t} = \frac{\log \left( \frac{b_t}{b_{t,\text{max}}} \right) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} = f \left( \frac{b_t}{b_{t,\text{max}}} \right)
\]
similarly we find for $d_{2,t}$:

$$d_{2,t} = \frac{\log \left( \frac{\tilde{b}_t/b_{t}^{max}}{\tilde{b}_t/b_t} \right) + \left( r - \sigma^2 \right) T}{\sigma \sqrt{T}} = \frac{\log \left( \frac{\tilde{b}_t/b_{t}^{max}}{b_t/b_t} \right) + \left( r - \sigma^2 \right) T}{\sigma \sqrt{T}} = g \left( \frac{\tilde{b}_t}{b_t}, \frac{b_{t}^{max}}{b_t} \right)$$

(2.64)

Hence we see that equations (2.62), (2.63) and (2.64) only depend on the ratios $b_{t}^{max}/b_t$ and $\tilde{b}_t/b_t$. We know the first ratio in steady state, so we can solve for the steady state ratio $\bar{\tilde{b}}/\bar{b}$. We also see that regarding the default function, it does not matter whether we calibrate on the number of bonds $b_t$ or the value of government liabilities $q_t b_t$, since the bond *ratios* are the variables that matter. Now we can find $\bar{\tilde{b}}/\bar{b}$, and hence we find $\bar{q}_b\bar{\tilde{b}} = \bar{q}_b\tilde{b} \left( \bar{\tilde{b}}/\bar{b} \right)$. Since we know $\bar{q}_b\tilde{b}$ we can find $\bar{r}_b$ from (2.54):

$$\bar{q}_b\tilde{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_b) \bar{q}_b\tilde{b} \implies$$

$$\bar{r}_b = \frac{\bar{q}_b\tilde{b} + \bar{\tau} - \bar{g}}{\bar{q}_b\tilde{b}} - 1$$

Now we can find the steady state bond price from (2.58):

$$1 + \bar{r}_b = \frac{r_c + \rho \bar{q}_b}{\bar{q}_b} \implies \bar{q}_b (1 + \bar{r}_b) = r_c + \rho \bar{q}_b$$

$$\bar{q}_b = \frac{r_c}{1 + \bar{r}_b - \rho}$$

after which we know $\bar{b}$, $\tilde{b}_{max}$ and $\tilde{\bar{b}}$. From (2.57) we can find the steady state default probability:

$$1 + \bar{r}_{b*} = \left( 1 - \bar{\Delta} \right) (1 + \bar{r}_b) \implies$$

$$\bar{\Delta} = 1 - \frac{1 + \bar{r}_{b*}}{1 + \bar{r}_b}$$
Chapter 3

The Macroeconomic Impact of Changing Capital Requirements

3.1 Introduction

The financial crisis of 2007-2009 almost led to the meltdown of the modern financial system. In response, governments around the world adopted the Basel III framework to prevent a new systemic crisis. A key element of this framework is the gradual implementation of higher capital requirements for commercial banks. A debate erupted not only on the macroeconomic consequences of higher capital ratios, but also on the structure of the requirements, which centered around the question whether capital requirements should be applied to risk-weighted assets or unweighted total assets. In this paper, we will focus on the macroeconomic impact of both the transition to higher capital ratios and the structure of capital requirements.

Changing the level and the structure of capital requirements on commercial banks can potentially affect aggregate investment, as it affects credit provision by commercial banks to the real economy, which accounts for 80% of debt-financing to non-financial corporations in the Eurozone (European Central Bank, 2015). In addition, commercial banks in the U.S. and Europe were undercapi-

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1This chapter is based on joint work with Sweder van Wijnbergen.
talized at the time of the adoption of the Basel III capital requirements (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). Higher (anticipated) capital requirements in such an environment can potentially force commercial banks to shrink the balance sheet, which has a negative effect on credit provision to the real economy when no new equity is raised.

As mentioned above, the proposed changes in capital requirements regime ignited two debates. The first debate concerned the effect of the transition to higher capital requirements on credit provision and economic growth, a debate in which widely differing views could be documented. Bankers, on the one hand, vehemently opposed the introduction of higher capital requirements, arguing that they would force commercial banks to reduce credit provision to the real economy, as the funding costs of credit increase because the required return on equity is higher than that on debt. Admati, DeMarzo, Hellwig, and Pfleiderer (2013), on the other hand, claim that higher capital requirements would not affect credit conditions adversely, as higher capital requirements reduce the risk-premium of equity, leading to a lower cost of equity.

Figure 3.1 suggests that the ways in which banks responded to the implementation of higher capital requirements markedly differed between the U.S. and Europe. While U.S. banks were forced by the government to meet higher capital requirements by raising capital levels under the Supervisory Capital Assessment Program (SCAP), European banks achieved higher capital ratios through a reduction in (risk-weighted) assets, documented by Hoshi and Kashyap (2014). This suggest that credit provision to the European economy was reduced in response to higher capital requirements.

At the same time, a second debate erupted on the macroeconomic effects of the structure of the capital requirements. Even though the European sovereign debt crisis clearly showed that sovereign debt is not riskless, the regulatory framework assigns a zero risk-weight to it, i.e. no bank capital needs to be held against holdings of sovereign debt. Private credit, in contrast, carries a much
3.1. INTRODUCTION

Sources of changes in bank capital ratios, end-2009 to end-2012

Normalised to percentage points of end-2009 risk-weighted assets

Figure 3.1. The graph shows the change in the ratios of common equity to risk-weighted assets at the (fiscal-year) end of 2009 and 2012, respectively, in percentage points. The overall change is shown by the red diamonds. All figures are weighted averages, using end-2012 assets as weights. Source: Cohen and Scatigna (2014).

Higher risk-weight. Hoshi and Kashyap (2014) document that European banks reduced risk-weighted assets, while unweighted assets did not decrease, which suggests a shift from high risk-weighted assets to low risk-weighted assets such as government bonds. This conclusion is supported by Figure 3.2, in which we see the aggregate risk exposure to different asset classes for 16 European global systemically important banks (G-SIBs).\(^2\) We see that the exposure to sovereign debt increased by EUR 550 billion between end-of-2010 and end-of-2012, while the risk exposure to corporates decreased by EUR 440 billion over the same period.

This second debate gained additional relevance when the European sovereign debt crisis erupted in May 2010. The interconnection between weak banks and weak sovereigns substantially complicated the resolution of the economic

\(^2\) Aggregate risk exposure refers to the part of the asset that exposes risk to the financial intermediary. Fitch Ratings (2013) reports for example, that the 16 G-SIBs represent end-2012 EUR 21 trillion in assets, which translates into to an aggregate risk exposure (EAD) of EUR 13.5 trillion.
Risk Exposure (EAD): Modest Reduction, Major Reallocation

![Graph showing change in EAD since End-2010 (EUR Bil.)](image)

Figure 3.2. Bank pillar 3 disclosures from a sample of 16 European global systemically important banks (G-SIBs). EAD denotes Aggregate Exposure at Default, a measure for aggregate risk exposure. Source: Fitch Ratings (2013).

troubles plaguing Europe’s periphery countries. Proponents of applying capital requirements against unweighted assets argued that higher risk-weights on sovereign debt would reduce the attractiveness for periphery banks to invest in periphery sovereign debt, which would lead to lower holdings of periphery sovereign debt and mitigate the spillovers from sovereign debt trouble to the banking sector. However, governments prefer the low risk-weight on sovereign debt under the current regime of risk-weighted capital requirements, as it reduces their funding costs since banks do not need capital against government bonds.

In this paper we look at the macroeconomic impact through the credit provision channel of a change in the level and structure of capital requirements in an environment where commercial banks are undercapitalized after a financial crisis. Do higher capital requirements in such an environment indeed have a contractionary impact on lending to the real economy, as bankers have claimed? And if so, are there policies that can mitigate this negative impact? A second question has to do with the regulatory treatment of different asset classes. Can
lower risk-weights on government bonds explain why commercial banks shift from private credit to government bonds in response to higher capital requirements? Can we mitigate this shift by changing risk-weights of certain asset classes? These are the questions we will focus on in the current paper.

To answer these questions, we build a DSGE model with commercial banks that finance private loans and government bonds. Commercial banks are balance-sheet-constrained through the introduction of capital requirements, which are modeled by forcing commercial banks to finance a fixed fraction of risk-weighted assets through net worth/equity. Private loans and government bonds have (potentially) different risk-weights. We abstain from an endogenous equity choice, a feature of several papers in this literature (Kiley and Sim, 2014; Van den Heuvel, 2008; Angeloni and Faia, 2013). In reality, commercial banks were very reluctant to issue new equity after the financial crisis of 2007-2009, as it would dilute the claims of existing shareholders and basically cover existing losses instead of financing new projects with positive Net Present Value. Not modeling outside equity therefore seems to be a realistic choice.

The innovation of our paper is that we introduce a portfolio choice between private loans and government bonds for commercial banks in combination with capital requirements based on risk-weighted assets, as is standard practice in the current regulatory framework. Our framework encompasses both the Capital-Adequacy-Ratio-regime (CAR-regime) in which different assets have different risk-weights, as well as the (Leverage-Ratio-regime) LR-regime when we set risk-weights on private loans and government bonds equal. This setup allows us to investigate the transition to a regime with higher capital ratios and, in addition, to contribute to the ongoing debate between CAR-proponents and LR-proponents by comparing the macroeconomic impact of the two regimes.

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3Throughout the paper, we will use ‘commercial banks’ and ‘financial intermediaries’ interchangeably to denote the same group of economic agents. This group of agents can be thought of to capture all kinds of credit institutions: commercial banks, savings banks, postbanks, specialized credit institutions, etc.
We find two effects in a financial crisis. First is the balance-sheet-effect. Since the amount of net worth/equity directly determines the size of the balance sheet, a fall in the level of net worth/equity forces commercial banks to shrink the balance sheet, leading to a contraction in credit to the real economy and a fall in investment and output.

Second is the asset-substitution-effect when the risk-weight on private credit is larger than that on government bonds. When commercial banks are forced to shrink total risk-weighted assets, they can shift from high risk-weight private loans to zero risk-weight government bonds, which allows commercial banks to deleverage in terms of risk-weighted assets without reducing unweighted assets by as much. The asset-substitution-effect therefore induces a shift from private credit to government bonds with negative consequences for investment and output. This effect is only present under the CAR-regime when risk-weights on private loans and government bonds are different.

Our paper connects to several strands of the literature. Many (European) commercial banks have been undercapitalized since the financial crisis of 2007-2009 (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). A key element of our analysis therefore involves the inclusion of balance sheet constrained financial intermediaries, see Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Gertler and Karadi (2013). An extension that creates a portfolio choice between private loans and government bonds is found in Gertler and Karadi (2013), Kirchner and van Wijnbergen (2012) and Van der Kwaak and Van Wijnbergen (2014). While these papers feature an endogenous balance sheet constraint based on an information asymmetry between bankers and depositors, we deviate by exogenously imposing how many (risk-weighted) assets can be financed by one unit of net worth/equity, which is in line with the current regulatory regime.

Our paper is related to the paper by Admati, DeMarzo, Hellwig, and Pfleiderer (2013), who argue that the claim that higher capital requirements adversely
affect credit market conditions is false. They claim that higher capital requirements will reduce the required return on equity because the equity risk-premium decreases, as banks become more safe. Average funding costs for equity decrease, thereby (partially) offsetting the higher average funding costs from a larger share of the balance sheet being financed by equity. In our model, credit intermediation is affected because commercial banks in our model cannot raise outside equity. When capital requirements increase, this automatically implies that commercial banks have to shrink (risk-weighted) assets, leading to a reduction in lending to the real economy. As argued before, this inability to raise outside equity is in line with the observation that commercial banks were reluctant to raise new equity unless forced by the government to do so (Hoshi and Kashyap, 2014), as was done in the U.S. under the SCAP.

Our paper connects to the literature on capital requirements for financial intermediaries within DSGE models. Most papers contain financial intermediaries only financing one asset. Higher capital requirements only have a balance-sheet-effect in that case, but not an asset-substitution-effect, for which intermediaries need a portfolio choice between private loans and sovereign debt. The effects of higher requirements in those papers are mixed: some papers find the balance sheet effect to reduce private credit provision when capital requirements are increased (Kiley and Sim, 2014; Van den Heuvel, 2008; Karmakar, 2013). Others find an expansionary effect from higher capital requirements (Begenau, 2015): since investors value safe and liquid assets (bank debt is included in the utility function), funding costs decrease as bank debt becomes more scarce, making lending more profitable which leads to an expansion of credit. While Begenau (2015) contains both a portfolio choice for banks between private credit and one period government debt as well as different risk-weights for private credit and government bonds, Begenau (2015) does not disentangle the role of the balance sheet effect and the asset substitution effect.
A last literature that we connect to is the literature on government bond accumulation by banks. Drechsler, Drechsel, Marques-Ilbanez, and Schnabl (2014) find that weakly capitalized banks actively invested in distressed sovereign debt after the start of the sovereign debt crisis in 2010. Crosignani (2014) finds within a general equilibrium setup that commercial banks shift into domestic bonds when they are undercapitalized. Domestic sovereign debt has the highest payoff in the good state of the world, which is the only state in the world which banks care about. Banks are protected by limited liability, and therefore do not care about the bad state of the world in which they are bankrupt. Becker and Ivashina (2014) document empirical evidence for a crowding out effect of private loans by government bonds, as well as financial repression by governments. Government bond accumulation by banks in our model, however, is caused by the regulatory structure, as reported by Hoshi and Kashyap (2014).

3.2 Model Description

We consider a standard New-Keynesian model, which includes commercial banks that are balance-sheet-constrained, as in Gertler and Karadi (2011), and have a portfolio choice between private loans and government bonds, as in Gertler and Karadi (2013). Assets are financed by net worth and deposits. In addition to commercial banks, private loans and government bonds are also intermediated through capital markets in which households participate. Households face transaction costs when purchasing private loans and government bonds, a feature which captures limited participation of households in asset markets.

In addition to commercial banks, the economy contains households, a production sector, and a government consisting of a central bank that follows a Taylor-rule and a fiscal authority. Each household consists of workers and bankers. Workers supply labor to intermediate goods producers, while bankers run the commercial banks. Intermediate goods producers borrow from commer-
cial banks to purchase physical capital from capital goods producers, who face convex adjustment costs. Labor, which is hired after the shocks realize in the beginning of the next period, and capital purchased in the previous period are used to produce the intermediate good. Intermediate goods producers sell their goods to retail goods producers and the used capital to the capital goods producers. Retail goods producers face monopolistic competition and price-stickiness, as in Calvo (1983) and Yun (1996). Final goods producers operate in a perfectly competitive market and purchase from retail goods producers while taking prices as given. The final good is sold to households for consumption, to capital goods producers as investment, and to the government. The government honors outstanding obligations, (potentially) provides financial sector support, and finances these expenditures by raising lump sum taxes and issuing (long-term) debt, in a way similar to Woodford (1998, 2001). Fiscal policy is determined via (exogenous) fiscal rules.

### 3.2.1 Financial Intermediaries

Financial intermediaries provide financing to the intermediate goods producers, and buy government debt. The balance sheet of a typical financial intermediary \( j \) is given by:

\[
q^k_t s^k_{j,t} + q^b_t s^b_{j,t} = n_{j,t} + d_{j,t} = p_{j,t}
\]  

(3.1)

where \( p_{j,t} \) denotes the total amount of assets on the balance sheet of the financial intermediaries, while \( q^k_t \) and \( q^b_t \) denote the price of capital and government bonds, respectively. The net worth next period can be found by the difference between the return on assets and the return that is paid on the deposits. In addition, net worth can be augmented by possible government support \( n^g_{j,t} \) or repayment of earlier government support \( \tilde{n}^g_{j,t} \), which are both linear in previous period net
worth $n_{j,t-1}$ by a factor $\tau^n_t$ and $\tilde{\tau}^n_t$, respectively, to be explained in section 3.2.2. New net worth $n_{j,t+1}$ is then given by:

$$n_{j,t+1} = (1 + r^k_{t+1}) q^k_t s^k_{j,t} + (1 + r^b_{t+1}) q^b_t s^b_{j,t} - (1 + r^d_{t+1}) d_{j,t} + n^g_{j,t+1} - \tilde{n}^g_{j,t+1}$$

where $r^i_{t+1}$ is the net return in period $t+1$ on asset $i = \{k, b\}$ that was purchased in period $t$. Both (expected) returns will be determined endogenously in equilibrium. We assume that there is a probability $\theta$ that the bankers running the financial intermediary can continue operating the intermediary next period. We introduce this finite horizon to prevent bankers from accumulating net worth to the point where they can circumvent the capital requirements, to be introduced shortly, by financing all assets by net worth.

Each period, there is a probability of $1 - \theta$ of having to shut down and terminate the financial intermediary. In that case, net worth is paid out to the households, the ultimate owners of the banks. The probability $\theta$ is exogeneously given, and i.i.d. both across banks and across time. Financial intermediaries are interested in maximizing the (expected) continuation value. Since financial intermediaries are owned by the households, they use the household’s stochastic discount factor to discount future cash flows. The continuation value of of financial intermediaries is therefore given by:

$$V_{j,t} = \max_{\{s^k_{j,t}, s^b_{j,t}, d_{j,t}\}} E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \theta) n_{j,t+1} + \theta V_{j,t+1} \right\} \right]$$

(3.3)

where $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$. Bankers, however, cannot perfectly elastic increase the balance sheet size because they face (legal) capital requirements. These capital requirements are set by a regulator/government, as will be explained below. Since bankers want to maximize the net worth that is paid out to the household upon a forced exit, they attract deposits to the point where the capital require-
3.2. MODEL DESCRIPTION

ments become binding. Capital requirements are modeled by assuming that a fraction $\bar{k}_t$ of risk-weighted assets must be financed by net worth. A risk-weight is basically a correction factor that should reflect the risk of a particular asset class. Hence total assets will not necessarily be the same as risk-weighted assets. The risk weight for asset class $s_{j,t}^a$ is denoted by $\omega_a$. Total risk-weighted assets $p_{j,t}^{rw}$ are now given by:

$$p_{j,t}^{rw} = \sum_a \omega_a q_t^a s_{j,t}^a,$$

(3.4)

where $q_t^a$ is the price of asset $s_{j,t}^a$. The relationship between net worth at the beginning of period $t$ and risk-weighted assets is given by:

$$n_{j,t} \geq \bar{k}_t p_{j,t}^{rw}.$$  

(3.5)

Whenever we talk about an increase in capital requirements, we refer to an increase in $\bar{k}_t$. We will assume that the capital requirement is binding in equilibrium for two reasons. First, a commercial bank has a finite life-time, hence they will not be able to accumulate enough net worth to circumvent the capital requirement. Second, since we study the effect of raising capital requirements after a financial crisis has occurred, it is reasonable to assume binding capital requirements.

Commercial banks in our model do not have the possibility to raise additional equity when a financial crisis depletes their net worth. In addition, the capital requirements are exogenously set at a fixed percentage of risk-weighted assets. The combination of these two features gives rise to a firesales externality: a reduction in commercial banks’ net worth leads to a tightening of the balance-sheet constraint. Commercial banks must shrink total assets by selling private loans and government bonds, which leads to a fall in asset prices, and a further deterioration of net worth. As a result, a second round of firesales starts, leading to a further fall in asset prices.
However, as the asset supply does not shrink at the same rate, demand and supply diverge, and an equilibrium cannot be attained. We therefore introduce capital markets in which both commercial banks and households can trade private loans and government bonds, which we will specify in section 3.2.3. Commercial banks are therefore able to sell assets to households when forced to shrink the banks’ balance sheet. The resulting fall in asset prices leads to an increased demand by households for assets, which counterbalances the downward pressure on asset prices from commercial banks selling assets.

**Bank optimization problem and F.O.C.’s**

Financial intermediaries try to maximize the continuation value (3.3) by choosing the size and composition of the balance sheet, subject to the capital requirements. The optimization problem of the intermediary can be formulated in the following way:

$$\max_{\{s_{jt}, s_{jt}, d_{jt}\}} V_{jt}, \quad \text{s.t.} \quad \eta_{jt} \geq k_{jt} \sum_a \omega_a q_a^d s_a^d = k_{jt} \left( \omega_k q_k^k s_k^k + \omega_b q_b^b s_b^b \right),$$

where \( a = \{k, b\} \). Solving the optimization problem is performed in appendix 3.A.1, and results in the following first order conditions:

$$E_t \left[ \Omega_{t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right] = \left( \frac{\omega_b}{\omega_k} \right) E_t \left[ \Omega_{t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right], \quad (3.6)$$

$$\mu_t k_t \omega_k = E_t \left[ \Omega_{t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right], \quad (3.7)$$

$$\eta_t = E_t \left[ \Omega_{t+1} \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \right) \right], \quad (3.8)$$

$$0 = \mu_t \left\{ n_{jt} - k_t \left[ \omega_k q_k^k s_{jt}^k + \omega_b q_b^b s_{jt}^b \right] \right\}, \quad (3.9)$$

$$\Omega_{t+1} = \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta (\mu_{t+1} + \eta_{t+1}) \right\}, \quad (3.10)$$

where \( \mu_t \) denotes the tightness of the bank balance sheet constraint (3.9), \( \eta_t \) the shadow value of an additional unit of net worth. \( \tau_{t}^n \) and \( \tilde{\tau}_{t}^n \), respectively relate financial sector support and repayment of earlier administered financial sector...
3.2. MODEL DESCRIPTION

support, respectively, to previous period bank net worth. We will discuss the support policy \( \{ \tau^n_t, \bar{\tau}^n_t \} \) in section 3.2.2.

Equation (3.6) determines the portfolio choice of the commercial bank between private loans and government bonds. The left hand side denotes the marginal benefit to the financial intermediary from acquiring an additional unit of government bonds, valued by the intermediaries’ stochastic discount factor. The right hand side reflects the marginal cost of giving up an additional unit of private loans, measured by the credit spread between private loans and the deposit rate, and corrected by the risk-weights \( \omega_b/\omega_k \) to reflect the fact that capital requirements are tighter for private loans, as more net worth needs to be held for one euro of private loans.

Equation (3.7) is the first order condition for private loans. The presence of a binding capital requirement, represented by the shadow value \( \mu_t \) in equation (3.9), limits the ability of commercial banks to arbitrage away the difference between the expected rate of return on private loans and deposits, since they cannot expand the balance sheet. A tighter capital requirement constraint, due to a fall in net worth, increases \( \mu_t \), and forces commercial banks to shift out of private loans so that the expected rate of return on capital can go up.

We can see an asset-substitution-effect from equations (3.6) and (3.7). Imagine a tightening of the capital requirements constraint, reflected by an increase in \( \mu_t \). A higher \( \mu_t \) increases the expected rate of return on private loans. But because the risk-weight on government bonds is usually lower than the risk-weight on private loans, the expected rate of return on government bonds increases by less. The required reduction in government bond holdings is therefore smaller than that for private loans, which leads to a relative portfolio shift from private loans to government bonds.

Equation (3.8) denotes the shadow value of an additional unit of net worth, which is equal to the gross return on deposits, augmented by the financial sector support per unit of net worth \( \tau^n_{t+1} \) and the repayment per unit of net worth \( \bar{\tau}^n_{t+1} \).
Equation (3.9) denotes the bank balance sheet constraint, which determines the size of the balance sheet in terms of current net worth $n_{j,t}$. A higher $\mu_t$ implies the balance sheet constraint is more binding, and the value of an additional unit of net worth is larger to the financial intermediary.

We can see from (3.9) that an increase in $\tilde{k}_t$ needs to be accompanied with an increase in $n_{j,t}$, or a decrease in asset holdings, i.e. the balance sheet constraint becomes more binding everything else equal. A second observation regarding an increase in $\tilde{k}_t$ can be inferred from equation (3.7): higher capital requirements will induce a shift from private loans, which will drive up the expected rate of return on capital.

**Aggregation financial sector**

The law of motion for aggregate net worth consists of the net worth of the bankers that continue to operate, together with the aggregate net worth given to new bankers, which is equal to a fraction $\chi$ of previous period net worth $n_{t-1}$. Together with net government support $n^g_t - \tilde{n}^g_t$, we obtain the following law of motion:

$$n_t = \theta \left[ \left( r^k_t - r^d_t \right) q^k_{t-1} s^k_t + \left( r^b_t - r^d_t \right) q^b_{t-1} s^b_t + \left( 1 + r^d_t \right) n_{t-1} \right] + \chi n_{t-1} + n^g_t - \tilde{n}^g_t \quad (3.11)$$

We define the aggregate leverage ratio by total (unweighted) aggregate assets $p_t$ over aggregate net worth $n_t$:

$$\phi_t = \frac{p_t}{n_t} = \frac{1}{\tilde{k}_t} \left( \frac{q^k_{t} s^k_t + q^b_{t} s^b_t}{\omega^k q^k_{t} s^k_t + \omega^b q^b_{t} s^b_t} \right) = \frac{1}{(\omega^k \tilde{k}_t)} \left( \frac{1 + \frac{q^b_{t} s^b_t}{q^k_{t} s^k_t}}{1 + \left( \frac{\omega^b}{\omega^k} \frac{q^b_{t} s^b_t}{q^k_{t} s^k_t} \right)} \right). \quad (3.12)$$

From equation (3.12) we see a possibility for financial intermediaries to increase leverage: increasing the fraction of government bonds at the expense of private loans will lead to a higher leverage ratio for financial intermediaries when we
3.2. MODEL DESCRIPTION

assume $\omega_b < \omega_k$, as is current practice under Basel regulation. When financial intermediaries face binding capital requirements in a financial crisis, and are forced to shrink the balance sheet, this clearly provides a way to shrink total assets by less than the amount by which risk-weighted assets have to shrink, a phenomenon we will refer to as the ‘asset substitution effect’. This possibility does not exist when both asset classes have equal risk-weights ($\omega_b = \omega_k$): in that case the leverage ratio is fixed and equal to $\phi_t = 1 / (\omega_k \bar{k}_t)$. Increasing leverage by changing the portfolio composition is no longer possible.

We introduce several definitions in order to facilitate the discussion of the several simulations we will perform. We first define CAR: Capital Adequacy Ratio ($\bar{k}_t$). This ratio is defined as net worth over total risk-weighted assets,

$$
\bar{k}_t = n_t / p_t^{rw}.
$$

(3.13)

Higher CAR implies that financial intermediaries need to hold more net worth $n_t$ for a given amount of risk-weighted assets $p_t^{rw}$. Next we define LR: Leverage Ratio ($\bar{k}_t^{LR}$). This ratio is defined as net worth over total (unweighted) assets, and is a special case encompassed in the CAR when $\omega_b = \omega_k$. The LR is given by:

$$
\bar{k}_t^{LR} = n_t / p_t = 1 / \phi_t = \omega_k \bar{k}_t.
$$

(3.14)

Risk-weights on private loans and government bonds differ, whenever we talk about a CAR-regime. When risk-weights are the same, we will talk about a LR-regime. Finally we define portfolio weight claims ($\omega_t$). Portfolio weight claims is defined as the fraction of total assets ($p_t = q_t^k s_t^k + q_t^b s_t^b$) that is invested in private loans,

$$
\omega_t = q_t^k s_t^k / p_t.
$$

(3.15)
3.2.2 Government

The government can be separated into a fiscal authority which is in charge of fiscal policy, and a central bank which is in charge of monetary policy.

Fiscal authority

The fiscal authority issues \( b_t \) bonds in period \( t \), and raises \( q_t^b b_t \) in revenue with \( q_t^b \) the market price of bonds. We parametrize the maturity structure of government debt like Woodford (1998, 2001): maturity is introduced by assuming that one government bond issued in period \( t \) pays out \( r_c \) units (in real terms) in period \( t + 1 \), \( \rho r_c \) real units in period \( t + 2 \), \( \rho^2 r_c \) real units in period \( t + 3 \) etc. This is equivalent to a payout of \( r_c \) plus \( \rho \) times one newly issued bond in period \( t + 1 \), with a value of \( r_c + \rho q_{t+1}^b \). So \( \rho \) pins down the maturity of government debt, and government debt service in period \( t \) is \( (r_c + \rho q_t^b) b_{t-1} \). The duration of public debt is \( 1/(1 - \beta \rho) \).

The fiscal authority also raises revenue by levying lump sum taxes on the households. Government purchases are constant in real terms: \( g_t = G \). Furthermore the fiscal authority may provide assistance to the financial intermediaries by injecting bank net worth \( n_t^g \), and it receives repayment of support administered in earlier periods \( \tilde{n}_t^g \). This gives rise to the following budget constraint:

\[
q_t^b b_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + \left(1 + r_t^b\right) q_{t-1}^b b_{t-1}, \tag{3.16}
\]

where \( r_t^b \) is the real return on government bonds:

\[
1 + r_t^b = \frac{r_c + \rho q_t^b}{q_{t-1}^b}. \tag{3.17}
\]

\[\text{Duration is defined as: } \sum_{j=1}^{\infty} \beta \rho^{j-1} r_c \]

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The tax rule of the fiscal authority is given by a rule which Bohn (1998) has shown secures sustainability:

$$\tau_t = \bar{\tau} + \kappa_b(b_{t-1} - \bar{b}) + \kappa_n n^g_t; \quad \kappa_b \in (0, 1], \quad \kappa_n \in [0, 1].$$ (3.18)

$\bar{b}$ is the steady state level of debt. $\kappa_n$ controls the way government transfers to the financial sector are financed. If $\kappa_n = 0$, support is financed by new debt. $\kappa_n = 1$ implies that the additional spending is completely financed by increasing lump sum taxes. We parametrize aggregate fiscal support as follows:

$$n^g_t = \tau^n_t n_{t-1},$$ (3.19)

$$\tau^n_t = \zeta \xi_{t-l}, \quad \zeta \leq 0, \quad l \geq 0.$$ (3.19)

Thus the fiscal authority provides funds to the financial sector in a financial crisis (represented by a capital quality shock to be introduced shortly) if $\zeta < 0$ (a negative shock $\xi_{t-l}$ to the quality of capital). Depending on the value of $l$, the fiscal authority can provide support instantaneously ($l = 0$), or with a lag ($l > 0$). Furthermore, $\vartheta$ indicates the extent to which the fiscal authority needs to be repaid by the financial sector:

$$\tilde{n}^g_t = \vartheta n^g_{t-e}, \quad \vartheta \geq 0, \quad e \geq 1.$$ (3.20)

$\vartheta = 0$ means the support is a gift from the fiscal authority. In case $\vartheta = 1$, the government aid is a zero interest loan, while a $\vartheta > 1$ implies that the financial intermediaries have to pay interest over the support received earlier. The parameter $e$ denotes the amount of time after which the government aid has to be paid back.

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5The case where $\vartheta > 1$ happened in the Netherlands, where financial intermediaries received government aid with a penalty rate of 50 percent. EU state support rules usually require financial intermediaries to repay state support at a penalty rate.
CHAPTER 3. MACROECONOMICS OF CAPITAL REQUIREMENTS

Monetary Policy

The central bank sets the nominal interest rate on deposits according to a standard Taylor rule, which aims to minimize output and inflation deviations. The central bank pursues an active monetary policy, i.e., an increase in the inflation rate is followed by an even larger increase in the nominal interest rate set by the central bank. In addition, there is interest rate smoothing, see appendix 3.A.4 for more details.

3.2.3 Household

There is a continuum of infinitely lived households. Like in Gertler and Karadi (2011), households consist of both bankers and workers. A fraction $f$ of the household members is running a financial intermediary, while the remaining fraction $1-f$ consists of workers. A fraction $1-\theta$ of the bankers is forced to leave the financial sector each period, where $\theta$ is i.i.d. both across time and the cross-section. We introduce a finite horizon for bankers to make sure that they do not accumulate enough net worth to fund all bank assets from their net worth. This would allow bankers to circumvent the capital requirements. The place of exiting bankers is filled by the same number of workers, who receive a starting net worth from the household. The income from the different household members is pooled at the end of the period, and is shared pro-rata among them.

Households use the available income in a period for consumption, savings and lump sum taxes. Households can save in three ways: bank deposits, private loans and government bonds. Households face transaction costs when purchasing private loans and government bonds. These transaction costs capture in a simple way limited household participation in asset markets. This modeling choice is in line with reality where not only financial intermediaries, but also capital markets provide financing to non-financial corporations and governments. Similar to Gertler and Karadi (2013), household transaction costs for
private loans and government bonds are quadratic in deviation from an exogenous level.

A technical reason for introducing these transaction costs is the fact that there is an excess return over deposits on private securities and government bonds held by commercial banks, as the size of the balance sheet of commercial banks is limited by the amount of net worth in the banks when the balance-sheet constraint (3.5) is binding. Without transaction costs, households would face an arbitrage opportunity and invest only in private securities and government bonds that have an excess return over the risk-free deposit rate. Transaction costs break this arbitrage opportunity, and allow an interior solution where households invest in private securities, government bonds and commercial bank deposits.

Households' income consists of wages, repayment of principal and interest on deposits from the previous period, the gross return on private loans and government bonds, and profits from both financial and non-financial firms owned by the household. A more elaborate description of the household problem and the derivation of the resulting first order conditions can be found in appendix 3.A.2.

3.2.4 Production sector

The production sector consists of capital producers and a standard three-layered New-Keynesian production sector (Calvo, 1983; Yun, 1996) that ultimately produces the final good in the economy and features price-stickiness and monopolistic competition to have real effects from monetary policy. As this part of the model is a relatively standard part of most monetary DSGE models, we shortly describe the model, and refer the reader to appendix 3.A.3 for a more detailed explanation and mathematical derivations.

Intermediate good producers

A continuum of intermediate good producers facing perfect competition acquire physical capital from capital producers. They pay for the capital by is-
CHAPTER 3. MACROECONOMICS OF CAPITAL REQUIREMENTS

suing private securities with a state-contingent payoff to commercial banks and households. Next, the shocks at the beginning of the new period arrive, after which intermediate good producers hire labor and start producing intermediate goods by combining labor and the effective physical capital stock with a standard Cobb-Douglas production technology. We introduce a capital quality shock as in Gertler and Karadi (2011), which creates a difference between the effective physical capital stock used in production and the capital stock acquired before the realization of the shocks.

Intermediate goods are then sold to retail firms, and the effective physical capital stock is sold (after depreciation) to the capital producers. Workers are paid a competitive wage, with the residual of the sales of goods and capital going to the holders of the private securities issued by the intermediate good producers in the period before the shocks arrived.

**Capital producers**

Intermediate good producers sell the effective capital stock (after depreciation) to capital producers, which use the ‘old’ capital together with final goods bought for investment to produce capital for next period’s production. Capital producers face convex adjustment costs when transforming final goods into physical capital, measured with respect to the level of previous period gross investment.

**Retail firms**

Price-stickiness is introduced at the level of retail firms as in Calvo (1983) and Yun (1996). Retail firms purchase intermediate goods, which they transform one-to-one into retail goods. Retail firms operate in a monopolistically competitive market, as each retail firm produces a differentiated good. Retail firms are therefore capable of charging a markup over the price of the interme-
3.2. MODEL DESCRIPTION

diate goods. Price-stickiness arises because each period only a fraction of retail firms is allowed to change prices, see Calvo (1983) and Yun (1996).

**Final Goods Producers**

Final goods producers purchase retail goods from different retail firms, and combine the different retail goods into the final good with a production technology that has constant elasticity of substitution between the different retail goods. Final goods producers face perfect competition, and take prices as given, they only decide how many goods to purchase from each retail firm.

### 3.2.5 Capital requirements

Capital requirements \( \bar{k}_t \) are specified by an exogenous process:

\[
\log \left( \frac{\bar{k}_t}{\bar{k}} \right) = \rho \bar{k} \log \left( \frac{\bar{k}_{t-1}}{k} \right) + u_t, \quad (3.21)
\]

\[
u_t = \rho u_{t-1} + \kappa \varepsilon_{k_t} + \varepsilon_{k,t}, \quad (3.22)
\]

**3.2.6 Equilibrium conditions**

Market clearing in the goods market implies that total output is divided between household consumption, investment in physical capital, government purchases, and the transaction costs that result from household participation in capital markets.

Market clearing in the markets for private securities and government bonds occurs when the supply of private securities and government bonds equals the demand from commercial banks and households.
3.3 Calibration

In this section we explain the calibration of the model, see Table 3.1 in appendix 3.B. We target key parameters to match Eurozone data. Several parameters are standard in the literature, or follow models with financial frictions that have become relatively standard since the start of the financial crisis, such as Gertler and Karadi (2011), Gertler and Kiyotaki (2010), or Gertler and Karadi (2013). Parameters which are taken from Gertler and Karadi (2011) are the subjective discount factor $\beta$, the degree of habit formation $\nu$, the relative utility weight of labor $\Psi$, the inverse Frisch elasticity $\phi$, the elasticity of substitution $\varepsilon$, the Calvo probability of keeping prices fixed $\psi$, the effective capital share $\alpha$, and the investment adjustment cost parameter $\gamma$. Portfolio adjustment costs are introduced to pin down the steady state division between assets placed at banks and households. Because there is no a priori reason to assume why capital markets would be less efficient in sovereign debt intermediation than commercial banks, we set the portfolio adjustment costs for government bonds $\gamma_{b,h}$ slightly larger than zero at 0.001. Commercial banks have an advantage over intermediation by capital markets in private loan provision, we set the coefficients on the portfolio adjustment costs of households for private loans $\gamma_{k,h}$ to be equal to 5 times $\gamma_{b,h}$. We adjust $\hat{k}_h$ and $\hat{b}_h$ to match an average fraction of private securities held by households equal to 20% of the total private securities, respectively 75% of the total stock of government debt in steady state (European Central Bank, 2015).

Risk-weights for non-financial corporates range from 20% for AAA rated debt to 150% for below B- (Basel Committee on Banking Supervision, 2004). We therefore choose risk-weights equal to $\omega_k = 0.8$ for private loans. We set the risk-weight on government bonds equal to $\omega_b = 0$, in line with current Basel regulation. We set the average number of survival quarters for bankers equal to 16 quarters. We set the steady state credit spread between private loans and deposits equal to 200 basis points on an annual basis, in line with the average
3.3. CALIBRATION

Euro Area interest rate difference between loans to non-financial corporations with maturity up to 5 years and overnight deposits.

One of the most important features of the new regulation was a gradual increase in capital requirements, from 4% of Tier-1 capital to 6%. We apply an initial capital requirement of 15%, which is significantly higher than under the envisaged Basel III regulation. The reason for doing so is the fact that our debt contracts are more like equity, i.e. part of non-financial corporations’ equity is within the banks. We therefore follow Gertler and Karadi (2013) by setting a higher capital requirement, namely 15% of net worth, as a proxy for Tier-1 capital. Simulating a gradual increase in capital requirements is implemented by setting $\bar{\rho}_k = 1$, i.e. a shock to the capital requirements is permanent, while setting the autoregressive component of the shock process $u_t$ at $\rho_u = 0.9$. Together with $\kappa_\xi = 0.52$, capital requirements are 19% of bank capital five years after starting to increase capital requirements. When $\kappa_\xi = 0$, capital requirements remain at 15%.

Government debt has an average maturity of 5 years, in line with a realistic maturity of European government debt. Investment to GDP and government spending to GDP ratios are both at 20% in the steady state, while the debt-GDP ratio is set at 90% of annual GDP, in line with Eurozone sovereign debt levels at the start of the sovereign debt crisis in 2011. The Taylor rule coefficients are standard, we apply an interest rate smoothing parameter of 0.4, to capture the fact that monetary policy is more aggressive in a financial crisis than in normal times, for which the smoothing parameter is usually set at 0.8.

A financial crisis is initiated as in Gertler and Karadi (2011) by having a one standard deviation capital quality shock of $\sigma_\xi = 0.050$, and an autoregressive component of $\rho_\xi = 0.66$. 

3.4 Results

In this section we look at the results from the simulations of our model. We consider the impact of a financial crisis on the real economy, which is initiated by a capital quality shock of 5% of steady state, as in Gertler and Karadi (2011).

3.4.1 The effects of a financial crisis: risk-weighted requirements vs. leverage ratio

First we turn to Figure 3.3 and 3.4, in which we compare the impact of a financial crisis on the real economy when risk-weights differ under the CAR-regime (Capital-Adequacy-Ratio-regime) and the LR-regime (Leverage-Ratio-regime) with equal risk-weights. The blue solid line depicts the case with different risk-weights ($\omega_k = 0.8$ vs. $\omega_b = 0$), while the red slotted line depicts the case with equal risk weights ($\omega_b = \omega_k = 0.8$).

The effect of a financial crisis is for both regimes very similar to standard DSGE models with financial frictions, such as Gertler and Karadi (2011): a deterioration in the quality of capital causes the ex-post return on private loans to fall, and net worth decreases. The bank balance-sheet constraint, as indicated by equation (3.9), tightens, which initiates a firesale as financial intermediaries are suddenly forced to shrink the balance sheet. We refer to this as the balance-sheet effect. The interest rate on new loans increases, which is reflected in a higher credit spread between private loans and deposits. The demand for private loans falls, and leads to a lower price of capital. Remember that the intermediate goods producers sell the capital after production, and use the revenue to repay financial intermediaries. A lower price of capital further reduces the ex-post return on private loans, and leads to a second round of capital losses, further deteriorating net worth. Bank balance sheets are impaired, as can be observed from an increase in the credit spread, and a fall in net worth of more than 50% with respect to the steady state. A tightening of the balance sheet constraint
3.4. RESULTS

Financial crisis impact, CAR-regime vs. LR-regime ($\omega_b = 0$ vs. $\omega_b = 0.8$)

Figure 3.3. Impulse response functions for the model without an increase in capital requirements comparing the CAR-regime ($\omega_b = 0$, blue, solid), and the LR-regime ($\omega_b = \omega_c = 0.8$, red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

not only forces intermediaries to reduce private credit provision, but also to sell government bonds. A lower demand for government bonds leads to a bond price decline, further damaging bank balance sheets.

Although households are not balance-sheet-constrained, they are less efficient in financial intermediation than commercial banks since they face intermediation costs. Households are therefore not capable to offset the balance sheet contraction by financial intermediaries. Total financial intermediation in the economy falls.

The repercussions for the real economy are severe: the deterioration of credit conditions due to higher interest rates leads intermediate goods firms to demand fewer loans: investment and physical capital fall by more than 5% of their respective steady state values. A lower capital stock reduces the real wage rate (not shown), and together with reduced dividends from the financial sector, leads to lower household income, thereby reducing output by approximately 2%.
CHAPTER 3. MACROECONOMICS OF CAPITAL REQUIREMENTS

Financial crisis impact, CAR-regime vs. LR-regime ($\omega_b = 0$ vs. $\omega_b = 0.8$)

![Impulse Response Functions](image)

**Figure 3.4.** Impulse response functions for the model without an increase in capital requirements comparing the CAR-regime ($\omega_b = 0$, blue, solid), and the LR-regime ($\omega_b = \omega_k = 0.8$, red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

Both the CAR-regime and the LR-regime suffer from a balance sheet effect, under which a reduction in net worth induces a firesale that forces intermediaries to shrink the balance sheet to meet capital requirements. However, the difference between the two regimes is the regulatory treatment of government bonds. Under the CAR-regime, the risk-weight on government bonds is zero, which introduces an asset substitution effect where financial intermediaries shift from private credit to government bonds. The additional reduction in credit provision to the real economy has severe macroeconomic consequences, as the trough in investment and output more than doubles. From a macroeconomic point of view, the LR-regime is clearly preferable to the CAR-regime.

The mechanism behind this additional fall is the following: since bonds have a smaller risk-weight than private loans, one euro invested in bonds adds less to risk-weighted assets than one euro invested in private loans. Because the CAR is applied to total risk-weighted assets, banks will (relatively) shift from
private loans to government bonds, as can be seen from the panel “Portfolio weight claims”, equation (3.15). Shifting from high risk-weighted assets to low risk-weighted assets allows financial intermediaries to shrink total risk-weighted assets while unweighted assets fall by less, which increases unweighted leverage as indicated by equation (3.14). We refer to this as the asset substitution effect, which is documented for Eurozone banks by Hoshi and Kashyap (2014). The large reduction in private credit leads to a large additional drop in investment and output. Note, however, that the higher credit spread, though, increases net worth accumulation under the CAR-regime, to such an extent that the level of net worth under the CAR-regime is higher than under the LR-regime after 15 quarters.

Bond prices, however, increase with respect to the LR-regime. This makes sense: the lower risk-weight on government bonds incentivizes financial intermediaries to load up on government debt, which increases demand for government bonds. This translates into higher bond prices and lower bond yields. Although the impact of a financial crisis on the real economy is much larger under the CAR-regime, government finances improve as a result of lower sovereign funding costs. A government that only cares about its own finances would clearly opt for a CAR-regime, which might have been an incentive for governments to keep the zero capital charge on domestic sovereign debt.

These results contribute to the discussion on whether or not to increase the risk-weights on sovereign debt, i.e. moving from a CAR-regime to an LR-regime. Under the current (CAR) regime the zero risk-weight on sovereign debt induces commercial banks to shift from private loans to government bonds, even though the zero risk-weight does not properly reflect the risk of these bonds. We find that the asset substitution effect is indeed an impediment to the recovery, and has very substantial negative effects on macroeconomic conditions, which operates through the credit provision channel when banks are undercapitalized.
CHAPTER 3. MACROECONOMICS OF CAPITAL REQUIREMENTS

3.4.2 Financial crisis impact and gradual increase in CAR

After the financial crisis of 2007-2009, Basel III capital regulations were adopted by most advanced economies, and entailed a gradual increase in CAR over several years. Figure 3.5 and 3.6 investigate the effect of higher capital requirements by comparing the CAR-regime from section 3.4.1 (blue solid line) with a gradual increase in CAR from 15% to 19% of net worth over a period of 20 quarters, or 5 years (red slotted line). Although the actual capital requirements increased by 50% from 4% of Tier-1 capital to 6% of Tier-1 capital, we implement a relative more conservative increase in capital requirements, which turns out to already have a significant macroeconomic impact.

In addition to the deleveraging because of lower levels of net worth, commercial banks are forced to hold fewer risk-weighted assets for each unit of net worth, resulting in a fall in financial intermediation everything else equal. But on top of the balance sheet effect, a tighter bank balance sheet constraint (3.9) leads to a further shift from private loans to government bonds, due to the asset substitution effect, see “Portfolio weight claims”. Both the balance sheet effect and the asset substitution effect become stronger and aggrerate credit conditions for non-financial corporations, which leads to fewer credit provision, and lower investment and output.

Also notice the time-pattern. A small increase in CAR in the initial quarters of the financial crisis hardly tightens the bank balance sheet constraint (3.9), see the credit spread, and has little macroeconomic impact. But as the CAR is increased further, the constraint becomes more binding, amplifying both the balance sheet effect and the asset substitution effect. Although larger credit spreads lead to higher net worth across the entire time-path, this is not enough to offset the balance sheet tightening due to a higher CAR. In fact, as the CAR approaches the new target level, credit spreads diverge even further from the case with no increase in CAR. As a consequence, the negative macroeconomic effects become especially pronounced after the initial quarters of the financial crisis.
3.4. RESULTS

Impact of gradual increase in CAR without recap

![Diagrams showing impulse response functions for quality of capital, output, consumption, investment, capital, and price of capital.]

Figure 3.5. Impulse response functions for the model without an increase in CAR (blue, solid), and a gradual increase in CAR of 15% to 19% over 20 quarters (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

crisis. At that point, capital and output continue to diverge from the case with no higher CAR.

A higher CAR is clearly an important impediment to the economic recovery, and seems to confirm the dire warnings by bankers that economic growth will be affected by more stringent capital requirements not only in the short run, but also in the long run. We saw in the previous section, though, that more than 50% of the drop in output can be attributed to the asset substitution effect, which arises under the CAR-regime but not under the LR-regime. We therefore compare an increase in capital requirements under the CAR-regime with an increase in capital requirements under the LR-regime in the next section.
CHAPTER 3. MACROECONOMICS OF CAPITAL REQUIREMENTS

Impact of gradual increase in CAR without recap

![Graph showing the impact of gradual increase in CAR without recap on Intermediary Net Worth, Intermediary Leverage, Credit Spread, Capital requirements, Portfolio weight claims, and Price of bonds.]

Figure 3.6. Impulse response functions for the model without an increase in CAR (blue, solid), and a gradual increase in CAR of 15% to 19% over 20 quarters (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

3.4.3 Financial crisis impact and gradual increase in LR

We have seen in the previous section that higher capital requirements have a negative impact on output when implemented after a financial crisis. In section 3.4.1, we saw that without higher capital requirements, the fall in output under the LR-regime is half the fall in output under the CAR-regime. The reason is that the asset-substitution effect is absent under the LR-regime.

Let us now investigate the macroeconomic consequences of raising capital requirements under the LR-regime, with $\omega_b = \omega_k = 0.8$ as in section 3.4.1. We therefore compare in Figure 3.7 and 3.8 the case where a financial crisis hits with no increase in capital requirements (blue, solid), and compare this with the case where the level of capital requirements $\bar{k}_t$ is increased from 15% to 19% of net worth over a period of 20 quarters, or 5 years (red, slotted).

Contrary to the results in section 3.4.2, we find that the short-run negative effects of higher capital requirements are largely absent under the LR-regime.
3.4. RESULTS

Figure 3.7. Impulse response functions for the model without an increase in LR (blue, solid), and a gradual increase in LR of 15% to 19% over 20 quarters (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

The decrease in the trough of output is negligible, which contrasts with the CAR-regime, where the trough in output falls by almost 0.5% of steady state output. Still, higher capital requirements also force commercial banks to finance fewer assets with one unit of net worth under the LR-regime. Hence long-run output falls as a result.

The main message, though, is that the short-run contractionary impact of higher capital requirements is largely absent under the LR-regime, which sharply contrasts with the short-run impact under the CAR-regime. Once again, the asset-substitution effect is the main culprit, as can be seen from the fraction of assets invested in private loans, see Panel “Portfolio weight claims”. This fraction increases under the LR-regime but decreases under the CAR-regime. This is the main reason why the fall in credit provision, and subsequently investment and output is mitigated under the LR-regime.
Gradual increase in LR ($\omega_b = 0.8$) without recap

Figure 3.8. Impulse response functions for the model without an increase in LR (blue, solid), and a gradual increase in LR of 15% to 19% over 20 quarters (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

A clear conclusion emerges from the simulations in section 3.4.1 and this section: from a macroeconomic perspective it is better to apply capital requirements to unweighted assets rather than risk-weighted assets, as the incentive to shift from private loans to government bonds is reduced under the LR-regime. This conclusion holds irrespective of whether capital requirements are increased or not.

The Great Recession of 2007-2009, however, was entered under the CAR-regime. Though the conclusions from the comparison between the CAR-regime and the LR-regime are relevant for the debate on future capital regulations, the situation in the aftermath of the Great Recession of 2007-2009 was one of an undercapitalized commercial banking system (at least in Europe) operating under the CAR-regime.

Given the output effects from higher capital requirements in section 3.4.2, the question is how this negative impact can be mitigated within the CAR-regime.
Since the amount of net worth is key to the tightness of the bank-balance-sheet-constraint (3.9) and the capacity to provide loans to the real economy, an obvious way to mitigate the macroeconomic impact from higher capital requirements is by increasing commercial banks’ net worth.\textsuperscript{6} We therefore turn to a direct recapitalization by the government in the next section.

\textsuperscript{6}A point of criticism could be that banks in our model do not have the possibility to issue new equity to replenish their net worth. In reality, new equity issue dilutes the claims of existing shareholders when banks are undercapitalized, which makes it hard for banks to raise new equity in times of financial crises: during the financial crisis of 2007-2009, banks did not issue new equity unless forced by governments.
3.4.4 Gradual increase in CAR: no policy vs. immediate recapitalization

It is clear from the previous sections that the LR-regime substantially reduces the fall in investment and output in comparison to the the fall under the CAR-regime, and would therefore be preferable from a macroprudential point of view. However, capital requirements were raised not under the LR-regime but under the CAR-regime. An alternative possibility to mitigate the contractionary impact from a higher CAR is to increase the levels of net worth, which is a key determinant for the size of the banks’ balance sheets.

We therefore investigate the effect of a gradual increase in the CAR in combination with an immediate recapitalization that restores net worth to pre-crisis levels (red, slotted line) as an alternative to just a gradual increase in the CAR (blue, solid line) in Figure 3.9 and 3.10. The increase in the CAR is the same as in the previous two sections, while a recapitalization equal to 6.25% of annual GDP is implemented at the moment the financial crisis hits (see the Panel “Transfers”), and is financed through additional debt issue by the sovereign. This particular size restores net worth to pre-crisis levels upon arrival of the financial crisis.

The recapitalization immediately raises intermediaries’ net worth and relaxes bank balance-sheet-constraints. As a result lending to firms increases, and leads to significantly higher levels of investment and output across the entire time-path. But the alleviation of the bank balance sheet constraint results in lower credit spreads, which slows down net worth accumulation in comparison with the no recap case. Net worth, credit spreads, investment, capital and output have converged after approximately 40 quarters. In the long run a unit of net worth allows banks to finance fewer risk-weighted assets, irrespective of whether or not a recap has been implemented.

The recapitalization, therefore, ameliorates the short run contractionary effects from a higher CAR, and shows that it is not so much the higher capital adequacy ratio that is hampering the short run recovery, but mostly the lack of
3.4. RESULTS

Gradual increase in requirements, no additional policy vs. immediate recapitalization

![Graphs showing impulse response functions for the model with a gradual increase in the CAR from 15% to 19% over 20 quarters. The blue, solid line depicts the case where no additional policy is implemented. The red, slotted line depicts the case where the financial intermediaries are in addition immediately recapitalized by an amount of 6.25% of annual GDP as the crisis hits. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.](image)

Figure 3.9. Impulse response functions for the model with a gradual increase in the CAR from 15% to 19% over 20 quarters. The blue, solid line depicts the case where no additional policy is implemented. The red, slotted line depicts the case where the financial intermediaries are in addition immediately recapitalized by an amount of 6.25% of annual GDP as the crisis hits. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

sufficient levels of bank capital. It is therefore no coincidence that the U.S., which together with the adoption of the Basel III capital requirements immediately recapitalized its banking system in early 2009, has recovered much faster than the Eurozone, where banks were not forced to raise capital levels. Our results indicate that Europe’s troubles could have been alleviated by requiring the commercial banking system to have raised new equity.
Gradual increase in requirements, no additional policy vs. immediate recapitalization

Figure 3.10. Impulse response functions for the model with a gradual increase in the CAR from 15% to 19% over 20 quarters. The blue, solid line depicts the case where no additional policy is implemented. The red, slotted line depicts the case where the financial intermediaries are in addition immediately recapitalized by an amount of 6.25% of annual GDP as the crisis hits. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

3.5 Conclusion

After the systemic crisis of 2007-2009, governments decided to increase capital requirements on commercial banks. The subsequent debate centered not only around the effects on long-run economic growth, but also on the structure of capital requirements. Current capital requirements compare banks’ capital levels with risk-weighted assets, which allows banks to keep high (unweighted) leverage ratios by substituting high risk-weighted assets for low risk-weighted assets. An alternative regime is one in which risk-weights are equal. We refer to the former as the Capital-Adequacy-Ratio-regime (CAR-regime), and to the latter as the Leverage-Ratio-regime (LR-regime).

We focus in our paper on the macroeconomic effects of higher capital requirements through the credit provision channel. We also contribute to the ongoing
debate between CAR-proponents and LR-proponents by comparing the macroeconomic impact of the two regimes. We build a DSGE model with banks that finance private loans and government bonds. Capital requirements are modeled by forcing banks to finance a fixed fraction of risk-weighted assets through net worth. Both private loans and government bonds are assigned a (potentially) different risk-weight. This setup encompasses both the CAR-regime, when private loans and government bonds have different risk-weights, as well as the LR-regime by setting risk-weights equal. A crucial assumption is that banks cannot issue new equity, which is in line with reality, as banks were reluctant to raise new equity to cover existing losses, which would have diluted existing shareholders’ claims in the process.

We simulate a financial crisis and find a balance-sheet effect under both the CAR-regime and the LR-regime: a drop in net worth makes the bank-balance-sheet-constraint more binding, and forces commercial banks to reduce credit provision to the real economy, with a contractionary effect on output. In addition, we find an asset-substitution effect under the CAR-regime: banks shift from private loans to government bonds since a given amount of net worth allows commercial banks to hold more government bonds than private loans. The asset substitution effect leads to a further credit contraction, which amplifies the recessionary impact of a financial crisis when compared with the LR-regime. This suggests that the current regulatory treatment of sovereign debt has aggravated the slow recovery in the Eurozone, by inducing undercapitalized European banks to substitute private credit for sovereign debt, as documented by Hoshi and Kashyap (2014). Risk-weights should be raised on sovereign debt, to reduce the impact of the asset-substitution effect.

When in addition capital requirements are increased, the recession deepens under the CAR-regime: a single unit of net worth can finance fewer risk-weighted assets, which leads to a further tightening of the bank balance sheet constraint. The balance-sheet effect and the asset-substitution effect become
more pronounced. We find that the performance difference with the case where capital requirements are not increased is increasing as the CAR increases. The negative effects of a higher CAR do not play out so much in the short term, but rather in the medium term, and seems to confirm the warnings by bankers that more stringent capital requirements lead to reduced economic growth. The negative consequences of higher capital requirements are much reduced when capital requirements are applied to unweighted assets instead of risk-weighted assets, as the asset-substitution effect disappears. The fall in investment and output is more than halved compared with the CAR-regime.

A forced recapitalization by the government increases banks’ net worth and mitigates both the balance-sheet effect and the asset-substitution effect in the short run. This pattern seems to be consistent with the different recoveries in the US and the Eurozone: while the US government forced the financial sector to recapitalize, restoring bank capital to pre-crisis levels, Eurozone banks had the freedom to meet a higher CAR by shrinking the balance sheet.

Our paper has two major conclusions that are relevant for policymakers. The first is that from a macroeconomic perspective, the focus of capital regulations should shift from risk-weighted assets to an unweighted leverage ratio. A first step has been undertaken in this direction, as an unweighted maximum leverage ratio has been introduced into the Basel III regulatory framework.

The second conclusion is that when capital requirements are raised after a financial crisis, regulators should not only focus on capital ratios, but in addition restore capital levels, which reduce the contractionary impact of higher requirements in the short run.
3.A Derivations

3.A.1 Elaborate model description

Bank Optimization Problem

First we derive the law of motion for net worth, which includes recapitalizations by the government and financial sector repayments:

\[
\begin{align*}
n_{j,t+1} &= \left(1 + r_{k,t+1}^d\right) q_{t}^k s_{j,t}^k + \left(1 + r_{b,t+1}^d\right) q_{t}^b s_{j,t}^b - \left(1 + r_{k,t+1}^d\right) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\
&= \left(1 + r_{k,t+1}^d\right) q_{t}^k s_{j,t}^k + \left(1 + r_{b,t+1}^d\right) q_{t}^b s_{j,t}^b \\
&\quad - \left(1 + r_{k,t+1}^d\right) d_{j,t} + \tau_{t+1}^n \left(q_{t}^k s_{j,t}^k + q_{t}^b s_{j,t}^b - d_{j,t}\right) - \bar{n}_{t+1} \left(q_{t}^k s_{j,t}^k + q_{t}^b s_{j,t}^b - d_{j,t}\right) \\
&= \left(1 + r_{k,t+1}^d + \tau_{t+1}^n - \bar{n}_{t+1}\right) q_{t}^k s_{j,t}^k + \left(1 + r_{b,t+1}^d + \tau_{t+1}^n - \bar{n}_{t+1}\right) q_{t}^b s_{j,t}^b \\
&\quad - \left(1 + r_{k,t+1}^d + \tau_{t+1}^n - \bar{n}_{t+1}\right) d_{j,t}
\end{align*}
\]

Now we remember the optimization problem of the financial intermediary:

\[
V_{j,t} = \max_{\{s_{j,t}^k, s_{j,t}^b, d_{j,t}\}} E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) n_{j,t+1} + \theta V_{j,t+1} \right\} \right],
\]

s.t.

\[
\begin{align*}
n_{j,t} &\geq \bar{k}_t \left( \omega_k q_{t}^k s_{j,t}^k + \omega_b q_{t}^b s_{j,t}^b \right), \\
n_{j,t} + d_{j,t} &\geq q_{t}^k s_{j,t}^k + q_{t}^b s_{j,t}^b, \\
n_{j,t} &\geq \left(1 + r_{k,t}^d + \tau_{t}^n - \bar{n}_{t}\right) q_{t-1}^k s_{j,t-1}^k + \left(1 + r_{b,t}^d + \tau_{t}^n - \bar{n}_{t}\right) q_{t-1}^b s_{j,t-1}^b \\
&\quad - \left(1 + r_{k,t}^d + \tau_{t}^n - \bar{n}_{t}\right) d_{j,t-1},
\end{align*}
\]
where we have abbreviated the value function of the financial intermediary by
\[ V_{j,t} = V \left( s_{j,t-1}^k, s_{j,t-1}^b, d_{j,t-1} \right) \]. We set up the accompanying Lagrangian of the problem:

\[ \mathcal{L} = E_t \left[ \beta \Lambda_{t,t+1} \{ (1 - \theta) \left( 1 + r_{t+1}^k + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) q_t^k s_{j,t}^k \right. \right. \\
+ \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) q_t^b s_{j,t}^b - \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) d_{j,t} \right] \\
+ \left. \theta V \left( s_{j,t}^k, s_{j,t}^b, d_{j,t} \right) \right] \\
+ \mu_t \left[ \left( 1 + r_{t}^k + \tau_{t}^n - \bar{\tau}_{t}^n \right) q_{t-1}^k s_{j,t-1}^k + \left( 1 + r_{t}^b + \tau_{t}^n - \bar{\tau}_{t}^n \right) q_{t-1}^b s_{j,t-1}^b \right] \\
- \left( 1 + r_{t}^d + \tau_{t}^n - \bar{\tau}_{t}^n \right) d_{j,t-1} - \tilde{k}_t \omega_k q_t^k s_{j,t}^k - \tilde{k}_t \omega_b q_t^b s_{j,t}^b \right] \\
+ \eta_t \left[ \left( 1 + r_{t}^k + \tau_{t}^n - \bar{\tau}_{t}^n \right) q_{t-1}^k s_{j,t-1}^k + \left( 1 + r_{t}^b + \tau_{t}^n - \bar{\tau}_{t}^n \right) q_{t-1}^b s_{j,t-1}^b \right] \\
- \left( 1 + r_{t}^d + \tau_{t}^n - \bar{\tau}_{t}^n \right) d_{j,t-1} - q_t^k s_{j,t}^k - q_t^b s_{j,t}^b + d_{j,t} \right) .

This gives rise to the following first order conditions:

\[ s_{j,t}^k : E_t \left[ \beta \Lambda_{t,t+1} \{ (1 - \theta) \left( 1 + r_{t+1}^k + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) q_t^k \right. \right. \\
+ \theta \frac{\partial}{\partial s_{j,t}^k} \left[ V \left( s_{j,t}^k, s_{j,t}^b, d_{j,t} \right) \right] \right] \right] \left[ \tilde{k}_t \omega_k q_t^k - \eta_t q_t^k \right] = 0, \quad (3.23) \\

\[ s_{j,t}^b : E_t \left[ \beta \Lambda_{t,t+1} \{ (1 - \theta) \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) q_t^b \right. \right. \\
+ \theta \frac{\partial}{\partial s_{j,t}^b} \left[ V \left( s_{j,t}^k, s_{j,t}^b, d_{j,t} \right) \right] \right] \right] \left[ \tilde{k}_t \omega_b q_t^b - \eta_t q_t^b \right] = 0, \quad (3.24) \\

\[ d_{j,t} : E_t \left[ \beta \Lambda_{t,t+1} \{ (1 - \theta) \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) (-1) \right. \right. \\
+ \theta \frac{\partial}{\partial d_{j,t}} \left[ V \left( s_{j,t}^k, s_{j,t}^b, d_{j,t} \right) \right] \right] \right] \left[ \eta_t \right] = 0, \quad (3.25) \]
with complementary slackness conditions:

\[
\mu_t : \left( (1 + r_t^k + \bar{\tau}_t^n - \bar{\tau}_t^i) q_{t-1}^{k,p}_{j,t-1} + (1 + r_t^b + \bar{\tau}_t^n - \bar{\tau}_t^i) q_{t-1}^{b,p}_{j,t-1} \right)
- \left( (1 + r_t^d + \tau_t^n - \bar{\tau}_t^i) d_{j,t-1} - \bar{\kappa}_t \omega_k q_{t-1}^{k,p}_{j,t-1} - \bar{\kappa}_t \omega_b q_{t-1}^{b,p}_{j,t-1} \right) \mu_t = 0, \quad (3.26)
\]

\[
\eta_t : \left( (1 + r_t^k + \tau_t^n - \bar{\tau}_t^i) q_{t-1}^{k,p}_{j,t-1} + (1 + r_t^b + \tau_t^n - \bar{\tau}_t^i) q_{t-1}^{b,p}_{j,t-1} \right)
- \left( (1 + r_t^d + \tau_t^n - \bar{\tau}_t^i) d_{j,t-1} - q_{t}^{k,p}_{j,t-1} - q_{t}^{b,p}_{j,t-1} + d_{j,t-1} \right) \eta_t = 0, \quad (3.27)
\]

Now we apply the envelope theorem to find the derivatives:

\[
\frac{\partial}{\partial s_{k,j,t-1}} \left[ V \left( s_{j,t-1}^{k}, s_{j,t-1}^{b}, d_{j,t-1} \right) \right] = (\mu_t + \eta_t) \left( 1 + r_t^k + \tau_t^n - \bar{\tau}_t^i \right) q_{t-1}^{k}, \quad (3.28)
\]

\[
\frac{\partial}{\partial s_{b,j,t-1}} \left[ V \left( s_{j,t-1}^{k}, s_{j,t-1}^{b}, d_{j,t-1} \right) \right] = (\mu_t + \eta_t) \left( 1 + r_t^b + \tau_t^n - \bar{\tau}_t^i \right) q_{t-1}^{b}, \quad (3.29)
\]

\[
\frac{\partial}{\partial d_{j,t-1}} \left[ V \left( s_{j,t-1}^{k}, s_{j,t-1}^{b}, d_{j,t-1} \right) \right] = - (\mu_t + \eta_t) \left( 1 + r_t^d + \tau_t^n - \bar{\tau}_t^i \right), \quad (3.30)
\]
CHAPTER 3. MACROECONOMICS OF CAPITAL REQUIREMENTS

Subsititution of the envelope conditions (3.28) - (3.30) with (3.23) - (3.25), we find the following relation between the different assets:

\[ s_{j,t}^k = \mu_t \bar{k}_t \omega_k + \eta_t = E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta (\mu_{t+1} + \eta_{t+1}) \right\} \left( 1 + r_{t+1}^k + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right], \]

(3.31)

\[ s_{j,t}^b = \mu_t \bar{k}_t \omega_b + \eta_t = E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta (\mu_{t+1} + \eta_{t+1}) \right\} \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right], \]

(3.32)

\[ d_{j,t} = \eta_t = E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta (\mu_{t+1} + \eta_{t+1}) \right\} \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right]. \]

(3.33)

Now we define the following variables:

\[ \nu_t^k = \mu_t \bar{k}_t \omega_k, \]

(3.34)

\[ \nu_t^b = \mu_t \bar{k}_t \omega_b. \]

(3.35)

Subsititution of (3.31) and (3.32) into (3.34) and (3.35) gives rise to the following first order conditions:

\[ \nu_t^k = E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta (\mu_{t+1} + \eta_{t+1}) \right\} \left( r_{t+1}^k - r_{t+1}^d \right) \right], \]

(3.36)

\[ \nu_t^b = E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta (\mu_{t+1} + \eta_{t+1}) \right\} \left( r_{t+1}^b - r_{t+1}^d \right) \right], \]

(3.37)

\[ \eta_t = E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \theta) + \theta (\mu_{t+1} + \eta_{t+1}) \right\} \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) \right]. \]

(3.38)
3.A.2 Household

There is a continuum of infinitely lived households. Like in Gertler and Karadi (2011), households consist of both bankers and workers. A fraction $f$ of the household members is running a financial intermediary, while the remaining fraction $1 - f$ is a worker. A fraction $1 - \theta$ of the bankers is forced to leave the financial sector each period, where $\theta$ is i.i.d. both across time and the cross-section. We introduce a finite horizon for bankers to make sure that they do not accumulate enough net worth to fund all bank assets from their net worth, which would allow the bankers to circumvent the capital requirements to be introduced shortly. The place of exiting bankers is filled by the same number of workers, who receive a starting net worth, and replace the bankers that have exited. The income from the different members is pooled at the end of the period, and is shared pro-rata.

Households use the available income in period $t$ for consumption, investment and lump sum taxes. There are three investment classes: bank deposits $d_t$, private loans $k^h_t$, which can be bought at market price $q^k_t$ and government bonds $b^h_t$, which can be purchased at market price $q^b_t$. We assume that households face transaction costs $f(k^h_t)$ and $g(b^h_t)$, respectively, for intermediation of private loans and government bonds, respectively. We motivate these transaction costs to capture in a simple way limited participation in asset markets. This modeling choice is in line with reality where not only financial intermediaries, but also capital markets provide financing to non-financial corporations and governments. Similar to Gertler and Karadi (2013), household transaction costs for private loans and government bonds, respectively, are quadratic in deviation from the exogenous number $\hat{k}_h$ and $\hat{b}_h$, respectively. Households earn income from wages, repayment of period $t - 1$ deposits $d_{t-1}$ with interest rate $r^d_t$, and a return $r^k_t$ respectively $r^b_t$ on private loans respectively government bonds. Finally, the households receive profits from the firms they own, both financial and non-financial.
A technical reason for introducing these transaction costs is the fact that there is an excess return over deposits on private securities and government bonds held by commercial banks, as the size of their balance sheet is limited by the amount of net worth in the banks. Without transaction costs, household would face an arbitrage opportunity, and invest only in private securities and government bonds. Transaction costs break this arbitrage opportunity, and allow an interior solution where households invest in private securities, government bonds and commercial bank deposits.

The household problem is given by:

\[
\max_{\{c_{t+i}, h_{t+i}, d_{t+i}, k_{t+i}, b_{t+i}\}} E_t \left[ \sum_{i=0}^{\infty} \beta^i \left\{ \log (c_{t+i} - \nu c_{t-1+i}) - \frac{\chi}{1 + \varphi} h_{t+i}^{1+\varphi} \right\} \right]
\]

s.t.

\[
c_t + q_t^k k_t^h + q_t^b b_t^h + d_t + f(k_t^h) + g(b_t^h) + \tau_t = w_t h_t + (1 + r_t^k) q_{t-1}^k k_{t-1}^h + (1 + r_t^b) q_{t-1}^b b_{t-1}^h + (1 + r_t^d) d_{t-1} + \Pi_t,
\]

where the cost functions \( f(k_t^h) \) and \( g(b_t^h) \) are given by:

\[
f(k_t^h) = \frac{\gamma_{k,h}}{2} (k_t^h - \hat{k}_h)^2,
\]

\[
g(b_t^h) = \frac{\gamma_{b,h}}{2} (b_t^h - \hat{b}_h)^2.
\]

We can now formulate the Lagrangian for the household’s optimization problem:

\[
\mathcal{L} = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \log (c_{t+i} - \nu c_{t-1+i}) - \frac{\chi}{1 + \varphi} h_{t+i}^{1+\varphi} \right] \right\}
\]

\[
+ E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \left[ w_{t+i} h_{t+i} + (1 + r_t^k) q_{t-1}^k k_{t-1}^h + (1 + r_t^b) q_{t-1}^b b_{t-1}^h + (1 + r_t^d) d_{t-1} + \Pi_{t+i} - c_{t+i} - q_t^k k_{t+i}^h - q_t^b b_{t+i}^h - d_{t+i} - f(k_{t+i}) - g(b_{t+i}) \right] \right\}
\]
Taking first order conditions, we find:

\[ c_t : \lambda_t = (c_t - \psi c_{t-1})^{-1} - \beta \nu E_t \left[ (c_{t+1} - \psi c_t)^{-1} \right], \quad (3.39) \]

\[ h_t : \chi h_t^\phi = \lambda_t \varpi_t, \quad (3.40) \]

\[ k_h^t : E_t \left[ \beta \Lambda_{t,t+1} \left( \frac{(1 + r_{t+1}^k) q_{t}^k}{q_{t}^k + f'(k_{t}^h)} \right) \right] = 1, \quad (3.41) \]

\[ b_h^t : E_t \left[ \beta \Lambda_{t,t+1} \left( \frac{(1 + r_{t+1}^b) q_{t}^b}{q_{t}^b + g'(b_{t}^h)} \right) \right] = 1, \quad (3.42) \]

\[ d_t : E_t \left[ \beta \Lambda_{t,t+1} \left( 1 + r_{t+1}^d \right) \right] = 1, \quad (3.43) \]

where \( \beta \Lambda_{t,t+1} = \beta \lambda_{t+1} / \lambda_t \) is the household’s stochastic discount factor, with \( \lambda_t \) the marginal utility of household consumption.

### 3.A.3 Production sector

**Intermediate Goods Producers**

There exists a continuum of intermediate goods producers indexed by \( i \in [0, 1] \). Each of these firms produce a differentiated good. The intermediate goods producers obtain funds from the financial intermediaries by pledging next period’s profits, so banks are exposed to downside risk. We assume that there are no financial frictions between the financial intermediaries and the intermediate goods producers. The securities issued by the intermediate goods producers are best considered as state-contingent debt, like in Gertler and Kiyotaki (2010).\(^7\) The price of the claims is equal to \( q_{t}^k, \) and pay a state-contingent net real return.

---

\(^7\)It is therefore better to think of the claims of financial intermediaries as equity. Occhino and Pescatori (2015) explicitly model loans to producers with a fixed face value, where the goods producers have the possibility of defaulting on the loans. We refrain from explicitly modelling this default possibility, and note the equity characteristics of debt in the real world when firms are short of funds to pay off the loans.
The production technology of the intermediate goods producers is given by:

\[ y_{i,t} = a_t (\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha}, \]

\[ \log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_{a,t}, \quad \log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t}. \]

Both (log of) total factor productivity \( a_t \) and capital quality \( \xi_t \) are AR(1) processes driven by random shocks \( \varepsilon_{a,t} \sim N(0, \sigma_a^2) \) and \( \varepsilon_{\xi,t} \sim N(0, \sigma_\xi^2) \). The intermediate goods producer acquires the capital at the end of period \( t-1 \) and uses it for production in period \( t \). The capital quality shock \( \xi_t \) occurs at the beginning of period \( t \), so \( \xi_t k_{i,t-1} \) is the effective stock of capital used for production in period \( t \). A negative realization of \( \varepsilon_{\xi,t} \) lowers the quality of the capital stock, hence the return on the claims of the financial intermediary will be lower. The intermediate goods producer hires labor \( h_{i,t} \) for a wage rate \( w_t \) after the shock \( \xi_t \) has been realized. When the firm has produced in period \( t \), the output is sold for price \( m_t \) to the retail firms. \( m_t \) is the relative price of the intermediate goods with respect to the price level of the final goods, i.e. \( m_t = P^m_t / P_t \). A fraction \( \delta \) of the capital stock \( \xi_t k_{i,t-1} \) is used up in the production process. The intermediate goods producing firms sell back what is left of the effective capital stock to the capital producers for the end-of-period price of \( q^k_t \) and thus receives \( q^k_t (1 - \delta) \xi_t k_{i,t-1} \). Hence period \( t \) profits are:

\[ \Pi_{i,t} = m_t a_t (\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha} + q^k_t (1 - \delta) \xi_t k_{i,t-1} - (1 + r^k_t) q^k_{t-1} k_{i,t-1} - w_t h_{i,t}. \]

The intermediate goods producing firms maximize expected current and future profits using the household’s stochastic discount factor \( \beta^s \Lambda_{t,t+s} \) (since they are owned by the households), taking all prices as given:

\[
\max_{\{k_{t+s}, h_{t+s}\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \Pi_{i,t+s} \right].
\]
The first order conditions belonging to this problem are given by:

\[ k_{i,t} : E_t \left[ \beta \Lambda_{t,t+1} q_{t+1}^k (1 + r_{t+1}^k) \right] = E_t \left[ \beta \Lambda_{t,t+1} \left( \alpha m_{t+1} y_{i,t+1} + q_{t+1}^k (1 - \delta) \xi_{t+1} \right) \right], \]

\[ h_{i,t} : w_t = (1 - \alpha) m_t y_{i,t} / h_{i,t}. \]

In equilibrium profits will be zero. By substituting the first order condition for the wage rate into the zero-profit condition \( \Pi_{i,t} = 0 \), we can find an expression for the ex-post return on capital:

\[ r_t^k = (q_{t-1}^k)^{-1} \left( \alpha m_t y_{i,t} / k_{i,t-1} + q_t^k (1 - \delta) \xi_t \right) - 1. \]

Now we rewrite the first order condition for labor and the expression for the ex-post return on capital to find the factor demands:

\[ k_{i,t-1} = \alpha m_t y_{i,t} / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t], \]

\[ h_{i,t} = (1 - \alpha) m_t y_{i,t} / w_t. \]

By substituting the factor demands into the production technology function, we get for the relative intermediate output price \( m_t \):

\[ m_t = \alpha^{-\alpha} (1 - \alpha)^{\alpha - 1} a_t^{-1} \left( w_t^{1-\alpha} [q_{t-1}^k (1 + r_t^k) \xi_t^{-1} - q_t^k (1 - \delta)]^{\alpha} \right). \tag{3.44} \]

**Capital Producers**

At the end of period \( t \), when the intermediate goods firms have produced, the capital producers buy the remaining stock of capital \( (1 - \delta) \xi_t k_{t-1} \) from the intermediate goods producers at a price \( q_t^k \). They combine this capital with goods bought from the final goods producers (investment \( i_t \)) to produce next period’s beginning of period capital stock \( k_t \). This capital is being sold to the intermediate goods producers at a price \( q_t^k \). We assume that the capital producers face convex
adjustment costs when transforming the final goods bought into capital goods, set up such that changing the level of gross investment is costly. Hence we get:

\[ k_t = (1 - \delta)\xi_t k_{t-1} + (1 - \Psi(t_t))i_t, \quad \Psi(x) = \frac{\gamma}{2}(x - 1)^2, \quad t_t = i_t/i_{t-1}. \] (3.45)

\( \xi_t \) represents a capital quality shock which will be discussed later. Profits are passed on to the households, who own the capital producers. The profit at the end of period \( t \) equals:

\[ \Pi_t^c = q_t^k k_t - q_t^k (1 - \delta)\xi_t k_{t-1} - i_t. \]

The capital producers maximize expected current and (discounted) future profits (where we substitute in (3.45)):

\[
\max_{\{i_{t+i}\}_{i=0}^{\infty}} E_t \left[ \sum_{i=0}^{\infty} \beta^i \Lambda_{t, t+i} \left( q_{t+i}^k (1 - \Psi(t_{t+i}))i_{t+i} - i_{t+i} \right) \right].
\]

Differentiation with respect to investment gives the first order condition for the capital producers:

\[ q_t^k (1 - \Psi(t_t)) - 1 - q_t^k t_t \Psi'(t_t) + \beta E_t \Lambda_{t, t+1} q_{t+1}^k t_{t+1}^2 \Psi'(t_{t+1}) = 0, \]

which gives the following expression for the price of capital:

\[
\frac{1}{q_t^k} = \frac{1}{q_t^k} \Psi'(t_t) - 1 - q_t^k t_t \Psi'(t_t) + \beta E_t \Lambda_{t, t+1} q_{t+1}^k t_{t+1}^2 \Psi'(t_{t+1}) = 0.
\]

Retail firms

Retail firms purchase goods \((y_{i,t})\) from the intermediate goods producing firms for a nominal price \(P_{i,t}^m\), and convert these into retail goods \((y_{f,t})\). These goods are sold for a nominal price \(P_{f,t}\) to the final goods producer. It takes one
intermediate goods unit to produce one retail good \((y_{i,t} = y_{f,t})\). All the retail firms produce a differentiated retail good by assumption, therefore operate in a monopolistically competitive market, and charge a markup over the input price earning them profits \((P_{f,t} - P_{t}^m)y_{f,t}\).

Each period, only a fraction \(1 - \psi\) of retail firms is allowed to reset their price, while the \(\psi\) remaining firms are not allowed to do so, like in Calvo (1983) and Yun (1996). The firms allowed to adjust prices are randomly selected each period. Once selected, they set prices so as to maximize expected current and future profits, using the stochastic discount factor \(\beta^s \Lambda_{t,t+s}(P_t/P_{t+s})\):

\[
\max_{P_{f,t}} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} (\beta \psi)^s \Lambda_{t,t+s}(P_t/P_{t+s}) \left[ P_{f,t} - P_{t+s}^m \right] y_{f,t+s} \right],
\]

where \(y_{f,t} = (P_{f,t}/P_t)^{-\varepsilon} y_t\) is the demand function. \(y_t\) is the output of the final goods producing firms, and \(P_t\) the general price level. Symmetry implies that all firms allowed to reset their prices choose the same new price \((P^*_t)\). Differentiation with respect to \(P_{f,t}\) and using symmetry then yields:

\[
\frac{P^*_t}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta \psi)^s \lambda_{t,s} P_{t+s}^\varepsilon m_{t+s} y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta \psi)^s \lambda_{t,s} P_{t+s}^\varepsilon m_{t+s} y_{t+s}}.
\]

Defining the relative price of the firms that are allowed to reset their prices as \(\pi^*_t = P^*_t/P_t\) and gross inflation as \(\pi_t = P_t/P_{t-1}\), we can rewrite this as:

\[
\pi^*_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\Xi_{1,t}}{\Xi_{2,t}},
\]

\[
\Xi_{1,t} = \lambda_t m_t y_t + \beta \psi E_t \pi_{t+1}^\varepsilon \Xi_{1,t+1},
\]

\[
\Xi_{2,t} = \lambda_t m_t y_t + \beta \psi E_t \pi_{t+1}^{1-\varepsilon} \Xi_{2,t+1}.
\]

The aggregate price level equals:

\[
P_t^{1-\varepsilon} = (1 - \psi)(P_t^*)^{1-\varepsilon} + \psi P_{t-1}^{1-\varepsilon}.
\]
CHAPTER 3. MACROECONOMICS OF CAPITAL REQUIREMENTS

Dividing by $P_t^{1-\varepsilon}$ yields the following law of motion:

$$(1 - \psi)(\pi_t^*)^{1-\varepsilon} + \psi \pi_t^{\varepsilon-1} = 1. \quad (3.50)$$

Final Goods Producers

Final goods firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by $y_t^{(\varepsilon-1)/\varepsilon} = \int_0^1 y_{f,t}^{(\varepsilon-1)/\varepsilon} df$, where $y_{f,t}$ is the output of the retail firm indexed by $f$. $\varepsilon$ is the elasticity of substitution between the intermediate goods purchased from the different retail firms. The final goods firms face perfect competition, and therefore take prices as given. Thus they maximize profits by choosing $y_{f,t}$ such that $P_t y_t - \int_0^1 P_{f,t} y_{f,t} df$ is maximized. Taking the first order conditions with respect to $y_{f,t}$, gives the demand function of the final goods producers for the retail goods. Substitution of the demand function into the technology constraint gives the relation between the price level of the final goods and the price level of the individual retail firms:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon} y_t,$$

$$P_t^{1-\varepsilon} = \int_0^1 P_{f,t}^{1-\varepsilon} df.$$

Aggregation

Substituting $y_{f,t} = y_{i,t} = y_t \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon}$ into the factor demands derived earlier yields:

$$h_{i,t} = (1 - \alpha)m_t y_{f,t}/w_t, \quad k_{i,t-1} = \alpha m_t y_{f,t}/[q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi].$$

Aggregation over all firms $i$ gives us aggregate labor and capital:

$$h_t = (1 - \alpha)m_t y_t D_t/w_t, \quad k_{t-1} = \alpha m_t y_t D_t/[q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi].$$
3. A. DERIVATIONS

where \( D_t = \int_0^1 \left( P_{f,t}/P_t \right)^{-\varepsilon} df \) denotes the price dispersion. It is given by the following recursive form:

\[
D_t = (1 - \psi)(\pi_t^*)^{-\varepsilon} + \psi \pi_t^E D_{t-1}.
\]

(3.51)

The aggregate capital-labor ratio is equal to the individual capital-labor ratio:

\[
k_{t-1}/h_t = \alpha(1 - \alpha)^{-1}w_t/[q_{t-1}^k(1 + r_t^k) - q_t^k(1 - \delta)\xi_t] = k_{i,t-1}/h_{i,t}.
\]

(3.52)

Now calculate aggregate supply by aggregating \( y_{i,t} = a_t(\xi_t k_{i,t-1})^{\alpha}h_{i,t}^{1-\alpha} \):

\[
\int_0^1 a_t(\xi_t k_{i,t-1})^{\alpha}h_{i,t}^{1-\alpha} di = a_t(\xi_t k_{i,t-1})^{\alpha}h_t^{1-\alpha},
\]

while aggregation over \( y_{i,t} \) gives:

\[
\int_0^1 y_{i,t} df = y_t \int_0^1 \left( P_{f,t}/P_t \right)^{-\varepsilon} df = y_t D_t.
\]

So we get the following relation for aggregate supply \( y_t \):

\[
y_t D_t = a_t(\xi_t k_{t-1})^{\alpha}h_t^{1-\alpha}.
\]

(3.53)

3.A.4 Monetary Policy

The central bank sets the nominal interest rate on deposits \( r_t^n \) following a standard Taylor rule, in order to minimize output and inflation deviations:

\[
r_t^n = (1 - \rho_r)(r^n + \kappa_\pi(\pi_t - \bar{\pi}) + \kappa_y \log(y_t/y_{t-1})) + \rho_r r_{t-1}^n + \varepsilon_{r,t},
\]

(3.54)

where \( \varepsilon_{r,t} \sim N(0, \sigma_\varepsilon^2) \), and \( \kappa_\pi > 0 \) and \( \kappa_y > 0 \). The parameter \( \bar{\pi} \) is the target inflation rate. We choose \( \kappa_\pi > 1, \kappa_y > 0 \), (leaning against the wind). The real interest rate on deposits then equals:

\[
1 + r_t^d = (1 + r_t^n)/\pi_t.
\]

(3.55)
3.A.5 Equilibrium conditions

In order to close the model, we need market clearing in the goods market, the capital market, and the bond market:

\[ y_t = c_t + i_t + g_t + \frac{\gamma_{k,h}}{2} (k^h_t - \hat{k}_h)^2 + \frac{\gamma_{b,h}}{2} (b^h_t - \hat{b}_h)^2, \]  
(3.56)

\[ k_t = s^k_t + k^h_t, \]  
(3.57)

\[ b_t = s^b_t + b^h_t. \]  
(3.58)

3.B Calibration parameters

The numerical values for the parameters can be found in Table 3.1.
### 3.B. Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.990</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.815</td>
<td>Degree of habit formation</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>3.6266</td>
<td>Relative utility weight of labor</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.276</td>
<td>Inverse Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\gamma_{k,h}$</td>
<td>0.005</td>
<td>Portfolio adjustment cost private loans</td>
</tr>
<tr>
<td>$\gamma_{b,h}$</td>
<td>0.001</td>
<td>Portfolio adjustment cost government bonds</td>
</tr>
<tr>
<td><strong>Financial Intermediaries</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_k$</td>
<td>0.8</td>
<td>Regulatory risk-weight on private loans</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>0</td>
<td>Regulatory risk-weight on government bonds</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.0140</td>
<td>Proportional transfer to entering bankers</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.9375</td>
<td>Survival rate of the bankers</td>
</tr>
<tr>
<td><strong>Intermediate good firms</strong></td>
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<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>4.176</td>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.779</td>
<td>Calvo probability of keeping prices fixed</td>
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<tr>
<td>$\alpha$</td>
<td>0.330</td>
<td>Effective capital share</td>
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<tr>
<td><strong>Capital good firms</strong></td>
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<tr>
<td>$\gamma$</td>
<td>1.728</td>
<td>Investment adjustment cost parameter</td>
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<tr>
<td>$\delta$</td>
<td>0.0592</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td><strong>Autoregressive components</strong></td>
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<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.95</td>
<td>Autoregressive component of productivity</td>
</tr>
<tr>
<td>$\rho_{\xi}$</td>
<td>0.66</td>
<td>Autoregressive component of capital quality</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.8</td>
<td>Autoregressive component of government spending</td>
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<tr>
<td>$\rho_r$</td>
<td>0.4</td>
<td>Interest rate smoothing parameter</td>
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<tr>
<td>$\rho_k$</td>
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<td>Autoregressive component of capital requirements</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>0.9</td>
<td>Persistence in cap. req. adjustment</td>
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<tr>
<td><strong>Policy</strong></td>
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<td></td>
</tr>
<tr>
<td>$r_c$</td>
<td>0.04</td>
<td>Real payment to government bondholder</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9596</td>
<td>Parameter government debt duration (5 yrs)</td>
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<td>$\kappa_b$</td>
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<td>Tax feedback parameter from government debt</td>
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<tr>
<td>$\kappa_\pi$</td>
<td>1.500</td>
<td>Inflation feedback on nominal interest rate</td>
</tr>
<tr>
<td>$\kappa_y$</td>
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<td>Output feedback on nominal interest rate</td>
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<tr>
<td><strong>Shocks</strong></td>
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<tr>
<td>$\sigma_z$</td>
<td>0.010</td>
<td>Standard deviation productivity shock</td>
</tr>
<tr>
<td>$\sigma_{\xi}$</td>
<td>0.050</td>
<td>Standard deviation capital quality shock</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.0025</td>
<td>Standard deviation interest rate surprise shock</td>
</tr>
</tbody>
</table>

Table 3.1. Model parameters.
Chapter 4

Financial Fragility and Unconventional Central Bank Lending Operations

4.1 Introduction

In December 2011 and February 2012, the European Central Bank (ECB) engaged in unconventional lending operations with the European commercial banking system to prevent a credit contraction, which would have had serious consequences for the macroeconomy (European Central Bank, 2012). These Longer-Term Refinancing Operations (LTROs) featured a long maturity (36 months) and a promise by the ECB to provide as much funding to the commercial banks as demanded against eligible collateral.¹ Commercial banks eventually took out more than 1000 billion EUR in ECB funding, making the LTROs the largest refinancing operation in the history of the ECB. The goal of this paper is to investigate the macroeconomic impact of such unconventional central bank lending by linking such a program to the credit transmission channel.

Such a large program has the potential to affect aggregate investment, as this funding can potentially be used to provide new credit to the real economy. Effects on private credit provision are relevant in the Eurozone, as 80% of debt-

¹Normally, the ECB provides cash loans to commercial banks under the Main Refinancing Operations (MROs) and the Longer-Term Refinancing Operations (LTROs). In exchange for cash loans, commercial banks must pledge eligible collateral. MROs have a maturity of one week, and LTROs of three months.
financing to non-financial corporations is intermediated by commercial banks (European Central Bank, 2015). In fact, expanding credit was one of the goals of the ECB, as stated by Mario Draghi, president of the ECB:

“...at least the banks will be more inclined to use the money - which was our prime expectation really - to expand credit to the real economy.” European Central Bank (2012).

It is not so clear, however, whether the ECB funding was used to provide new loans to the private sector. European commercial banks were undercapitalized at the time (International Monetary Fund, 2011; Hoshi and Kashyap, 2014), which made it more difficult for them to expand the balance sheet to provide new credit. Empirical evidence suggests that lending to the real economy did not expand in Italy, Spain and Portugal, which took out more than 50% of LTRO funding (Bruegel, 2015), while (domestic) bondholdings sharply increased. In addition, Drechsler, Drechsel, Marques-Ilbanez, and Schnabl (2014) find that weakly capitalized banks in the Eurozone borrowed more from the ECB, pledged riskier collateral, and actively invested in distressed sovereign debt after the European sovereign debt crisis began in 2010. Crosignani, Faria-e Castro, and Fonseca (2015) find that Portuguese banks purchased domestic bonds during the LTRO allotment. It is therefore not clear from an empirical point of view whether the funding was used to provide new credit to the real economy.

In this paper I investigate the impact of unconventional central bank lending operations on credit provision to the real economy and, through that channel, on output. Following up on this question, I compare the effectiveness of this policy with that of a recapitalization by the fiscal authority.

2Throughout the paper, I use ‘commercial banks’ and ‘financial intermediaries’ interchangeably to denote the same group of economic agents, which capture all kinds of credit institutions: commercial banks, savings banks, postbanks, and specialized credit institutions, among others.
4.1. INTRODUCTION

European commercial banks have been undercapitalized since the financial crisis of 2007-2009 (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). The Gertler and Karadi (2011) framework enables me to capture this feature, which I extend in two directions. First, commercial banks have a portfolio choice between government bonds and private loans, which are used by non-financial corporations to purchase productive ‘physical’ capital. For details, see Gertler and Karadi (2013) and Bocola (2015). Second, I introduce the possibility of collateralized central bank lending, which represents an alternative form of funding for commercial banks besides net worth and deposits. To obtain central bank funding, commercial banks have to pledge collateral, for which only government bonds are eligible. In line with LTRO policy at the time, the central bank has two instruments for its lending policy: the nominal interest rate on central bank funding and the collateral or haircut policy, which determines how much central bank funding is obtained for one euro of collateral. The haircut is the difference between the two. I model the LTRO by temporarily decreasing the interest rate on central bank funding with respect to the interest rate on deposits.

The main contribution of my paper is linking private credit provision and central bank lending operations by including a collateral constraint that determines how much central bank funding is obtained for one euro of government bonds. The requirement to pledge government bonds as collateral affects the portfolio decision between private loans and government bonds. This setup allows me to investigate the effect of unconventional central bank lending to commercial banks, such as the LTROs of December 2011 and February 2012. Within this framework I am also able to investigate the impact of the haircut policy on the credit provision channel and to explore the role of financial fragility in such operations.

My first result is that unconventional central bank lending to balance-sheet-constrained commercial banks has a contractionary short-run effect and an expansionary long-run effect. I distinguish two competing effects. First is the
collateral effect, which has a contractionary effect on output. The LTRO increases the collateral value of government bonds. Commercial banks shift into government bonds and, consequently, must shed private loans. Credit provision to the real economy falls, with a contractionary effect on output.

Second is the subsidy effect, which has an expansionary effect on output. Because the interest rate on central bank funding is below the interest rate on regular deposits, the LTRO reduces funding costs and increases commercial bank profits. Banks’ balance sheets recover more quickly, leading to a credit expansion. The collateral effect dominates the short-run effect, but disappears once the LTRO has ended. The subsidy effect is still present and dominates the long-run effect, which leads to an expansion of output.

My second and main result is that unconventional central bank lending does not have a cumulative effect on output. This result holds for different values of the haircut parameter. Interestingly, the haircut policy does have a pronounced effect on the time-pattern of output. A smaller haircut leads to a stronger collateral effect and a steeper short-run output contraction. At the same time, the smaller haircut leads to a stronger economic recovery in the long run through the subsidy effect. The mechanism is as follows. The smaller haircut increases the collateral value and leads to a larger shift into government bonds. More government bonds allow commercial banks to obtain more low-interest-rate central bank funding, thereby increasing commercial bank profits. Through the resulting faster balance sheet recovery, I obtain the expansionary effect that offsets the contractionary collateral effect.

A third result is that the provision of central bank funding, which amounts to an indirect recapitalization of the financial sector, is less effective in stimulating output than a direct recapitalization by the fiscal authority. A direct recapitalization provides commercial banks with new net worth and alleviates banks’ balance sheets constraints. Contrary to central bank lending operations, a direct recap does not involve crowding out private loans by government bonds.
4.1. INTRODUCTION

My paper is related to several strands of the literature. First, it relates to recent papers that study the transmission to the macroeconomy of shocks to the balance sheets of financial intermediaries (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011, 2013). In all these models, financial intermediaries face an endogenous leverage constraint that limits the size of the balance sheet for a given amount of net worth. Kirchner and van Wijnbergen (2012) extend this framework by introducing a portfolio choice between private loans and government debt. A debt-financed fiscal expansion increases commercial banks’ bondholdings, which crowds out private credit. My framework also features crowding out through an alternative mechanism: the provision of central bank funding at an interest rate below that on deposits increases the collateral value of government bonds and makes government bonds more attractive.

Gertler and Kiyotaki (2010) and Bocola (2015) allow for central bank lending to financial intermediaries, in an otherwise similar setup as my paper. Bocola (2015), who explicitly looks at the LTRO, finds a small positive effect on lending and output because central bank funding relaxes bank balance sheet constraints in this setup. My paper differs in two dimensions: first, central bank funding does not directly relax bank balance sheet constraints, but does so only indirectly by providing debt financing at an interest rate below that on regular deposit funding. Second, commercial banks must pledge government bonds as collateral, giving rise to the collateral effect, which is absent in Gertler and Kiyotaki (2010) and Bocola (2015). Contrary to Bocola (2015), I find a contractionary short-run effect on private credit provision and output due to the collateral effect.

Contrary to Gertler and Kiyotaki (2010) and Bocola (2015), Schabert (2015) and Hörmann and Schabert (2015) include the requirement to pledge collateral to access central bank lending facilities. These papers find that a smaller haircut has an expansionary effect on output. My paper differs along two dimensions: first, Schabert (2015) and Hörmann and Schabert (2015) explicitly incorporate money holdings, as households face a cash-in-advance constraint for which they
can borrow at the central bank. Instead, I consider a cashless economy in which central bank lending is an alternative form of debt financing at an interest rate (possibly) below that on deposits. Second, instead of providing central bank funding to unconstrained households, commercial banks in my model are the only agents in the economy with access to central bank funding. Because commercial banks are balance-sheet-constrained, forcing them to pledge collateral to obtain central bank funding affects the portfolio decision between private loans and government bonds, giving rise to the collateral effect. Contrary to Schabert (2015) and Hörmann and Schabert (2015), I find that the haircut policy does not have a cumulative impact on output, because the expansionary subsidy effect is offset by the short-run collateral effect.

Finally, my paper connects to the literature on government bond accumulation by commercial banks. Becker and Ivashina (2014) find empirical evidence for the crowding out of private loans by increased holdings of government bonds and for financial repression by governments. Crosignani (2014) develops a general equilibrium model in which banks shift into domestic sovereign debt when undercapitalized. Domestic sovereign debt has the highest payoff in the good state of the world, which is the only state banks care about. Banks have limited liability, and therefore do not care about being bankrupt in the bad state of the world. Hence, government bond accumulation by commercial banks arises because of risk shifting. Government bond accumulation in my model arises because commercial banks need government bonds for collateral purposes when the central bank engages in special lending operations with commercial banks.

I describe some stylized facts in section 4.2. The model description can be found in section 4.3, while section 4.4 discusses the calibration of my model. Section 4.5 presents the results from my simulations, and section 4.6 concludes the paper.
4.2 Stylized facts

Credit institutions in the Eurozone play a crucial role in the provision of credit to the real economy, as they intermediate approximately 80% of debt financing to non-financial corporations (European Central Bank, 2015). The LTROs of December 2011 and February 2012 provided credit institutions with fresh liquidity amounting to approximately 10% of Eurozone GDP. However, did the credit institutions use this funding to provide new loans to the real economy? Did they expand their balance sheets? In this section, I present some stylized facts regarding the aggregated balance sheets of Monetary Financial Institutions (MFIs) at the time of the LTROs of December 2011 and February 2012.

Data about the refinancing operations of the ECB were collected from Bruegel (2015), while balance sheet data of MFIs were collected from the ECB’s statistical warehouse (European Central Bank, 2015). The time series have a monthly frequency. Balance sheet data of MFIs, excluding the European System of Central Banks, are available at a country level.3 The vast majority of Euro Area MFIs are credit institutions (i.e., commercial banks, savings banks, postbanks, specialized credit institutions, among others) (European Central Bank, 2011b).

Country use of ECB funding

Figure 4.1 shows the stock of total refinancing operations in the Eurozone, as well as the country use by MFIs in Italy, Spain and Portugal. Total refinancing operations consist of Main Refinancing Operations (MROs) and Longer-Term Refinancing Operations (LTROs), which differ in their respective maturity. MROs have a maturity of one week, while regular LTROs feature a maturity of three months. Contrary to regular LTROs, the LTROs of December 2011 and

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3MFIs include “credit institutions and non-credit institutions (mainly money market funds) whose business is to receive deposits from entities other than MFIs and to grant credit and/or invest in securities” (European Central Bank, 2011b).
February 2012 featured a maturity of 36 months. I show the sum of MROs and LTROs because part of the LTROs of December 2011 and February 2012 were used to repay outstanding MROs and/or roll over outstanding LTROs.

Figure 4.1 suggests three main observations. First, the stock of total refinancing operations increased by approximately 350 billion EUR during the period in which the two unconventional LTROs were undertaken. The net increase is smaller than the gross increase of 1000 billion EUR, which was mentioned in the introduction, because part of the gross increase has been used to repay outstanding MROs and/or roll over outstanding LTROs. Second, a disproportionate share of the funding went to MFIs in Italy, Spain and Portugal. By March 1st, 2012,
more than 50% of total ECB funding had been borrowed by MFIs from these
countries, while their cumulative share in Eurozone GDP is less than one-third.
Apparently, the LTROs of December 2011 and February 2012 were especially
attractive for MFIs in Italy, Spain and Portugal. Third, the use of ECB funding
by MFIs from these three countries amounted to a large share of their respec-
tive GDP. On March 1st, 2012, ECB funding accounted for 200 billion EUR
of debt funding for Italian MFIs, which is approximately 10% of Italian GDP.
For Spain, this number is as high as 400 billion EUR, which amounts to 40% of
Spanish GDP. To sum up, ECB funding constituted a significant source of debt
funding for MFIs in Italy, Spain and Portugal.

**Balance sheet composition MFIs**

Figure 4.2 shows the total asset holdings of MFIs in Italy, Spain and Portugal.
I normalize total asset holdings to 100 on December 1st, 2011 which is one
week before the unconventional LTROs were announced by the ECB (European
Central Bank, 2011a). The figure suggests a small expansion in total MFI assets
during the period in which the two LTROs took place.

![Total assets MFIs in Italy, Spain and Portugal](image)

**Figure 4.2.** Total assets of Monetary Financial Institutions (MFI's) excluding the European System of Central Banks in Italy (IT), Spain (ES), and Portugal (PT) from January 2011 to January 2013. Levels were rescaled with respect to total assets on December 1st, 2011, which has a value of 100 in all three plots. **Source:** ECB.
A more important question from a macroeconomic point of view is whether credit to non-financial corporations and households expanded with doubts raised by Figure 4.3, which depicts private credit to the real economy as a percentage of total MFI assets: credit fell by one to two percentage points in all three countries during the period in which the two LTROs took place. There seems to be a clear break between credit levels (as a percentage of MFI assets) in the period before December 2011 and those in the period after March 2012 in Portugal and Spain. Although credit as a percentage of total MFI assets also fell in Italy during this period, credit had already been falling in the months before the first intervention, and it is less clear whether the fall can be attributed to the two LTROs. Given that balance sheets expanded very little (Figure 4.2), the data suggest that private credit intermediation measured in euros fell. The data clearly indicate that a credit expansion, as envisaged by ECB President Draghi, did not occur.

At the same time, Figure 4.4 shows an increase in domestic government bondholdings of one to one-and-a-half percentage points of total MFI assets for all three countries during the period, amounting to a striking increase of 30% in euros of government bondholdings. This result is in line with the findings of Drechsler, Drechsel, Marques-Ilbanez, and Schnabl (2014) and Crosignani, Faria-e Castro, and Fonseca (2015). There is a clear break between government bondholdings in the period before December 2011 and those in the period after March 2012.

These figures give a mixed message: while balance sheets slightly expanded in the direct aftermath of the LTROs, MFIs shifted from private credit to government bonds, given the fraction of total assets invested in a particular asset class. One possible explanation for this finding is that government bonds can easily be pledged as collateral to obtain funding from the ECB. In addition, haircuts, i.e., the amount of central bank funding obtained for one euro of collateral, are usually small for government bonds. Loans to non-financial corporations and
4.2. STYLIZED FACTS

Balance sheet composition MFIs in Italy, Spain and Portugal

Figure 4.3. Loans to non-financial corporations (NFC) and households (HH) as a percentage of total assets of Monetary Financial Institutions (MFIs) excluding the European System of Central Banks in Italy (IT), Spain (ES), and Portugal (PT) from January 2011 to January 2013. The two dashed vertical lines refer to December 1st, 2011 and March 1st, 2012, respectively, which mark the beginning and the end of the period in which the two LTROs took place, respectively. Source: ECB.

households, however, are usually difficult to convert into eligible collateral, and usually come with a larger haircut.
Figure 4.4. Domestic government bondholdings as a percentage of total assets of Monetary Financial Institutions (MFIs) excluding the European System of Central Banks in Italy (IT), Spain (ES), and Portugal (PT) from January 2011 to January 2013. The two dashed vertical lines refer to December 1st, 2011 and March 1st, 2012, respectively, which mark the beginning and the end of the period in which the two LTROs took place, respectively. Source: ECB.

4.3 Model

4.3.1 Model overview

I consider a standard New-Keynesian model, which includes commercial banks that are balance-sheet-constrained, as in Gertler and Karadi (2011), and have a portfolio choice between private loans and government bonds, as in Gertler and Karadi (2013). The asset side is financed by net worth, deposits and central bank funding. Commercial banks need to pledge collateral in the form of government bonds to obtain funding from the central bank, while private loans are not eligible as collateral. The central bank determines the interest rate on both regular deposits and central bank funding and how much funding
commercial banks receive for one euro in government bonds, i.e., the haircut applied to collateral. The central bank can decrease the interest rate on central bank funding in times of financial crises, compared with the interest rate on regular household deposits, and finances its lending operations by issuing deposits to households. Central bank profits and losses are transferred period by period to the fiscal authority.

In addition to commercial banks and a central bank, the economy contains households, a production sector, and a fiscal authority. Each household consists of workers and bankers. Workers supply labor to intermediate goods producers, while bankers run the commercial banks. Intermediate goods producers borrow from commercial banks to purchase physical capital from capital goods producers, who face convex adjustment costs. Labor is hired the next period, together with capital used for the production of the intermediate good. Intermediate goods producers sell their goods to retail goods producers and sell the used capital back to the capital goods producers. Retail goods producers face monopolistic competition and price-stickiness, as in Calvo (1983) and Yun (1996). Final goods producers operate in a perfectly competitive market and purchase from retail goods producers while taking prices as given. The final good is sold to households for consumption, to capital goods producers as investment, and to the government. The government honors outstanding obligations, (potentially) provides financial sector support, covers possible central bank losses, and finances these expenditures by raising lump-sum taxes and issuing (long-term) debt, in a way similar to Woodford (1998, 2001). Fiscal policy is determined via (exogenous) fiscal rules.

4.3.2 Households

A continuum of households with measure one are infinitely lived, and exhibit identical preferences and asset endowments. Each household consists of bankers and workers. There is perfect consumption insurance within the household,
which allows me to keep the representative agent representation. Households obtain utility from consumption $c_t$, while labor $h_t$ provides disutility. Households receive income from labor at wage rate $w_t$. Households can invest in one period debt $a_t$, which consists of deposits $d_t$ and loans to the central bank (CB) $d_t^{cb}$, which can be treated as perfect substitutes from the household’s point of view by assumption, i.e. $a_t = d_t + d_t^{cb}$. Investment of $a_{t-1}$ in period $t-1$ yields repayment of principal $a_{t-1}$ and interest $r^d_t$ in period $t$. Households can also invest in government bonds with return $r^b_t$ on their holdings $q_{t-1}^{b,h}$, where $q_{t-1}^{b,h}$ is the bond price in period $t-1$, and $s_{t-1}^{b,h}$ the number of bonds purchased in period $t-1$. Besides that, they receive income from profits $\Pi_t$ from the production sector and the financial sector. Income is used for consumption $c_t$, investment in one period debt $a_t$ and investment in government bonds $s_{t}^{b,h}$ at price $q_t^b$. Households pay a cost for the intermediation of government bonds, which is quadratic in the deviation of the number of bonds from the level $\hat{s}_{b,h}$ to capture in a simple way the limited participation in asset markets by households (Gertler and Karadi, 2013). The government levies lump sum taxes $\tau_t$. Household members derive utility from consumption and leisure, with habit formation in consumption to more realistically capture consumption dynamics, as in Christiano, Eichenbaum, and Evans (2005). Households maximize expected life-time utility subject to the budget constraint:

$$\max \left\{ c_{t+i}, h_{t+i}, a_t, s_{t+i}^{b,h} \right\}_{i=0}^\infty E_t \left\{ \sum_{i=0}^\infty \beta^i \left[ \log (c_{t+i} - \nu c_{t-1+i}) - \Psi \frac{h_{t+i}^{1+\phi}}{1+\phi} \right] \right\}$$

s.t.

$$c_t + \tau_t + a_t + q_t^b s_{t}^{b,h} + \frac{\kappa_{s_{b,h}}}{2} \left( s_t^{b,h} - \hat{s}_{b,h} \right)^2 = w_t h_t + \left( 1 + r^d_t \right) a_{t-1} + \left( 1 + r^b_t \right) q_{t-1}^{b,h} s_{t-1}^{b,h} + \Pi_t$$
This will give rise to the following first order conditions:

\[ c_t : \lambda_t = (c_t - \nu c_{t-1})^{-1} - \beta \nu E_t \left[ (c_{t+1} - \nu c_t)^{-1} \right], \quad (4.1) \]
\[ h_t : \Psi h_t^\phi = \lambda t w_t, \quad (4.2) \]
\[ a_t : E_t \left[ \beta \Lambda_{t,t+1} \left( 1 + r_{t+1}^d \right) \right] = 1, \quad (4.3) \]
\[ s^{b,h}_t : E_t \left\{ \beta \Lambda_{t,t+1} \left[ \frac{1 + r_{t+1}^b}{q_t^b + \kappa_{sb,h} (s^{b,h}_t - \hat{s}_{b,h})} \right] \right\} = 1, \quad (4.4) \]

where \( \lambda_t \) is the marginal utility of consumption. The household’s stochastic discount factor is \( \beta \Lambda_{t,t+1} = \beta \lambda_{t+1} / \lambda_t \). Equation (4.1) denotes the marginal utility from an additional unit of consumption, while equation (4.2) equates the marginal cost from an additional unit of labor in the form of disutility with the marginal benefit from additional wage income. Equation (4.3) and (4.4) weigh the benefit from an additional unit of consumption tomorrow from investing in deposits respectively government bonds, with the cost of lower consumption today.

### 4.3.3 Financial intermediaries

**Optimization problem**

Financial intermediaries channel funds from savers to borrowers. They invest in two asset classes: private loans to intermediate goods producers \( s^{k,p}_{j,t} \) and government bonds \( s^{b,p}_{j,t} \). Total assets \( p_{j,t} \) are given by:

\[ p_{j,t} = q^k_t s^{k,p}_{j,t} + q^b_t s^{b,p}_{j,t}, \]
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

where $q^k_t$ is the price of private loans, and $q^b_t$ the price of government bonds. Intermediaries fund their assets through net worth $n_{j,t}$, risk-free household deposits $d_{j,t}$ and central bank funding $d_{cb,j,t}$:

$$p_{j,t} = n_{j,t} + d_{j,t} + d_{cb,j,t}^c.$$ 

New net worth is the difference between the return on assets and the return on debt funding. Net worth can be increased through financial sector support by the government, which is proportional to previous period net worth ($n^g_{j,t} = \tau_t n_{j,t-1}$). Net worth decreases when the financial sector has to repay previously administered support, which is also proportional to previous period net worth, ($\tilde{n}^g_{j,t} = \tilde{\tau}_t n_{j,t-1}$). $\tau_t$ and $\tilde{\tau}_t$ will be defined in section 4.3.5. The law of motion for individual net worth is given by:

$$n_{j,t+1} = \left(1 + r^k_{t+1}\right) q^{k,p}_{t} s_{j,t}^{k,p} + \left(1 + r^b_{t+1}\right) q^{b,p}_{t} s_{j,t}^{b,p} - \left(1 + r^d_{t+1}\right) d_{j,t} - \left(1 + r^{cb}_{t+1}\right) d_{cb,j,t}^c + n^g_{j,t+1} - \tilde{n}^g_{j,t+1} = \left(1 + r^k_{t+1} + \tau_{t+1} - \tilde{\tau}_{t+1}\right) q^{k,p}_{t} s_{j,t}^{k,p} + \left(1 + r^b_{t+1} + \tau_0 n_{j,t-1} - \tilde{\tau}_0 n_{j,t-1}\right) q^{b,p}_{t} s_{j,t}^{b,p} - \left(1 + r^d_{t+1} + \tau_{t+1} - \tilde{\tau}_{t+1}\right) d_{j,t} - \left(1 + r^{cb}_{t+1} + \tau_{t+1} - \tilde{\tau}_{t+1}\right) d_{cb,j,t}^c,$$

where $r^k_t$ is the net real return on private loans in period $t$, $r^b_t$ the net real return on government bonds, $r^d_t$ the net real return on deposits and $r^{cb}_t$ the net real return on central bank funding. Following Gertler and Karadi (2011), I assume intermediaries are forced to shut down with probability $\sigma$, which is i.i.d. and exogenous, both in time and the cross-section. When the intermediary is forced to stop operating, all net worth is paid out to the household, the ultimate owners

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of the financial intermediary. The financial intermediary maximizes expected future discounted profits:

\[
V \left( s_{k,p}^{j,t-1}, s_{b,p}^{j,t-1}, d_{j,t-1}, a_{j,t-1} \right) = \max E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) n_{j,t+1} + \sigma V \left( s_{j,t}^{k,p}, s_{j,t}^{b,p}, d_{j,t}, a_{j,t} \right) \right] \right\}
\]

Households face an agency problem when deciding on the amount of funds to save through deposits, as in Gertler and Karadi (2011, 2013): financial intermediaries have the capability to divert assets when moving from the current to the next period. Depositors can force the intermediary into bankruptcy in that case, but will only recoup a fraction \(1 - \lambda_a\) of asset class \(a = \{k, b\}\). The remaining fraction \(\lambda_a\) of each asset class is paid out as a dividend to the household owning the intermediary. Depositors, however, anticipate this possibility, and will in equilibrium only provide deposits up to the point where the continuation value of the intermediary is larger or equal than the opportunity cost of diverting the assets.

\[
V \left( s_{k,p}^{j,t-1}, s_{b,p}^{j,t-1}, d_{j,t-1}, a_{j,t-1} \right) \geq \lambda_k q_k s_{j,t}^{k,p} + \lambda_b q_b s_{j,t}^{b,p}. \tag{4.5}
\]

Financial intermediaries have access to central bank funding \(d_{j,t}^{cb}\), but are required to pledge collateral in the form of government bonds. The commercial bank remains the legal owner of the bond, and will receive the interest payments from the bond after repayment of \(d_{j,t}^{cb}\) to the central bank in period \(t + 1\), unless the financial intermediary becomes insolvent, a case I abstain from through a proper calibration. The collateral constraint has the following functional form:

\[
d_{j,t}^{cb} \leq \theta q_t s_{j,t}^{b,p}, \tag{4.6}
\]

where \(\theta\) is the haircut parameter that regulates the collateral policy of the central bank, see section 4.3.5. A low \(\theta\), or a high haircut, indicates that a commercial
bank will obtain little central bank financing for one euro of government bonds. The optimization problem of the financial intermediary can now be formulated:

\[
V_{j,t} = \max_{\{s_{j,t}^{k,p}, s_{j,t}^{b,p}, d_{j,t}, d_{j,t}^{cb}\}} E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) n_{j,t+1} + \sigma V_{j,t+1} \right] \right\},
\]

s.t.

\[
V_{j,t} \geq \lambda_k q_{j,t}^{k,p} s_{j,t}^{k,p} + \lambda_b q_{j,t}^{b,p} s_{j,t}^{b,p},
\]

\[
n_{j,t} + d_{j,t} + d_{j,t}^{cb} = q_{j,t}^{k,p} s_{j,t}^{k,p} + q_{j,t}^{b,p} s_{j,t}^{b,p},
\]

\[
n_{j,t} = \left( 1 + r_t^k + \bar{\alpha}_t^n - \bar{\alpha}_t^n \right) q_{t-1}^{k,p} s_{j,t-1}^{k,p} + \left( 1 + r_t^b + \bar{\alpha}_t^n - \bar{\alpha}_t^n \right) q_{t-1}^{b,p} s_{j,t-1}^{b,p} - \left( 1 + r_t^d + \bar{\alpha}_t^n - \bar{\alpha}_t^n \right) d_{j,t-1} - \left( 1 + r_t^{cb} + \bar{\alpha}_t^n - \bar{\alpha}_t^n \right) d_{j,t-1}^{cb},
\]

\[
\theta q_{j,t}^{b,p} s_{j,t}^{b,p} \geq d_{j,t}^{cb},
\]

where I have abbreviated the value function of the financial intermediary by:

\[
V_{j,t} = V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t-1}^{cb} \right).
\]

**First order conditions**

Appendix 4.A.1 shows that the problem of the financial intermediary leads to the following first-order conditions:

\[
\frac{\lambda_b}{\lambda_k} E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right] = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right] + \theta \left( \frac{\psi_t}{1 + \mu_t} \right), (4.7)
\]

\[
\lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right], (4.8)
\]

\[
\frac{\psi_t}{1 + \mu_t} = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^d - r_{t+1}^{cb} \right) \right], (4.9)
\]

\[
\eta_t = E_t \left[ \Omega_{t,t+1} \left( 1 + r_{t+1}^d + \bar{\alpha}_t^n - \bar{\alpha}_t^n \right) \right], (4.10)
\]

where \( \mu_t \) is the Lagrangian multiplier on the bank’s balance sheet constraint (4.5), and \( \psi_t \) the Lagrangian multiplier on the collateral constraint (4.6). \( \eta_t \) de-
notes the shadow value of an additional unit of net worth. The intermediaries’ stochastic discount factor is given by
\[ \Omega_{t,t+1} = \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right], \]
and can be interpreted as the household’s stochastic discount factor \( \beta \Lambda_{t,t+1} \), augmented by an additional term to incorporate the effect of the financial frictions.

First order condition (4.7) pins down the bank’s portfolio choice on the asset side of the balance sheet. The left hand side denotes the marginal benefit to the financial intermediary from investing an additional unit of private loans, valued by the intermediaries’ stochastic discount factor, and corrected by the term \( \lambda_b/\lambda_k \) to reflect the fact that the financial friction is more severe for private loans than for government bonds. The right hand side denotes the marginal cost of giving up an additional unit of government bonds, measured by the credit spread between government bonds and the deposit rate. But government bonds also derive value from the fact that they serve as collateral with which intermediaries can obtain central bank funding, which is reflected by the second term on the right hand side of equation (4.7).

Equation (4.8) is the first order condition for private loans. We clearly see that the presence of a binding bank balance sheet constraint (\( \mu_t > 0 \) in equation (4.5)) limits the ability of commercial banks to arbitrage away the difference between the expected rate of return on private loans and deposits, since commercial banks cannot expand the balance sheet.

The portfolio choice on the liabilities side of the balance sheet is given by equation (4.9): an increase in the credit spread between deposits and central bank funding \( r^d_{t+1} - r^{cb}_{t+1} \) increases the collateral value of government bonds \( \psi_t \), everything else equal, and leads to a shift into government bonds, see (4.7), which allows the commercial bank to increase the amount of central bank funding.

**Proposition 1:** central bank lending and the collateral policy only have a first order effect when both the bank balance sheet constraint (4.5) is binding
and the interest rate on deposits $r^d_{t+1}$ is not equal to the interest rate on central bank funding $r^{cb}_{t+1}$.

**Lemma 1:** If the bank balance sheet constraint (4.5) is not binding, then there is no first order effect from central bank lending on private credit.

**Proof:** Consider optimal choices for the financial intermediary. When equation (4.5) is not binding, $\mu_t$ is equal to zero. From equation (4.8), we see that the expected return on private loans must equal the expected rate on deposits. The term on the left hand side of equation (4.7) is zero, and is not affected by $\psi_t$, which captures the effects from central bank lending.

**Lemma 2:** When the interest rate on deposits $r^d_{t+1}$ and central bank funding $r^{cb}_{t+1}$ are equal, there is no effect from central bank lending and/or the haircut parameter $\theta$.

**Proof:** When $r^d_{t+1} = r^{cb}_{t+1}$, we find from equation (4.9) that $\psi_t = 0$. This implies that the second term on the right hand side of equation (4.7) drops out. This is the only first order condition where $\psi_t$, which measures the collateral value of government bonds, and the haircut parameter $\theta$ show up. Hence, central bank lending does not affect the portfolio decision between private loans and government bonds.

**Proof of Proposition 1:** To have an effect from central bank lending, lemma 2 shows that the interest rate on deposits and central bank funding has to differ. Lemma 1 shows that even when the interest rate on deposits and central bank funding is different (captured by $\psi_t > 0$), there is no first order effect unless the bank balance sheet constraint is binding ($\mu_t > 0$).

Note that when the interest rate on deposits and central bank funding is the same ($\psi_t = 0$), commercial banks are indifferent between the two funding sources, since the collateral value of government bonds is now zero. Deposits
and central bank funding are perfect substitutes. Hence I can take the collateral constraint to be binding in my simulations, irrespective of the difference between the interest rate on deposits and that on central bank funding.

Because of the proposition, and the fact that European commercial banks have been undercapitalized since the financial crisis of 2007-2009 (International Monetary Fund, 2011; Hoshi and Kashyap, 2014), I will take both the bank’s balance sheet constraint (4.5) and the collateral constraint (4.6) to be binding in my simulations. When the bank’s balance sheet constraint (4.5) is binding, I can rewrite it into the following equation:

\[(1 + \mu_t) \eta_t n_{j,t} = \lambda_k q_{t}^{k,p} s_{j,t} + \lambda_b q_{t}^{b,p} s_{j,t}, \quad (4.11)\]

It is clear from equation (4.11) that the size of the balance sheet is limited by the amount of current net worth \(n_{j,t}\).

**Aggregate law of motion net worth**

The law of motion for aggregate net worth consists of the net worth of the bankers that are allowed to continue operating, together with the aggregate net worth given to new bankers, which is equal to a fraction \(\chi\) of previous period assets \(p_{t-1}\). Together with net government support \(n_{g,t} - \tilde{n}_{g,t}\), I obtain the following law of motion:

\[n_t = \sigma \left[ \left( r_t^k - r_t^d \right) q_{t-1}^k s_{t-1}^k + \left( r_t^b - r_t^d \right) q_{t-1}^b s_{t-1}^b + \left( r_t^d - r_t^{cb} \right) d_{t-1}^{cb} \right. \right. \]
\[+ \left. \left( 1 + r_t^d \right) n_{t-1} \right] + \chi p_{t-1} + n_{g,t} - \tilde{n}_{g,t}. \quad (4.12)\]

I introduce the variable \(\omega_t^k\) to denote the fraction of assets invested in private loans:

\[\omega_t^k = q_t^k s_t^k / p_t. \quad (4.13)\]
4.3.4 Production sector

The production factor is modeled in standard New-Keynesian fashion. I will shortly outline the setup below, with a more detailed exposition in appendix 4.A.2.

Intermediate Goods Producers

A continuum of intermediate goods producers, that face perfect competition, acquire capital $k_{i,t-1}$ from capital producers at the end of period $t - 1$ for a price $q_{i-1}^k$ through a state-contingent loan $s_{i,t-1}^k = k_{i,t-1}$ from the financial intermediaries. Next period’s profits can credibly be pledged to the intermediaries, as in Gertler and Kiyotaki (2010). After realization of the shocks, the producers hire labour $h_{i,t}$ at a wage $w_t$, and start producing intermediate goods with previous period capital $k_{i,t-1}$ and labor $h_{i,t}$ as input. After production, the intermediate goods producers pay a state-contingent net real return $r_t^k$ over claims issued in period $t$, with the following production technology:

$$y_{i,t} = z_t (\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha}.$$  

Quality of capital $\xi_t$ and total factor productivity $z_t$ are driven by exogenous AR(1) processes.

Output $y_{i,t}$ is sold to retail firms for a price $m_t$. The effective capital stock (after depreciation) is sold to the capital producers for a price $q_t^k$ and the proceeds are used to pay back the loans and a net return to the financial intermediaries, after paying wages set in a perfectly competitive labor market:

$$w_t = (1 - \alpha) m_t y_{i,t} / h_{i,t}, \quad (4.14)$$

$$1 + r_t^k = \frac{\alpha m_t y_{i,t} / k_{i,t-1} + q_t^k (1 - \delta) \xi_t}{q_{t-1}^k}.$$  

(4.15)
A capital quality shock $\xi_t$ decreases the return on capital for two reasons: production decreases, as capital becomes less productive, reducing the first term of (4.15). But the capital price $q_t^k$ will fall, leading to a further decrease in the return on capital because of the second term in (4.15).

**Capital Producers**

Capital producers purchase the effective capital stock that is left after production (including depreciation), $(1 - \delta)\xi_t k_{t-1}$, from the intermediate goods producers. They also purchase an amount $i_t$ of final goods, and convert the old capital stock and newly purchased final goods into new capital. The newly produced capital stock $k_t$ is subsequently sold to the intermediate goods producers at the same price $q_t^k$ that was paid for the capital after production. The capital producers face convex adjustment costs, so that for every unit $i_t$ only $1 - \Psi(t_t)$ units of capital are produced, with $t_t = i_t/i_{t-1}$ representing the change in the investment level. The expression for the capital stock after the capital producers have produced (or output of capital producers) is then:

$$k_t = (1 - \delta)\xi_t k_{t-1} + [1 - \Psi(t_t)] i_t,$$

with $\Psi(t_t) = \frac{\gamma}{2} (t_t - 1)^2$.

**Retail Firms**

A continuum of differentiated retail firms indexed by $i \in [0, 1]$ transform intermediate goods $y_{i,t}$ into differentiated retail goods $y_{f,t} = y_{i,t}$ under monopolistic competition. Each period, only a random portion $(1 - \psi)$ of retail firms is allowed to reset their prices $P_{f,t}$, while the other firms must keep their prices fixed, see Calvo (1983) and Yun (1996). Retail firms face the demand function $y_{f,t} = (P_{f,t}/P_t)^{-\varepsilon} y_t$, with $\varepsilon > 1$ and price index $P_t^{1-\varepsilon} = \int_0^1 P_{f,t}^{-\varepsilon} df$. 
Final Goods Producers

Final goods producers purchase the differentiated retail goods $y_{f,t}$ to produce final goods. They face the following technology constraint:

$$ y_t^{(\varepsilon-1)/\varepsilon} = \int_0^1 y_{f,t}^{(\varepsilon-1)/\varepsilon} df, $$

where $\varepsilon$ represents the elasticity of substitution between goods bought from the retail firms. Final good producers operate in a perfectly competitive market. Hence they take prices as given, and sell their goods for the same price $P_t$. Final goods are sold to households and government for consumption, and to capital producers as input for investment.

4.3.5 Government

Fiscal authority

The government issues $b_t$ long term bonds in period $t$, and raises $q_t^b b_t$ in revenue, with $q_t^b$ the market price of bonds. I parametrize the maturity structure of government debt like Woodford (1998, 2001). A bond issued in period $t-1$ pays a cash flow $r_c$ in period $t$, $\rho r_c$ in period $t+1$, $\rho^2 r_c$ in period $t+2$, etc. The rate of return $r_t^b$ on a bond purchased in period $t-1$ is given by:

$$ 1 + r_t^b = \frac{r_c + \rho q_t^b}{q_{t-1}^b}, $$

(4.16)

where $\rho$ pins down the maturity\(^4\) of government debt, see for more details Van der Kwaak and Van Wijnbergen (2014). The government also raises revenue by levying lump sum taxes $\tau_t$ on the households and receives profits $\Pi_t^{cb}$ from the central bank. Government purchases are constant in real terms: $g_t = G$. Furthermore, the government may provide assistance to the financial sector by

\(^4\)The duration of the bond is equal to $\sum_{t=1}^{\infty} b_t^{t}(\rho^{t-1} r_c) / \sum_{t=1}^{\infty} b_t^{t}(\rho^{t-1} r_c)$.
injecting new net worth \( n_t^g \). The government also receives repayment from previously administered support \( \tilde{n}_t^g \). The budget constraint is given by:

\[
q_t^b b_t + \tau_t + \tilde{n}_t^g + \Pi_t^{cb} = g_t + n_t^g + \left( 1 + r_t^b \right) q_{t-1}^b b_{t-1}.
\] (4.17)

The tax rule of the government is given by a rule which makes sure the intertemporal government budget constraint is satisfied (Bohn, 1998):

\[
\tau_t = \bar{\tau} + \kappa_b (b_t - \bar{b}) + \kappa_n n_t^g, \quad \kappa_b \in (0, 1], \quad \kappa_n \in [0, 1].
\] (4.18)

where \( \bar{b} \) is the steady state level of debt. \( \kappa_n \) controls the way government transfers to the financial sector are financed. If \( \kappa_n = 0 \), support is financed by new debt. \( \kappa_n = 1 \) implies that the additional spending is completely financed by increasing lump sum taxes. I parametrize government support as follows:

\[
\begin{align*}
n_t^g &= \tau_t n_{t-1}, \quad \zeta \leq 0, \quad l \geq 0, \\
\tau_t^l &= \zeta \varepsilon_{\xi, t-l}.
\end{align*}
\] (4.19)

Thus the government provides funds to the financial sector if \( \zeta < 0 \) (a negative shock \( \varepsilon_{\xi, t-l} \) to the quality of capital). Depending on the value of \( l \), the government can provide support instantaneously \( (l = 0) \), or with a lag \( (l > 0) \). Furthermore, \( \vartheta \) indicates the extent to which the government needs to be repaid:

\[
\tilde{n}_t^g = \vartheta n_{t-e}^g, \quad \vartheta \geq 0, \quad e \geq 1.
\] (4.20)

\( \vartheta = 0 \) means the support is a gift from the government. In case \( \vartheta = 1 \), the government aid is a zero interest loan, while a \( \vartheta > 1 \) implies that the financial intermediaries have to pay interest over the support received earlier.\(^5\) The pa-

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\(^5\)The case where \( \vartheta > 1 \) happened in the Netherlands, where financial intermediaries received government aid with a penalty rate of 50 percent. EU state support rules usually require financial intermediaries to repay previously received state support with a penalty rate.
parameter \( e \) denotes the amount of time after which the government aid has to be paid back.

**Central Bank Conventional Interest Rate Policy**

The Central Bank sets the nominal interest rate on deposits \( r^n_t \) according to a standard Taylor rule, in order to minimize output and inflation deviations:

\[
    r^n_t = (1 - \rho_r) [\rho^n + \kappa_\pi (\pi_t - \bar{\pi}) + \kappa_y \log (y_t / y_{t-1})] + \rho_r r^n_{t-1} + \epsilon_{r,t},
\]

(4.21)

where \( \epsilon_{r,t} \sim N(0, \sigma^2_r) \), and \( \kappa_\pi > 1 \) and \( \kappa_y > 0 \) (active monetary policy). The parameter \( \bar{\pi} \) is the target inflation rate. The term \( \kappa_\xi (\xi_t - \bar{\xi}) \) can be interpreted as a conventional monetary stimulus in times of financial crises. The real interest rate on deposits then equals:

\[
    1 + r^d_t = (1 + r^n_t) / \pi_t.
\]

(4.22)

**Central Bank Balance Sheet Policy**

Besides conventional interest rate policy, the central bank lends \( d^c_b_t \) to the commercial banking system. In order to obtain access to this facility, commercial banks are required to provide collateral in the form of government bonds, see equation (4.6). Collateral is needed in order for the creditor to recoup the principal in case of a debtor’s bankruptcy. A haircut is applied to protect the creditor from capital losses on the collateral.

The central bank has two instruments for its balance sheet policy. It controls the nominal interest rate \( r^{n,cb}_t \) on central bank funding provided to commercial banks, and the haircut parameter \( \theta \) applied to the collateral. For a given interest rate \( r^{n,cb}_t \) and haircut parameter \( \theta \), the central bank provides as much funding as demanded by the commercial banks, in line with the Fixed Rate Full Alotment
4.3. MODEL

policy of the ECB after October 2008. The central bank can lower the nominal interest rate $r_{n,cb}$ in times of crisis with respect to the interest rate on regular deposit funding $r_{n}$ by increasing the spread $\Gamma_{cb}^c$:

$$r_{n,cb} = r_{n} - \Gamma_{cb}^c.$$  (4.23)

The spread $\Gamma_{cb}^c$ between the nominal rate on deposits and the nominal rate on central bank funding is given by:

$$\Gamma_{cb}^c = \tilde{\Gamma}_{cb} + \kappa_{cb}(\xi_t - \bar{\xi}).$$  (4.24)

where $\tilde{\Gamma}_{cb}$ is the steady state spread. When $\kappa_{cb} < 0$, a financial crisis increases the interest rate spread between deposits and central bank funding, decreasing funding costs for commercial banks.

The second policy instrument is the haircut parameter $\theta$ that is applied to the collateral. From equation (4.6) we see that a commercial bank delivering $q_{j,s,j,t}^b$ in government bonds, provides the commercial bank $\theta q_{j,s,j,t}^b$ units of central bank funding. Hence the collateral haircut is $1 - \theta$. A higher value of $\theta$ allows the commercial bank to obtain more central bank funding for the same number of government bonds, which increases the collateral value of government bonds.

The asset side of the central bank balance sheet consists of loans $d_{cb}$ to commercial banks on which it receives a nominal interest rate $r_{n,cb}$. The liabilities consist of household deposits on which it pays the nominal deposit rate $r_{n}$. Cen-

---

6Before October 2008, the ECB used to auction a given amount of funding against eligible collateral, with the interest rate being determined in the auctioning process. In October 2008, the ECB switched to a Fixed Rate Full Alotment policy, under which the ECB sets the collateral haircuts and the interest rate, and provides commercial banks with the funding demanded (European Central Bank, 2015).

7Central banks in most advanced economies are not directly financed through deposits, but through interest bearing commercial bank reserves. I could model this by including commercial bank reserves on the asset side of commercial banks. Commercial banks would not be capable of diverting commercial bank reserves in such a setup because the central bank is in charge of the reserve system. Commercial banks are not balance-sheet-constrained in financing commercial bank reserves in such a setup. Hence explicitly modelling commercial bank reserves is equivalent to letting the central bank be financed directly by household deposits, see also Gertler and Karadi (2011).
Central bank profits (or losses) are passed on to the fiscal authority period by period. Hence central bank net worth is zero, and liabilities consist solely of household deposits. Real central bank profits (or actually losses, since \( r_{t-1}^{n,cb} \leq r_{t-1}^{n} \)), are given by:

\[
\Pi_{t}^{cb} = \left( \frac{1 + r_{t-1}^{n,cb}}{\pi_{t}} \right) d_{t-1}^{cb} - \left( \frac{1 + r_{t-1}^{n}}{\pi_{t}} \right) d_{t-1}^{cb} = \left( r_{t}^{cb} - r_{t}^{d} \right) d_{t-1}^{cb},
\]

(4.25)

where \( \pi_{t} \) is the gross inflation rate, and the net real return on central bank funding \( r_{t}^{cb} \) is given by:

\[
1 + r_{t}^{cb} = \frac{1 + r_{t-1}^{n,cb}}{\pi_{t}}.
\]

(4.26)

Note that without the central bank intervention, commercial banks would have to pay the interest rate \( r_{t}^{d} \). Hence the central bank losses \( \Pi_{t}^{cb} \) can be interpreted as the subsidy given by the central bank to the commercial banks.

### 4.3.6 Market clearing

In equilibrium, the total number of private loans \( k_{t} \) must equal the total number of loans provided by the financial intermediaries. Similarly, the total bond supply must equal the bonds purchased by the household sector and financial intermediaries:

\[
k_{t} = s_{t}^{k,p},
\]

(4.27)

\[
b_{t} = s_{t}^{b,h} + s_{t}^{b,p}.
\]

(4.28)

The aggregate resource constraint is given by:

\[
y_{t} = c_{t} + i_{t} + g_{t} + \frac{1}{2} \kappa_{s_{t}^{b,h}} \left( s_{t}^{b,h} - \hat{s}_{b,h} \right)^{2}.
\]

(4.29)
4.4. CALIBRATION

4.3.7 Equilibrium

The resulting equilibrium definition can be found in appendix 4.D, which gives a complete description of the model including the standard first order conditions that have been left out in the main text.

4.4 Calibration

I calibrate the model on a quarterly frequency. I solve the model using a perturbation method that solves for the policy function with a first-order approximation around the non-stochastic steady state. The parameter values can be found in Table 4.1. Most of the parameter values are common in the literature on DSGE models, or frequently used in models containing financial frictions. I follow Gertler and Karadi (2011) for these parameters. Standard parameter values are the subjective discount factor $\beta$, the habit formation parameter $\upsilon$, the inverse Frisch elasticity $\varphi$, the elasticity of substitution for final goods producers $\varepsilon$, the Calvo probability of keeping prices fixed $\psi$, the capital share in output $\alpha$, the investment adjustment parameter $\gamma$, and the smoothing parameters for production $\rho_z$ and the quality of capital $\rho_\xi$. Parameters regarding conventional monetary policy are set at relatively standard values. The interest rate smoothing parameter $\rho_r$ is set at 0.4, to reflect a more aggressive response of conventional monetary policy, which is in normal times in the range of 0.8 – 0.9.

Other coefficients are calibrated to match specific targets: the relative utility weight of labor $\Psi$ is calibrated to have the steady state labor supply equal 1/3. A crucial parameter in my analysis is the intermediation cost for households on bond holdings $\kappa_{s,b,h}$. This affects the elasticity of household demand for government bonds in response to changes in the bond price. When commercial banks want to increase their bond holdings because of central bank funding provided at an attractive interest rate, this parameter indicates how willing households are to sell government bonds to commercial banks. I set $\kappa_{s,b,h}$ equal to 0.0025. For
### Table 4.1. Model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.990</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\upsilon$</td>
<td>0.815</td>
<td>Degree of habit formation</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>3.6023</td>
<td>Relative utility weight of labor</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.276</td>
<td>Inverse Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\kappa_{s,b,h}$</td>
<td>0.0025</td>
<td>Constant portfolio adjustment cost function</td>
</tr>
<tr>
<td>$\delta_{b,h}$</td>
<td>1.6656</td>
<td>Reference level portfolio adjustment cost function</td>
</tr>
<tr>
<td><strong>Financial Intermediaries</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_k$</td>
<td>0.3861</td>
<td>Fraction of private loans that can be diverted</td>
</tr>
<tr>
<td>$\lambda_b$</td>
<td>0.1930</td>
<td>Fraction of gov’t bonds that can be diverted</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.0026</td>
<td>Proportional transfer to entering bankers</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.95</td>
<td>Survival rate of the bankers</td>
</tr>
<tr>
<td><strong>Intermediate good firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>4.176</td>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.779</td>
<td>Calvo probability of keeping prices fixed</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.330</td>
<td>Effective capital share</td>
</tr>
<tr>
<td><strong>Capital good firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.728</td>
<td>Investment adjustment cost parameter</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0592</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td><strong>Autoregressive components</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.95</td>
<td>Autoregressive component of productivity</td>
</tr>
<tr>
<td>$\rho_\xi$</td>
<td>0.66</td>
<td>Autoregressive component of capital quality</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.4</td>
<td>Interest rate smoothing parameter</td>
</tr>
<tr>
<td><strong>Policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_c$</td>
<td>0.04</td>
<td>Real payment to government bondholder</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.96</td>
<td>Parameter government debt duration (5 yrs)</td>
</tr>
<tr>
<td>$\kappa_b$</td>
<td>0.050</td>
<td>Tax feedback parameter from government debt</td>
</tr>
<tr>
<td>$\kappa_\pi$</td>
<td>1.500</td>
<td>Inflation feedback on nominal interest rate</td>
</tr>
<tr>
<td>$\kappa_y$</td>
<td>0.125</td>
<td>Output feedback on nominal interest rate</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.010</td>
<td>Standard deviation productivity shock</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>0.050</td>
<td>Standard deviation capital quality shock</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.0025</td>
<td>Standard deviation interest rate surprise shock</td>
</tr>
</tbody>
</table>
this parameter value, a decrease in the interest rate on central bank funding of 50 basispoints on impact with respect to the interest rate on deposits leads to an increased recourse to central bank funding of approximately 5% of annual steady state GDP, which is on the conservative side in comparison with the cumulative take up of LTRO funding in December 2011 and February 2012, as was shown in Section 4.2.

Other parameters are calibrated to match data from the periphery of the Eurozone, where most LTRO funding was taken up.\footnote{The Eurozone periphery consists of Greece, Ireland, Italy, Portugal, and Spain in my calibration} I target the steady state investment-GDP ratio and the steady state government spending-GDP ratio, and set both to 0.2, in line with long term average values in the periphery. The depreciation rate $\delta$ is calibrated to target the steady state investment-GDP ratio and a steady state credit spread $\Gamma_k$ between private loans and deposits of 50 basispoints (quarterly), which is the average difference between the interest rate on total loans to non-financial corporations and total overnight deposits to households and non-profit institutions in the periphery of the Eurozone over the period from July 2010 to June 2011.

The steady state leverage ratio is targeted by taking monthly country level data on Monetary Financial Institutions (MFIs) excluding the European System of Central Banks (ESCB) from the European Central Bank (2015). I aggregate total MFI assets from the periphery countries, as well as MFI capital and reserves. I divide aggregate MFI assets over aggregate MFI capital and reserves to find a monthly time-series for the average MFI leverage ratio, which was equal to 12 in the beginning of 2011. Due to the fact that the cash flows from private loans are the residual after wages have been paid out, private loans are more like equity than debt in my model. Taking a leverage ratio of 12 would therefore overstate fluctuations in net worth. Hence I reduce the steady state leverage ratio to 6, a procedure also applied by Gertler and Karadi (2013).
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

The parameter $\sigma$ is set in such a way that the average survival period for bankers is 20 quarters. To make sure that this parameter value is not driving the model results, I investigate the model response for different values of $\sigma$ in appendix 4.E. I calibrate the diversion parameter for government bonds $\lambda_b$ to equal 0.5 times the diversion rate for private loans $\lambda_k$, as in Gertler and Karadi (2013). To make sure that this value does not drive my results, I investigate the model response for different values of $\lambda_b$ in appendix 4.E. The haircut parameter $\theta$ is set to 0.95, implying a haircut of 5% on government bonds pledged to the central bank as collateral, which is in line with the haircut on periphery sovereign debt at the time. The steady state spread $\Gamma_{cb}$ between the interest rate on deposits and central bank lending facilities is equal to zero, reflecting the fact that these facilities only have value to commercial banks in times of financial crisis, and are otherwise perfect substitutes.

I calibrate the steady state government liabilities $\bar{q}_b\bar{b}$ to be equal to 100% of annual steady state output, in line with the average debt level in the periphery at the start of the crisis in 2011. The average duration of government bonds is 5 years and can be calculated from the stress tests performed by the European Banking Authority (2011). The cash flow payment to the bondholder $r_c$ is set to 0.04, in line with coupon payments on long term government debt. The tax feedback parameter $\kappa_b$ is set to 0.05 in order to guarantee intertemporal solvency of the budget constraint of the fiscal authority. The effect of this parameter is limited, because taxes are lump-sum in my model. We set the steady state fraction of government bonds placed at commercial banks to be equal to 25% of total government bonds, approximately equal to the average fraction of periphery bonds placed at periphery banks. A financial crisis is initiated as in Gertler and Karadi (2011) by having a one standard deviation capital quality shock of $\sigma_\xi = 0.050$, and an autoregressive component of $\rho_\xi = 0.66$.

I conduct several robustness checks in appendix 4.E. I will perform this check along two dimensions. First, I will change some of the key parameters of the
model. A second line of robustness checks is along the dimension of model specification. I perform these checks to make sure that my model results do not depend on some arbitrary parameter choice or a particular model specification.

4.5 Results

In this section I discuss the results from the model simulations. I first simulate a financial crisis with no additional support measures to explain the general mechanism of the model. I then proceed to look at the case where the interest rate on central bank funding is decreased with respect to the interest rate on regular deposits, capturing the LTROs of December 2011 and February 2012. I continue by investigating the impact of different haircut parameters, and I conclude by comparing the effect of the LTRO with the effect of an immediate debt-financed recapitalization by the fiscal authority.

4.5.1 Financial crisis impact, no additional policy

I start by inspecting the response of the economy to a financial crisis in Figure 4.5 and 4.6, where the financial crisis is initiated through a capital quality shock $\xi_t$ of 5% compared with the steady state, as in Gertler and Karadi (2011). For now, I abstain from a policy intervention by the central bank. The capital quality shock deteriorates the return on private loans, and hence commercial banks suffer losses, as can be seen from equation (4.15). Net worth falls, and (commercial) bank balance sheet constraints tighten. Credit spreads and (expected) interest rates increase, reducing the demand for private loans and capital, which leads to a drop in the price of capital. Remember that the capital is sold by intermediate goods producers after production, and the proceeds are used to repay the loan from the commercial bank. Hence a lower capital price leads to additional losses on the outstanding loans to the intermediate goods producers. Net
worth of commercial banks falls further, leading to a second round of interest rate increases.

Financial crisis impact, no additional policy intervention

Balance sheet tightening of commercial banks does not only affect credit spreads on private loans, but also induces commercial banks to sell government bonds. Bond prices go down, and impose capital losses on existing bondholders, resulting in an additional fall in commercial banks’ net worth on top of the losses on private loans, further tightening bank balance sheet constraints. Since the interest rate on deposits and central bank funding are equal, it follows from equation (4.9) that the collateral value of government bonds is zero. Commercial banks shrink their balance sheet by selling government bonds to the household sector, thereby increasing the fraction of the balance sheet invested in private credit, see panel “Fraction of assets in private loans”. As previously mentioned, banks can only sell government bonds to households, but not private loans. A lower bond price makes it more attractive for households (who are not balance-
4.5. RESULTS

Financial crisis impact, no additional policy intervention

Figure 4.6. Impulse response functions for the case with no additional policy. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

sheet-constrained) to purchase additional government bonds. When commercial banks offload government bonds to households, central bank lending falls along by approximately 80% with respect to the steady state.

Lower private credit provision adversely affects the real economy: investment drops by 8% with respect to steady state, and a lower capital stock leads to lower wages and reduced household income. The wealth effect causes consumption to fall, generating, together with the fall in investment, a drop in output of almost 3%.
4.5.2 No additional policy vs. unconventional monetary policy

In this section I will derive my first result. Consider the effect of an unconventional central bank intervention in Figures 4.7 and 4.8. I compare the no intervention case from section 4.5.1 (blue, solid) with a policy that entails an increase in the interest rate spread $\Gamma^c_{bt}$ between the nominal interest rate on deposits and the central bank lending facilities of 50 quarterly basis points on impact (red, slotted) to simulate the LTRO intervention by the ECB in December 2011 and February 2012.9

Financial crisis impact, no additional policy vs. unconventional monetary policy

![Figure 4.7. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.](image)

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9Under the LTRO, commercial banks were allowed to borrow for 3 years at a nominal interest rate equal to the MRO rate, which is the short term policy rate of the ECB. The difference with the 1-year Euribor, a measure of unsecured interbank funding is at least 1%, see also Figure 4.13 in appendix 4.C. I therefore take an annual spread between central bank lending facilities and deposits of 2% since the LTRO has a maturity of 3 years, compared with a 1 year maturity of the 1-year Euribor. This gives a quarterly spread of 50 basis points.
Financial crisis impact, no additional policy vs. unconventional monetary policy

Figure 4.8. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

The lower interest rate induces commercial banks to shift from funding through regular deposits to central bank funding, which increases net worth everything else equal. It follows from equation (4.9) that the (collateral) value of a government bond to commercial banks increases: not only does a bond produce a cash flow in the future, it also allows the intermediary to gain immediate access to additional central bank funding. However, since intermediaries are balance-sheet-constrained, purchasing more government bonds also forces them to shift out of private loans ("Fraction of assets in private loans"), which are not eligible as collateral. Demand for private loans is reduced, and pushes down the price of capital further, leading to additional capital losses on private loans. Net worth falls further, and tightens bank balance sheet constraints, leading to a second round of interest rate increases on private loans and additional capital losses. The credit spread between private loans and deposits increases on impact by...
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75 basis points compared with the no intervention case. Net worth drops with respect to the no intervention case on impact. The shift out of private loans initially leads to a substantial drop in investment and pushes down the trough of output by almost 0.5%. Note, however, that the low-interest-rate central bank funding allows an expansion of commercial banks’ balance sheets (“Total bank assets”). The extra space on the balance sheet, though, is not used to expand lending to the real economy, but to purchase additional government bonds.

The lower interest rate on central bank funding, however, increases commercial bank profits, and leads (except in the initial period) to higher net worth compared with the no intervention case. But as long as the interest rate on central bank funding is below that on deposit funding, intermediaries will invest a larger fraction of their balance sheet in government bonds, thereby crowding out private credit. Commercial banks, however, are better capitalized after the period of lower interest rates on central bank funding has ended, and have more space on their balance sheets to finance capital purchases by the intermediate goods producers. Investment increases with respect to the no intervention case, leading to a faster recovery along the entire time path once the intervention has ended.

In the short run, these results contradict the regular narrative concerning a monetary expansion, in which lower interest rates lead to higher output. Instead, the collateral effect dominates the subsidy effect. The key reason is the fact that commercial banks are balance sheet constrained, and cannot increase their holdings of government bonds, necessary as collateral, without shedding private loans. In the long run, central bank lending at an interest rate below that on deposits amounts to an indirect recapitalization of commercial banks, which leads to a stronger economic recovery.

My results seem to be in line with the data surrounding the period in which the LTRO was undertaken by the ECB: just as in the data, we see a portfolio shift from private loans to government bonds. This would explain why credit to
the real economy did not expand in the aftermath of the LTRO, contrary to the expectations of ECB President Draghi, alluded to in the introduction.

The results clearly introduce a trade-off for policymakers: if they are concerned about the short run, unconventional central bank lending operations such as the LTRO do not seem to be a good idea, due to the contractionary short run impact. But when they are concerned about the long run, the intervention is beneficial, as it supports the long run recovery by delivering a commercial banking system that is better capitalized. This raises two questions. First, is the cumulative effect of the central bank intervention positive or negative? Second, can the haircut policy, indicated by $\theta$, affect the outcome, and if so how?
4.5.3 The cumulative intervention effect and the role of the haircut policy

In this section I will derive the main result of my paper. We saw in the previous section that the collateral effect dominates the short run effect, while the subsidy effect dominates the long run effect. This leads us to the question whether the cumulative effect of increased central bank lending is positive or negative. I therefore introduce the cumulative discounted multiplier $\mu_D$:

$$\mu_D = \frac{\sum_s \beta^s \left( y_{t+s}^{cb} - y_{t+s}^{np} \right)}{\sum_s \beta^s \left( d_{t+s}^{cb,cb} - d_{t+s}^{cb,np} \right)},$$

where $x_{t+s}^{cb}$ denotes the value of variable $x$ in period $t + s$ under the central bank intervention, and $x_{t+s}^{np}$ the value of variable $x$ in period $t + s$ under the no policy case. Figure 4.9 displays $\mu_D$ versus the haircut parameter $\theta$. We clearly see that $\mu_D$ is basically zero, and hardly changes as I vary $\theta$. This results in two conclusions: first, cumulatively, the subsidy effect and the collateral effect offset each other, with a net effect that is zero. Second, $\mu_D$ hardly changes as the haircut parameter $\theta$ is varied. This suggests that the collateral policy of the central bank has no macroeconomic effects. This seems counterintuitive, given the way the central bank intervention affected the real economy in the previous section, see Figures 4.7 and 4.8.

We therefore turn to Figure 4.10, which is the result from the same comparison between no additional policy and unconventional central bank lending as in Figures 4.7 and 4.8 but for several values of the haircut parameter $\theta$. Figure 4.10 shows the difference between the two policies, expressed as a percentage of steady state output. The upper panel displays the difference in central bank lending, while the lower panel displays the difference in output.

Contrary to the results in Figure 4.9, we see that the haircut parameter $\theta$ has a large effect on output and central bank lending. A smaller haircut (larger $\theta$)
leads to a stronger collateral effect and a steeper short-run output contraction. In the long run, however, a larger $\theta$ leads to a stronger subsidy effect and, through this effect, to a stronger economic recovery.

This can be explained in the following way: the smaller haircut leads to a larger collateral value: commercial banks obtain more low-interest-rate central bank funding for one euro of collateral. The higher collateral value induces more crowding out of private loans by government bonds and leads to a stronger collateral effect. Commercial bank profits, however, increase, as a larger share of the balance sheet is financed through low-interest-rate central bank funding. The recovery of bank balance sheets is accelerated, and leads to a stronger subsidy effect and a long-run credit expansion. Long-run output increases, and offsets the stronger initial contractionary impact.

Figures 4.9 and 4.10 lead to a clear conclusion regarding the effectiveness of providing low-interest-rate central bank funding: the cumulative effect from
the intervention compared with the no intervention case is basically zero. Hence the policy is not very effective in stimulating output, irrespective of the haircut parameter $\theta$. This rather ineffective policy, however, does not imply that the central bank cannot affect the real economy. Quite the contrary, the haircut parameter $\theta$ allows the central bank to influence the time path of the recovery, and shift output losses between periods. Setting a high $\theta$ induces commercial banks to shift into government bonds (with a stronger collateral effect and a short run contractionary impact) but indirectly recapitalizes commercial banks because of a stronger subsidy effect. Reducing $\theta$ mitigates the short run contractionary
impact of the central bank policy, but leaves commercial banks weaker capitalized once the central bank intervention has ended. But no matter what haircut policy \( \theta \) the central bank pursues, the cumulative impact with respect to the no intervention cases is basically zero. This leaves the question whether other policies are more effective in stimulating private credit provision and output. I will address this question in the next section.
4.5.4  Unconventional monetary policy vs. immediate recapitalization

We have seen in the previous section that unconventional central bank lending operations are rather ineffective, despite the fact that the commercial banking sector is indirectly recapitalized through the provision of central bank funding at an interest rate below that on deposits. I therefore investigate in Figures 4.11 and 4.12 whether a direct recapitalization by the fiscal authority is more effective. The blue solid line refers to the central bank intervention from section 4.5.2, while the red slotted line refers to an immediate debt-financed recapitalization of 1.25% of annual steady state output, see equation (4.19).

Financial crisis impact: unconventional monetary policy vs. immediate recap

![Graphs showing impulse response functions](image)

Figure 4.11. Impulse response functions for the case from section 4.5.2 with a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (blue, solid) vs. the case where the commercial banking system receives new net worth equal to 1.25% of annual GDP (red, slotted) through an immediate debt-financed recapitalization by the fiscal authority. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

We clearly see that the debt-financed recap has a positive effect compared with the central bank intervention, which is most striking in the initial quarters.
4.5. RESULTS

Financial crisis impact: unconventional monetary policy vs. immediate recap

Figure 4.12. Impulse response functions for the case from section 4.5.2 with a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (blue, solid) vs. the case where the commercial banking system receives new net worth equal to 1.25% of annual GDP (red, slotted) through an immediate debt-financed recapitalization by the fiscal authority. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

...mitigated. There are two key reasons why the recap seems to be more effective. First, commercial banks immediately receive new net worth, in contrast to the indirect recap by the central bank, in which commercial banks are gradually recapitalized through lower interest rates on central bank funding. A direct recap therefore immediately alleviates bank balance sheet constraints, allowing commercial banks to expand the balance sheet at once.

The second reason is the fact that, contrary to the central bank intervention, a direct recap does not increase the collateral value of government bonds. The interest rate on household deposits and the central bank lending facilities is the
same under the direct recap policy, and hence the collateral value, indicated by $\psi_t$ in equation (4.9) is zero. Commercial banks do not have an incentive to load up on government bonds, as is the case under the unconventional central bank intervention. The collateral effect is therefore absent under the direct recap policy, see also equation (4.7), and only the subsidy effect is present.

Even though the initial drop in net worth is almost 20% larger under the unconventional central bank policy, net worth levels have converged after approximately 10 quarters. The credit spread between private loans and deposits, which measures not only the tightness of the bank balance sheet constraint, but also the profitability of private loans if no new shocks arrive, is much larger under the unconventional central bank policy. As long as the credit spread is larger under the unconventional monetary policy case, net worth accumulation is faster, resulting in a convergence of net worth. The collateral value of government bonds is zero under both policies after approximately 10 quarters, and net worth and other credit market conditions are basically the same, resulting in similar long run macroeconomic outcomes.
4.6 Conclusion

The European Central Bank (ECB) conducted in December 2011 and February 2012 the largest refinancing operation in its history by providing the European commercial banking system with more than 1000 billion EUR in funding. One of the goals of the program, as stated by ECB president Draghi, was to expand credit to the real economy. Commercial banks within the Eurozone play a crucial role in this respect, as more than 80% of debt financing to non-financial corporations is intermediated by commercial banks. Empirical evidence, though, suggests that rather than expanding private credit, the ECB funding was used to purchase additional government bonds. In this paper I investigate whether unconventional central bank lending operations can expand credit provision to the real economy, and through that channel stimulate output.

I construct a DSGE model with balance-sheet-constrained commercial banks that have a portfolio choice between private loans and government bonds. The central bank provides funding to commercial banks, for which commercial banks pledge collateral in the form of government bonds. I model the LTRO intervention of the ECB by decreasing the interest rate on central bank funding with respect to that on regular deposit funding. I contribute to the literature by linking private credit provision and central bank lending operations through a collateral constraint which determines how much central bank funding is obtained for one euro of government bonds pledged as collateral. Government bonds therefore have a collateral value, which affects the portfolio decision of the commercial bank.

I find a contractionary short-run effect, but an expansionary long-run effect on credit provision and output when the central bank engages in unconventional lending operations with a balance-sheet-constrained commercial banking system. The provision of central bank funding at an interest rate below that on deposits induces commercial banks to shift from private credit to government
bonds, an effect referred to as the collateral effect, which has a contractionary short-run effect on output. Bank balance sheets recover faster, however, as profits increase due to lower interest rate payments, which increases credit provision and output in the long run, referred to as the subsidy effect.

My main result is that there is no cumulative effect on output with respect to the no intervention case. This result is independent of the haircut policy of the central bank. A smaller haircut provides commercial banks with more central bank funding for one euro of government bonds pledged as collateral, and induces a shift from private loans to government bonds, which increases the contractionary impact of the collateral effect. At the same time, the shift into government bonds allows the commercial banks to finance a larger fraction of their balance sheet through central bank funding, thereby decreasing interest rate payments and increasing commercial bank profits: the stronger collateral effect implies a stronger long-run subsidy effect, which offsets the collateral effect. Even though the cumulative impact on output is zero, the haircut policy allows the central bank to shift output gains and losses over time.

I find that an immediate debt-financed recapitalization is more effective in expanding private credit than unconventional central bank lending operations. The recap instantaneously increases net worth of commercial banks and alleviates bank balance sheet constraints, similar to the unconventional lending operations, but does not induce commercial banks to shift into government bonds. Credit to the real economy expands, and leads to an increase in output across the entire time path.

My model explains why banks did not expand credit to the real economy in response to the LTROs of December 2011 and February 2012. The LTROs, which were particularly attractive for banks in Italy, Spain and Portugal, increased the collateral value of government bonds, as they could be used to get additional LTRO funding. To buy more government bonds, banks had to shift from private credit, as they were undercapitalized at the time (Hoshi and
Kashyap, 2014). To avoid this collateral effect, governments could have recapitalized the banking sector directly or through European funds, which is more likely to have expanded credit provision to the real economy than the LTROs.

4.A Derivations

4.A.1 Financial intermediaries

In the main text, the collateral constraint is given by $d_{j,t}^{cb} \leq \theta q_t^{b,p} s_{j,t}^{b,p}$. In this appendix I will apply a more general formulation, namely $d_{j,t}^{cb} \leq \theta \kappa_t s_{j,t}^{b,p}$, where $\kappa_t$ can be equal to:

$$\kappa_t = \begin{cases} 
q_t^b & \text{“Regular collateral constraint”;} \\
1 & \text{“No risk-adjustment collateral constraint”}.
\end{cases}$$

I also include the possibility of financial sector net worth support by the government $n_{j,t}^{g} = \tau_t^n n_{j,t-1}$ and repayment of earlier provided support $\tilde{n}_{j,t}^{g} = \bar{\tau}_t^n n_{j,t-1}$. 

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The law of motion for net worth, which includes recapitalizations by the government and financial sector repayments is given by:

\[
n_{j,t+1} = \left(1 + r^k_{t+1}\right) q^k_t s_{j,t}^p + \left(1 + r^b_{t+1}\right) q^b_t s_{j,t}^p - \left(1 + r^d_{t+1}\right) d_{j,t} - \left(1 + r^{cb}_{t+1}\right) d_{j,t}^{cb} + n^g_{j,t+1} - \bar{n}_{j,t+1}
\]

\[
= \left(1 + r^k_{t+1}\right) d_{j,t} s_{j,t}^p + \left(1 + r^b_{t+1}\right) q^b_t s_{j,t}^p - \left(1 + r^d_{t+1}\right) d_{j,t} - \left(1 + r^{cb}_{t+1}\right) d_{j,t}^{cb} + n^n_{j,t+1} - n^n_{t+1} n_{j,t}
\]

\[
= \left(1 + r^k_{t+1}\right) d_{j,t} s_{j,t}^p + \left(1 + r^b_{t+1}\right) q^b_t s_{j,t}^p - \left(1 + r^d_{t+1}\right) d_{j,t} - \left(1 + r^{cb}_{t+1}\right) d_{j,t}^{cb}
\]

Now we remember the optimization problem of the financial intermediary:

\[
V_{j,t} = \max_{\left\{ s_{j,t}^p, s_{j,t}^b, d_{j,t}, d_{j,t}^{cb} \right\}} E_t \left\{ \beta \lambda \left[ \left(1 - \sigma \right) n_{j,t+1} + \sigma V_{j,t+1} \right] \right\},
\]

s.t.

\[
V_{j,t} \geq \lambda_k q^k_t s_{j,t}^p + \lambda_b q^b_t s_{j,t}^p,
\]

\[
n_{j,t} + d_{j,t} + d_{j,t}^{cb} \geq q^k_t s_{j,t}^p + q^b_t s_{j,t}^p,
\]

\[
n_{j,t} = \left(1 + r^k_t + \tau^n_t - \tau^n_{t+1}\right) q^k_t s_{j,t}^p + \left(1 + r^b_t + \tau^n_t - \tau^n_{t+1}\right) q^b_t s_{j,t}^p - \left(1 + r^d_t + \tau^n_t - \tau^n_t\right) d_{j,t} - \left(1 + r^{cb}_t + \tau^n_t - \tau^n_{t+1}\right) d_{j,t-1}^{cb},
\]

\[
\theta \kappa_s s_{j,t}^b \geq d_{j,t}^{cb}
\]
where we have abbreviated the value function of the financial intermediary by $V_{j,t} = V\left(s^{k,p}_{j,t-1}, s^{b,p}_{j,t-1}, d_{j,t-1}, d^{cb}_{j,t-1}\right)$. We set up the accompanying Lagrangian of the problem:

$$
\mathcal{L} = (1 + \mu_t) E_t \left\{ \beta A_{t+1} \left[ (1 - \sigma) \left( \left( 1 + r^{k}_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1} \right) q^{k,p}_{t} s^{k,p}_{j,t} \right. \right. \\
+ \left. \left. \left( 1 + r^{b}_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1} \right) q^{b,p}_{t} s^{b,p}_{j,t} \right. \right. \\
- \left. \left. \left( 1 + r^{cb}_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1} \right) d^{cb}_{j,t} \right) + \sigma V\left(s^{k,p}_{j,t}, s^{b,p}_{j,t}, d_{j,t}, d^{cb}_{j,t}\right) \right\} \\
- \mu_t \lambda_k q^{k,p}_{t} s^{k,p}_{j,t} - \mu_t \lambda_b q^{b,p}_{t} s^{b,p}_{j,t} \\
+ \chi_t \left[ \left( 1 + r^{k}_{t} + \tau^n_{t} - \tilde{\tau}^n_{t} \right) q^{k,p}_{t-1} s^{k,p}_{j,t-1} + \left( 1 + r^{b}_{t} + \tau^n_{t} - \tilde{\tau}^n_{t} \right) q^{b,p}_{t-1} s^{b,p}_{j,t-1} \right. \\
- \left. \left( 1 + r^{d}_{t} + \tau^n_{t} - \tilde{\tau}^n_{t} \right) d_{j,t-1} - \left( 1 + r^{cb}_{t} + \tau^n_{t} - \tilde{\tau}^n_{t} \right) d^{cb}_{j,t-1} \right. \\
- \left. d^{k,p}_{t} s^{k,p}_{j,t} - d^{b,p}_{t} s^{b,p}_{j,t} + d_{j,t} + d^{cb}_{j,t} \right] + \psi_t \left( \theta \kappa_t s^{b,p}_{j,t} - d^{cb}_{j,t} \right). 
$$
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This gives rise to the following first order conditions:

\[
\begin{align*}
\dot{s}_{j,t}^{k,p} & : (1 + \mu_t) E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) \left( 1 + r_{t+1}^{k} + \tau_{t+1}^{n} - \tau_{t+1}^{c} \right) q_{t}^{k} \right. \right. \\
& \left. \left. + \frac{\partial V \left( s_{j,t}^{k,p}, s_{j,t}^{b,p}, d_{j,t}, d_{j,t}^{cb} \right)}{\partial s_{j,t}^{k,p}} \right) \right\} - \mu_t \lambda_k q_{t}^{k} - \chi_t q_{t}^{k} = 0, \quad (4.30) \\
\dot{s}_{j,t}^{b,p} & : (1 + \mu_t) E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) \left( 1 + r_{t+1}^{b} + \tau_{t+1}^{n} - \tau_{t+1}^{c} \right) q_{t}^{b} \right. \right. \\
& \left. \left. + \frac{\partial V \left( s_{j,t}^{k,p}, s_{j,t}^{b,p}, d_{j,t}, d_{j,t}^{cb} \right)}{\partial s_{j,t}^{b,p}} \right) \right\} - \mu_t \lambda_b q_{t}^{b} - \chi_t q_{t}^{b} + \psi_t \theta \kappa_t = 0, \quad (4.31) \\
\dot{d}_{j,t} & : (1 + \mu_t) E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) \left( 1 + r_{t+1}^{d} + \tau_{t+1}^{n} - \tau_{t+1}^{c} \right) \right] \right. \\
& \left. \left. (-1) + \frac{\partial V \left( s_{j,t}^{k,p}, s_{j,t}^{b,p}, d_{j,t}, d_{j,t}^{cb} \right)}{\partial d_{j,t}} \right) \right\} + \chi_t = 0, \quad (4.32) \\
\dot{d}_{j,t}^{cb} & : (1 + \mu_t) E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) \left( 1 + r_{t+1}^{cb} + \tau_{t+1}^{n} - \tau_{t+1}^{c} \right) \right] \right. \\
& \left. \left. (-1) + \frac{\partial V \left( s_{j,t}^{k,p}, s_{j,t}^{b,p}, d_{j,t}, d_{j,t}^{cb} \right)}{\partial d_{j,t}^{cb}} \right) \right\} + \chi_t - \psi_t = 0, \quad (4.33)
\end{align*}
\]

with complementary slackness conditions:

\[
\begin{align*}
\mu_t & : \left[ V \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}, d_{j,t-1}^{cb} \right) - \lambda_k q_{t-1}^{k} s_{j,t-1}^{k,p} - \lambda_b q_{t-1}^{b} s_{j,t-1}^{b,p} \right] \mu_t = 0 \tag{4.34} \\
\chi_t & : \left[ \left( 1 + r_{t}^{k} + \tau_{t}^{n} - \tau_{t}^{c} \right) q_{t}^{k} s_{j,t-1}^{k,p} + \left( 1 + r_{t}^{b} + \tau_{t}^{n} - \tau_{t}^{c} \right) q_{t}^{b} s_{j,t-1}^{b,p} \right. \\
& \left. \left. - \left( 1 + r_{t}^{d} + \tau_{t}^{n} - \tau_{t}^{c} \right) d_{j,t-1} - \left( 1 + r_{t}^{cb} + \tau_{t}^{n} - \tau_{t}^{c} \right) d_{j,t-1}^{cb} \right) \chi_t = 0, \tag{4.35} \\
\psi_t & : \left( \theta \kappa_t s_{j,t}^{b,p} - d_{j,t}^{cb} \right) \psi_t = 0. \tag{4.36}
\end{align*}
\]
Now we apply the envelope theorem to find the derivatives:

\[
\frac{\partial V}{\partial s_{j,t}^{k,p}} \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^c, d_{j,t-1}^{cb} \right) = \chi_t \left( 1 + r_t^k + \tau_t^n - \tau_t^n \right) q_{t-1}^k, \tag{4.37}
\]

\[
\frac{\partial V}{\partial s_{j,t}^{b,p}} \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^c, d_{j,t-1}^{cb} \right) = \chi_t \left( 1 + r_t^b + \tau_t^n - \tau_t^n \right) q_{t-1}^b, \tag{4.38}
\]

\[
\frac{\partial V}{\partial d_{j,t-1}} \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^c, d_{j,t-1}^{cb} \right) = -\chi_t \left( 1 + r_t^d + \tau_t^n - \tau_t^n \right), \tag{4.39}
\]

\[
\frac{\partial V}{\partial d_{j,t-1}^{cb}} \left( s_{j,t-1}^{k,p}, s_{j,t-1}^{b,p}, d_{j,t-1}^c, d_{j,t-1}^{cb} \right) = -\chi_t \left( 1 + r_t^{cb} + \tau_t^n - \tau_t^n \right). \tag{4.40}
\]

Substitution of the envelope conditions (4.37) - (4.40) in (4.30) - (4.33), we find the following relation between the different assets:

\[
s_{j,t}^{k,p} = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \chi_{t+1} \right] \left( 1 + r_{t+1}^k + \tau_{t+1}^n - \tau_{t+1}^n \right) \right\} \tag{4.41}
\]

\[
s_{j,t}^{b,p} = \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} - \left( \frac{\kappa_t}{q_t^b} \right) \theta \left( \frac{\psi_t}{1 + \mu_t} \right) = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \chi_{t+1} \right] \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \tau_{t+1}^n \right) \right\} \tag{4.42}
\]

\[
d_{j,t} = \frac{\chi_t}{1 + \mu_t} = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \chi_{t+1} \right] \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \tau_{t+1}^n \right) \right\} \tag{4.43}
\]

\[
d_{j,t}^{cb} = \frac{\chi_t}{1 + \mu_t} - \frac{\psi_t}{1 + \mu_t} = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \chi_{t+1} \right] \left( 1 + r_{t+1}^{cb} + \tau_{t+1}^n - \tau_{t+1}^n \right) \right\}. \tag{4.44}
\]
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Now we define the following variables:

\[ \eta_t = \frac{\chi_t}{1 + \mu_t}, \]  
(4.45)

\[ \nu_t^k = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \eta_t, \]  
(4.46)

\[ \nu_t^b = \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} - \left( \frac{\kappa_t}{\psi_t} \right) \theta \left( \frac{\psi_t}{1 + \mu_t} \right) \]  
(4.47)

\[ \eta_t^{cb} = \frac{\chi_t}{1 + \mu_t} - \frac{\psi_t}{1 + \mu_t} = \eta_t - \frac{\psi_t}{1 + \mu_t} \]  
(4.48)

We remember that we can rewrite (4.45) into \( \chi_t = (1 + \mu_t) \eta_t \). Substitution of (4.45) - (4.48) into (4.41) - (4.44). This gives rise to the following first order conditions:

\[ \nu_t^k = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right] \left( 1 + r^k_{t+1} + \tau^n_{t+1} - \tau^n_{t+1} \right) \right\}, \]  
(4.49)

\[ \nu_t^b = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right] \left( 1 + r^b_{t+1} + \tau^n_{t+1} - \tau^n_{t+1} \right) \right\}, \]  
(4.50)

\[ \eta_t = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right] \left( 1 + r^d_{t+1} + \tau^n_{t+1} - \tau^n_{t+1} \right) \right\}, \]  
(4.51)

\[ \eta_t^{cb} = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right] \left( 1 + r^{cb}_{t+1} + \tau^n_{t+1} - \tau^n_{t+1} \right) \right\}. \]  
(4.52)

Now we assume a particular function for the value function, and will later check whether our first order conditions are consistent with it:

\[ V_{j,t} = V \left( s^{k,p}_{j,t-1}, s^{b,p}_{j,t-1}, d_{j,t-1}, d^{cb}_{j,t-1} \right) = V_t q_t^{k,p} s_{j,t}^{k,p} + V_t q_t^{b,p} s_{j,t}^{b,p} - \eta_t d_{j,t} - \eta_t^{cb} d_{j,t}^{cb} \]  

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Substitution of the first order conditions (4.46) - (4.48) in the value function of the typical financial intermediary gives the following expression:

\[
V_{j,t} = V_t^k q_t^k s_{j,t}^k + V_t^b q_t^b s_{j,t}^b - \eta_t d_{j,t} - \eta_t^{cb} d_{j,t}^{cb}
\]

\[
= \left[ \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \eta_t \right] q_t^k s_{j,t}^k + \left[ \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \eta - \left( \frac{\kappa_t}{d_t} \right) \theta \left( \frac{\psi_t}{1 + \mu_t} \right) \right] q_t^b s_{j,t}^b - \eta_t d_{j,t} - \eta_t^{cb} d_{j,t}^{cb}
\]

\[
= \eta_t \left( q_t^k s_{j,t}^k + q_t^b s_{j,t}^b - d_{j,t} - d_{j,t}^{cb} \right) + \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b \right)
\]

\[
+ - \frac{\psi_t}{1 + \mu_t} \left( \theta \kappa_t s_{j,t}^b q_t^b - d_{j,t}^{cb} \right)
\]

\[
= \eta_t n_{j,t} + \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b \right)
\]

(4.53)

where the term with \( \psi_t \) drops out because of the slackness condition (4.36). We can now rewrite the leverage constraint:

\[
\eta_t n_{j,t} + \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b \right) \geq \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b \quad \Rightarrow
\]

\[
\left( 1 - \frac{\mu_t}{1 + \mu_t} \right) \left( \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b \right) \leq \eta_t n_{j,t} \quad \Rightarrow
\]

\[
\lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b \leq (1 + \mu_t) \eta_t n_{j,t}.
\]

(4.54)

We can now rewrite the value function of the financial intermediary (4.53) in the following way in case the leverage constraint (4.54) is binding:

\[
V_{j,t} = \eta_t n_{j,t} + \frac{\mu_t}{1 + \mu_t} \left( \lambda_k q_t^k s_{j,t}^k + \lambda_b q_t^b s_{j,t}^b \right)
\]

\[
= \eta_t n_{j,t} + \frac{\mu_t}{1 + \mu_t} \left( 1 + \mu_t \right) \eta_t n_{j,t} = (1 + \mu_t) \eta_t n_{j,t}
\]

(4.55)
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Now we substitute the expressions for the value function of the financial intermediary (4.55) in the expression for the expected discounted profits of the financial intermediary to obtain the following expression:

\[
V_{j,t} = \max E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) n_{j,t+1} + \sigma V_{j,t+1} \right] \right\}
= E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) n_{j,t+1} + \sigma \left( 1 + \mu_{t+1} \right) \eta_{t+1} n_{j,t+1} \right] \right\}
= E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] n_{j,t+1} \right\}
= E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left[ 1 + r_{t+1}^k + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right] d_{k,s,j,t}^{k,p}
+ \left( 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) q_{t,s,j,t}^{b,p} - \left( 1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) d_{j,t}
- \left( 1 + r_{t+1}^{cb} + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right) d_{j,t}^{cb} \right\}
\]

Comparing with the initial guess for the solution, we obtain the following first order conditions:

\[
\nu_t^k = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left[ 1 + r_{t+1}^k + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right] \right\},
(4.56)
\]

\[
\nu_t^b = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left[ 1 + r_{t+1}^b + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right] \right\},
(4.57)
\]

\[
\eta_t = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left[ 1 + r_{t+1}^d + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right] \right\},
(4.58)
\]

\[
\eta_t^{cb} = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right] \left[ 1 + r_{t+1}^{cb} + \tau_{t+1}^n - \bar{\tau}_{t+1}^n \right] \right\}.
(4.59)
\]
We see that the solutions (4.49) - (4.52) and (4.56) - (4.59) coincide, and hence that our initial guess for the value function is correct. The law of motion for aggregate net worth is given by:

\[
\nu_t = \sigma \left[ (r^k_t - r^d_t) q^{k,p}_{t-1} s_{t-1} + (r^b_t - r^d_t) q^{b,p}_{t-1} s_{t-1} \right] + \left( r^d_t - r^{cb}_t \right) d_{t-1} + \left( 1 + r^d_t \right) n_{t-1} + \chi p_{t-1} + n^g_t - \tilde{n}^g_t \quad (4.60)
\]
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Financial Sector First Order Conditions

The resulting first order conditions for the financial sector are now given by:

\[ \nu_k^t = \lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} \]  \hspace{2cm} (4.61)

\[ \nu_b^t = \lambda_b \left( \frac{\mu_t}{1 + \mu_t} \right) + \frac{\chi_t}{1 + \mu_t} - \left( \frac{\kappa_t}{q_t^b} \right) \theta \left( \frac{\psi_t}{1 + \mu_t} \right) \]  \hspace{2cm} (4.62)

\[ \eta_t = \frac{\chi_t}{1 + \mu_t}, \]  \hspace{2cm} (4.63)

\[ \eta^{cb}_t = \frac{\chi_t}{1 + \mu_t} - \frac{\psi_t}{1 + \mu_t} \]  \hspace{2cm} (4.64)

\[ \nu_k^t = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right] \left( 1 + r_{t+1}^k + \tau^k_{t+1} - \bar{\tau}^k_{t+1} \right) \right\}, \]  \hspace{2cm} (4.65)

\[ \nu_b^t = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right] \left( 1 + r_{t+1}^b + \tau^b_{t+1} - \bar{\tau}^b_{t+1} \right) \right\}, \]  \hspace{2cm} (4.66)

\[ \eta_t = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right] \left( 1 + r_{t+1}^d + \tau^p_{t+1} - \bar{\tau}^n_{t+1} \right) \right\}, \]  \hspace{2cm} (4.67)

\[ \eta^{cb}_t = E_t \left\{ \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \right] \left( 1 + r_{t+1}^{cb} + \tau^{cb}_{t+1} - \bar{\tau}^n_{t+1} \right) \right\}. \]  \hspace{2cm} (4.68)

\[ (1 + \mu_t) \eta_t n_{j,t} \geq \lambda_k q_t^{k,p} s_{j,t}^{k,p} + \lambda_b q_t^{b,p} s_{j,t}^{b,p} \]  \hspace{2cm} (4.69)

\[ n_t = \sigma \left[ \left( r_t^k - r_t^d \right) q_{t-1}^{k,p} s_{t-1}^{k,p} + \left( r_t^b - r_t^d \right) q_{t-1}^{b,p} s_{t-1}^{b,p} \right. \]  \hspace{2cm} (4.70)

\[ + \left( r_t^d - r_t^{cb} \right) d_t^{cb} + \left( 1 + r_t^d \right) n_{t-1} + \chi p_{t-1} + n_t^g - \bar{n}_t^g \]  \hspace{2cm} (4.70)

\[ d_t^{cb} = \theta \kappa_t s_t^{b,p} \]  \hspace{2cm} (4.71)
Further simplification of the F.O.C.’s for mathematical proofs

Now we combine some of the F.O.C.’s found in section 4.A.1 to obtain a better economic understanding and more intuition. We start by combining (4.61) and (4.62), while substituting (4.63) for $\chi_t/(1+\mu_t)$ to obtain:

$$\nu_t^b - \eta_t = \frac{\lambda_b}{\lambda_k} \left( \nu_t^k - \eta_t \right) - \left( \frac{\kappa_t}{q_t^b} \right) \theta \left( \frac{\psi_t}{1+\mu_t} \right)$$

Substitution of the expressions (4.65), (4.66) and (4.67) results in the following equation:

$$\frac{\lambda_b}{\lambda_k} E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right] = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right] + \left( \frac{\kappa_t}{q_t^b} \right) \theta \left( \frac{\psi_t}{1+\mu_t} \right),$$

where $\Omega_{t,t+1} = \beta \Lambda_{t,t+1} [(1-\sigma) + \sigma (1+\mu_{t+1}) \eta_{t+1}]$ refers to the stochastic discount factor of the financial intermediaries, which is equal to the household’s stochastic discount factor, augmented to incorporate the financial frictions.

Now we combine (4.63) and (4.64) to obtain the following relation between the shadow value on deposit funding and central bank funding:

$$\frac{\psi_t}{1+\mu_t} = \eta_t - \eta_t^{cb}.$$

Substitution of (4.67) and (4.68) gives rise to the following relation:

$$\frac{\psi_t}{1+\mu_t} = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^d - r_{t+1}^{cb} \right) \right].$$

(4.73)
By these substitutions, we have removed the four shadow values $\nu^k_t$, $\nu^b_t$, $\eta^{cb}_t$, and $\chi_t$. Thus we get the following set of F.O.C.’s:

$$\frac{\lambda_b}{\lambda_k} E_t \left[ \Omega_{t,t+1} \left( r^k_{t+1} - r^d_{t+1} \right) \right] = E_t \left[ \Omega_{t,t+1} \left( r^b_{t+1} - r^d_{t+1} \right) \right]$$

$$\lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) = E_t \left[ \Omega_{t,t+1} \left( r^k_{t+1} - r^d_{t+1} \right) \right], \quad (4.75)$$

$$\Psi_t \frac{1}{1 + \mu_t} = E_t \left[ \Omega_{t,t+1} \left( r^d_{t+1} - r^{cb}_{t+1} \right) \right], \quad (4.76)$$

$$\eta_t = E_t \left[ \Omega_{t,t+1} \left( 1 + r^d_{t+1} + \tau^m_{t+1} - \tilde{\tau}^n_{t+1} \right) \right], \quad (4.77)$$

$$(1 + \mu_t) \eta_t n_{j,t} \geq \lambda_k q^k_{j,t} s^k_{j,t} + \lambda_b q^b_{j,t} s^b_{j,t}, \quad (4.78)$$

$$n_t = \sigma \left[ (r^k_t - r^d_t) q^k_{t-1} s^{k,p}_{t-1} + (r^b_t - r^d_t) q^b_{t-1} s^{b,p}_{t-1} \right.$$ \n
$$\left. + (r^d_t - r^{cb}_t) d_{t-1}^c + \left( 1 + r^d_t \right) n_{t-1} \right]$$

$$n_t + \chi p_{t-1} + n^g_t - \tilde{n}^g_t$$

$$d^c_{t} = \theta \kappa_s s^{b,p}_{t}, \quad (4.80)$$

$$\Omega_{t,t+1} = \beta \Lambda_{t,t+1} \left[ (1 - \sigma) + \sigma \left( 1 + \mu_{t+1} \right) \eta_{t+1} \right]. \quad (4.81)$$

### 4.A.2 Production Process

**Capital Producers**

At the end of period $t$, when the intermediate goods firms have produced, the capital producers buy the remaining stock of capital $(1 - \delta)\xi_t k_{t-1}$ from the intermediate goods producers at a price $q^k_t$. They combine this capital with goods bought from the final goods producers (investment $i_t$) to produce next period’s beginning of period capital stock $k_t$. This capital is being sold to the intermediate goods producers at a price $q^k_t$. We assume that the capital producers face convex
adjustment costs when transforming the final goods bought into capital goods, set up such that changing the level of gross investment is costly. Hence we get:

\[ k_t = (1 - \delta)\xi_t k_{t-1} + [1 - \Psi(t)]i_t, \quad \Psi(x) = \frac{\gamma}{2}(x - 1)^2, \quad \tau_t = i_t/i_{t-1}. \]  

(4.82)

\( \xi_t \) represents a capital quality shock which will be discussed later. Profits are passed on to the households, who own the capital producers. The profit at the end of period \( t \) equals:

\[ \Pi^c_t = q^k_i k_t - q^k_i (1 - \delta)\xi_t k_{t-1} - i_t. \]

The capital producers maximize expected current and (discounted) future profits (where we substitute in (4.82)):

\[
\max_{\{i_{t+i}\}_{i=0}^{\infty}} E_t \left[ \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left( q^k_{t+i} (1 - \Psi(t_{t+i})) i_{t+i} - i_{t+i} \right) \right].
\]

Differentiation with respect to investment gives the first order condition for the capital producers:

\[
q^k_i (1 - \Psi(t)) - 1 - q^k_i \tau_t \Psi'(t) + \beta E_t \Lambda_{t,t+1} q^k_{t+1} i_{t+1}^2 \Psi'(t_{t+1}) = 0,
\]

which gives the following expression for the price of capital:

\[
\frac{1}{q^k_i} = 1 - \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 - \frac{\gamma i_t}{i_{t-1}} \left( \frac{i_t}{i_{t-1}} - 1 \right) + \beta E_t \left[ \Lambda_{t,t+1} \frac{q^k_{t+1}}{q^k_t} \left( \frac{i_{t+1}}{i_t} \right)^2 \gamma \left( \frac{i_{t+1}}{i_t} - 1 \right) \right].
\]

(4.83)

**Intermediate Goods Producers**

We remember that period \( t \) profits are given by:

\[
\Pi_{i,t} = m_t z_t(\xi_{t} i_{t,t-1})^{\alpha} h_{i,t}^{1-\alpha} + q^k_i (1 - \delta)\xi_t k_{i,t-1} - (1 + \delta) q^k_{t-1} k_{i,t-1} - w_i h_{i,t}.
\]
The intermediate goods producing firms maximize expected current and future profits using the household’s stochastic discount factor $\beta^s \Lambda_{t,s} \Lambda_{t+1,s}$ (since they are owned by the households), taking all prices as given:

$$\max_{\{k_{t+s}, h_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,s} \Pi_{t,s+1} \right].$$

The first order conditions belonging to this problem are given by:

\begin{align*}
  k_{i,t} & : E_t \left[ \beta \Lambda_{t+1} q_t^k (1 + r_t^k) \right] = E_t \left[ \beta \Lambda_{t+1} \left( \alpha m_{t+1} y_{i,t+1} / k_{i,t} + q_{t+1}^k (1 - \delta) \xi_{t+1} \right) \right], \\
  h_{i,t} & : w_t = (1 - \alpha) m_t y_{i,t} / h_{i,t}.
\end{align*}

In equilibrium profits will be zero. By substituting the first order condition for the wage rate into the zero-profit condition $\Pi_{i,t} = 0$, we can find an expression for the ex-post return on capital:

$$r_t^k = (q_{t-1}^k)^{-1} \left( \alpha m_t y_{i,t} / k_{i,t-1} + q_t^k (1 - \delta) \xi_t \right) - 1.$$

Now we rewrite the first order condition for labor and the expression for the ex-post return on capital to find the factor demands:

\begin{align*}
  k_{i,t-1} & = \alpha m_t y_{i,t} / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t], \\
  h_{i,t} & = (1 - \alpha) m_t y_{i,t} / w_t.
\end{align*}

By substituting the factor demands into the production technology function, we get for the relative intermediate output price $m_t$:

$$m_t = \alpha^{-\alpha} (1 - \alpha)^{\alpha-1} q_{t-1}^k \left( w_t^{1-\alpha} \left[ q_{t-1}^k (1 + r_t^k) \xi_t^{-1} - q_t^k (1 - \delta) \right] \right)^{\alpha} \tag{4.84}$$

**Retail firms**

Retail firms purchase goods $(y_{i,t})$ from the intermediate goods producing firms for a nominal price $P_{m}^n$, and convert these into retail goods $(y_{f,t})$. These
4.A. DERIVATIONS

Goods are sold for a nominal price $P_{f,t}$ to the final goods producer. It takes one intermediate goods unit to produce one retail good ($y_{i,t} = y_{f,t}$). All the retail firms produce a differentiated retail good by assumption, therefore operate in a monopolistically competitive market, and charge a markup over the input price earning them profits $(P_{f,t} - P^m_t)y_{f,t}$.

Each period, only a fraction $1 - \psi$ of retail firms is allowed to reset their price, while the $\psi$ remaining firms are not allowed to do so, like in Calvo (1983) and Yun (1996). The firms allowed to adjust prices are randomly selected each period. Once selected, they set prices so as to maximize expected current and future profits, using the stochastic discount factor $\beta^s \Lambda_{t, t+s} (P_t/P_{t+s})$:

$$\max_{P_{f,t}} E_t \left[ \sum_{s=0}^{\infty} (\beta \psi)^s \Lambda_{t, t+s} (P_t/P_{t+s}) [P_{f,t} - P^m_{t+s}] \right] y_{f,t+s},$$

where $y_{f,t} = (P_{f,t}/P_t)^{-\epsilon} y_t$ is the demand function. $y_t$ is the output of the final goods producing firms, and $P_t$ the general price level. Symmetry implies that all firms allowed to reset their prices choose the same new price ($P^*_t$). Differentiation with respect to $P_{f,t}$ and using symmetry then yields:

$$\frac{P^*_t}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta \psi)^s \lambda_{t+s} P^e_{t+s} P_t^{-\epsilon} m_{t+s} y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta \psi)^s \lambda_{t+s} P^e_{t+s} P_t^{-\epsilon} y_{t+s}}.$$

Defining the relative price of the firms that are allowed to reset their prices as $\pi^*_t = P^*_t / P_t$ and gross inflation as $\pi_t = P_t / P_{t-1}$, we can rewrite this as:

$$\pi^*_t = \frac{\epsilon}{\epsilon - 1} \frac{\Xi_{1,t}}{\Xi_{2,t}}, \quad (4.85)$$

$$\Xi_{1,t} = \lambda_t m_t y_t + \beta \psi E_t \pi_t^{\epsilon} \Xi_{1,t+1}, \quad (4.86)$$

$$\Xi_{2,t} = \lambda_t y_t + \beta \psi E_t \pi_t^{\epsilon-1} \Xi_{2,t+1}. \quad (4.87)$$

The aggregate price level equals:

$$P_t^{1-\epsilon} = (1 - \psi)(P^*_t)^{1-\epsilon} + \psi P_t^{1-\epsilon}.$$
Dividing by $P_t^{1-\varepsilon}$ yields the following law of motion:

$$(1 - \psi)(\pi_t^*)^{1-\varepsilon} + \psi \pi_t^{\varepsilon-1} = 1.$$  \hspace{1cm} (4.88)

**Final Goods Producers**

Final goods firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by $y_t^{(\varepsilon-1)/\varepsilon} = \int_0^1 y_{f,t}^{(\varepsilon-1)/\varepsilon} df$, where $y_{f,t}$ is the output of the retail firm indexed by $f$. $\varepsilon$ is the elasticity of substitution between the intermediate goods purchased from the different retail firms. The final goods firms face perfect competition, and therefore take prices as given. Thus they maximize profits by choosing $y_{f,t}$ such that $P_t y_t - \int_0^1 P_{f,t} y_{f,t} df$ is maximized. Taking the first order conditions with respect to $y_{f,t}$, gives the demand function of the final goods producers for the retail goods. Substitution of the demand function into the technology constraint gives the relation between the price level of the final goods and the price level of the individual retail firms:

$$y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon} y_t,$$

$$P_t^{1-\varepsilon} = \int_0^1 P_{f,t}^{1-\varepsilon} df.$$

**Aggregation**

Substituting $y_{f,t} = y_{i,t} = y_t \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon}$ into the factor demands derived earlier yields:

$$h_{i,t} = (1 - \alpha)m_t y_t / w_t, \quad k_{i,t-1} = \alpha m_t y_t / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t].$$

Aggregation over all firms $i$ gives us aggregate labor and capital:

$$h_t = (1 - \alpha)m_t y_t D_t / w_t, \quad k_{t-1} = \alpha m_t y_t D_t / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t].$$
where \( D_t = \int_0^1 (P_{f,t}/P_t)^{-\varepsilon} df \) denotes the price dispersion. It is given by the following recursive form:

\[
D_t = (1 - \psi)(\pi_t^*)^{-\varepsilon} + \psi \pi_t^\varepsilon D_{t-1}. \tag{4.89}
\]

The aggregate capital-labor ratio is equal to the individual capital-labor ratio:

\[
k_{t-1}/h_t = \alpha(1 - \alpha)^{-1}w_t/[q_{t-1}^k(1 + r_t^k) - q_t^k(1 - \delta)\bar{\xi}_t] = k_{i,t-1}/h_{i,t}. \tag{4.90}
\]

Now calculate aggregate supply by aggregating \( y_{i,t} = z_t(\bar{\xi}_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha} \):

\[
\int_0^1 z_t(\bar{\xi}_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha} di = \int_0^1 h_{i,t} di = z_t(\bar{\xi}_t k_{t-1})^\alpha h_t^{1-\alpha},
\]

while aggregation over \( y_{i,t} \) gives:

\[
\int_0^1 y_{i,t} df = y_t \int_0^1 (P_{f,t}/P_t)^{-\varepsilon} df = y_t D_t.
\]

So we get the following relation for aggregate supply \( y_t \):

\[
y_t D_t = z_t(\bar{\xi}_t k_{t-1})^\alpha h_t^{1-\alpha}. \tag{4.91}
\]
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Parameter | Value | Definition
---|---|---
Households | $\bar{h}$ | 1/3 Labor supply
Financial intermediaries | $\bar{\phi}$ | 6 Leverage ratio
 | $\Gamma_k$ | 0.0050 Quarterly credit spread $E[r^k - r^d]$ 
 | $\Gamma_{cb}$ | 0 Quarterly credit spread $E[r^d - r^{cb}]$
 | $\theta$ | 0.95 Haircut parameter
 | $\lambda_b/\lambda_k$ | 0.5 Diversion rate bonds over private loans
Government policy | $\bar{i}/\bar{y}$ | 0.2 Investment-output ratio
 | $\bar{g}/\bar{y}$ | 0.2 Gov’t spending-output ratio
 | $\bar{q}_b\bar{b}/\bar{y}$ | 4 Gov’t liabilities-output ratio (quarterly)
 | $\bar{s}_b/\bar{b}$ | 0.25 Fraction of gov’t financing by banks

Table 4.2. Calibration targets for the NK version of the model.

4.B Calibration

Some of the targets for the calibration of the model can be found in Table 4.2.
4.C Additional Figures

![Money market rates graph](image)

Figure 4.13. The ECB MRO (Main Refinancing Operation) rate is charged on collateralized loans with a maturity of 1 week from the ECB. The MRO rate is the ECB’s main instrument for conducting monetary policy. EURIBOR (Euro InterBank Offered Rate) rates are money market rates at which commercial banks can obtain unsecured funding in the European interbank market. Source: European Central Bank (2015).
4.D First Order Conditions & Equilibrium

4.D.1 First Order Conditions

The household’s first order conditions are given by:

\[ \lambda_t = (c_t - \nu c_{t-1})^{-1} - \beta \nu E_t \left[ (c_{t+1} - \nu c_t)^{-1} \right] , \quad (4.92) \]

\[ \Psi T^H = \lambda_t w_t , \quad (4.93) \]

\[ 1 = E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \left( 1 + r_{t+1}^d \right) \right] , \quad (4.94) \]

\[ 1 = E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{(1 + r_{t+1}^b) q_t^b}{q_t^b + \kappa_{s_t,h} (s_{t, h}^b - \hat{s}_{b,h})} \right) \right] . \quad (4.95) \]
First order conditions for financial intermediaries are given by:

\[
\frac{\lambda_b}{\lambda_k} E_t \left[ \Omega_{t,t+1} \left( r^k_{t+1} - r^l_{t+1} \right) \right] = \frac{E_t}{\eta_t} \left[ \Omega_{t,t+1} \left( r^b_{t+1} - r^d_{t+1} \right) \right] + \left( \frac{\varphi_t}{q^p_t} \right) \theta \left( \frac{\psi_t}{1 + \mu_t} \right),
\]

(4.96)

\[
\lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) = \frac{E_t}{\eta_t} \left[ \Omega_{t,t+1} \left( r^k_{t+1} - r^l_{t+1} \right) \right],
\]

(4.97)

\[
\frac{\psi_t}{1 + \mu_t} = \frac{E_t}{\eta_t} \left[ \Omega_{t,t+1} \left( r^d_{t+1} - r^c_b \right) \right],
\]

(4.98)

\[
\eta_t = \frac{E_t}{\eta_t} \left[ \Omega_{t,t+1} \left( 1 + r^d_{t+1} + \tau^n_{t+1} - \varpi_{t+1} \right) \right],
\]

(4.99)

\[
(1 + \mu_t) \eta_t n_{j,t} = \lambda_k q^k_{t} s^k_{j,t} + \lambda_b q^b_{t} s^b_{j,t},
\]

(4.100)

\[
n_{t} = \sigma \left[ (r^k - r^d) q^k_{t-1} s^k_{t-1} + (r^b - r^d) q^b_{t-1} s^b_{t-1} \right] + \left( r^d - r^c_b \right) q^{cb}_{t-1} t + \left( 1 + r^d_{t} \right) n_{t-1} \]

\[
+ \chi (\tau - 1) n_{t} - \bar{n}_t,
\]

(4.101)

\[
d^c_b = \theta \eta^b_t t_{s^b_p},
\]

(4.102)

\[
p_t = q^k_t s^k_t + q^b_t s^b_t,
\]

(4.103)

\[
p_t = n_t + d_t + d^c_b,
\]

(4.104)

\[
\phi_t = p_t / n_t,
\]

(4.105)

\[
\omega^k_t = q^k_t s^k_t / p_t,
\]

(4.106)

where \( \Omega_{t,t+1} = \beta \Lambda_{t,t+1} \{ (1 - \sigma) + \sigma (1 + \mu_{t+1}) \eta_{t+1} \} \). Price setting conditions are given by:

\[
\pi^*_t = \frac{\varepsilon}{\varepsilon - 1} \Xi_{1,t},
\]

(4.107)

\[
\Xi_{1,t} = \lambda_t m_t y_t + \beta \psi E_t \pi^e_{t+1} \Xi_{1,t+1},
\]

(4.108)

\[
\Xi_{2,t} = \lambda_t y_t + \beta \psi E_t \pi^e_{t+1} \Xi_{2,t+1},
\]

(4.109)

\[
1 = (1 - \psi) (\pi^*_t)^{-1 - \varepsilon} + \psi \pi^e_{t-1},
\]

(4.110)

\[
D_t = (1 - \psi) (\pi^*_t)^{-\varepsilon} + \psi \pi^e_{t-1},
\]

(4.111)
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

Production equations are given by:

\[ m_t = \alpha^{-\alpha}(1 - \alpha)^{\alpha - 1}z_t^{-1}\left[w_t^{1-\alpha}\left[q_{t-1}^k(1 + r_t^k)\xi_t^{-1} - q_t^k(1 - \delta)^{\alpha}\right]\right], \]  
(4.112)

\[ k_{t-1}/h_t = \alpha(1 - \alpha)^{-1}w_t/[q_{t-1}^k(1 + r_t^k) - q_t^k(1 - \delta)\xi_t], \]  
(4.113)

\[ k_t = (1 - \delta)\xi_t k_{t-1} + \left(1 - \frac{\gamma}{2}\left[i_t/i_{t-1} - 1\right]^2\right)i_t, \]  
(4.114)

\[ \frac{1}{q_t^k} = 1 - \frac{\gamma}{2}\left(i_t/i_{t-1} - 1\right)^2 - \gamma\frac{q_t}{i_{t-1}}(i_t/i_{t-1} - 1) \]
\[ + \beta E_t\left[\Lambda_{t,t+1}\left(i_{t+1}/i_t\right)^2\gamma\left(i_{t+1}/i_t - 1\right)\right], \]  
(4.115)

\[ y_t D_t = z_t(\xi_t k_{t-1})^{\alpha}h_t^{1-\alpha}. \]  
(4.116)

The first order conditions for the fiscal authority are given by:

\[ 1 + r_t^b = \frac{r_c + \rho q_t^b}{q_{t-1}^b}, \]  
(4.117)

\[ q_t^b b_t + \tau_t + \bar{n}_t^g + \Pi_t^{cb} = g_t + n_t^g + \left(1 + r_t^b\right)q_{t-1}^b b_{t-1}, \]  
(4.118)

\[ g_t = G, \]  
(4.119)

\[ \tau_t = \tau + \kappa_b(b_{t-1} - \bar{b}) + \kappa_t n_t^g, \]  
(4.120)

\[ n_t^g = n_t^g n_{t-1}, \]  
(4.121)

\[ \tau_t^l = \tau_{t-1}/n_t^g, \]  
(4.122)

\[ \bar{n}_t^g = \bar{n}_t^g n_{t-1}, \]  
(4.123)

\[ \tau_t^u = n_t^g / n_{t-1}. \]  
(4.124)
4.D. FIRST ORDER CONDITIONS & EQUILIBRIUM

The first order conditions for the central bank are given by:

\[ r^n_t = (1 - \rho_r)(r^n + \kappa_r [\pi_t - \bar{\pi}] + \kappa_y \log [y_{t-1}/y_t]) + \rho_r r^n_{t-1} + \epsilon_{r,t}, \tag{4.125} \]

\[ 1 + r^d_t = (1 + r^n_{t-1})/\pi_t, \tag{4.126} \]

\[ r^{n,cb}_t = r^n_t - \Gamma^{cb}_t, \tag{4.127} \]

\[ 1 + r^{cb}_t = (1 + r^{n,cb}_{t-1})/\pi_t, \tag{4.128} \]

\[ \Gamma^{cb}_t = \hat{\Gamma}^{cb}_t + \varsigma^{cb}_t (\xi_t - \bar{\xi}), \tag{4.129} \]

\[ \Pi^{cb}_t = (r^{cb}_t - r^d_t) a^{cb}_{t-1}, \tag{4.130} \]

Market clearing conditions are given by:

\[ k_t = s^k_t, \tag{4.131} \]

\[ b_t = s^b_t + s^{b, h}_t, \tag{4.132} \]

\[ a_t = d_t + a^{cb}_t, \tag{4.133} \]

\[ y_t = c_t + i_t + g_t + \frac{1}{2} \kappa^{b, h}_{s, h} (s^{b, h}_t - \hat{s}^{b, h}_t)^2. \tag{4.134} \]

And finally, exogenous processes are given by:

\[ \log (z_t) = \rho_z \log (z_{t-1}) + \epsilon_{z,t}, \tag{4.135} \]

\[ \log (\xi_t) = \rho_\xi \log (\xi_{t-1}) + \epsilon_{\xi,t}. \tag{4.136} \]

4.D.2 Equilibrium Conditions

Let \( c_{t-1}, s^{b, h}_{t-1}, a_{t-1}, s^{k, p}_{t-1}, n_{t-1}, a^{cb}_{t-1}, p_{t-1}, k_{t-1}, i_{t-1}, b_{t-1}, y_{t-1}, \Omega_{t-1}, r^n_{t-1}, r^{n, cb}_{t-1}, z_t, \xi_t \) be the state vector. A recursive competitive equilibrium is a sequence of quantities and prices \( c_t, \lambda_t, h_t, s^{b, h}_t, a_t, \eta_t, \mu_t, \psi_t, \phi_t, n_t, s^k_t, s^b_t, p_t, d_t, a^{cb}_t, \Omega_k^t, q^k_t, q^b_t, r^k_t, r^b_t, r^d_t, r^{cb}_t, w_t, m_t, \pi_t, \pi^*_t, \Xi^1_t, \Xi^2_t, \Omega_t, i_t, k_t, y_t, b_t, g_t, n^g_t, \tilde{n}_t, \tau_t, \tau^n_t, \bar{\tau}_t, r^n_{t, cb}, r^{n, cb}_t, \Gamma^{cb}_t, \Pi^{cb}_t, z_t, \xi_t \) such that:

i. Households optimize taking prices as given: (4.92) - (4.95).
ii Financial intermediaries optimize taking prices as given: (4.96) - (4.106).

iii Capital producers optimize taking prices as given: (4.114) - (4.115).

iv Intermediate goods producers optimize taking prices as given: (4.112) - (4.113)

v Retail goods producers that are allowed to change prices optimize taking input prices $m_t$ as given: (4.107) - (4.111).

vi Final goods producers optimize taking prices as given: (4.116).

vii Asset markets clear: (4.131) - (4.133).

viii The goods market clears: (4.134).

ix The fiscal variables evolve according to: (4.117) - (4.124).

x The monetary variables evolve according to: (4.125) - (4.130).

xi Productivity and capital quality evolve according to: (4.135) and (4.136).

4.E Robustness checks

To check whether my results are robust, I conduct a robustness check. I will perform this check along two dimensions. First, I will change some of the key parameters of the model. A second line of robustness checks is along the dimension of model specification. I perform these checks to make sure that my model results do not depend on some arbitrary parameter choice or a particular model specification.

4.E.1 Parameters

Since the substitution of private loans for government bonds is driving the results of the unconventional monetary policy operation, I change parameters that are related to government bond holdings, or the willingness to hold government bonds. I start by adjusting the household’s portfolio adjustment costs
for government bond holdings $\kappa_{sb,h}$, since this parameter determines the elasticity with which households are willing to buy and sell government bonds when the demand for government bonds by commercial banks changes in response to the unconventional central bank lending. Because the initial calibration shows a rather conservative increase in the recourse to low-interest-rate central bank funding, I decrease $\kappa_{sb,h}$ from 0.0025 to 0.0010 in Figures 4.15 and 4.16.

Second I investigate the relative diversion rate $\lambda_b/\lambda_k$. An increase in $\lambda_b/\lambda_k$ increases the steady state spread between bonds and deposits, and makes it harder for commercial banks to expand the balance sheet through an increase in bond holdings, since bankers can divert a larger fraction of bondholdings. I investigate both a decrease in $\lambda_b/\lambda_k$ from 0.5 to 0.25 (Figures 4.17 and 4.18), as well as an increase in $\lambda_b/\lambda_k$ from 0.5 to 0.75 (Figures 4.19 and 4.20).

Third, a parameter that influences the volatility of net worth, and hence of the tightness of bank balance sheet constraints, is the bankers’ survival rate $\sigma$. I look at both a decrease in the average survival period from 20 to 8 quarters (Figures 4.21 and 4.22), as well as an increase from 20 to 28 quarters (Figures 4.23 and 4.24).

### 4.E.2 Model specification

A second line of robustness checks is along the dimension of model specification. One obvious robustness check is to allow households to intermediate private loans. Another specification allows for commercial banks to use private loans for collateral purposes in addition to government bonds. The last specification is the implementation of the zero lower bound (ZLB).

**Household financing of private loans**

Commercial banks are in reality not the only financiers of private loans to non-financial corporations. I abstain, however, from introducing household intermediation of private loans in the main part of the paper for two reasons.
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

First, bank financing of credit to non-financial corporations accounts for approximately 80% in the Eurozone. Second, I construct a model that includes household financing of private loans, and find that the results are not significantly affected by the introduction of household intermediation of private loans.

Private loans yield a return \( r_k \) on their holdings \( q_{k,t-1}^{k,h} \), where \( s_{t-1}^{k,h} \) is the number of private loans purchased in period \( t-1 \). Households are less efficient in financial intermediation than commercial banks, hence they incur financial intermediation costs, which is quadratic in the deviation from the number of private loans \( \hat{s}_{k,h} \). The household’s problem changes into:

\[
\max \left\{ c_t + \sum_{i=0}^{\infty} \beta^i \left[ \log (c_{t+i} - \psi c_{t-1+i}) - \Psi_{t}^{1+\phi} \right] \right\}
\]

s.t.

\[
c_t + \tau_t + a_t + q_t^{k,h} + q_t^{b,h} = \frac{\kappa_{s_{k,h}}}{2} \left( s_t^{k,h} - \hat{s}_{k,h} \right)^2 + \frac{\kappa_{s_{b,h}}}{2} \left( s_t^{b,h} - \hat{s}_{b,h} \right)^2
\]

\[
= w_t h_t + \left( 1 + r_{t}^d \right) a_{t-1} + \left( 1 + r_{t}^k \right) q_{t-1}^{k,h} s_{t-1}^{k,h}
\]

\[
+ \left( 1 + r_{t}^b \right) q_{t-1}^{b,h} s_{t-1}^{b,h} + \Pi_t,
\]

which give rise to an additional first order condition for private loans \( s_t^{k,h} \), next to the first order conditions from the main text:

\[
s_t^{k,h} : E_t \left[ \beta^{t+1} \left( \frac{1 + r_{t+1}^k q_t^k}{q_t^k + \kappa_{s_{k,h}} \left( s_t^{k,h} - \hat{s}_{k,h} \right)} \right) \right] = 1. \quad (4.137)
\]

This introduces a new parameter, namely \( \kappa_{s_{k,h}} \), into the model. It is reasonable to assume that transaction costs for households are larger for private credit intermediation than for sovereign debt intermediation. We therefore set \( \kappa_{s_{k,h}} = 0.1 \), which is four times larger than \( \kappa_{s_{b,h}} \). The market clearing condition (4.27) for private loans changes into:

\[
k_t = s_t^{k,p} + s_t^{k,h}, \quad (4.138)
\]
while the aggregate resource constraint (4.29) changes into:

\[ y_t = c_t + i_t + g_t + \frac{1}{2} \kappa_{s_k,h} \left( s_{t}^{k,h} - \hat{s}_{k,h} \right)^2 + \frac{1}{2} \kappa_{s_b,h} \left( s_{t}^{b,h} - \hat{s}_{b,h} \right)^2. \] (4.139)

The qualitative results of the unconventional central bank lending operations do not change upon introduction of household intermediation of private loans, see Figures 4.25 and 4.26.

**Private loans under the collateral constraint**

In reality, government bonds are not the only security that can be pledged as collateral in refinancing operations at the ECB. Alternative collateral classes include covered and uncovered bank bonds, asset-backed securities (ABS), corporate bonds and some other securities. Private loans to non-financial corporations and households are usually hard to collateralize, and most of these loans are therefore not used in refinancing operations with the ECB. To test whether the inclusion of private credit under the collateral constraint affects my result, I change the collateral constraint into:

\[ d_{c,b}^{k,b} \leq \theta_{k} q_{t}^{k,b} s_{j,t}^{k,b} + \theta_{b} q_{t}^{b,b} s_{j,t}^{b,b}. \] (4.140)

where \( \theta_{k} \), respectively \( \theta_{b} \) denote the haircut parameter on private loans, respectively government bonds. In line with actual ECB policy, I take \( \theta_{k} \) to be smaller than \( \theta_{b} \), i.e. the ECB applies a larger haircut on private loans, which are considered to be more risky than government bonds.
The first order conditions for the portfolio choice between private loans and government bonds, as well as the first order condition for private loans, are adjusted:

\[
\frac{\lambda_b}{\lambda_k} E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right] = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^b - r_{t+1}^d \right) \right] + \left( \theta_b - \frac{\lambda_b}{\lambda_k} \right) \left( \frac{\psi_t}{1 + \mu_t} \right),
\]

\[
\lambda_k \left( \frac{\mu_t}{1 + \mu_t} \right) = E_t \left[ \Omega_{t,t+1} \left( r_{t+1}^k - r_{t+1}^d \right) \right] + \theta_k \left( \frac{\psi_t}{1 + \mu_t} \right),
\]

(4.141)

(4.142)

Just as in the previous section, I include household intermediation of private loans under this specification. We immediately see that the inclusion of private credit under the collateral constraint is not capable to offset the effect of the collateral requirement on the portfolio decision of the commercial banks. There are two reasons for this: first, government bonds usually carry a smaller haircut than private loans, i.e. \( \theta_b > \theta_k \). Hence a commercial bank obtains more central bank funding for one euro of government bonds than for one euro of private loans. Hence government bonds are more attractive collateral than private loans. At the same, the balance sheet constraint is more binding for private loans than for government bonds, as reflected by the fact that \( \lambda_k > \lambda_b \). Hence it is harder to expand the balance sheet by a unit of private loans to obtain more funding from the central bank than by buying a government bond. This is a second reason why buying government bonds is more attractive when commercial banks want to obtain more central bank funding. Offsetting the distortion in the portfolio decision induced by the central bank requires \( \theta_b = \left( \frac{\lambda_b}{\lambda_k} \right) \theta_k \). This requires \( \lambda_b > \lambda_k \) if \( \theta_b > \theta_k \), which implies that it is easier to expand the balance sheet by buying private loans than by buying government bonds, which is clearly not realistic. If \( \lambda_k > \lambda_b \), we need \( \theta_k > \theta_b \), i.e. a smaller haircut for private loans than for government bonds, which is in sharp contrast with current central bank
policies at the ECB and the Fed. When we take the realistic case where $\lambda_k > \lambda_b$ and $\theta_b > \theta_k$, we see that $\theta_b - (\lambda_b/\lambda_k) \theta_k > 0$. Hence including private loans under the collateral constraint still induces commercial banks to shift out of private loans and into government bonds, although the effect will be muted with the case where private loans can not be pledged as collateral, i.e. $\theta_k = 0$.

To summarize, there are two reasons why government bonds remain more attractive as collateral. First it is easier to expand the balance sheet by one unit of government bonds than by one unit of private loans, as the balance sheet constraint is less binding for government bonds than for private loans, since $\lambda_b < \lambda_k$. Second, the haircut on government bonds is smaller than on private loans, i.e. $\theta_b > \theta_k$. Hence more central bank funding is obtained for a unit of government bonds as collateral than for a unit of private loans as collateral.

Figures 4.27 and 4.28 show the result from a simulation where the haircut on private loans is 50%, i.e. $\theta_k = 0.50$, and the elasticity of household demand for private loans $\kappa_{s_k,h}$ is equal to 0.1, as in the version with household intermediation of private loans, but no private loans under the collateral constraint. Of course the shift out of private loans and into government bonds is reduced when compared with the case where private loans are not eligible for collateral purposes, but the short term contractionary effect remains.
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

Zero Lower Bound

In this section I implement the zero lower bound by setting the interest rate smoothing parameter at $\rho_r = 0.999$ in Figure 4.14. This is strictly speaking not the same as the zero lower bound, but captures the fact that endogenous changes in output and inflation do not cause an adjustment in the nominal interest rate.

Variation in haircut parameter $\theta$ at zero lower bound.

![Graph showing the difference in central bank lending and output](image)

**Figure 4.14.** Both panels display the difference between the case where the nominal interest rate on central bank lending facilities is lowered by 50 basis points with respect to the nominal interest rate on regular deposit funding on impact (LTRO) and the no intervention case for different steady state values of the haircut parameter $\theta$. The blue solid line refers to $\theta = 0.20$, the red slotted line to $\theta = 0.45$, the green dotted line to $\theta = 0.70$, and the black dashed line to $\theta = 0.95$. The upper panel displays the difference in central bank (CB) lending, while the lower panel displays the difference in output. The zero lower bound is implemented by setting $\rho_r = 0.999$.
4.E. ROBUSTNESS CHECKS

Financial crisis impact, no additional policy vs. unconventional monetary policy for $\kappa_{Sb,b} = 0.0010$.

Figure 4.15. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

Financial crisis impact, no additional policy vs. unconventional monetary policy for $\kappa_{b,h} = 0.0010$.

Figure 4.16. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
4.E. ROBUSTNESS CHECKS

Financial crisis impact, no additional policy vs. unconventional monetary policy for \( \lambda_b / \lambda_k = 0.25 \).

Figure 4.17. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

Financial crisis impact, no additional policy vs. unconventional monetary policy for $\lambda_b/\lambda_k = 0.25$.

Figure 4.18. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
4.E. ROBUSTNESS CHECKS

Financial crisis impact, no additional policy vs. unconventional monetary policy for \( \lambda_b/\lambda_k = 0.75 \).

![Impulse response functions](image)

**Figure 4.19.** Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy vs. unconventional monetary policy for \( \lambda_b/\lambda_k = 0.75 \).

Figure 4.20. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy vs. unconventional monetary policy for average survival time bankers 8 quarters.

Figure 4.21. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

Financial crisis impact, no additional policy vs. unconventional monetary policy for
average survival time bankers 8 quarters.

Figure 4.22. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy vs. unconventional monetary policy for average survival time bankers 28 quarters.

Figure 4.23. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy vs. unconventional monetary policy for average survival time bankers 28 quarters.

Figure 4.24. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy vs. unconventional monetary policy with household intermediation of private loans.

Figure 4.25. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy vs. unconventional monetary policy with household intermediation of private loans.

Figure 4.26. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Financial crisis impact, no additional policy vs. unconventional monetary policy with private loans as collateral.

Figure 4.27. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
CHAPTER 4. UNCONVENTIONAL CENTRAL BANK LENDING

Financial crisis impact, no additional policy vs. unconventional monetary policy with private loans as collateral.

![Graphs](image)

Figure 4.28. Impulse response functions for the case from section 4.5.1 with no additional policy (blue, solid) vs. the LTRO, represented by a decrease in the nominal interest rate on central bank lending facilities of 50 basis points on impact with respect to the nominal interest rate on regular deposit funding (red, slotted). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
Chapter 5

Summary of “The Macroeconomics of Banking”

This thesis has investigated the macroeconomic effectiveness of fiscal and monetary policy in an environment where commercial banks are undercapitalized after a financial crisis, and have large amounts of (risky) sovereign debt on their balance sheet. An undercapitalized banking system implies that banks have little net worth, which prevents them from perfectly elastically expanding their balance sheet for a given amount of net worth. In that case, banks have to choose whether an additional euro of funding is used to provide new credit to the real economy or to expand government bond holdings.

It is clear that the European sovereign debt crisis erupted in such an environment in the beginning of 2010 in Greece. First of all, European commercial banks were undercapitalized after the financial crisis of 2007-2009 (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). Second, Southern-European commercial banks had large amounts of domestic sovereign debt on their balance sheet. Stress-tests by the European Banking Authority (2011) revealed that domestic sovereign debt holdings of Spanish commercial banks amounted to 150% of aggregate Spanish Tier-1 capital. For Italian banks this percentage increased to 200% of aggregate Italian Tier-1 capital, while Greek banks had an exposure of 250% of aggregate Tier-1 capital to Greek sovereign
CHAPTER 5. CONCLUSION

debt (European Banking Authority, 2011). With such a large exposure to domestic sovereign debt, losses on these holdings can potentially have a destabilizing effect on the financial system, as it can completely wipe out Tier-1 capital, or commercial banks’ net worth.

The most important result from this thesis is that the presence of (risky) government bonds on the balance sheet of undercapitalized commercial banks decreases the macroeconomic effectiveness of monetary and fiscal policy. Two channels can be identified through which the effectiveness is decreased.

The first channel is through capital losses on long-term government bonds held by commercial banks. Capital losses arise when the interest rate on these bonds increases, for example because of an increase in sovereign default risk. In that case the price of government bonds falls, which results in losses on government bond holdings that reduce commercial banks’ Tier-1 capital or net worth. As the size of commercial banks’ balance sheets is limited by the amount of net worth, lower net worth forces commercial banks to reduce credit provision to the real economy, which results in lower aggregate investment in the macroeconomy.

The second channel constitutes an asset substitution effect, whereby commercial banks have an incentive to shift from private credit to government debt. Such an incentive can arise from regulation, such as for example a low risk-weight on government bonds compared with the risk-weights on other bank assets. As the net worth of commercial banks limits the size of the balance sheet, the asset substitution effect leads to a reduction of credit to the real economy, and therefore has a negative effect on aggregate investment and output.

In chapter 1 I investigate the effectiveness of a debt-financed recapitalization of the commercial banking system in an environment where commercial banks are undercapitalized, sovereigns are subject to default risk, and commercial banks with large amounts of domestic risky sovereign debt on their balance sheet. I show that in such an environment the effectiveness of a debt-
financed recapitalization is reduced because the additional debt issue leads to lower bond prices which negatively affect commercial banks’ balance sheets as bank capital/net worth is reduced, and leverage constraints are tightened. As a response, commercial banks charge higher interest rates on private credit. Arbitrage between private credit and sovereign debt leads to higher interest rates on sovereign debt, which deteriorates the fiscal position of the sovereign, and increases sovereign default risk, leading to a second round of capital losses on government debt. Higher sovereign default risk limits the space for government intervention, and limits the options available to the government to aid the financial sector.

This conclusion is in striking contrast with the standard macroeconomic policy prescription, which says that the government should recapitalize an undercapitalized financial sector. An example of a failed recapitalization attempt is the case where the Spanish government announced a debt-financed recapitalization of the Spanish banking system in May 2012. Contrary to expectations, the market for Spanish government debt collapsed upon the announcement and prevented the government from executing the recapitalization. Instead, the Spanish government had to apply for funding from European institutions.

In chapter 2 I look at the effectiveness of debt-financed fiscal stimuli in a similar environment as in chapter 1. The debate on the effectiveness of fiscal stimuli has received considerable attention after the Great Recession of 2007-2009, an environment in which the policy rate is at the zero lower bound, and inflation and economic growth are sclerotic.

The conclusion of this chapter is that the effectiveness of fiscal stimuli is much reduced in an environment where commercial banks are undercapitalized after a financial crisis, the sovereign subject is to default risk, and commercial banks with large amounts of risky sovereign debt on their balance sheet. In such an environment, a fiscal stimulus can become counterproductive because of a feedback effect through lower bond prices. Lower bond prices lead to a reduc-
CHAPTER 5. CONCLUSION

tion of commercial banks’ net worth, which reduces the ability of commercial banks to expand lending to either the government or the corporate sector. As a result, credit provision to the real economy falls when losses on government bonds arise, which leads to lower aggregate investment in the macroeconomy. An example of such an environment is Southern-Europe after the start of the European sovereign debt crisis. The effectiveness of fiscal policy improves when banks are better capitalized, so Capital-Adequacy-Ratios (CARs) have to go up.

In chapter 3 I investigate the macroeconomic impact of the structure and the transition to higher capital requirements for commercial banks through its impact on credit provision to the real economy. Higher capital requirements were introduced to prevent a new financial crisis, and require commercial banks to hold more bank capital/net worth for one euro of bank assets. The introduction of higher capital requirements, however, initiated a debate on the macroeconomic consequences of these higher requirements.

In addition, a second debate erupted on the structure of current Basel-III capital requirements, which are applied to so-called risk-weighted assets. Risk-weighted assets refer to a correction that is applied to a bank’s assets to capture the risk that a particular asset poses to the balance sheet of the bank. Some assets pose a bigger risk to the balance sheet of commercial banks than others. Different assets therefore have different risk-weights. A commercial bank, for example, needs to have more bank capital/net worth for a loan to the real economy than for holding a government bond.

Chapter 3 shows that higher capital requirements have a negative effect on credit provision to the real economy, in an environment where commercial banks are undercapitalized after a financial crisis. The effects can be ameliorated through a recapitalization that increases the level of bank capital, as happened in the U.S. in 2009, as more net worth allows banks to expand the size of the balance sheet.
A second conclusion is that the structure of capital requirements has large macroeconomic consequences: in contrast to applying capital requirements to unweighted assets, applying them to risk-weighted assets induces asset substitution by commercial banks, whereby they switch from assets with a high risk-weight, such as loans to the real economy, to assets with a low risk-weight, such as government bonds in response to higher capital requirements.

In chapter 4 I investigate the macroeconomic impact of unconventional Longer-Term Refinancing Operations (LTROs) initiated at the end of 2011 and the beginning of 2012 in which the European Central Bank (ECB) lent approximately 1,000 billion EUR to the European commercial banking system. The ECB loans came at a low interest rate from the ECB, but required commercial banks to pledge collateral at the ECB, for which government bonds are the most attractive asset-class.

My most important results are (i) that the unconventional LTROs had a negative short-run effect on credit provision to the real economy because undercapitalized commercial banks shifted from private credit to government bonds (asset substitution), which could be pledged as collateral to obtain ECB funding, and (ii) that the collateral policy of the ECB did not affect the cumulative impact on output of the LTROs.

The provision of low-interest-rate credit to the commercial banking system can be interpreted as an indirect recapitalization of the commercial banking system by the ECB. I therefore compare the LTRO-policy with a direct recap by the fiscal authority, and find that a direct recap is more effective in stimulating output than the LTRO-policy, as it does not induce a shift from private credit to government bonds.

The most obvious response to improve the effectiveness of traditional macroeconomic policy is by recapitalizing the commercial banking system. I have shown in chapter 2, 3 and 4 that a recapitalization of the commercial banking system improves macroeconomic outcomes by alleviating commercial banks’
balance sheet constraints, which increases credit provision to the real economy. In chapter 1, however, I showed that the effectiveness of a debt-financed recapitalization also deteriorates when commercial banks are undercapitalized and have large amounts of risky sovereign debt on their balance sheets.

Policymakers will have to think about alternative policy measures in such an environment to reduce or avoid the above-mentioned negative effects. A recapitalization of commercial banks, for example, could be executed by an external party to avoid the negative feedback from lower bond prices on commercial banks’ net worth as an alternative policy to the one investigated in chapter 1. Another possibility to recapitalize commercial banks is by doing a debt-equity swap in which debt of commercial banks is written-off and replaced by equity. Such an operation occurred in Cyprus in 2013, but also led to a capital flight. Investigation of these alternative policies is a future avenue for research.

Another line of research to be pursued is to investigate the effects of quantitative easing programs. In the last couple of years, central banks have massively expanded their balance sheets by purchasing assets, which is referred to as quantitative easing. This has resulted in a large increase in reserves held by the commercial banking system. An important question is why the commercial banks, despite having large amounts of liquidity on their balance sheets, have not expanded lending to the real economy. Could the fact that commercial banks are undercapitalized play a role in this liquidity hoarding? The framework that has been employed in this thesis, namely one of undercapitalized commercial banks, can be used to investigate questions as to why this expansion in liquidity does not lead to a credit expansion to the real economy.
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Nederlandse Samenvatting van “The Macroeconomics of Banking”

Dit proefschrift onderzoekt de macro-economische effectiviteit van fiscaal en monetair beleid in een omgeving waarin commerciële banken ondergekapitaliseerd zijn na een financiële crisis, en een grote hoeveelheid (risicovol) staatspapier op hun balans hebben staan. Wanneer banken ondergekapitaliseerd zijn betekent dat dat ze relatief weinig eigen vermogen hebben, en om deze reden hun balans niet onbeperkt kunnen verruimen voor een gegeven hoeveelheid eigen vermogen. In dat geval moeten banken kiezen of ze een extra euro aan financiering aanwenden voor extra kredietverlening aan de reële economie of om de hoeveelheid staatsobligaties uit te breiden.

Het is duidelijk dat de Europese staatsschuldencrisis losbarstte in een dergelijke omgeving begin 2010 in Griekenland. Allereerst waren (en zijn) de Europese banken ondergekapitaliseerd na de financiële crisis van 2007-2009 (International Monetary Fund, 2011; Hoshi and Kashyap, 2014). Ten tweede hebben de commerciële banken in Zuid-Europa een grote hoeveelheid staatsobligaties van hun eigen overheid op de balans staan. Bij stresstesten in 2011 constateerde de European Banking Authority (2011) dat de banken in Spanje voor 150% van hun Tier-1 capital (eigen vermogen) aan Spaans staatspapier op de balans hadden staan. In Italië was dit percentage 200% van Tier-1 capital, en in Griekenland 250% (European Banking Authority, 2011). Verliezen op staatspapier kunnen destabiliserend op het financiële systeem werken in een dergelijke
situatie, omdat dergelijke verliezen het complete eigen vermogen van de bank kunnen wegvagen.

Het belangrijkste resultaat van mijn proefschrift is dat de aanwezigheid van (risicovol) staatspapier op de bankbalans van ondergekapitaliseerde banken ervoor zorgt dat de macro-economische effectiviteit van monetair en fiscaal beleid vermindert. Ik onderscheid twee kanalen waaronder de effectiviteit kan afnemen.

Het eerste kanaal is via kapitaalsverliezen op lange-termijn staatsobligaties die op de bankbalans staan. Wanneer de rente op deze staatsobligaties omhoog gaat omdat bijvoorbeeld het risico op een overheidsfaillisement toeneemt, gaan de prijzen van de staatsobligaties omlaag. Dit resulteert in kapitaalsverliezen en een aantasting van het eigen vermogen van de banken. Aangezien de grootte van de bankbalans beperkt wordt door het eigen vermogen van de bank, zorgt een lager eigen vermogen ervoor dat banken de kredietverlening aan de reële economie terugschroeven, wat tot lagere investeringen in de macro-economie leidt.

Het tweede kanaal betreft een zogenaamd “asset substitution effect”, waarbij het voor banken aantrekkelijk is om kredietverlening aan de reële economie te verkleinen, en in de plaats daarvan meer staatsobligaties te kopen. Dergelijke prikkels kunnen ontstaan ten gevolge van regelgeving, zoals bijvoorbeeld een lagere risico-gewicht voor staatsobligaties in vergelijking met andere bank-activa. Omdat het eigen vermogen de grootte van de bankbalans beperkt, zorgt het “asset substitution effect” ervoor dat op deze manier de kredietverlening aan de reële economie teruggestroefd wordt, waardoor de investeringen omlaag gaan, met een negatief effect op het Bruto Binnenlands Product (BBP) tot gevolg.

In hoofdstuk 1 kijk ik naar de effectiviteit van een schuldgefinancierde herkapitalisatie van commerciële banken in een omgeving waarin banken ondergekapitaliseerd zijn, overheden te maken hebben met een significant risico op faillisement, en banken een grote hoeveelheid risicovolle staatsobligaties op hun
balans hebben staan. We laten zien dat in een dergelijke omgeving de effectiviteit van een schuldgefinancierde herkapitalisatie omlaag gaat omdat de additieve schulduitgifte ter financiering van de herkapitalisatie leidt tot lagere prijzen voor staatsobligaties, wat een negatief effect heeft op het eigen vermogen van de banken. Dientengevolge brengen de commerciële banken een hogere rente op bedrijfssleningen. Arbitrage tussen bedrijfssleningen en staatsobligaties zorgt ervoor dat ook de rente op staatsobligaties omhoog gaat, wat leidt tot een verslechtering van de fiscale positie van de overheid, en een hoger risico op een overheidsfaillissement. Rentes gaan verder omhoog ter compensatie van het hogere faillisementsrisico. Een groter risico op een overheidsfaillissement beperkt de ruimte voor overheidsingrijpen, en beperkt de mogelijkheden van de overheid om de financiële sector te helpen.

Deze conclusie contrasteert met de standaard macro-economische beleidsaanbeveling dat de overheid de financiële sector dient te herkapitaliseren. Een praktijk voorbeeld van een gefaalde herkapitalisatie is Spanje, waar de Spaanse overheid in mei 2012 aankondigde de Spaanse banken te herkapitaliseren. De markt voor staatsobligaties stortte echter in elkaar na aankondiging van de herkapitalisatie, en voorkwam dat de overheid de herkapitalisatie kon implementeren. In plaats daarvan moest de Spaanse overheid echter steun zoeken bij de Europese instituties.

In hoofdstuk 2 kijk ik naar de effectiviteit van een schuldgefinancierde bestedingsimpuls door de overheid in een vergelijkbare omgeving als die in hoofdstuk 1. Het debat over de effectiviteit van stimulering door middel van extra overheidsuitgaven heeft een prominente plek gekregen na de kredietcrisis van 2007-2009, een omgeving waar de beleidsrente op nul staat, en inflatie en economische groei beperkt blijven.

De conclusie van dit hoofdstuk is dat de effectiviteit van een fiscale bestedingsimpuls omlaag gaat in een omgeving waarin commerciële banken ondergekapitaliseerd zijn, de overheid bloot staat aan faillisementsrisico, en banken een
grote hoeveelheid staatsobligaties op hun balans hebben staan. In een dergelijke
omgeving kan een publieke bestedingsimpuls contraproducentief uitpakken van-
wege terugkoppelingseffecten in de vorm van een lagere prijs voor staatsobli-
gaties. Lagere prijzen voor staatsobligaties leiden tot een daling van het eigen
vermogen, wat ervoor zorgt dat de capaciteit van commerciële banken om te
lenen aan de overheid of het bedrijfsleven terugloopt. Dientengevolge loopt de
kredietverlening aan de reële economie terug wanneer er verliezen op staatsobli-
gaties optreden, wat leidt tot lagere investeringen in de macro-economie. Een
voorbeeld van een dergelijke omgeving is Zuid-Europa na het uitbreken van de
Europese staatsschuldencrisis. De boodschap van dit hoofdstuk is dat de effec-
tiviteit van een een fiscale bestedingsimpuls door de overheid in een dergelijke
omgeving terugloopt en zelfs contraproducentief kan zijn. Het fiscale beleid wordt
effectiever als commerciële banken beter gekapitaliseerd zijn. Daarom zullen
kapitaalratios (Capital-Adequacy-Ratios, CARs) omhoog moeten.

In hoofdstuk 3 kijk ik naar de macroeconomische impact van de structuur
en de transitie naar hogere kapitaalseisen voor commerciële banken via de kre-
dietverlening aan de reële economie. Om een nieuwe kredietcrisis te voorkomen
zijn er na 2009 hogere kapitaalseisen ingevoerd waarbij banken gedwongen wer-
den meer kapitaal (eigen vermogen) aan te houden per euro aan bankactiva. De
invoering van deze hogere eisen heeft echter een debat op gang gebracht over de
macroeconomische gevolgen van hogere eisen.

Daarnaast is een tweede debat op gang gebracht over de macroeconomis-
sche impact van de huidige structuur van kapitaalseisen, die toegepast worden
op zogeheten risico-gewogen activa. Risico-gewogen activa verwijzen naar de
activa van de bank, waarbij een correctie wordt toegepast voor het risico dat
een bepaalde klasse van activa voor de balans van commerciële banken vormt.
Sommige activa vormen namelijk een groter risico voor de bank balans dan
andere activa. Verschillende activa hebben om die reden verschillende risico-
gewichten. Voor een lening aan het bedrijfsleven, bijvoorbeeld, dient meer ka-
pitaal aangehouden te worden dan voor een lening met dezelfde hoofdsom aan de overheid.

Hoofdstuk 3 laat zien dat hogere kapitaalseisen een negatief effect hebben op kredietverlening aan de reële economie in een omgeving waarin banken ondergekapitaliseerd zijn na een financiële crisis. De gevolgen kunnen beperkt worden door een herkapitalisatie die het eigen vermogen van de banken verhoogt, zoals gebeurd is in de V.S. in 2009.

Een tweede belangrijke conclusie is dat de structuur van kapitaalseisen grote macro-economische gevolgen heeft: in tegenstelling tot wanneer kapitaalseisen worden toegepast op ongewogen activa, zorgt het toepassen van kapitaalseisen op risico-gewogen activa ervoor dat banken de hoeveelheid activa met een hoog risico-gewicht reduceren, zoals bijvoorbeeld krediet aan de reële economie, ten gunste van activa met een laag risico-gewicht, zoals staatsobligaties wanneer de kapitaalseisen verhoogd worden.

In hoofdstuk 4 kijk ik naar het effect van onconventioneel monetair beleid, en specifiek naar de onconventionele Longer-Term Refinancing Operations (LTROs) van eind 2011, begin 2012, waarin de Europese Centrale Bank (ECB) bijna 1.000 EUR miljard aan het Europese banksysteem uitleende. Ondergekapitaliseerde commerciële banken kunnen tegen een lage rente geld lenen van de ECB, maar moeten onderpand inbrengen bij de ECB, waarvoor staatsobligaties het meest aantrekkelijk zijn.

Mijn belangrijkste resultaten zijn (i) dat de onconventionele LTROs een negatief korte-termijn effect op de kredietverlening aan de reële economie hebben gehad, omdat ondergekapitaliseerde banken de kredietverlening terugschroeven om ruimte te creëren voor staatsobligaties die als onderpand kunnen dienen voor ECB financiering, en (ii) dat het onderpandbeleid van de ECB geen invloed heeft op de cumulatieve impact op het BBP van de LTRO-operatie.

Het verschaffen van goedkoop krediet aan de commerciële banken door de ECB kan geïnterpreteerd worden als een indirecte herkapitalisatie van de com-
merciële banken door de ECB. Ter vergelijking implementeer ik een directe herkapitalisatie door de fiscale autoriteit, en concludeer dat dit een effectiever beleid is om de economie te stimuleren dan de onconventionele LTROs, omdat banken niet verleid worden om kredietverlening aan de reële economie te reduceren ten gunste van staatsobligaties.

De meest logische beleidsreactie om de effectiviteit van het traditionele macro-economische beleid te verbeteren is door middel van een herkapitalisatie van de commerciële banken. In hoofdstuk 2, 3 en 4 laat ik zien dat een herkapitalisatie macro-economische uitkomsten verbetert door het verlichten van de leverage constraint van de commerciële banken, waardoor de kredietverlening aan de reële economie toeneemt. In hoofdstuk 1 heb ik echter laten zien dat de effectiviteit van een schuldgefinancierde herkapitalisatie ook afneemt wanneer commerciële banken ondergekapitaliseerd zijn en een grote hoeveelheid risicovolle staatsobligaties op hun balans hebben staan.

In een dergelijke omgeving zullen overheden en centrale banken over alternatieve beleidsmaatregelen na moeten denken om de bovengenoemde negatieve effecten terug te dringen danwel te neutraliseren. Een herkapitalisatie van banken zal in een dergelijke situatie door bijvoorbeeld een externe partij plaats moeten vinden, zodat de negatieve terugkoppeling op de prijs van staatsobligaties achterwege blijft als voorbeeld van een alternatief beleid in hoofdstuk 1. Een alternatieve mogelijkheid is om banken te herkapitaliseren door middel van een debt-equity swap, waarbij schuld van commerciële banken wordt afgeschreven en omgezet in eigen vermogen. Een dergelijke operatie vond plaats in Cyprus in 2013, maar leidde ook tot een kapitaalsvlucht. Meer onderzoek is echter noodzakelijk om alternatief beleid te evalueren.

Een andere onderzoeksrichting is om de effecten van de zogenaamde kwantitatieve verruiming programma’s te onderzoeken. In de afgelopen jaren hebben centrale banken massaal hun balans vergroot door het aankopen van activa, wat ook wel bekend staat als quantitative easing. Dit heeft geresulteerd in een sub-
stantiële vergroting van de reserves die door commerciële banken worden aangehouden. Een belangrijke vraag is waarom de commerciële banken, ondanks een grote hoeveelheid liquiditeit op hun balans, de kredietverlening aan de reële economie niet hebben vergroot. Speelt het feit dat commerciële banken ondergekapitaliseerd zijn een rol bij het achterwege blijven van een kredietexpansie aan de reële economie? De opzet in dit proefschrift, waarbij commerciële banken ondergekapitaliseerd zijn, kan gebruikt worden om te onderzoeken waarom een vergroting van de liquiditeit niet de reële economie bereikt.
The Tinbergen Institute is the Institute for Economic Research, which was founded in 1987 by the Faculties of Economics and Econometrics of the Erasmus University Rotterdam, University of Amsterdam and VU University Amsterdam. The Institute is named after the late Professor Jan Tinbergen, Dutch Nobel Prize laureate in economics in 1969. The Tinbergen Institute is located in Amsterdam and Rotterdam. The following books recently appeared in the Tinbergen Institute Research Series:

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