Incomplete cartels and antitrust policy : incidence and detection

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The Economics of Incomplete Cartels

"Where is the rest of me?" – Kings Row[1]

2.1 Introduction

This chapter provides an overview and discussion of economic theories of incomplete cartels. There exists an abundance of literature on industrial collusion, but theoretical analyses of incomplete cartels are relatively scarce and infrequent. A possible explanation for the modest interest in incomplete cartels is that assuming an all-inclusive cartel typically greatly simplifies the analysis and taking account of cartel size is often not required to study the research question at hand properly. Also, incomplete cartels have never been a major topic of debate. Rather, in many studies, the issue of cartel size is touched upon only marginally and contributions that are explicitly concerned with incomplete cartels have been published irregularly. The main objective of this chapter is to survey and discuss this literature and to highlight some key issues with an eye on future research.

To begin, we might ask: What could be reasons for a cartel not to encompass all firms in the industry? The literature offers at least two main explanations, which both provide an interesting parallel with the most prominent causes of cartel failure. First, the full cartel could be unachievable for reasons that have to do with the internal organization of the cartel and second, cartel size is partly determined by factors external to the cartel.

[1] Drake McHugh (Ronald Reagan) in the movie Kings Row (1942).

Prospective cartel members face an incentive problem and a coordination problem.[2] The incentive problem concerns the potential incentives of cartel members not to abide by the collusive arrangement, while the coordination problem is about reaching consensus on the content of the cartel contract. Real-world examples of cartels reveal that the incentive problem is non-trivial, but perhaps not as important as the coordination problem.[3] Indeed, many studies suggest that bargaining problems and not secret cheating was actually the main cause of cartel failure.[4] In fact, the failure to reach consensus may even temporarily lead to more severe market competition.[5] These internal organization issues form a possible explanation for the existence of incomplete cartels. For instance, there might be too many producers in the market to reach an agreement among all parties.[6] Arguably, it is easier to reach consensus in smaller groups, all else unchanged. Hence, when establishing the all-inclusive cartel appears to be impossible, there might still be an opportunity to install an effective cartel arrangement among a subset of sellers. Also, the cartel may initially be all-inclusive, but one or more members may have decided to secretly cheat on the agreement. Opportunistic behavior by cartel participants could result in an incomplete cartel when the remaining firms still find it in their interest to adhere to the collusive arrangement.

Whether or not a cartel is effective not only depends on these organizational matters, but also on the market environment in which the cartel operates. Indeed, the ability of a cartel to anticipate (a change in) market conditions is pivotal to its success. The effectiveness of a cartel arrangement will partly depend on factors like government interventions, technological advances and market entry. Thus, even when a cartel is able to solve organizational issues, it may not be successful due to external causes.[7] Incomplete cartels may emerge because of structural changes in the market. For example, the cartel may initially encompass all firms, but its collusive gains could attract additional production.[8] If new suppliers neither join the conspiracy nor lead to the

[2] See, for instance, Whinston (2006).

[3] The electrical-equipment conspiracy among twenty-nine U.S. manufacturers in the 1950s is a prominent example of a cartel in which participants persistently cheated. For a formal analysis of cheating on collusive agreements see, for instance, Slade (1990).

[4] See Levenstein and Suslow (2004), which compares many cartel studies and concludes that "About one quarter of the cartel episodes ended because of bargaining problems. Bargaining issues affected virtually every industry studied." An experimental study conducted by Goppelsroeder (2008) also suggests that coordination problems can be significant.

[5] See Levenstein (1996) who uses the term 'bargaining price wars' to describe the price wars that sometimes follow conflicts in the cartel bargaining phase. Bargaining price wars are reported to have occured for example in the Bromine and Tea industries. See Gupta (1997).

[6] In some instances it may be more accurate to talk about the number of decision makers instead of the number of firms. See Cyert *et al.* (1995).

[7] The nineteenth century cartel among U.S. salt producers is illustrative in this respect. Despite a very sophisticated organizational structure it was only modestly succesful, because there were not sufficient barriers to entry. See Levenstein (1995). A similar conclusion is drawn by Clay and Troesken (2002) who analyze collusion in the market for distilled alcohol in the late nineteent century. Also, as noted by Pindyck (1979), even if a cartel is able to solve organizational issues, it may not have sufficient market power to rise price well above the competitive level. For example, market power is limited if the demand curve is highly elastic.

[8] For instance, the successful mercury cartel in the 1950s and 1960s induced entry. See MacKie-Mason and Pindyck (1987).

demise of the cartel, then market entry results in a cartel with less than one hundred percent market share.

A first major issue to consider is then under what conditions incomplete cartels are viable. An incomplete cartel is viable when it enables participants to earn substantially more profits than they would have earned in absence of collusion.[9] A second core issue is whether or not firms have an incentive to form a particular cartel, which we may loosely label 'the participation problem'. This is especially relevant in a discussion of incomplete cartels, because then only a subset of firms should find it in their interest to collude. The participation problem further concerns questions about how many and which firms have an incentive to join the conspiracy.

In this chapter, these and related issues are analyzed from an empirical and a theoretical point of view. We first examine quite a few real-world examples of incomplete cartels and distill four 'stylized facts'. The examples listed suggest that (i) cartels are often not all-inclusive, (ii) incomplete cartels tend to have a dominant position in the market, (iii) the market share of a cartel tends to decline over time, and (iv) incomplete cartels often comprise the larger firms in the industry.

We also address these issues in reference to five basic oligopoly models. The settings that we consider are:

- Simultaneous Bertrand competition with homogeneous products;
- Collusive price leadership;
- Simultaneous Bertrand competition with product differentiation;
- Simultaneous Cournot competition with homogeneous products;
- Collusive quantity leadership.

The choice for these models is not arbitrary. Except for the first, the vast majority of theoretical studies of incomplete cartels uses one of these settings. In fact, we show that no viable incomplete cartel exists in the first model. Nevertheless, it is very useful in illustrating the main concepts and it helps in clarifying the analyses of incomplete cartels in the other settings. The literature reveals that (i) viable incomplete cartels exist in all settings, but the first, provided that cartel members are sufficiently patient, (ii) it is unclear whether or not firms have an incentive to engage in an incomplete cartel in price setting games, (iii) In the simultaneous Cournot model, firms have an incentive to form an incomplete cartel with approximately 80-90% market share, and (iv) In a setting of collusive quantity leadership, firms will form an incomplete cartel with roughly 50% market share.

In the remainder of the chapter, we briefly discuss coalition formation with positive externalities, theories of incomplete cartels with heterogeneous firms and incomplete bidding rings. The five oligopoly models are useful to explore the incentives of firms to form an incomplete cartel. However, more often than not, the actual cartel formation

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[9] In principle, a profitable cartel may have negative earnings. For example, prices may fall even in the presence of a cartel when demand is also declining.

process is not considered. There is a separate strand of literature that deals with the actual cartel formation process. In addition, theoretical contributions on incomplete cartels typically assume identical firms. Studies that assume heterogeneous firms allow to address the question of what type of firms have a stronger incentive to join a cartel. Finally, quite a few incomplete cartels have been discovered in auctions. We briefly survey the main theoretical contributions on incomplete bidding rings.

This chapter proceeds as follows. In Section 2, we present some real-world examples of incomplete cartels and distill some ‘stylized facts’. Section 3 lays out the basics of cartel theory and the standard model that is presented is used as benchmark in subsequent sections. Section 4 and 5 discusses the profitability and sustainability of incomplete cartels respectively. The sustainability of incomplete cartels is studied in the context of an infinitely repeated game. In Section 6 and 7 we consider the participation problem. In contrast to analyses that are concerned with the incentive problem of incomplete cartels, the participation problem is typically studied in static models. In the five oligopoly settings, Section 6 explores the incentives of firms to participate in an incomplete cartel. Section 7 discusses cartel formation games with externalities. Theories of incomplete cartels with firm heterogeneity are surveyed in Section 8. Section 9 discusses economic research on incomplete bidding rings. Section 10 concludes with a brief discussion and some unresolved issues regarding theories of incomplete cartels are pointed out.

2.2 Incomplete Cartels in Practice

Due to the secret nature of many cartels, cartel research necessarily relies quite heavily on, as Ronald Coase once phrased it, “blackboard economics”.[10] This is not to say that we should limit ourselves to a theoretical discourse. In order to arrive at a better understanding of incomplete cartels, we believe it is instructive to first examine some real-world examples. It must be noted that cartel data are only scarcely available. One particular problem is that, more often than not, no information exists about the (combined) market share of cartel members so that the inclusivity of a cartel is unknown. Still, some illustrative evidence of incomplete cartels can be found in both descriptive cartel studies and antitrust cases. We briefly discuss our main findings in this section.

First, however, a word of caution is in order. Most of the evidence presented is, directly or indirectly, derived from antitrust cases. It is not unthinkable and even quite probable that this sample is biased. For instance, some industries have been under closer scrutiny for political reasons, which is likely to have increased the probability of discovery in these sectors substantially. We therefore do not know to what extent the results provide a reliable picture of incomplete cartels. This may lead some to conclude that the use of cartel data is misleading. Be that as it may, it will arguably be equally misleading to completely ignore this empirical knowledge. After all, antitrust cases are the prime source of information on cartels that operated “beyond the boundaries

[10] Coase (1991), p. 5.

of a blackboard". Moreover, we can take some comfort in the fact that the empirical evidence that was found appears to point in the same direction.

Cartels have a very long history. They date back at least as far as one thousand years before Christ. In ancient times, like today, cartels did not necessarily encompass all sellers. For example, the Phoenicians, which are among the first merchants recorded in history, maintained their market power by establishing cartel arrangements in which Greek rivals took no part. Piotrowski (1933, p. 87) writes:

> "The towns of Phoenicia, in particular Carthage, maintained during whole centuries their monopolistic position owing to the cartel contracts with the neighbors protecting their markets against Greek competition. These agreements had at the time excluded competition between the contracting parties themselves by strict division of the markets, exactly as is done by the district cartels to-day."

Around the year 1900, quite a few historical cartel studies were published that report on less than all-inclusive cartels.[11] A recurring theme in these works is "the purpose of a cartel". Most authors seem to agree that 'full monopoly' has been the ideal for many cartels, but, at the same time, it is concluded that the overwhelming majority of cartels had never succeeded to attain this ideal.[12] However, it was generally felt that the monopoly position was not required for a cartel to be effective. Controlling or monopolizing the market is not equivalent to having a 'full monopoly', that is. These studies suggest that cartels do not necessarily have to encompass all firms in the industry as long as the coalition has sufficient power to control the market and raise industry prices well above competitive rates. Broadly speaking, cartels seem not to be too worried about outside production as long as the competitive pressure is limited.

Yet, one particular problem for less than all-inclusive cartels is that outsiders often have an incentive to expand their output levels. For example, Genesove and Mullin (2001) reports with respect to the sugar cartel in the 1930s that the sugar market was bigger than the national market and that foreign suppliers increased their exports to the U.S. once the national sugar cartel was installed. Fringe members that are minor players initially may therefore become formidable competitors over time and ultimately result in a cartel breakdown. This problem is particularly severe when outsiders are unaware of the cartel arrangement. When the competitive fringe silently cooperates with the cartel, it is not unthinkable that fringe investments remain limited. If, however, the incentive to expand production capacity is too strong this ultimately could lead to the termination of the cartel.

The global incomplete cartels *Vitamin C* and *Citric acid* are illustrative in this respect. Both cartels were confronted with Chinese non-participants that expanded

[11] A detailed description of all these works is beyond the scope of this thesis. For a detailed discussion and references the reader is referred to Piotrowski (1933) and Liefmann (1977).

[12] See, for example, Grunzel (1902).

their production and these producers captured an ever bigger share of the market.[13] Ultimately, the Chinese rivals undermined the stability of both cartels. For example, the case description of *Citric acid* reads,

> "During the second period, from mid-1993 until the ending of the cartel in May 1995, it became increasingly difficult for the participating companies to sustain the price levels, in no small measure due to a dramatic increase of citric acid imports from China, particularly into the European market. Accusations of cheating on the agreement, especially against Jungbunzlauer, became rife and the level of trust between cartel members deteriorated."[14]

and,

> "The falling market share of the cartel members was also a matter of concern. From 1991 to 1993 the cartel members' world market share in terms of total sales had fallen from around 70% to less than 60% and continued to fall to 52% in 1994. This continuous decline meant that the size of the 'pie' being shared out between the companies in the cartel was steadily decreasing, a factor that led to increased tension between them."[15]

There were instances in which the cartel faced severe competitive pressure from the start. An illustrative example is a case called *District heating pipes*. This Danish incomplete cartel dominated the European market for district heating systems, but saw itself confronted with one major competitor, Powerpipe in Sweden. According to the European Commission, the cartel systematically attempted to drive Powerpipe out of the market after it refused to join the cartel. In particular, a collective boycott was organized after Powerpipe had won a major project in Germany. The idea behind the boycott was to prevent Powerpipe from getting essential supplies. Yet, this strategy appeared unsuccessful and as a consequence the cartel lost some valuable projects to Powerpipe.

> "Unlike some other smaller producers, Powerpipe not only rejected pressure to join the club: it incurred the wrath of the cartel by systematically underbidding the favourite [the member designated by the cartel to win a particular project] and winning a series of major projects in Germany."[16]

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[13]See, for example, de Roos (2001), who applies a structural dynamic model to the *Vitamin C* cartel. In particular, his analysis suggests that a cartel will persist only if fringe competitors remain small.

[14]See case description at (91). *Official Journal of the European Union*, L 239/18, 6.9.2002. Case No COMP/E-1/36 604.

[15]See case description at (118). *Official Journal of the European Union*, L 239/18, 6.9.2002. Case No COMP/E-1/36 604.

[16]See *Competition Policy Newsletter*, number 1, February, 1999, 27-28.

In general, it has been quite common for cartels that attempted to monopolize the market to take reprisals against unwilling outsiders indirectly by influencing the environment of the competitive fringe. A prime example are the so-called "exclusive trading" clauses. The key element of this method is to either cut off supply (e.g., raw materials), demand (customers) or both.[17] Direct purchasers are forced to buy exclusively from cartel members and raw material suppliers are urged to deliver exclusively to the cartel. Such contracts are typically enforced by means of a boycott. The cartel is simply not selling or buying when sellers or buyers do not respect the clause. In a few instances a modified version is used by asking for a so-called 'loyalty rebate'. Purchasers that do not respect the contract are charged a higher price. The additional earnings are then divided among those who do respect the contract. Evidently, these methods only work if the cartel is sufficiently powerful.[18] Notice that these type of methods secure stability of the cartel and, moreover, render entry of new competition difficult.[19]

A cartel might also decide to take direct reprisals against fringe members. For instance, a cartel may express predatory behavior if it has sufficiently 'deep pockets'. The central idea is to lower combined prices in the short-run until outside producers are knocked out. They are then given the option either to close down or to join the cartel. Another method is buying-up competing firms. Cartels may create common pools to finance the take-over and rival works are often closed down after buying them up. In some instances the cartel has better access to technology required for producing the products. One simple strategy to limit non-cartel supply is then not to share this technology with fringe members. This is what happened in the *sorbates* cartel and the *graphite electrodes* cartel.[20] In light of antitrust enforcement, however, it may be quite risky to take measures against outsiders. For instance, installing a boycott might induce non-cartel members to file a complaint with the antitrust agency and this may significantly increase the probability of discovery.

Occasionally, incomplete cartels are supported by legal institutions and the government. In 1890, for instance, a German bookseller's cartel was challenged by a bookstore not in the ring. Among other things, this outsider had been subject to a boycott. Nevertheless, the court ruled in favor of the cartel and not only that, it also established a precedent against interference by nonconspirators.[21] An example of governmental support for cartels are the Japanese export cartels, which were mandated by government decrees to extend to all firms in the industry.[22] However, governmental support does not always guarantee effectiveness of the cartel. Pindyck and Rubinfeld (2001)

[17] An example is a cartel in seafood processors in the U.S. in the 1980s. The cartel faced external competition by Viking Seafood Co. of Malden. Although not successful, the conspirators responded by trying to cut off Viking's supply of processed fish blocks, used to make fish sticks.

[18] For example, Feldenkirchen (1992) argues that, although German cartels in iron and steel had influence and often used it against opposing outsiders their success was limited.

[19] An illustrative example is a railroad cartel in the U.S. at the end of the nineteenth century, which in cooperation with the Standard Oil Company raised rivals' costs. In particular, the cartel arrangement functioned as a significant barrier to entry. See Granitz and Klein (1996).

[20] See Harrington (2006).

[21] See Kinghorn and Nielsen (2004).

[22] See Dick (2004).

report on an incomplete cartel in the market for milk in the United States. The cartel is called the Northeast Interstate Dairy Compact and is exempt from antitrust laws. The cartel comprises the producers of milk in the Northeastern corner of the U.S. (New England), which together make agreements on the minimum wholesale price. Prices, however, are not much higher than the competitive milk prices, which is due to significant fringe supply from dairy farmers located in the surrounding states.

In addition, there exists some scarce evidence on what type of firms tend to take part in the cartel. The above discussion suggests that larger firms are more prone to collusion than smaller firms. This is confirmed by Asch and Seneca (1975) which analyzes a sample of 101 large manufacturing companies for the period 1958-1967. The sample consists of 51 convicted price-fixers and 50 randomly selected 'non-colluders'. The samples are used to determine what characteristics may distinguish collusive from non-collusive firms. It is important to note that the paper does not contain any information about the inclusivity of the cartels. Yet, the authors repeatedly find that low profit rates induce firms, independent of size, to collude, but that firms with high market shares have a stronger incentive to collude than firms with a relatively low market share.

All the above finds support in Hay and Kelly (1974), which is one of the most well-known empirical cartel studies. They analyzed a limited sample of 65 horizontal cartel cases dealt with by the Antitrust Division of the U.S. Department of Justice between 1963 and 1972. In 20 cases, the inclusivity of the cartel could not be determined. Of the remaining 45 cases, 32 dealt with a cartel that was less than all-inclusive. The incomplete cartels consisted on average of 10-11 participants. However, this average is somewhat misleading in the sense that 26 partial cartels consisted of less than ten firms. Information about the combined market share of cartel members was available in 20 cases.[23] The average market share is approximately equal to 88%.[24] The lowest combined market share that is reported equals 65% and the market share of the incomplete cartel exceeded 90% in 14 cases.[25]

To receive an impression of the size distribution of firms in the industry we may use the widely applied CR4 market concentration ratio in combination with the number of firms in the industry. The CR4 is the percentage of the value of total sales accounted for by the four largest firms in an industry. Hay and Kelly (1974) were able to compute the CR4 in 22 cases in which the cartel is less than all-inclusive. The average CR4 for these 22 cases is approximately 73%. The number of firms in the market was known in only 14 of these 22 cases. The average number of firms is around 12, while the average CR4 is 75%. This result suggests that market shares were unevenly spread in these industries. Hay and Kelly (1974, p. 21) further remarks that,

[23] In 27 cases, data were available to determine the combined market share of the cartel, seven of which were estimated to be all-inclusive.

[24] More precisely, the average is equal to 88.75%. However, in one of these 20 cases the market share was reported as "< 91", which is the reason we rounded the average to the lowest integer.

[25] For the sake of completeness, the combined market share was between 70 and 79% in 2 cases and between 80% and 89% in 3 cases.

> "...it is not necessary for a conspiracy to include all of the firms in the market to exist, or indeed, to be successful...In general, they [non-conspirators] seem to be the smallest competitors. It is difficult to believe that these non-conspirators were unaware of the conspiracy. It might be assumed that they were willing silent accomplices living under the price umbrella provided by the conspirators."

In summary, the results listed above lend some support to the view that:

- Cartels are often incomplete;
- Incomplete cartels tend to dominate the industry in which they operate;
- Incomplete cartels tend to lose market share over time;
- Incomplete cartels often comprise the larger firms in the industry.

2.3 Foundations of Cartel Theory

In this section, we lay out the basics of cartel theory. The main objective is to introduce two key concepts that play a central role in a theoretical discussion of incomplete cartels: The incentive constraint and the participation constraint. A cartel is viable only when none of its members has an incentive to defect from the agreement. Hence, the incentive constraint is important to assess whether or not a particular cartel contract is feasible. The participation constraint indicates whether or not firms have an incentive to form a particular cartel. In the following, we closely follow the literature by analyzing the incentive problem and participation problem separately, because there exists hardly no contributions that take both problems into account. Moreover, discussing both concepts separately is helpful in identifying the main issues that are involved in theories of incomplete cartels. After we have introduced these two concepts, we briefly discuss why cartels are considered bad.

To begin, we might ask: what drives firms to engage in a cartel? To explore this issue, consider a single market with three identical and perfectly informed sellers that compete in price and produce a homogeneous commodity. Suppose that unit costs are constant and that market demand is a linear decreasing function of price.[26] This setting is visualized in the following diagram.

In Figure 2.1, price and cost are depicted as the dependent variable and the x-axis represents total market output Q. Demand and corresponding marginal revenue curve are respectively denoted by D and MR. In this setting, the average and marginal cost curve coincide, which is indicated by the solid horizontal line $AC = MC$.

Competition between the three rivals exerts a downward pressure on prices and margins. Consequently, the competitive Nash equilibrium is such that every firm sets a price equal to marginal cost and none of the competitors is making economic profits.

[26] For simplicity and without loss of generality, we assume there are no fixed costs.

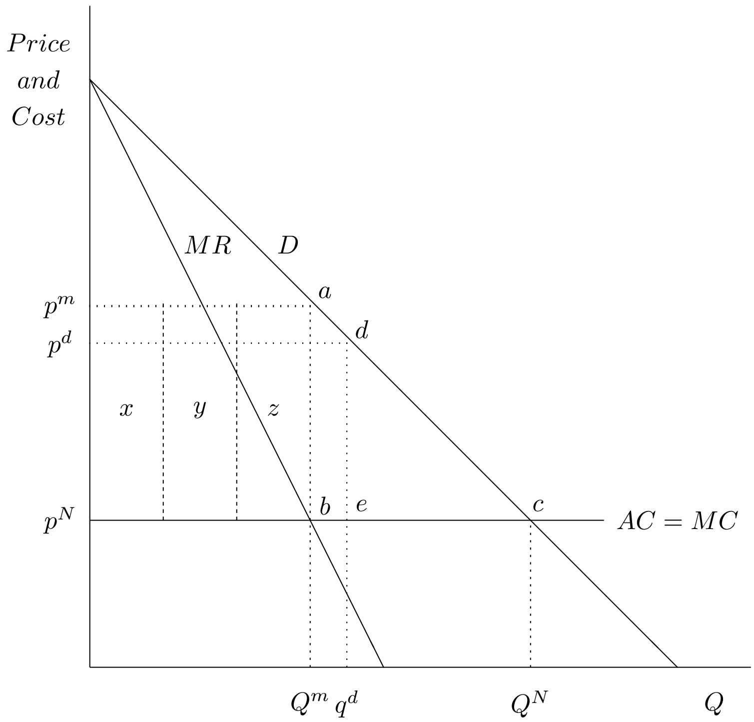

Figure 2.1 Incentives to collude illustrated.

The competitive market price is indicated by p^N. Demand at p^N is denoted by Q^N and we may safely assume that every seller produces one third of the competitive output. By comparison, a single supplier would optimally produce Q^m, at the point where MR intersects with MC, and sell these products at the monopoly price p^m. As a result, a profit-maximizing monopolist would make economic profits. The monopoly profits are indicated by the area $p^m abp^N$ in Figure 2.1.

In this setting, sellers can improve their situation by forming a cartel. The most profitable cartel arrangement is one in which firms mimic a monopolist. That is to say, cartel profits are maximal when the three sellers operate as if they are a single multiplant undertaking. To achieve this market outcome it is required that firms reduce their production levels in such a way that cumulative outputs are equal to Q^m. The three undertakings might therefore fix the price at p^m and/or install output quota so that total cartel outputs equal Q^m. Such an all-inclusive hard-core cartel arrangement will yield total cartel profits equal to monopoly profits indicated by the area $p^m abp^N$. The profits per cartel member are given by areas x, y and z in Figure 2.1 under the assumption that firms receive an equal share of total profits. Observe that cartel profits are higher than competitive profits for all firms, which illustrates the persistent incentive to collude.

2.3.1 The Incentive Constraint

Based on the analysis so far, it may appear that cartels are likely to be the rule rather than the exception. After all, every firm is better off than in competition so that forming a cartel agreement should not be too difficult. Yet, even in this simple framework, a cartel arrangement is all but trivial. The reason is that cartel participants have a strong incentive not to abide by the agreement. To see this, consider the perfect cartel in Figure 2.1. At the cartel price p^m, the three firms all produce $1/3Q^m$. Note that at this point marginal revenue exceeds marginal cost and, as a result, all participants have an incentive to increase their production level.

To illustrate, suppose one member lowers its price slightly to p^d. This allows the chiseling firm to increase its production level to q^d, given that the other members still adhere to the agreement. This deviating strategy would yield profits equal to the area $p^d dep^N$, which is clearly larger than any of the areas x, y, z. It is noteworthy that the incentive to defect is positively correlated with the number of participants. The reason is that maximum industry profit, i.e., the square $p^m abp^N$, is independent of the number of firms, which implies that every member receives a smaller share of cartel profits the larger is the number of participants. All-inclusive cartels are therefore, *ceteris paribus*, more likely to emerge in concentrated industries.

A major challenge for firms is then to form a cartel that is sustainable and often extra measures are required to make a cartel agreement effective. For a cartel to be sustainable, none of the cartel participants should find it profitable to change its strategy given the strategies adopted by the other sellers. However, as illustrated in Figure 2.1, every member typically has an incentive to lower its price and increase its production under the assumption that fellow members abide by the cartel contract. The problem of collusion is therefore very much akin to the prisoner's dilemma. That is to say, any collusive strategy is strictly dominated by some alternative strategy available to firms. Consequently, the cartel is not viable in the static model discussed above.

Yet, cooperation might be sustainable if the players believe that the game goes on forever, i.e., if the one-shot game is repeated for an infinite number of periods. Collusive outcomes can be commonly achieved by means of punishment strategies. The essential difference with the static setting is that in the repeated version of the game cheating might come with a price. Even though cheating may still be profitable in the short-run, it could cause the cartel to dissolve for a number of periods when fellow members retaliate, which implies a loss. Clearly, deviating from the agreement is unattractive when future losses outweigh short term gains.

One particular punishment strategy that is often applied in theoretical studies of collusion is the so-called grim-trigger strategy. Simply put, this strategy prescribes that in the event of cheating all firms compete in all periods following the period of defection. The seminal work is due to Friedman (1971) who showed that, given that firms adopt 'trigger-strategies', there always exists a discount factor for which (full) collusion is sustainable. Formally, a collusive outcome can be supported as an equilibrium outcome of the game if the following condition holds,

$$\delta \geq \delta^* = \frac{\pi^d - \pi^c}{\pi^d - \pi^N}. \tag{2.1}$$

Competitive and collusive profits per firm are respectively given by π^N and π^c. The one-period maximum profit that can be obtained by a chiseling firm is denoted π^d and $\delta \in (0, 1)$ is the actual common discount factor used to value future income. Cartel participants will find it profitable to adhere to the cartel agreement when the actual discount factor δ (weakly) exceeds the critical discount factor δ^*. In the literature this condition is referred to as the incentive compatibility constraint (ICC).

It is important to realize that in the homogeneous goods Bertrand game introduced above an incomplete cartel might be sustainable even though all incomplete coalitions are unprofitable in this particular setting. That is, firms are indifferent between collusion and competition as they earn zero profits in both situations. By contrast, profitable cartels are sustainable in the context of an infinitely repeated game only under the assumption that cartel members are sufficiently patient.

2.3.2 The Participation Constraint

If the incentive constraint is satisfied for all cartel members, then none of the participants has an incentive to deviate from the cartel agreement. The incentive constraint therefore guarantees the stability of a cartel arrangement. However, it tells us nothing about the incentives of firms to take part in a cartel. For a particular coalition to form, it must be individually rational for every (potential) member to join that cartel. Likewise, no firm outside the coalition should be willing to join the coalition. This notion is captured by the 'internal stability condition' and 'external stability condition' as introduced originally by d'Aspremont *et al.* (1983).

Let π^c and π^o denote the profit of a cartel member and an outsider respectively. Formally, a firm has no incentive to leave a cartel formed by k firms when the following condition holds,

$$\pi^c(k) \geq \pi^o\left(k-1\right). \tag{2.2}$$

In a similar fashion, no fringe member has an incentive to join a cartel formed by k firms when,

$$\pi^o\left(k\right) \geq \pi^c\left(k+1\right). \tag{2.3}$$

If both conditions hold simultaneously, then a cartel is called *stable*. In what follows, we refer to (2.2) in conjunction with (2.3) as the *participation constraint.*[27]

In the example presented above and ignoring the special case of a cartel formed by a single firm, the participation constraint only holds for a cartel comprising all three firms. Notice that the full cartel is trivially externally stable, because there are

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[27] Note that with these conditions, a firm only considers the changes after it has made its choice. A more sophisticated notion of this participation constraint, when firms are farsighted and take account of all participation choices of other firms, has been analyzed in Diamantoudi (2005). See also Thoron (1998) which studies coalition-proof stable cartels.

no outsiders. The all-inclusive cartel is internally stable if $\pi^c(3) \geq \pi^o(2) = 0$, which always holds. Firms therefore do have an incentive to form the full cartel in this setting. Moreover, the full cartel was shown to be viable when firms are sufficiently patient. Hence, in the Bertrand homogeneous goods example, the all-inclusive cartel is the only viable cartel and firms do have an incentive to engage in this cartel agreement.

2.3.3 Why are Cartels Bad?

Nowadays, the vast majority of capitalist societies has declared cartels illegal. A legitimate question to ask is, why? As Figure 2.1 reveals, an immediate consequence of a hard-core cartel is higher prices and lower outputs. An effective cartel arrangement therefore leads producers to earn more profits. Yet, this comes with a price. Customers with a sufficiently high willingness to pay have to spend a larger part of their budget to obtain the product. Additionally, some customers that would have bought the product in absence of collusion do not buy the product anymore. Cartels therefore cause a re-allocation of welfare from consumers to producers and a welfare loss for society at large. The latter is referred to as 'allocative inefficiency'. The term used for inefficiencies associated with a misallocation of resources is 'dead-weight welfare loss'. In Figure 2.1, the dead-weight welfare loss is indicated with the triangle *abc*.[28]

In addition to allocative inefficiencies, cartels are likely to generate so-called X-inefficiencies.[29] Firms that shelter from competitive pressure too much are widely believed to lose the incentives to produce their products or services as efficient as possible. Not operating efficiently might also not be needed to survive, because supra-normal profits are more than sufficient to make up for higher costs. Moreover, depending on the cost structure, a cartel may lead to technical inefficiencies in the short run when the reduction of outputs yields an increase in unit costs. Note, however, that when unit costs are increasing in output a cartel may actually yield technical efficiencies.[30]

Cartels are sometimes believed to cause dynamic inefficiencies. Firms that take part in a cartel are thought to have less incentives to innovate, because there is less need to improve the position in the market. However, there are two counterarguments to the idea that collusion is likely to result in fewer product and process innovations. First, a cartel member has typically more means available to innovate, because more profits are earned per period.[31] The additional collusive gains may in principle allow for more R&D expenditures. Second, when innovative activities are successful, a firm may benefit more from its inventions when competition is not too fierce. For example, additional gains due to R&D efforts can be limited when rivals can relatively easy implement the product or process innovation. Thus, collusion might enhance dynamic efficiency. It is unclear which of these opposing forces dominates and the economic literature is inconclusive regarding this issue.

The economic damage caused by cartels is substantial. In real-world examples, it has been estimated that modern international cartels were responsible for an average

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[28] Note that such 'allocative inefficiency' is absent when demand is perfectly inelastic.

[29] See Leibenstein (1966).

[30] I thank Jan Tuinstra for pointing this out.

[31] Schumpeter (1912) argues that a monopolist has stronger incentives to invest in R&D.

price increase of around 28%, but it might well be substantially higher.[32] The graphite electrode cartel, for instance, raised prices by more than 60% in the U.S. and affected 1.7 billion dollars in U.S. commerce alone. More generally, empirical studies suggest it is not uncommon for cartels to increase prices by more than 20%.[33] Yet, with a lower price increase, economic harm can still be significant. For example, in the well-known international lysine cartel, U.S. customers paid on average approximately 17% more than what they would have paid in absence of the cartel. This corresponds to an overcharge of more than 75 million dollars in the United States and 200 million dollars worldwide.[34] Another example is the vitamin cartel which made customers in the United States pay an additional amount of at least 1.2 billion dollars.[35] Baker (2003) provides plenty of other examples. Moreover, he loosely estimates total costs to the U.S. economy of exercising market power to exceed 100 billion dollars annually. It is unclear, however, what share can be attributed to cartel practices.

In addition, cartels may cause other types of harm. In the year 2001, a detailed study of the global price-fixing cartel *lysine* by John M. Connor was published. It is titled: *Global Price-Fixing: "Our Customers are the Enemy"*. The phrase "our customers are the enemy" is a quote from one of the participating managers who were secretly taped by an FBI undercover agent during one of their cartel meetings. This statement reflects the attitude a cartel might have towards direct purchasers of their products. More generally, the title suggests that cartels can have consequences that are not merely economic. Indeed, it could be argued that firms ought not to steal from customers and society. Competition in the market place is distinct from ultimate rivalry, because it takes place within legal boundaries. These 'rules of competition' are established in a more or less democratic fashion and as such can be viewed a fundamental part of the social contract. To put it differently, firms that operate in breach of competition laws behave contrary to what should be done, at least according to the majority of the people.

Cartels, like many other concentrations of power, might lead to abuse and other spin-offs that overall can be considered unwanted. The infamous international *DeBeers Diamonds* cartel is illustrative in this respect. This cartel formed in South Africa at the end of the nineteenth century and is arguably one of the most successful cartels in history.[36] The diamond producer DeBeers takes a leading role in this cartel and, in fact, imposes collusive behavior on the other members by use of coercive tactics. In the early 1980s, Zaire, which is the worlds largest supplier of industrial diamonds defected. Only two months later the market was flooded by industrial diamonds, which caused the price of Zairian diamonds to drop significantly. It is a public secret that DeBeers was responsible by supplying an enormous amount of stockpiled industrial diamonds. Of course, this is a very costly strategy to discipline fellow members, but also very effective. Ultimately, Zaire had to stop the fight, because its pockets were not deep enough to hold out against DeBeers. By 1983, Zaire was allowed to join the cartel

[32] See Connor (2003).
[33] See Bolotova (2006).
[34] See Connor (2001).
[35] See Connor (2001).
[36] See Spar (1994) for a detailed discussion of the international Diamond Cartel.

again, but, not surprisingly, on less favorable terms. DeBeers clearly made its point and one executive posed the following question in a published interview: 'Anyone want to follow Zaire?'[37]

Part of the success of the cartel is, no doubt, due to the fact that the power of DeBeers reaches far beyond the diamond production. It is also involved in the distribution and marketing of diamonds. One of the diamond dealers is said to have said: "DeBeers is like the Mafia... but it is good for the trade"[38] Also, a broker vividly stated:[39]

> "The Syndicate [DeBeers] knows when you sneeze or take a leak. They have a spy system that would put the CIA to shame. They know everything there is to know about anyone of any significance in the diamond world."

Clearly, cartels which are the result of a coercive sort of cooperation could easily lead to practices that are harmful in non-economic terms.

It must be noted, however, that there might be an exception to the rule that cartels are indeed bad. These are the so-called structural crisis cartels. During a business cycle downturn, competition potentially threatens the existence of some firms. One might argue that the duty of saving a company from bankruptcy and consequently protect jobs weighs more heavily than strict compliance with antitrust laws. The implication of a severe economic crisis forcing firms out of business is that fewer firms operate in the market once the economy recovers. The reduced competitive pressure may yield inefficiencies that potentially outweigh the cost of collusion during the crisis.[40]

2.4 On the Profitability of Incomplete Cartels

In this section, we discuss the profitability of incomplete cartels. The key difference between a full cartel and an incomplete cartel is that the latter faces outside competition, which potentially limits the ability to charge a price above competitive levels. In the extreme, suppliers that do not take part in the conspiracy undermine the effectiveness of a cartel. To see this, consider the example depicted in Figure 2.1 above. Suppose, however, that two out of three sellers attempt to collude by fixing a price $p^c \in (c, p^m)$. For such a cartel to be profitable, the non-conspirator must charge a price $p^o \geq p^c$, because cartel demand would be zero with any other price. However, given p^c, the best response strategy of the outsider is typically to slightly undercut p^c. As a result, all customers will choose to buy from the fringe supplier. It follows

[37] See Spar (1994)

[38] See Spar (1994).

[39] See Koskoff (1981).

[40] The European Union and the U.S. have a different attitude towards structural crisis cartels. In the U.S. such cartels are prohibited and considered a *per se* violation of the Sherman Act. By contrast, in Europe structural crisis cartels are sometimes permitted. See, for example, Neumann (2001).

that there exists no profitable incomplete cartel in this simple setting.[41] Therefore, the main question is under what conditions incomplete cartels are profitable.

To begin, we address the question: What are factors that affect the ability of a cartel to raise price above competitive levels? To that end, consider a given industry in which n identical firms produce a homogeneous product and simultaneously decide on their output level q. The profit function of firm i is given by,

$$\pi_i = (p(Q) - c)q_i, \tag{2.4}$$

with c being marginal production cost and $Q = \sum_{i=1}^{n} q_i$. In order to determine the profit-maximizing output level for firm i, we take the first-order derivative of (2.4) with respect to q_i and level to zero,

$$\frac{d\pi_i}{dq_i} = p - c + q_i p' \left(1 + \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{dq_j}{dq_i} \right) = 0. \tag{2.5}$$

Let $s_i = \frac{q_i}{Q}$ denote the market share of firm i and, for the sake of simplicity, we assume firms have equal market shares, i.e, $s_i = s = \frac{1}{n}$. Furthermore, let $\varepsilon = \frac{-p}{Qp'}$ be the price elasticity of demand and define $r_{ij} = \frac{dq_j}{dq_i}$ as the conjectural variation. The conjectural variation measures the expected change in output of firm j to a change in output of firm i. Using some basic calculations, we can rearrange (2.5) to,

$$L_i = \frac{p - c}{p} = \frac{s\left(1 + \sum_{\substack{j=1 \\ j \neq i}}^{n} r_{ij}\right)}{\varepsilon}, \tag{2.6}$$

which is the Lerner index of firm i. The Lerner index has an upper bound equal to 1 and higher values of L_i indicate more market power. Due to symmetry, all firms have a Lerner index with a similar structure, i.e., $L_i = L$. Observe that the price level (and mark-up) is positively correlated with the market share of a firm and negatively with the price elasticity of demand. The profitability of a firm further depends on beliefs about the strategy of competitors. For example, in Cournot competition, firm i does not expect the others to change outputs in response to a change in its own output level, i.e, $\sum_{\substack{j=1 \\ j \neq i}}^{n} r_{ij} = 0$. Hence, $L_i = \frac{s}{\varepsilon} = \frac{1}{n\varepsilon}$ when firms compete in quantity. By contrast, in Bertrand competition firm i expects a change in its production level to be exactly offset by its rivals, i.e., $\sum_{\substack{j=1 \\ j \neq i}}^{n} r_{ij} = -1$. As a result, $L_i = 0$ when firms compete in price.

Now consider the case of collusion. As in competition, the ability of cartel members to raise price above unit production cost depends on the elasticity of demand, the market share and beliefs about the strategies of rivals. Perfect collusion requires every firm to perfectly match each others output reduction, i.e., $\sum_{\substack{j=1 \\ j \neq i}}^{n} r_{ij} = n - 1$.

[41] This conclusion closely resembles the result that became known as the 'Bertrand paradox'. See Tirole (1988) for a more extensive explanation and a formal proof.

Consequently, $L_i = \frac{ns}{\varepsilon} = \frac{1}{\varepsilon}$, which corresponds to the market power of a monopolist and is the maximum market power that can be obtained by a cartel. To see what happens when the cartel is not all-inclusive suppose that k firms form a hard-core cartel agreement and together reduce outputs. The market power of this incomplete cartel then depends on how the $n-k$ outsiders respond. If the outsiders do not perfectly follow the cartel policy the Lerner index of cartel members will be in the following interval $[0; \frac{1}{\varepsilon})$. For example, when the cartel believes fringe members will not adapt their output levels when participants lower their production then the Lerner index of a cartel member equals $L_i^c = \frac{k}{n\varepsilon}$. Clearly, the k cartel members have more market power than in competition, i.e., $\frac{k}{n\varepsilon} > \frac{1}{n\varepsilon}$ for $k > 1$. Yet, the market power is less than with a full cartel, i.e., $\frac{k}{n\varepsilon} < \frac{1}{\varepsilon}$ for $k < n$. It can be concluded that the ability to raise price above marginal cost positively depends on the number of firms that take part in the cartel, all else equal. Note, however, that when firms compete in price, outsiders perfectly compensate the output reduction of the cartel. As a result, no incomplete cartel is viable. If and to what extent an incomplete cartel is profitable therefore depends heavily on the assumptions one is working with.

Some empirical studies attempt to compute the Lerner index for cartels. Eckbo (1976) shows that in 19 out of 51 cartel agreements the cartel was able to raise price more than 200 percent above unit cost. Griffin (1989) analyzes 54 international cartels, 53 of which were less than all-inclusive. The average market share equals approximately 60%. For all 54 cartels, he computes the Lerner index. Among other things, he obtains that the highest Lerner index is equal to 0.80 for an incomplete cartel in the rubber industry that lasted from 1923-1928. The incomplete wheat cartel from 1933-1934 was computed to have the lowest Lerner index (-0.12). It must be noted, however, that this cartel was active in a period following the Great Depression of 1929 and therefore still may have been reasonable successful in relative terms.

The economic literature on collusion offers a variety of models that allow us to further explore the profitability of incomplete cartels. In the following, we briefly discuss four of the most prominent ones. All these theories are closely related to the basic setting laid out in the previous section. The difference lies in the fact that either one or two of the core assumptions are modified.

2.4.1 Collusive price leadership

Incomplete cartels are traditionally studied in a model of collusive price leadership.[42] In this setting, it is assumed that a limited number of large firms engages in a price-fixing cartel. This cartel is supposed to operate as a price leader followed by a large number of small firms that take the cartel price as given.[43] Hence, the key difference with the basic setting described above is that in this model outsiders are considered

[42] One of the first detailed discussions of collusive price leadership is Markham (1951). For a modern and advanced theoretical analysis see Deneckere and Kovenock (1992).

[43] The asymmetry in firm size assumed in this model is commonly observed in oligopolistic markets. We can think of various reasons for why some firms might grow big. Examples include, more efficient production technologies, inventions that are temporarily protected by patents and advantageous geographic locations.

price takers. The competitive fringe is indeed competitive in the sense that profit maximization requires fringe firms to produce the quantity for which price equals marginal cost.

In order to explain the workings of this model consider the following diagram:

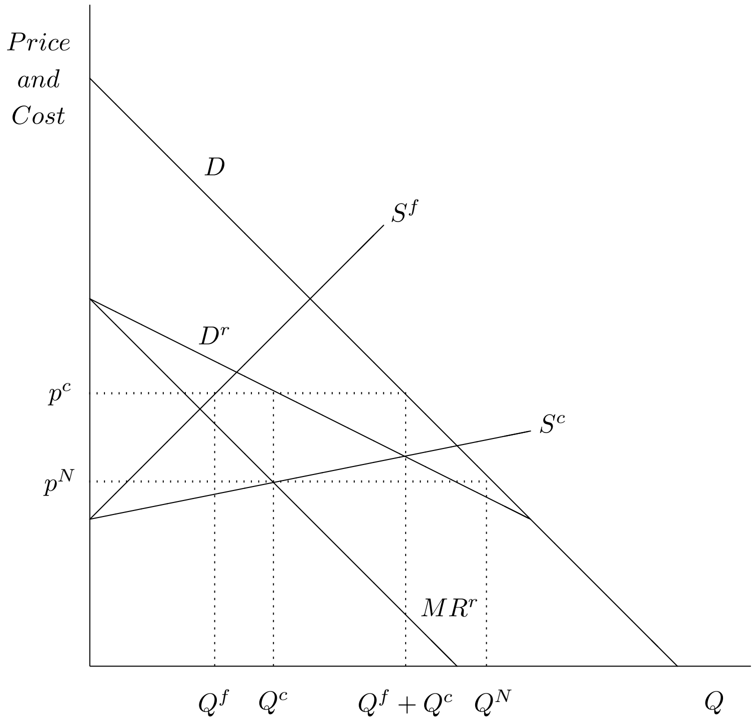

Figure 2.2 Collusive price leadership equilibrium.

The situation depicted in Figure 2.2 is very much similar to the situation sketched in Figure 2.1. An important difference with the previous model is that in the current setting it is assumed that marginal costs are not constant, but increasing in production. Prior to the cartel, all firms set price equal to marginal cost and determine the optimal output accordingly. Like before, the competitive output and price are respectively indicated by Q^N and p^N.[44]

Suppose now that a subset of firms forms a price-fixing cartel, which attempts to maximize joint cartel profits. The cartel supply function is the horizontal sum of the marginal cost curves of members and indicated by S^c. In a similar fashion, one can construct the supply function of the competitive fringe, which is denoted S^f. The demand for cartel output depends both on industry demand and total fringe

[44] For simplicity, the competitive supply curve is omitted. The market supply curve is the horizontal sum of firm's marginal cost curves and intersects the market demand curve D at point (p^N, Q^N).

production. Residual cartel demand D^r is constructed by substracting fringe supply from industry demand at any given price. The corresponding residual marginal revenue curve is denoted by MR^r.

The cartel maximizes joint profits by restricting outputs so that joint marginal costs of members equals MR^r. In Figure 2.2, the cartel optimally sets a price p^c and produces Q^c. As mentioned, the competitive fringe takes p^c as given and optimally produces total fringe output Q^f for which price equals marginal cost. Notice that in equilibrium total fringe supply plus cartel output equals market demand at the cartel price. Note further that, although price is the choice variable and products are homogeneous, an incomplete cartel is profitable when the cartel operates as a price leader. In particular, the assumption that fringe producers sell their products at the cartel price allows the cartel to increase prices without losing all its customers to producers not included in the price-fixing conspiracy. Finally, an interesting observation is that, due to the presence of outside competition, the residual demand (D^r) is more elastic than market demand (D). As a result, an incomplete cartel typically sets a lower price than an all-inclusive cartel would.

2.4.2 Differentiated goods

In the previous setting, the standard framework was modified by changing the cost structure and the order of moves. Alternatively, it may be assumed that products are less than perfect substitutes. With product heterogeneity, low price fringe members may not attract all customers and potentially leave part of the market to the cartel. As a result, incomplete cartels might be profitable when products are sufficiently different. In this section, we briefly discuss the setting discussed in Section 2.3 under the assumption that the price elasticity of demand is less than infinite.

Such a study has been conducted by Deneckere and Davidson (1985).[45] They consider a setting in which firms face the following Shubik-Levitan demand function,

$$q_i = a - p_i - \gamma(p_i - \frac{1}{n}\sum_{j=1}^{n} p_j). \tag{2.7}$$

Here, a is a demand scale parameter and n indicates the number of (symmetric) firms. The direct price effect and the cross-price effect are respectively given by $1+\gamma(1-\frac{1}{n})$ and by $\frac{\gamma}{n}\sum_{j\neq i} p_j$. The common parameter $\gamma \geq 0$ measures the degree of product differentiation. If $\gamma = 0$, goods are unrelated and collusion would make no sense. By contrast, if γ is large, goods are close substitutes and price competition becomes fierce.[46] We therefore naturally suppose γ to be neither very large nor very small.

For simplicity, we normalize marginal cost to zero, i.e, $c = 0$. Suppose that k firms engage in a price-fixing cartel with the objective to maximize joint profits. The profit-maximizing cartel price is equal to,

[45] This paper is primarily concerned with mergers, but as the authors remark in a footnote the analysis could be equally well applied to cartels.

[46] Perfect substitutability corresponds to $\gamma \to \infty$.

$$p^c = \frac{a[2n + \gamma(2n-1)]}{4n + 2\gamma(3n-k-1) + \gamma^2(\frac{n-k}{n})(2n+k-2)}, \tag{2.8}$$

and independent outsiders optimally charge,

$$p^o = \frac{a[2n + \gamma(2n-k)]}{4n + 2\gamma(3n-k-1) + \gamma^2(\frac{n-k}{n})(2n+k-2)}. \tag{2.9}$$

Observe that $p^c > p^o$ for $k > 1$, which always holds. Hence, independent of the size of the coalition, a cartel member always sets a higher price than a fringe firm.[47] As a consequence, cartel members lose market share, i.e., outsiders become the larger firms in the industry.

Concomitant profits are respectively,

$$\pi^c = a^2[\frac{2n + \gamma(2n-1)}{4n + 2\gamma(3n-k-1) + \gamma^2(\frac{n-k}{n})(2n+k-2)}]^2(1 + \gamma\frac{(n-k)}{n}), \tag{2.10}$$

and,

$$\pi^o = a^2[\frac{2n + \gamma(2n-k)}{4n + 2\gamma(3n-k-1) + \gamma^2(\frac{n-k}{n})(2n+k-2)}]^2(1 + \gamma\frac{(n-1)}{n}). \tag{2.11}$$

It is easy to show that profits of cartel members are positively correlated with the number of participants, i.e., larger coalitions are more profitable than smaller ones. In particular, any incomplete cartel is profitable.

2.4.3 Quantity competition

A third way in which we can adapt the standard framework is by assuming that firms have output rather than price as their strategic variable. The key difference with the previous models is that in quantity setting games reaction functions are typically downward sloping, while these are often upward sloping in price setting games.[48] As we have seen in the Bertrand differentiated goods model, a price increase by cartel members leads to a price increase of fringe firms. In this particular setting, a price increase leads to an output reduction by both the cartel and outsiders. By contrast, when firms compete in quantity, an output reduction by cartel members leads to an *increase* of fringe output. Thus, given that the quantities supplied by firms function

[47] In addition, note that both the cartel price and the fringe price are higher than the competitive price, $p^N = \frac{a}{2+\gamma\frac{n-1}{n}}$.

[48] Note, however, that under some conditions reaction functions may have upward sloping portions in quantity setting games even when products are substitutes. Likewise, reaction functions may in some settings not be upward sloping over the relevant range in price setting games. Yet, the positive slope of reaction functions in price setting games is felt to be natural for a broad class of situations. See, for example, Martin (2002) for an extensive discussion.

as strategic substitutes, an incomplete cartel will be profitable only when the output reduction of members is more than compensated for by a price increase. As before, we consider a situation in which firms make their decisions simultaneously and a situation in which the cartel moves first.

2.4.3.1 Simultaneous play

The question when incomplete collusion is profitable in a static Cournot setting with linear demand has been addressed by Salant *et al.* (1983). To explain their main findings, let N denote a set of n identical profit-maximizing firms, which choose their production level simultaneously. Products are homogeneous and produced at constant marginal and average cost equal to $c > 0$. Suppose that the inverse demand function is linear and given by,

$$p(Q) = a - bQ, \tag{2.12}$$

where p is the market price and $Q = \sum_{i=1}^{N} q_i$ is total market output. In order to guarantee positive production levels, we naturally suppose that $a > c$. In this symmetric setting, every firm earns the following competitive Cournot-Nash equilibrium profits,

$$\pi^N = \frac{(a-c)^2}{b(n+1)^2}. \tag{2.13}$$

Now suppose that a number of k firms collude. Every cartel member then makes a profit equal to,

$$\pi^c = \frac{(a-c)^2}{bk(n-k+2)^2}, \tag{2.14}$$

whereas independent outsiders act according to their best response function and earn,

$$\pi^o = \frac{(a-c)^2}{b(n-k+2)^2}. \tag{2.15}$$

As can be observed, outsiders make more profits than in competition for all cartel sizes. In addition, a fringe member benefits more from collusion than a cartel member for all $k > 1$. Cartel members reduce their production levels and thereby create incentives for outsiders to increase their output. This is due to the assumption about quantities being strategic substitutes. As a result, the cartel creates a positive externality for firms that remain independent outsiders, i.e., fringe members enjoy the benefits of a higher market price, but do not incur the cost associated with a reduction of output. Furthermore, it is noteworthy that total industry profits always increase in the presence of a cartel.

A second key result of the analysis is that collusion is only (weakly) profitable if,

$$\frac{k}{n} \geq 1 + \frac{3 - \sqrt{4n+5}}{2n}.$$

This implies that at least 80% of the firms in the industry must participate in the cartel to ensure that members are better off than in competition.[49] At first glance, this result may appear surprising, because in principle the multiplant firm is always able to copy the competitive strategy. This, however, is not an equilibrium following the cartel, because the best response strategy *given* unchanged outputs of outsiders is to reduce total cartel production. In fact, the possibility of lower profits for cartel members arises precisely because outsiders increase their output levels. The price increase resulting from the cartel may therefore not be sufficient to offset the loss in market share.[50] In sum, an incomplete cartel might be profitable if it takes a sufficiently dominant position in the market.

2.4.3.2 Sequential play

Shaffer (1995) and Martin (2002) study a model in which a cartel operates as a Stackelberg leader. To formalize, consider the same setting as discussed in the previous subsection. A cartel typically reduces outputs, which in equilibrium induces a fringe firm to adapt its output level according to the following best response function,

$$q^o = \frac{a - c - bQ^c}{b(n-k+1)}, \tag{2.16}$$

where Q^c denotes total cartel output. Given that fringe firms act accordingly, the optimal output level of a cartel member equals,

$$q^c = \frac{a-c}{2bk}. \tag{2.17}$$

This yields the following per firm equilibrium profits,

$$\pi^c = \frac{(a-c)^2}{4bk(n-k+1)}, \tag{2.18}$$

and,

$$\pi^o = \frac{(a-c)^2}{4b(n-k+1)^2}. \tag{2.19}$$

To examine whether or not an incomplete cartel is profitable we compare (2.18) with the competitive Cournot profits as given by (2.13). Cartel members are (weakly) better off than in competition when the following condition holds,

$$(n+1)^2 \geq 4k\,(n-k+1)\,. \tag{2.20}$$

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[49] This minimum is reached for $n = 5$ and $k = 4$, i.e., $\frac{k}{n} = \frac{4}{5} = 0.8$.

[50] The original analysis of Salant *et al.* (1983) focuses on the profitability of mergers instead of cartels. However, as Perry and Porter (1985) rightly remarks, mergers are conceptually not well defined in this setting. The reason is that the new collusive equilibrium implicitly assumes that the merger does not differ from fringe firms, while in fact it is larger. Moreover, the number of firms will be lower post-merger.

The right-hand side of (2.20) reaches its maximum at $k = \frac{n+1}{2}$ and for this value (2.20) holds with equality. Hence, any cartel that operates as a Stackelberg leader in the presence of a Cournot fringe is profitable.

2.4.4 Comparison

At this point, it is useful to summarize and compare the findings discussed above. In this section, we considered five different oligopoly models and examined which incomplete cartels are profitable. The main conclusions are summarized in the following table.

Table 2.1: Profitable incomplete cartels.

Strategic variable	Order of moves	Homogeneous goods	Heterogeneous goods
Price	Simultaneous	none	all
	Sequential	all	-
Quantity	Simultaneous	dominant cartels	-
	Sequential	all	-

Clearly, whether or not an incomplete cartel is profitable very much depends on the assumptions one is working with. As Table 2.1 reveals, incomplete cartels are often profitable. In particular, in three out of five models, incomplete cartels of any size are profitable. This is not the case in simultaneous move games in which firms produce a homogeneous commodity. Under these assumptions, an incomplete cartel must take a dominant position in a market in which firms compete in quantity, while no incomplete coalition is profitable when firms compete in price.

2.5 On the Sustainability of Incomplete Cartels

In the previous section, it has been shown that incomplete cartels are profitable in a variety of settings. In this section, we explore the sustainability of these profitable coalitions. Recall that every cartel member has an incentive to defect from the agreement given that its fellow members abide by the cartel contract. Consequently, none of the profitable incomplete coalitions is sustainable in the one-shot version of the games discussed so far. As we have explained in Section 2.3, a cartel is potentially sustainable in an infinitely repeated version of the static games. A cartel can be sustained when the incentive compatibility constraint is satisfied for all participants, i.e., when cartel members are sufficiently patient. It is important to realize that the incentive constraint is not necessarily violated for unprofitable coalitions. For example, firms might not have an incentive to defect from an unprofitable cartel agreement in the simultaneous Bertrand game with homogeneous products, because deviating yields no additional gains. However, we are naturally interested in cartels that are profitable. As mentioned, all profitable cartels are sustainable in the context of an infinitely repeated game under the assumption that cartel members are sufficiently patient.

In the following, we discuss the infinitely repeated version of the remaining four models. In particular, we explore if the critical discount factor is increasing or decreasing in the number of cartel participants. This is of special interest, because if the critical discount factor is increasing in the number of participants, then the full cartel might not be sustainable. Alternatively, a full cartel might be sustainable only when firms set a lower price than the unconstrained profit-maximizing price. This could yield lower per-firm collusive gains compared to an incomplete cartel that is sustainable even when it charges the unconstrained profit-maximizing price and/or output. In other words, when δ^* is increasing in k, there might exist a range of values for δ for which incomplete cartels are the preferred subgame perfect equilibrium.

2.5.1 Collusive price leadership

A cartel that operates as a price leader chooses its price such that the joint marginal costs of its members equals marginal revenue corresponding to the residual demand curve. Given that all cartel participants are identical, this implies that every participant produces a quantity for which marginal cost equal the marginal revenue derived from residual demand. In principle, therefore, none of the members has an incentive to deviate *given the cartel price*. That is, increasing production is not a profitable strategy when the cheating firm does not alter its price. Deviating from the agreement, however, is potentially very profitable when a cartel member secretly undercuts the cartel price slightly and produces a quantity such that the deviation price equals marginal costs.

However, determining what is the optimal deviating strategy in the basic standard collusive price leadership model is difficult. The reason for this is that the demand structure is such that firm demand does not depend directly on the prices set by rivals. Hence, to derive an explicit equation for the critical discount factor one needs to take a slightly different approach. For instance, one could assume that products are less than perfect substitutes. Posada (2001) makes an attempt in this direction. He finds that for a market with Bertrand competition, the critical discount factor is an increasing function of the number of cartel members. Consequently, when the actual discount factor is not too high, the only sustainable cartels are less than all-inclusive. Overall, however, this contribution illustrates that an analysis of incomplete cartels in repeated Bertrand settings very easily becomes intractable.

2.5.2 Differentiated goods

Above, it has been shown that when firms compete in price and produce differentiated goods all cartel sizes are profitable. As a result, there always exists a δ for which any incomplete cartel is sustainable. However, for a certain cartel to be sustainable it is required that in the event of cheating the non-cheating members can credibly commit themselves to dissolve the cartel. Eaton and Eswaran (1998) considers the situation in which non-cheating members may find it profitable to continue to operate as a cartel and refrain from implementing the punishment strategy. In other words, they assume the rest of the cartel to remain after a firm defected from the agreement. The authors refer to this assumption as 'stacked reversion' in contrast to 'Nash reversion'.

Eaton and Eswaran (1998) shows with a numerical example that the critical discount factor is increasing in the number of cartel participants. However, under the assumption of 'stacked reversion', it increases much more rapidly. The reason is that the punishment for deviating from the agreement is less severe when the remaining members adhere to the agreement. Consequently, some cartels that are sustainable with 'Nash reversion' are not sustainable with 'stacked reversion'. The main finding of the study is that under the assumption of 'stacked reversion' only cartels comprising a small fraction of the industry might be viable. This result is driven by the fact that cartel participants may have an incentive to leave the cartel as long as non-cheating members find it profitable to adhere to the agreement.

2.5.3 *Quantity competition*

As before, we also consider the possibility that firms compete in quantity. In the following, a distinction is made between a simultaneous move game and a setting in which the cartel has a first mover advantage.

2.5.3.1 Simultaneous play

Friedman (1971) shows that when firms play a simultaneous Cournot game with an infinite horizon there always exists a δ for which the monopoly outcome can be sustained. In a similar setting, Escrihuela-Villar (2004) analyzes the possibility that not every firm participates in the cartel. Consider the simultaneous Cournot game discussed above. The maximum amount to be earned by a chiseling firm is,

$$\pi^d = \frac{(k+1)^2}{4k}\pi^c. \tag{2.21}$$

Observe that any cartel is inherently unstable, because $\pi^d > \pi^c$ for $k > 1$. Combining with (2.13) and (2.14) and normalizing $\frac{(a-c)^2}{b} = 1$ we get,

$$\delta \geq \delta^* = \frac{(k-1)^2(n+1)^2}{(k+1)^2(n+1)^2 - 4k^2(n-k+2)^2}. \tag{2.22}$$

This critical discount factor δ^* is increasing in the number of firms. Hence, the more firms are operating in a given industry the higher will be the critical discount factor and the less likely it is that the actual discount factor is high enough to allow for effective collusion, all else equal. At the same time, the critical discount factor is strictly decreasing in the number of cartel participants. That is, whenever a cartel of k firms is sustainable, i.e., spans at least 80% of the market and $\delta \geq \delta^*$, then larger coalitions are sustainable too.

The latter result is not obvious and, in fact, the result of a trade-off. On the one hand, the larger the number of participants, the higher is the profit level. On the other hand, the incentive to cheat is positively correlated with the number of cartel members. Yet, the first effect dominates. It must be noted that this result in part depends on the type of punishment strategy. For example, when firms adopt a more optimal punishment scheme as in Abreu (1986, 1988), the result is partly reversed.

That is, the critical discount factor is decreasing in the number of cartel participants, but only for certain parameter values.

2.5.3.2 Sequential play

Martin (1990, 2002) examines a setting in which the cartel operates as a Stackelberg leader. In every period, cartel profits per member are therefore given by (2.18). He further assumes that in the event of cheating all firms compete à la Cournot in all periods following the period of defection. The static Nash profits π^N are given by (2.13). It remains to determine what a cartel member can maximally earn when it decides to defect on the cartel arrangement. Given that $k-1$ firms stick to the collusive output and $n-k$ firms play their fringe outputs, a chiseling member optimally sets the following output level,

$$q^d = \frac{(a-c)(n+1)}{4bk(n-k+1)}, \tag{2.23}$$

with concomitant profits,

$$\pi^d = \frac{(a-c)^2(n+1)^2}{16b(k(n-k+1))^2}. \tag{2.24}$$

Collusion can therefore be sustained if and only if,

$$\delta \geq \delta^* = \frac{(n+1)^2}{4k(n-k+1)}. \tag{2.25}$$

It is easy to verify that δ^* depends non-monotonically on k. In fact, δ^* is decreasing in k as long as $k < \frac{n-1}{2}$ and the reverse holds for larger values of k. It can be concluded that no cartel is sustainable whenever an incomplete cartel of size $k = \frac{n-1}{2}$ is not sustainable.

2.5.4 *Comparison*

A sustainable cartel is profitable, but the converse is not necessarily true. In the previous section, we have shown that there exists a variety of profitable coalitions in the five scenarios that were considered. All these cartels, however, are not self-enforcing in the static setting. One way of explaining how firms can sustain some level of collusion is by assuming that the static games are repeated for an infinite number of periods. In this section, we have shown that when the static games become part of a supergame with infinite horizon many coalitions are sustainable as subgame perfect equilibrium of the repeated game. In fact, when firms are sufficiently patient, any profitable coalition is sustainable. Clearly, punishment strategies are a very effective means to sustain collusion, but in some sense they are too successful. Indeed, the number of subgame perfect equilibria in these games is, as noted by Tirole (1988), 'an embarrassment of riches'.

It should be emphasized that an all-inclusive cartel could be less effective than an incomplete cartel. Particularly, it might occur that when δ is sufficiently low the ICC for members of the full cartel is binding, while it is not binding for some incomplete cartel. Thus, for certain values of δ an incomplete cartel in principle could yield more profits per member than a cartel that encompasses all firms in the industry. Whether or not this is the case depends in part on the setting under consideration. For the five oligopoly models, we might ask for what cartel size(s) the critical discount factor reaches its minimum. The following table shows for what cartel size δ^* is lowest.

Table 2.2: Cartel size (k/n) for which δ^* is lowest.

Strategic variable	Order of moves	Homogeneous goods	Heterogeneous goods
Price	Simultaneous	1	$2/n$
	Sequential	$2/n$	-
Quantity	Simultaneous	1	-
	Sequential	$\pm 1/2$	-

As Table 2.2 reveals, for what cartel size the critical discount factor reaches its minimum varies per setting. In the simultaneous models with homogeneous products, δ^* is lowest when all firms take part in the cartel. In other words, full collusion is sustainable whenever some collusion can be sustained. An all-inclusive cartel is therefore always preferred by cartel members. By contrast, when δ is relatively low, an incomplete cartel could be more effective when the cartel has a first mover advantage or when products are differentiated. In these scenarios, the critical discount factor does not reach its minimum for a cartel that comprises the entire industry. As a result, there might exist ranges of values for δ for which an incomplete cartel yields higher profits per member than the full cartel.

2.6 The Participation Puzzle

We have established that a variety of incomplete cartels is viable provided that cartel members are sufficiently patient. So far, however, we did not address the question how such a cartel can emerge. That is to say, we did explore under what conditions none of the participants finds it profitable to deviate from the agreement, but did not consider how these firms became part of the cartel. This issue is of particular interest if one is concerned with cartels that are not all-inclusive, because then only a subset of firms should have an incentive to engage in an anticompetitive arrangement. In this section, we discuss the incentives of firms to collude or to become an outsider to a cartel formed by rivals instead.

The discussion in the previous sections reveals that a cartel functions as a public good. A group of firms that collectively reduces outputs and increases prices creates a positive externality for firms that do not take part in the conspiracy. The best response of fringe members is often to increase both their production levels and their prices, which implies windfall profits. However, the main issue is not so much that outsiders benefit from the cartel, but that outsiders typically benefit *more* than insiders. As a consequence, there exists a strong incentive to free-ride on a cartel formed by competitors.[51] The ensuing "participation-puzzle" has been clearly formulated by George Stigler in a discussion on horizontal mergers. He stated that,

> "...the major difficulty in forming a *merger* is that it is more profitable to be outside a *merger* than to be a participant. The outsider sells at the same price but at a much larger output at which marginal cost equals price. Hence, the promoter of a *merger* is likely to receive much encouragement from each firm - almost every encouragement, in fact, except participation..."

This reasoning applies equally well to (incomplete) cartels. It is therefore especially difficult to understand how less than all-inclusive cartels can emerge in equilibrium. Although all firms in the industry would benefit from the cartel, the very fact that the choice for fringe membership often strictly dominates the choice of becoming a cartel member may prevent the formation of (incomplete) cartels.

Firms may have an incentive to form a particular cartel when the participation constraint is satisfied, i.e., when the cartel is stable. Recall that for a cartel to be stable it is required that none of the firms wants to change membership. To begin, consider the Bertrand homogeneous goods model. For this model, it was shown that the only sustainable coalition is one comprising the entire industry. Notice that the full cartel is trivially externally stable, because there are no outsiders. The all-inclusive cartel is internally stable if $\pi^c(n) \geq \pi^o(n-1) = 0$, which holds for all n. Firms therefore do have an incentive to form the full cartel in this setting. In the following, we apply the participation constraint to the remaining four oligopoly models.

2.6.1 Collusive price leadership

D'Aspremont *et al.* (1983) apply their notion of stable cartels to a basic model of collusive price leadership. To explain its workings, we assume that the inverse demand function is given by,

$$p = \frac{1}{bn}(an - Q), \tag{2.26}$$

and further suppose that firms have identical cost functions of the form,

$$C(q) = \frac{1}{2}cq^2. \tag{2.27}$$

Recall that in a setting of collusive price leadership fringe firms take the cartel price as given and produce quantities so that price equals marginal cost. We therefore have that $cq^o = p^c$, which determines the output for an individual fringe member $q^o = \frac{p^c}{c}$.

[51] A separate strand of literature focuses on how a firm or group of firms might raise rivals' costs. An incomplete cartel could attempt to increase the costs of outsiders and thereby reduce free-rider incentives. Potential cost raising scenarios will not be discussed in detail here and the reader is referred to Salop and Scheffman (1983, 1987) for analyses and discussion.

With a number of $n-k$ outsiders this yields a residual demand for the cartel that amounts to,

$$kq^c = n(a - bp^c) - (n-k)\left(\frac{p^c}{c}\right). \tag{2.28}$$

The joint-profit maximizing price of the cartel is then equal to,

$$p^c = \frac{an(1-c)}{\left(bn + \frac{n-k}{c}\right)(2-c)}, \tag{2.29}$$

which is strictly higher than the competitive price and which yields per firm cartel profits equal to,

$$\pi^c(k) = \frac{ca^2}{2\left((n+bc)^2 - k^2\right)}. \tag{2.30}$$

Outsiders charge the same price for their products, but produce more. Their individual profits amount to,

$$\pi^o(k) = \frac{ca^2(n+bc)^2}{2\left((n+bc)^2 - k^2\right)^2}. \tag{2.31}$$

This analysis yields several interesting results. First, cartel profits per member are strictly increasing in the number of cartel participants. Second, outsiders earn more profits than insiders for all $k > 1$ so that the 'participation puzzle' is fully applicable.[52]

The main result of d'Aspremont *et al.* (1983) is that, under the reasonable assumption that n is finite, there always exists a coalition for which the participation constraint is satisfied. It must be noted, however, that the proof does not exclude the coalition consisting of only one firm and that the stable cartel may as well comprise the entire industry.[53] Also, the fraction of firms in a stable cartel is decreasing in the size of the industry. That is, if n increases the number of firms in the stable cartel tends to zero.

2.6.2 *Differentiated goods*

Applying the participation constraint to the Bertrand model with differentiated goods yields intractable expressions. Observe, however, that outsiders always earn more profits than insiders, i.e., (2.11) is larger than (2.10) for all $k > 1$. Clearly, for all cartel sizes, firms prefer to be outside the cartel. However, whether or not firms have an

[52] Note that $\pi^o(k) > \pi^c(k)$ holds for all k does not imply that there exists no internally stable coalition. That is, $\pi^o(k) > \pi^c(k)$ is not inconsistent with $\pi^c(k) > \pi^o(k-1)$.

[53] Donsimoni *et al.* (1986) shows that, in this setting, the stable cartel may not be unique when firms are cost-efficient relative to market demand. In particular, they show that there exist industry sizes for which two stable cartels exist, a cartel that is incomplete and a cartel that is all-inclusive.

incentive to change membership cannot be determined *per se.* In all likelihood, this will depend heavily on the parameter specifications.[54]

2.6.3 *Quantity competition*

Clearly, the participation constraint can also be applied to games in which firms have output rather than price as their choice variable. Like before, the simultaneous and sequential move games are discussed respectively.

2.6.3.1 Simultaneous play

Consider the Cournot market with homogeneous goods and linear demand analyzed above. A minimum requirement for firms to have an incentive to form a cartel of k firms in this setting is that none of the k members finds it in their interest to be an outsider to a cartel consisting of $k-1$ firms. To begin, recall that any cartel for which $k < n + \frac{3}{2} - \frac{1}{2}(4n+5)^{\frac{1}{2}}$ is not profitable in a sense that members are worse off than in competition. Clearly, such a cartel is never internally stable, because fringe profits are always (weakly) larger than competitive profits. In other words, firms never have an incentive to form a cartel with a market share less than 80%.

We now focus on the situation in which $k \geq n + \frac{3}{2} - \frac{1}{2}(4n+5)^{\frac{1}{2}}$ so that any cartel member is (weakly) better off than in competition. Formally, the cartel is internally stable if,

$$\pi^c(k) = \frac{(a-c)^2}{bk(n-k+2)^2} \geq \pi^0(k-1) = \frac{(a-c)^2}{b(n-k+3)^2}.$$

It is easy to verify that this condition is violated for all $k > 1$ as long as $n \geq 3$. The condition does hold for $n = 2$ and $k = 2$. It can be concluded that cartel members have an incentive to leave the cartel as long as the remaining coalition is (weakly) profitable. Accordingly, firms have an incentive to form a cartel of size $k \simeq n + \frac{3}{2} - \frac{1}{2}(4n+5)^{\frac{1}{2}}$. Such a cartel is externally stable, because outsiders always earn higher profits than insiders. The reason that no member has an incentive to leave the cartel is that this would lead to Cournot-Nash profits that are (weakly) lower.[55]

2.6.3.2 Sequential play

Shaffer (1995) applies the participation constraint to the sequential Cournot model.[56] Using the expressions,

$$\pi^c = \frac{(a-c)^2}{4bk(n-k+1)}, \tag{2.32}$$

and,

$$\pi^o = \frac{(a-c)^2}{4b(n-k+1)^2}, \tag{2.33}$$

[54] See, for instance, Bloch (2002).
[55] See also Escriuhela-villar (2004, 2008a).
[56] See also Martin (2002).

it can be shown that the cartel is (weakly) internally stable if,

$$k \leq \frac{(n-k+2)^2}{(n-k+1)}. \tag{2.34}$$

Hence, none of the firms is willing to leave the cartel only if the number of conspirators is not too big. The intuition behind this result is that a coalition between a small number of firms gives a relative big share of cartel profits to participants. At the same time, outside competition is fierce, which makes joining the fringe not too attractive. In contrast, if the number of cartel participants is large, cartel profits are shared among many, while the number of outsiders is small. Leaving the cartel is attractive now, because the fringe profits are relatively high compared to cartel profits. Moreover, note that for some combinations (n, k),

$$\frac{(a-c)^2}{4b(n-k+1)^2} \geq \frac{(a-c)^2}{4bk(n-k+1)} \geq \frac{(a-c)^2}{4b(n-k+2)^2}, \tag{2.35}$$

implying that the cartel is (weakly) internally stable even though outsiders earn higher profits. This boundary result emerges when joining the fringe lowers the profits of outsiders just enough so that staying in the cartel is preferred.

The cartel is (weakly) externally stable if,

$$k \geq n-k+1+\frac{1}{n-k}. \tag{2.36}$$

Hence, the number of conspirators must be sufficiently large to make joining the cartel not attractive. In other words, although joining the cartel always increases total cartel profits, it lowers the per firm cartel profits due to an increasing number of participants.

The (subgame) perfect equilibrium of this Stackelberg game yields the following stable cartels for $n \geq 3$,

- $n > 4 \Longrightarrow k = \frac{1}{2}n + 1$, for even n,
- $n > 4 \Longrightarrow k = \frac{1}{2}(n+1) + 1$, for odd n,
- $n = 3 \Longrightarrow \; k = 3$
- $n = 4 \Longrightarrow k = 3 \vee k = 4$

Note that, apart from $n = 4$, the equilibrium number of cartel participants is uniquely determined. In particular, the equilibrium number of cartel participants facing a Cournot fringe is just over half the firms in the industry regardless of the total number of firms. Finally, note that outsiders in equilibrium earn more profits than insiders.[57] Nevertheless, under certain conditions, insiders may be better off than outsiders when the cartel operates as a quantity leader. Salant (1976), for example, analyzes the world oil market with a quantity setting game in which the cartel is a

[57] When $k < \frac{n+1}{2}$, insiders earn more profits than outsiders. Note, however, that this will never occur in equilibrium, i.e., such a small cartel is not stable.

dominant firm. Participation is beneficial because the cartel builds up more reserves by restricting outputs, while the stock of fringe firms will be exhausted sooner leaving the cartel a monopoly position in the future.

2.6.4 Comparison

The incentive to form a particular cartel in the various models are summarized in Table 2.3.

Table 2.3: Profitable stable incomplete cartels.

Strategic variable	Order of moves	Homogeneous goods	Heterogeneous goods
Price	Simultaneous	none	unknown
	Sequential	unknown	-
Quantity	Simultaneous	$k \simeq n + \frac{3}{2} - \frac{1}{2}(4n+5)^{\frac{1}{2}}$	-
	Sequential	$k \simeq \frac{1}{2}n$	-

Based on the studies discussed in this section, we can draw two main conclusions. First, when firms compete in price, it might be that no profitable stable incomplete cartel exists. This certainly holds when firms decide on their choice variable simultaneously and produce homogeneous commodities. Whether or not this conclusion carries over to a Bertrand setting with differentiated goods and a model of collusive price leadership is uncertain. Bloch (2002) shows numerically that, in a Bertrand setting with differentiated goods, profits per cartel member are increasing in cartel size. Yet, it remains unclear whether or not firms have an incentive to form a particular cartel. Given that these type of numerical analysis are very sensitive to parameter specifications, results, if any, are likely to be only of limited value. Second, a stable cartel always exists when firms compete in quantity. When the cartel is assumed to take a leading role in the industry, firms have an incentive to engage in a cartel that comprises about half the industry. Instead, when firms take their strategic decisions simultaneously, firms have an incentive to form a cartel that controls approximately 80-90% of industry supply.

It is important to emphasize that the existence of stable incomplete cartels does not completely solve the participation puzzle. We have shown that only a subset of firms might find it in their interest to collude, but we did not say anything about how firms arrive at a particular equilibrium cartel arrangement. That is, the existence of a stable incomplete cartel reveals that firms have an incentive to engage in an incomplete coalition, but nothing is said about how such a coalition actually forms. This issue is discussed in the next section.

2.7 Coalition Formation with Positive Externalities

In the previous section, we have discussed the incentives of firms to form an incomplete cartel, but did not consider the actual coalition formation process. A separate strand of literature focuses on the manner in which a cartel is formed. This literature typically assumes firms to play a two-stage game. In the first stage, players form a coalition according to the specific rules of the game. Given the established coalition structure, the game is played noncooperatively in the second stage. The oligopoly models analyzed above are examples of games played in the second stage. Recall that in this type of models a coalition creates positive externalities, i.e., non-participants benefit from the cartel.[58] Hence, we are naturally interested in coalition formation games with positive externalities. In this section, we review some of the main studies that deal with cartel formation explicitly.[59]

To begin, it must be noted that we limit ourselves to a discussion of the formation of a single coalition with independent outsiders. The possibility of forming multiple coalitions is only marginally considered. In particular, we do not survey the literature on "equilibrium binding agreements" in which coalitions are allowed to split up in smaller coalitions.[60] Moreover, this section is organized slightly differently from the previous sections. As before, we do categorize the games with respect to the timing of decision-making. However, we do not discuss the five oligopoly models separately. Alternatively, we make a distinction between, on the one hand, bargaining or "exclusive membership" games and, on the other hand, participation or "open membership" games. In settings of "exclusive membership" an undertaking is only allowed to join the cartel when all members agree. In "open membership" games, by contrast, players have the freedom to join whatever coalition they like.

2.7.1 Simultaneous cartel formation

In simultaneous coalition formation games, firms do know the strategy space of the game, but have no knowledge about the actual participation decision of competitors. Arguably, this approach is somewhat restrictive within the context of cartel formation. After all, a cartel is commonly initiated by one firm or a small group of firms which operates as the cartel ringmaster. The ringmaster typically contacts other competitors with the request to become part of the ring.[61] This implies that firms often have some knowledge about the participation decision of rivals. Nevertheless, the assumption of firms taking formation decisions simultaneously allows for a deeper understanding of what type of cartels are likely to emerge. We can distinguish between simultaneous bargaining models and simultaneous participation models, which are subsequently discussed.

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[58] In contrast, a coalition could create negative externalities. A well-known example is a R&D joint-venture that creates efficiency gains for participating firms. These forms of cooperation may yield a competitive disadvantage for outsiders. See, for example, Bloch (1995).

[59] For an extensive overview and comparitive analyses of stable coalition structures with externalities see Yi (1997). For traditional coalition formation literature that does not consider externalities across coalitions see, for example, Aumann and Dreze (1974) and Shenoy (1979).

[60] See Ray and Vohra (1999).

[61] Occasionally, firms are more or less forced to join the cartel. See the brief description of the international diamond cartel in this chapter.

2.7.1.1 Simultaneous bargaining

A simultaneous bargaining game can be considered a game of exclusive membership. In such a setting, all players simultaneously propose a coalition in which they are willing to engage. There are two well-known variations.[62] In one version, a cartel is formed only when all participants proposed this particular coalition. For example, the coalition $C = \{i, j, z\}$ will form if players i, j and z all three propose C. The second version is less stringent and rules that a coalition will emerge if part of the players reach consensus. For instance, if C is proposed by i and j, but not by z, then i and j will form a cartel.

It is important to emphasize that players base their proposals on the expected gain from participating in a particular coalition. Players may be aware, however, that their *ex ante* expectations of participating may not coincide with *ex post* earnings. Hart and Kurz (1983), for instance, analyzes a non-cooperative setting in which players simultaneously announce coalitions. In addition, however, they examine the bargaining process within a coalition once it is formed and analyze what coalition structures are stable given the *ex ante* value of a player.

2.7.1.2 Simultaneous participation

Probably the first research that took this approach is the game-theoretic work *where four are few and six are many* by Selten (1973). The main aim of this contribution is to analyze the conjecture by Chamberlin (1933) and Stigler (1964) that joint profit maximization among the few is likely, while impossible in markets with many firms. Selten analyzes a three-stage simultaneous Cournot game. In the first stage, firms simultaneously decide if they want to participate in the cartel or not, the second stage determines the cartel output quota, which in turn are implemented in the third stage. The main finding is that in markets with four or less players all firms will participate in a cartel, while in industries with six or more firms no cartels will be formed leaving five as the intermediate case.

In the game considered by Selten (1973), cartel size is taken to be endogenous. He calculates the probability of a particular coalition to emerge in equilibrium. In an industry with four or fewer firms all-inclusive cartels form with probability one. Hence, in such concentrated industries incomplete cartels cannot be explained as a subgame perfect equilibrium. In an industry with five sellers, the probability of an incomplete cartel consisting of four firms is approximately 18%. For markets with more than five firms this probability is positive but very small. It is below 1.3% and tends to zero quickly for larger industries. Therefore, in markets with five or more undertakings the probability of incomplete collusion to occur is positive, but very small.

A similar approach has been conducted by Prokop (1999) who investigates the case of a dominant cartel with price leadership. Recall, that in this setting outsiders earn always higher profits than insiders independent of the number of participants. One may, therefore, *a priori* expect all firms to decide for the outsider position if such decisions are supposed to be taken independently and simultaneously. Indeed, the

[62] See, for example, Bloch (2005).

situation is similar to that of the well-known prisoner's dilemma. Yet, at the same time we know all firms benefit if some cartel nevertheless emerges. The outcome of the game depends on parameter values.

In particular, if costs are sufficiently high, a complete cartel will form in industries with $n \leq 3$ with certainty, while in industries with $n \geq 4$ each firm joins the cartel with a positive probability. By contrast, if firms in the industry are sufficiently efficient a complete cartel will form with certainty for $n \leq 5$. For $n > 5$ the uniqueness of a pure strategy equilibrium, i.e., all firms are willing to participate, cannot be guaranteed and very much depends on the value of the cost parameters. It is further shown, that both the participation probability as well as the expected size of the cartel are declining in the industry size.[63] The reason is that within this setting the stable cartel size is independent of n, which makes the free-rider effect more severe when more firms are operating on the market. It is noteworthy that the expected result does not coincide with the stability of the cartel. So, even though such a stable cartel always exist, it may be difficult to reach this outcome when firms take their participation decision simultaneously.

The case of a cartel that operates as quantity leader has been examined by Hviid (1992). This contribution is particularly concerned with the question how private information (e.g., about a common parameter in the demand function) affects the incentives to form a cartel. Note that the Stackelberg assumption implies that part of the information is either directly disseminated to the fringe or indirectly, for example, because it is reflected in the output decision of the cartel that is known by fringe members with certainty. By assumption, however, the analysis solely focuses on equilibria that comprise all firms in the industry. Instead, changes in industry size are used as an indicator of how private information is likely to affect the incentive to form cartels. The main conclusion is that if outsiders can infer the information of the cartel, it will, *ceteris paribus*, lead to smaller cartels, i.e., the existence of private information yields a disincentive to cartel formation. What drives this result is that, apart from the existing free-rider effect, outsiders also benefit from correct inference of the cartel's output choice and the attempt of the cartel to manipulate this inference is shown to have an adverse effect on cartel profits.

2.7.2 Sequential cartel formation

In sequential cartel formation games, players take a decision or make a proposal in a particular order. This implies that some players have more information about the participation decisions of rivals than others. Arguably, a sequential approach more accurately reflects cartel formation in practice. After all, a manager that has to decide on whether or not to take part in the conspiracy typically has some information about the plans of (part of) the other firms. For instance, receiving an invitation to attend a meeting in a "smoke-filled room" at least partly reveals the intentions of the other firms that are invited.

[63] Note the similarity with the symmetric Cournot game analyzed in Selten (1973).

However, assuming that participation decisions are taken sequentially comes with a price. The outcomes of sequential formation games tend to be very sensitive to the often arbitrary assumptions about the order in which firms take decisions. Clearly, firm a taking its decision before firm b could lead to an outcome that might differ substantially from a situation in which firm b decides first. Obviously, however, this drawback is far less severe under the assumption that firms are identical.

In sequential formation games, every decision maker has (perfect) knowledge about the history of the game, because otherwise it would be *de facto* simultaneous. At the same time, players anticipate the consequence of their decision for (the optimal) decisions made by subsequent players, i.e., players are farsighted. Moreover, for such a formation setting to work, it is important that once a decision is made it is binding for the rest of the formation stage. That is, a sequential formation game is only fundamentally different from a simultaneous formation game when firms commit to their choices. As before, we distinguish between bargaining and participation models, which are subsequently discussed.

2.7.2.1 Sequential bargaining

Bloch (1996) proposes a method that allows coalitions to form in a non-cooperative sequential process. Given a certain order of players, the first player proposes a coalition, which is formed when accepted by all prospective members. The first player to reject the proposal becomes the initiator in the next round. An important property of this method is that firms which successfully form a cartel are removed from the game so that the rest of the game is played only by the remaining players. Moreover, the focus is exclusively on the formation of coalitions given a certain sharing-rule of the value of a coalition. Hence, players *ex ante* know what their earnings will be in a given coalition.

The author applies the method to the simultaneous Cournot setting discussed above. Recall that a cartel must at least consist of 80% of the firms in order to be profitable and that outsiders benefit more from the cartel than insiders. He finds that the unique equilibrium of the formation game is such that the first $n-k$ firms remain independent, while the last k firms form a cartel. Given the number of firms in the industry the unique k is given by the following equation,

$$k \geq n + \frac{3 - \sqrt{4n+5}}{2}. \tag{2.37}$$

That is, the first integer that "solves" (2.37) gives the equilibrium cartel size. The reason that the last $n - k$ firms form a cartel even though outsiders are better off is that their next best alternative is competition and competitive profits are lower than cartel profits given that the 80%-requirement is met.

In the previous section, it was shown that firms have an incentive to form this particular cartel. Due to the free-rider incentive, the unique cartel size is the minimum cartel size required to sustain some collusion. Yet, although this result is theoretically sound, it is difficult to give the formation mechanism a practical interpretation. That is, it is difficult to see how a couple of firms start the formation process by announcing "no", leaving the remaining firms in a situation in which they are willing to say "yes".

Part of this problem is due to the fact that commitment to the participation decision is not credible.

2.7.2.2 Sequential participation

In a sequential participation game firms, one after the other, take a binary decision, which is either to participate in a cartel or to remain an independent outsider instead. Prokop (1999) analyzes the process of cartel formation in relation to collusive price leadership as studied by d'Aspremont *et al.* (1983). In particular, he attempts to find an answer if such stable cartels will emerge if the participation decision is made endogenous. He finds that within the sequential-move game the first $n-k$ firms refuse to participate leaving the task for the last k firms to form the cartel. It is important to emphasize that, in contrast to the analysis of this model with simultaneous participation decisions, the cartel that is formed is also stable. The intuition underlying this result is similar to that in Bloch (1996). The first firms understand that saying "no" leaves a situation that is internally stable, but not externally stable for firms later on in the sequence of decision makers. Hence, after $n-k$ firms said "no" it is in the self-interest of the next firm to say "yes" and all the others that follow, given that a cartel of k firms is indeed stable.

2.8 Incomplete Cartels and Firm Heterogeneity

The theoretical studies of incomplete cartels that we have discussed up until now assume identical firms. This assumption has the major advantage that it often allows the researcher to obtain clean analytical results. In many instances, calculations are tedious and can easily become intractable without this assumption. However, assuming identical firms is potentially problematic when studying incomplete cartels, because it is *a priori* unclear how identical firms would take non-identical decisions, which is trivially needed for an incomplete cartel to emerge. That is, assuming identical firms implies that firms lack identity in the model and therefore we cannot analyze why a particular firm decided to join the cartel or became an independent outsider instead. In other words, nothing can be said about which or what type of firms are more inclined to join a cartel. Moreover, any collusive equilibrium is unique only up to permutations.

There are a few theoretical studies that analyze incomplete cartels when firms are heterogeneous, which are briefly surveyed in this section. We will focus exclusively on studies that assume firms to differ *ex ante*, i.e., prior to cartel formation. For example, the possibility that participants enjoy a cost advantage, as may be the case with a merger, is excluded. This is based on the belief that cartel organizations will create substantially less synergies than mergers.[64]

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[64] Perry and Porter (1985) and Farrell and Shapiro (1990) analyze the ability of firms to create synergies in horizontal merger settings. The central idea is that cooperation may yield a significant reduction in production costs so that *ex post* insiders may be better off than outsiders. It seems difficult to convincingly defend the case of cost-saving cartels. On the contrary, restricting outputs in

Heterogeneity among firms can take many forms. For instance, a firm can have an information advantage over its rivals, access to more financial resources or control more production capacity. Also, some sellers may enjoy the benefits of having a more advantageous geographical location. Under a regime of basing-point pricing, for example, a group of firms located relatively close together potentially could use a distant firm as a natural focal point to determine a collusive base location. This not only allows for a stable agreement, but at the same time prevents outsiders from profitably entering local markets. Indeed, basing-point pricing is well-known to facilitate incomplete cartels.[65]

Differences between sellers might explain why full collusion sometimes seems to be an unattainable ideal. For example, when firms have different cost levels it might be difficult to establish a cartel contract that is acceptable to all parties. Low-cost firms typically prefer a lower cartel price than high-cost firms.[66] We can imagine that it is easier to form a cartel agreement between firms that are more or less identical. That is, substantial differences in production costs could hurt the negotiation process and, as a result, full collusion could be difficult to arrange.[67] In addition, the stability of the cartel may depend on relative cost efficiencies of firms as is shown in Rothschild (1999).

Donsimoni (1985) studies incomplete cartels when firms differ in terms of unit production cost in a model of collusive price leadership as discussed above. Cost functions are identical among firms of the same type, but allowed to differ across types. The main question is what type of firms are more inclined to become a price leader, i.e., what type of firms have a stronger incentive to join the cartel. Among other things, she finds that the most efficient firms will be in the cartel, while less efficient firms remain independent outsiders. Collusion among firms that differ in terms of production costs is also studied by Cramton and Palfrey (1990). The authors analyze a static model with a finite number of firms and a continuum of cost types and find that a group of low-cost firms might find it in their interest to bribe high cost firms not to produce. However, the main purpose of this study is to analyze if heterogeneity in costs prevents firms from obtaining the monopoly outcome. The main result is that with sufficiently many firms in the industry, the monopoly outcome cannot be obtained. This is in line with the conventional wisdom that collusion becomes increasingly difficult the higher the number of firms.

A different form of asymmetry is introduced by McAfee (1994). He analyzes a particular advertising model to study cartel formation. Firms differ in their availability

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the presence of potential scale economies cartel members are more likely to have higher production costs.

[65] Chapter 5 provides an in-depth discussion of basing-point pricing.

[66] This is not to say that collusion among firms with different efficiency levels yields relatively low cartel prices. For example, as is shown in Harrington (1991) the optimal collusive price might exceed the monopoly price of a low-cost firm given that the discount factor is sufficiently high.

[67] Alexander (1997) analysis of the 1930s pasta industry suggests that cost heterogeneity is a problem to form a cartel. In particular, the large firms had low costs and therefore a different optimal collusive price than high costs small firms.

rate, which is defined as the probability that a consumer receives a price offer from a particular firm. He finds that there exists an equilibrium in which only the largest firms participate in the cartel. Moreover, cartels typically comprise at least two, but generally not every firm in the industry.

The studies discussed so far in this section use a static model and cartel agreements are therefore *de facto* binding. That is, firms face a participation constraint, but no incentive constraint. To the best of our knowledge, there is no contribution that considers incomplete cartels with firm heterogeneity in an infinitely repeated game setting. The effect of firm heterogeneity on collusion has been studied in a dynamic setting, but only under the assumption that the cartel is all-inclusive. Harrington (1989), for example, considers a setting in which firms differ in terms of their discount factors. Among other things, he finds that firms with a relatively low discount factor receive a relatively large share of collusive profits, which is required to prevent cheating.

There exist a few studies that explore how the size distribution of firms in the industry affects collusion. Kühn and Motta (1999) analyzes a model in which firms produce a variety of products. The authors find that small firms, i.e., firms with only one or a few type of products, have a stronger incentive to defect from the cartel agreement. In addition, implementing a punishment strategy is more costly for larger firms, i.e., firms with many type of products. Hence, a more equal distribution of product varieties facilitates collusion. Compte *et al.* (2002) and Vasconcelos (2005) analyze a supergame in which firms differ in terms of production capacity. Like Kühn and Motta (1999), both find that the highest critical discount factor is lowest when firms are of equal size. Under the assumption of an all-inclusive cartel, all three papers then find support for the idea that more symmetry among firms facilitates collusion.

2.9 Incomplete Bidding Rings

So far, we have restricted ourselves to a discussion of incomplete collusion among sellers. A substantial part of economic activity, however, is organized through auctions and procurements. These exchange mechanisms allow a group of bidders to compete for an object or contract offered by a seller. Like in almost every market there is a natural incentive for competitors to reduce competitive pressure among them with the aim to increase (short-run) gains from trade. There exists both formal and informal evidence that bidder collusion is a prevalent phenomenon and that the rings formed may or may not include all bidders.[68] Incomplete cartels in auctions have their own characteristics and are not necessarily comparable to incomplete cartels in more regular markets.[69]

[68] See, for example, Froeb (1988) who reports that between 1979-1988 there are 319 Sherman Act Section 1 criminal cases filed by the U.S. DOJ, 81% of which concerned auctions. Feinstein *et al.* (1985), for instance, analyzes a real-world example of an incomplete bidding ring in the highway construction industry

[69] For example, as mentioned by Porter and Zona (1993,1999), some participants may purposely decide not to submit bids. It is difficult to see how some cartel members that operate in more regular markets could temporarily “disappear”, although participants may refuse to sell to certain groups of buyers.

In the following, a brief overview is provided of the theoretical economic literature on incomplete bidding rings.

Like in the discussion above, we may distinguish between the feasibility of a cartel and cartel formation. As regards to the sustainability of bidding rings there is a variety of methods available to guarantee cartel stability. For instance, the cartel may appoint an enforcer who is given the task to punish observed chiselers. Simultaneously, when bidders interact frequently, the ring may adopt punishment strategies to ensure compliance. A bidder who does not adhere to the cartel policy is confronted with lower competitive payoffs in future auctions. A bidding ring is then effective when discounting is sufficiently low. In short, bidding rings often can be sustained in much the same way as cartel arrangements in more regular markets.

In a recent contribution, Marshall and Marx (2007) develop a framework in which the ring not only faces competition from outside bidders, but is also confronted with potential stability problems. They conclude that collusion is less effective in a first-price rather than a second-price auction. The authors explain the difference in the incentives to chisel in a first- and second-price auction as follows,

> "At a second-price auction, a ring must suppress the bids of all members except the bidder with highest value. The ring member with the highest value goes to the auction and bids as he would were he acting non-cooperatively. Any ring member who thinks of breaking ranks and competing at the auction faces the highest ring bidder and the highest non-ring bidder, each submitting bids that are the same as if all were acting non-cooperatively. Thus, there is no gain to deviant behavior. The first-price auction is quite different. In order to secure a collusive gain, the ring member with the highest value must lower his bid below what he would have bid acting non-cooperatively, and other ring members must suppress their bids. But when the highest-valuing ring member lowers his bid, the opportunity is created for a non-highest-valuing cartel member to enter a bid at the auction, either on his own or through a shill, and secure the item. This possibility jeopardizes the feasibility of a cartel at a first-price auction."[70]

Among other things, they show that when a ring adopts a so-called 'bid coordination mechanism' it cannot eliminate all ring competition at a first-price auction.[71] Furthermore, the work illustrates that incomplete cartels in auctions quite generally are profitable.

Also, the 'participation puzzle' might well be different in auctions. More accurately, unlike for incomplete coalitions in regular markets, cartel formation may be less of a problem for bidding rings. The reason is that the incentive to free-ride on a cartel

[70] See Marshall and Marx (2007), pp. 375-376.

[71] In a bid coordination mechanism (BCM), the cartel can install a side-payment scheme and recommend bids to all members. However, it lacks the power to control the bids of the ring members.

might be weaker or even absent. Indeed, whether or not an incomplete bidding ring creates externalities for non-conspirators typically depends on the type of auction. To illustrate, consider the following two settings.

McAfee and McMillan (1992) analyzes a first-price auction with n bidders of which k collude. Bidders have valuation v, which is assumed to follow a discrete distribution so that $v = 0$ or $v = 1$. Let $p = \Pr\{v = 1\}$ and assume the reserve price to be zero. The expected profit of a cartel member and an independent outsider are then respectively given by,[72]

$$\pi^c = (1-p)^{n-k}\left(\frac{1}{k}\right)\left[1-(1-p)^k\right],$$

and

$$\pi^o = (1-p)^{n-k}\,p.$$

Observe that outsiders always earn more profits than insiders independent of cartel size.[73] Moreover, in line with the notion of stable cartels discussed above, it can be shown that a stable cartel always exists. The cartel size can be (almost) uniquely predicted, assuming that the cartel is formed through an open membership game. Following the participation constraint as given by (2.2) and (2.3), the optimal cartel size k^* is implied by the following inequality constraints,

$$\frac{1-(1-p)^{k^*}}{k^*} \geq p\,(1-p) \geq \frac{1-(1-p)^{k^*+1}}{k^*+1}.$$

The number of bidders participating in a stable ring is increasing in p and is always larger than three.

In contrast, it can be shown that the free-riding effect is absent in second-price auctions. To see this, assume that valuations are drawn independently from a common distribution function F with density f. The gains for respectively cartel members and outsiders are given by,

$$\pi^c = \frac{1}{k}\int_0^\infty \int_0^x (x-y)\,(n-k)\,F(y)^{n-k-1}f(y)kF(x)^{k-1}f(x)dydx,$$

and

$$\pi^o = \int_0^\infty \int_0^x (x-y)\,(n-1)\,F(y)^{n-2}f(y)f(x)dydx.$$

As can be observed, the expected gain of outside bidders is independent of k. Moreover, the expected gain of outsiders is always lower than the expected profits of participants for all values of k. Clearly, it is always beneficial to join the cartel and the unique stable cartel is all-inclusive.

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[72] It is common in the auction literature to refer to "utility" of bidders instead of profits. For convienence, we have chosen to keep notation in line with the previous sections.

[73] Here, we only present some of the main results. For a detailed description of the game as well as derivations the reader is referred to the original article.

The reason that free-riding incentives are absent in second-price auctions is that it is typically optimal for the ring to submit a bid equal to the bid that its highest-valuing member would submit in competition. It is well-known that in second-price auctions submitting a bid equal to the personal valuation is a dominant noncooperative strategy for each bidder. Mailath and Zemsky (1991) show that in second-price private value auctions any subset of bidders can efficiently collude, but their analysis also indicates that an all-inclusive ring is most likely even when bidders are heterogeneous. Incomplete bidding rings in second-price auctions are therefore not easily explained as the result of strong free-rider incentives. Yet, the authors remark that,

> "...one would expect that in many environments it will not be feasible to extend membership to all potential bidders. For example, legal considerations might lead the ring to limit membership to avoid detection. Or there might be a large number of potential bidders each with only a small probability of having a value above the reservation price so that the costs of coordinating the large ring exceeded the benefits."[74]

Bidder heterogeneity in first-price auctions has been studied by Marshall *et al.* (1994). This type of analysis becomes easily analytically intractable and, as a result, the authors have limited themselves to a numerical analysis. They propose an algorithm that proves to be particularly useful in the analysis of incomplete cartels that face competition from independent outsiders. The study conducted suggests that the expected revenue is higher in first-price auctions, but also that cartel formation is more complicated than in English auctions. As mentioned above, this is due to the fact that in first-price auctions a less than all-inclusive cartel creates positive externalities for bidders not part of the ring. Furthermore, a noteworthy result is that small all-inclusive rings are often feasible, but quite unlikely when the number of (potential) bidders is large.

The absence of free-rider and deviation incentives in second-price auctions forms a sound explanation for the prevalence of bidder collusion. It must be noted, however, that often some additional measures are required to make the ring effective. For instance, quite frequently only one or a few cartel members attend the main auction and consequently have a positive chance of winning the object or contract. Clearly, the remaining participants want their share of the pie and this may give rise to several organizational problems. Indeed, the problem of how to divide the (extra) rents among members can be quite severe.[75] This in particular implies that bidding rings, more often than not, form examples of overt collusion. A notable exception are multi-object auctions. Brusco and Lopomo (2002) consider a multi-object English auction and establish that in this setting side contracts may not be needed to sustain some level of collusion. The reason is that in such open ascending bid auctions bidders can signal what object they value most, e.g., by abstaining from competing over certain objects. However, tacit collusion in these type of auctions becomes difficult when the number of (potential) bidders grows large.

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[74] See Mailath and Zemsky (1991), p. 485.

[75] Eckbo (1976) shows empirical evidence that in a sample of international cartels almost half collapsed due to disagreement on how to share the profits.

There exists some real-world evidence of bidding rings that have developed sophisticated methods to cope with these type of problems. Graham and Marshall (1987), for example, reports that bidding rings address the issue of how to allocate collusive gains through "nesting", i.e., forming rings within rings.[76] Interestingly, it appears that some rings have adopted a scheme in which every participant is awarded its "Shapley-value", i.e., every member receives its own average marginal contribution. Graham *et al.* (1990) builds on this work and theoretically explores these so-called secondary auctions or knockouts. Among other things, they establish that in equilibrium side-payments to ring members are indeed equal to their Shapley value in a variety of settings.

2.10 Discussion

In this chapter, we have provided an overview of available theories of incomplete cartels. Three issues have been discussed in reference to five basic oligopoly models: The profitability and sustainability of incomplete cartels and the incentives of firms to form a particular cartel. It has been shown that incomplete cartels are often profitable and sustainable provided that coalition members are sufficiently patient. When firms compete in quantity, there is an incentive to form an incomplete cartel and, given industry size, the size of the incomplete cartel is uniquely determined. By contrast, in Bertrand competition conclusions are less clear cut and whether or not firms have an incentive to form a less than all-inclusive cartel is unknown.

In the remainder of the chapter, we have briefly explored cartel formation with positive externalities, incomplete cartels with heterogeneous firms and incomplete bidding rings. In simultaneous-move formation games, no incomplete cartels will form in concentrated industries, i.e., when the number of firms is weakly smaller than four. In industries with five or more sellers, firms might form an incomplete cartel, but the probability of an incomplete cartel to emerge in equilibrium is typically low. By contrast, incomplete coalitions are quite likely to emerge in sequential formation games. There is only a modest number of studies that assumes non-identical firms. These analyses suggest that larger firms are more inclined to join a cartel. Finally, we have discussed incomplete bidding rings. One of the key observations is that both the incentive problem and the participation problem are typically absent in second-price auctions. As a result, incomplete cartels in second-price auctions cannot be explained by free-rider incentives. Still, the optimal collusive arrangement might not include all bidders when the number of competitors is large, because with many (potential) bidders coordination costs can be formidable.

At this point, it is interesting to note that the Cournot models seem to better explain the stylized facts that were distilled from real-world examples of incomplete cartels. Indeed, taking account of both the incentive constraint and the participation con-

[76] The authors list several 'stylized facts' that illustrate the organization of rings. For instance, ring members tend not to bid meaningfully against each other and the ring attempts to conceal its existence from the auctioneer.

straint, cartels are typically not all-inclusive and tend to take a dominant position in the market when firms compete in quantity. So far, the economic literature on incomplete cartels remains largely silent about what happens when firms compete in price. Also, independent of the strategic variable, the literature that was discussed cannot fully explain how incomplete cartels lose market share over time and why incomplete cartels often comprise the larger firms in the industry. The latter is suggested by some studies, but these contributions do not take account of the incentive problem of the cartel. To provide a rational for these 'stylized facts' one necessarily needs to take a dynamic approach in which firms differ in at least one respect.

To address these issues, one could, for example, take a structural dynamic approach as in Fershtman and Pakes (2000). In particular, altering the structure of the game potentially could shed light on how and why firms take different decisions. An example of a structural analysis applied to an incomplete cartel is deRoos (2001) which examines the vitamin c market that was part of the global *vitamin* cartel. The vitamin cartel was confronted by a group of Chinese producers. At first, the cartel decided to accommodate the emergence of Chinese vitamin c production, but eventually, after a considerable growth of fringe production it caused the demise of the cartel. The model as developed endogenizes decisions of firms on investment, entry, exit and collusion and allows for a deeper understanding of why the cartel initially accepted fringe production, but ultimately could not, or did not wish to, maintain the cartel organization anymore. Among other things, it is found that the cartel tends to accept fringe competition as long as competitive pressure remains small. In addition, less effort is taken to deter entry when it is likely that the cartel could be maintained in light of entry. Interestingly, the entrant tends to invest heavily in the presence of a cartel, which broadly speaking can be understood as the result of the free-rider effect emphasized above. It must be noted, however, that a major drawback of a structural dynamic analysis is that it typically yields no analytical results. Consequently, one has to settle for numerical solutions, which are quite sensitive to parameter specifications.

The strategic dynamic approach as discussed in this chapter is more suited for obtaining analytical solutions. In order to provide a rational basis for the existence of incomplete cartels one should take into account both the incentive problem and the participation problem of firms. As noted, this at a minimum requires a dynamic setting in which firms differ in at least one respect. However, introducing firm heterogeneity in a repeated-game setting significantly complicates the analysis. The study of incomplete cartels forms no exception. As remarked by Motta (2004, p. 181), "Unfortunately, the analysis of partial collusion raises several difficulties, as one should model a situation where a group of firms collude whereas others simply best respond. Solving such a model analytically is not easy, and further work is needed on this issue." In the next chapter, we make an attempt in this direction.