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Incomplete cartels and antitrust policy : incidence and detection

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3

A Theory of Incomplete Cartels with Heterogeneous Firms

“There are two kinds of people: those who do the work, and those who take the credit. Try to be in the first group; there is less competition there.”
– Indira Gandhi¹

3.1 Introduction

Most economic theories on collusion predict that the optimal cartel arrangement is one in which all sellers in the industry participate.² To explain how firms could fail to eliminate all market competition it is typically argued that an all-inclusive cartel might not be stable and therefore may be too high of a goal. The potential instability is due to the incentive of firms to cheat on a cartel. Such cheating may occur both *ex ante* and *ex post*. Cheating *ex ante* refers to the incentive of firms to free-ride on a cartel formed by competitors. The output reduction and price increase of cartel members allows nonconspirators to raise their prices and expand their production, which yields windfall profits. The incentive to cheat *ex post* results from the fact that a cartel member commits to a production level for which marginal revenue exceeds marginal cost. As a consequence, all members feel a temptation to increase outputs. Yet, if every firm expects its rivals to collude, no cartel will emerge. Cartels are therefore the result of a complex trade-off between, on the one hand, incentives to collude and, on the other hand, incentives to cheat.

¹See <http://quotationsbook.com>.

²This chapter is in part based on joint work with J.E. Harrington. See Bos and Harrington (2008).

This chapter provides a rational basis for the existence of incomplete cartels. We contribute to the existing literature in at least two important ways. First, we focus on explicit cartel agreements as opposed to tacit collusion. Communication between firms makes that cartelizing is costly and taking into account these cost of colluding could provide an additional explanation for why many discovered cartels were less than all-inclusive. The intuition is that excluding one or more firms from the cartel may significantly reduce the cost of colluding so that cartel profits per member are higher for an incomplete cartel. Second, we take into account both the incentive to cheat *ex ante* and the incentive to cheat *ex post*. Indeed, the main challenge in building a theory of incomplete cartels is to model a situation in which a group of sellers finds it in their interest to collude, whereas others prefer to remain outside the cartel. That is to say, in order to understand how incomplete cartels emerge, a subset of firms must have no incentive to cheat, neither *ex ante* nor *ex post*, and the remaining firms should lack the incentive to join the cartel. At a minimum, this requires a model in which firms face both an incentive problem and a participation problem. In addition, we want firms to differ in at least one respect in order to address the question: What (type of) firms take part in a cartel and what (type of) firms remain independent outsiders?

To that end, we analyze a price setting supergame in which firms differ in terms of capacity stock, which is taken as a proxy for firm size.³ There are two reasons for choosing this setting. First, it is a simple and convenient way to deal with firm heterogeneity. In particular, it allows us to derive analytical solutions that are difficult to obtain in other type of models. Second, the descriptive cartel studies surveyed in Chapter 2 suggest that firm size is an important determinant in a firm's decision on whether or not to take part in a cartel. There exist quite a few studies that analyze collusion in price setting supergames with capacity constraints. Examples include Brock and Scheinkman (1985), Benoit and Krishna (1987), Staiger and Wolak (1992) and more recently Compte *et al.* (2002).⁴ All these studies assume the cartel to be all-inclusive. In the supergame literature on collusion, this assumption can sometimes be defended on the ground that a full cartel is sustainable whenever some collusion is sustainable. In more technical terms, the critical discount factor might be decreasing in the number of cartel participants, all else equal.⁵ However, whether or not firms have an incentive to form an industry-wide cartel is a different matter. The main difference with our analysis is that these studies focus exclusively on the incentive problem of the cartel and that only opportunities to collude tacitly are considered.⁶

³In this chapter, the terms capacity stock, capacity and production capacity are used interchangeably. All refer to the maximum number of products that a firm can supply in a given period.

⁴Brock and Scheinkman (1985) consider the role of production capacity in enforcing collusion among a fixed number of identical firms. They establish that there exists no monotone relationship between the number of firms and the maximum sustainable profits. Benoit and Krishna (1987) allows firms to choose both capacity and price. Among other things, they find that colluding firms carry excess capacity. Staiger and Wolak (1992) introduce stochastic demand in this type of setting and find support that excess capacity may lead collusion to breakdown. See Chapter 2 of this dissertation for a discussion of Compte *et al.* (2002).

⁵See Chapter 2 of this thesis.

⁶The standard supergame literature on collusion often fails to discriminate between tacit and overt collusion. One convenient and popular way to tailor the analysis to explicit cartel agreements is by

The profound questions to be addressed are (i) under what conditions is the optimal cartel size less than all-inclusive?, (ii) given that the cartel is not all-inclusive, what (type of) firms have the strongest incentive to join a cartel?, (iii) do firms have an incentive to form the cartel for which total cartel profits are highest?, and (iv) how does a change in the size distribution of firms due to a merger affect the impact of a cartel?

The price setting supergame with asymmetric capacity constraints that we consider is closest to the model analyzed in Compte *et al.* (2002).⁷ Our analysis differs in at least three aspects. First, they exclusively focus on how firm heterogeneity affects the sustainability of an *all-inclusive* cartel and free-riding incentives are therefore not considered. As documented in the previous chapter, there exists a separate strand of literature that focuses on the participation problem of firms. In order to take account of the incentives of firms to free-ride on a cartel, we extend the price setting supergame with a participation stage. Second, although we do analyze the possibility that cartels emerge tacitly, our main focus is on explicit cartel agreements.⁸ Third, Compte *et al.* (2002) analyze how a redistribution of capacity, for example due to a merger, affects the sustainability of an all-inclusive cartel. Instead, we examine the relationship between mergers and the composition of the (incomplete) cartel. For example, two firms not part of a cartel might find it in their interest to join the cartel post-merger. The main question therefore is what type of mergers have the strongest coordinated effects.

The main results of our analysis are as follows. We find that the optimal cartel size might not be all-inclusive when colluding is costly and the smallest firms are sufficiently small. We establish that the incentive to take part in a cartel is positively related to firm size and that very small firms have no incentive to take part in any cartel. Also, the most profitable cartel is formed by the largest firms in the industry and firms do have an incentive to form this cartel whenever its smallest member is sufficiently large. Furthermore, we show that sellers lack the incentive to merge in absence of collusion. In particular, firms only have an incentive to engage in a merger when they are part of the cartel post-merger. Our analysis further suggests that the strongest coordinated effects may come from a merger between moderate-sized firms.

The chapter proceeds as follows. Section 2 presents the model and derives some benchmark results. Section 3 explores what is the optimal cartel size both under the assumption that colluding is costless and when colluding is costly. Here, we do not yet consider the incentives of firms to take part in a cartel. Section 4 extends the analysis of the previous section by introducing a participation stage. In this section, we examine what type of firms have a stronger incentive to collude. In Section 5, we

introducing extra costs in the model (e.g., cartels that require members to communicate are subject to antitrust enforcement and participants therefore face an additional cost equal to the probability of being discovered times the level of punishment.). See, for instance, McCutcheon (1997), in which it is assumed that collusion requires communication, but the communication process is not modelled explicitly.

⁷The main difference with their model is that we impose an upper bound on the production capacity of the largest firm(s). We further assume demand to be downward-sloping instead of unit demand.

⁸It must be noted, however, that the model presented in this chapter in principle applies equally well to tacit collusion.

examine if and when firms have an incentive to form the cartel for which total cartel profits are highest. In Section 6, we explore how the change in the size distribution of firms due to a merger affects collusion. We analyze merger incentives and potential coordinated effects of a merger. Section 7 concludes.

3.2 A Model of Collusion with Asymmetric Capacity Constraints

In a given industry, let N denote the set of firms containing a fixed number of n profit-maximizing sellers that simultaneously set prices, which is assumed to be their single strategic variable. Commodities are homogeneous and produced at common unit cost $c > 0$. For simplicity, fixed costs are normalized to zero. The market demand function is denoted $D(p)$, which we assume to be twice continuously differentiable with $D'(p) < 0$ and $D''(p) \leq 0$. We naturally suppose that society values the production of the first unit, i.e., $D(c) > 0$. Firms differ in terms of capacity stock. The production capacity of firm i is denoted k_i and is taken to be fixed for all $i \in N$. Without loss of generality, we index the firms so that,

$$k_1 \geq k_2 \geq \dots \geq k_n.$$

Let firm i 's demand be $D_i(p_i, \mathbf{p}_{-i})$, $\forall i \in N$, which is a function of the price charged by firm i (p_i) and the prices set by all its rivals (\mathbf{p}_{-i}). In what follows, let $\Delta(p) \equiv \{j \in N : p_j < p\}$ denote the set of sellers that price strictly below p and let $\Omega(p) \equiv \{j \in N : p_j = p\}$ be the set of firms that price at p . Assuming the efficient rationing rule, we impose the following condition on individual demand,

$$D_i(p_i, \mathbf{p}_{-i}) \leq \max \left\{ D(p_i) - \sum_{j \in \Delta(p_i)} k_j, 0 \right\}, \forall i \in N.$$

Basically, this condition ensures that customers prefer cheaper products. In particular, it implies that firm i has positive demand only when firms that price below p_i have excess demand.

We make several additional assumptions on firm demand.

- Assumption 1:

- (i) If $0 < D(p) - \sum_{j \in \Delta(p)} k_j < \sum_{j \in \Omega(p)} k_j$, then $0 < D_i(p_i, \mathbf{p}_{-i}) < k_i$, $\forall i \in \Omega(p)$.
- (ii) If $\sum_{j \in \Omega(p)} k_j < D(p) - \sum_{j \in \Delta(p)} k_j$, then $k_i < D_i(p_i, \mathbf{p}_{-i})$, $\forall i \in \Omega(p)$.

Assumption 1(i) means that if firms, which price at p , have combined capacity that exceeds residual demand, then demand is allocated such that all firms that price at p have positive demand and excess capacity. Assumption 1(i) therefore imposes a lower and upper bound on the production level of firms that charge the same price. Note that,

within these boundaries, market shares of firms can vary substantially. Assumption 1(ii) mirrors the first part. It states that, when residual demand exceeds combined capacity of firms that price at p , demand is allocated such that all firms that price at p produce up to capacity. Assumption 1 therefore imposes some symmetry across firms.

The next two assumptions put some limits on the market power of the largest firm(s).

- Assumption 2: $D(p^m) > k_i, \forall i \in N$, i.e., none of the firms has sufficient capacity to meet monopoly demand $D(p^m)$.
- Assumption 3: $\sum_{j \in N \setminus \{i\}} k_j \geq D(c), \forall i \in N$, i.e., any combination of $n - 1$ firms has combined capacity that is sufficient to satisfy maximum demand.

Arguably, the last two assumptions are somewhat restrictive, because it rules out the possibility of firms being very large in *absolute* terms. Yet, it still allows firms to be very large in *relative* terms. Also, an immediate consequence of Assumption 2 in conjunction with Assumption 3 is that duopolistic market structures are excluded. However, the condition $n \geq 3$ is only mildly restrictive, because our focus is on cartels that are not all-inclusive.

For technical reasons, firms will choose a price from the set,

$$\{0, \varepsilon, \dots, c - \varepsilon, c, c + \varepsilon, \dots\},$$

with ε denoting the smallest monetary unit, which we typically assume to be sufficiently small.⁹

3.2.1 Static Nash Equilibrium

In competition, firm i 's profit function is given by,

$$\pi_i = (p_i - c) D_i(p_i, \mathbf{p}_{-i}). \quad (3.1)$$

The following result establishes the static Nash equilibrium of the game.

Proposition 3.1 *As $\varepsilon \rightarrow 0$, all firms price at marginal cost in competition.*

Proof. Suppose that all firms price at marginal cost, which implies $\pi_i = 0, \forall i \in N$. Given that $p_j = c, \forall j \neq i$, firm i may attempt to increase its profits by charging $p_i \neq c$. Setting $p_i < c$ would yield $\pi_i < 0$, which is clearly not an improvement. Alternatively, firm i may set a price $p_i > c$, but this means that firm i will lose all its customers due to Assumption 3, which implies $\pi_i = 0$. Hence, $p_i = c, \forall i \in N$, constitutes a (symmetric) Nash equilibrium of the game.

Next, suppose without loss of generality that $p_i \geq p_j, \forall j \in N$ and $i \neq j$ and further assume that all firms charge a price that is weakly higher than $c + \varepsilon$. If $p_i > c + \varepsilon$, then

⁹The reason for choosing a discrete price distribution instead of a continuous price distribution is that with the latter the best response functions of firms is not well-defined.

$D_i = 0$ or $D_i > 0$. If $D_i = 0$ firm i earns zero profits, which is clearly not optimal. For instance, $p_i = c + \varepsilon$ would always yield $\pi_i > 0$ due to Assumption 1. We therefore focus on the situation in which $p_i > c + \varepsilon$ and $D_i > 0$. Then, in light of Assumption 3 there must exist at least one firm j that charges $p_j \geq p_i$, which yields a contradiction unless $p_i = p_j$. However, given that firm i does not produce up to capacity at p_i , it can increase its demand by charging $p_i - \varepsilon < p_j$, which yields higher profits for ε sufficiently small. To show that firm i does not experience excess demand at p_i we will assume the opposite and derive a contradiction. Suppose that firm i does produce up to capacity at p_i . Then, in light of Assumption 1 (ii), all firms that price at p_i face excess demand. That is, $D(p_i) - \sum_{j \in \Delta(p_i)} k_j \geq \sum_{j \in \Omega(p_i)} k_j \implies D(p_i) \geq \sum_{j \in N} k_j$, which violates Assumption 3.

We have established that all firms charge a price strictly higher than $c + \varepsilon$ with zero probability. Note that a further reduction in price to c is not optimal, because this would yield zero profits, while at $c + \varepsilon$ profits are positive. All firms charging a price equal to $c + \varepsilon$ therefore constitutes a (symmetric) Nash equilibrium. It was already shown that $p = c$ constitutes an equilibrium. Yet, as $\varepsilon \rightarrow 0$, this difference disappears, which leaves $p_i = c, \forall i \in N$, as the competitive Nash equilibrium. ■

The above result implies that in competition none of the sellers is making economic profits. Clearly, it is in the interest of all undertakings to reduce competitive pressure and raise industry prices. Denote $\Gamma \subseteq N$ a (sub)set of firms that form a price-fixing cartel. The cartel is assumed to operate as a multiplant unit, which sets a single cartel price $p^c > c + \varepsilon$. Obviously, such a coalition is profitable only if cartel demand is strictly positive. This is naturally the case for an all-inclusive cartel. If the cartel is incomplete, however, its demand will depend in part on the characteristics and behavior of independent outsiders. The next result establishes the optimal behavior of firms that do not take part in the price-fixing conspiracy.

Lemma 3.1 *If a profitable cartel sets a price $p^c > c + \varepsilon$, then the best response of an individual profit-maximizing outsider j is to sell k_j units at a price $p_j^o = p^c - \varepsilon$, for ε sufficiently small.*

Proof. First, suppose that the cartel price is the (weakly) lowest price in the industry and consider the case in which $p_i^o > p^c, \forall i \in N \setminus \Gamma$. Without loss of generality, let p_j^o denote the highest price in the industry. Then, following Assumption 3, firm j has zero demand if it charges the strictly highest price, which is clearly not optimal, e.g., charging $p_j^o = p^c$ would yield $\pi_j^o > 0$. Suppose therefore that $D_j > 0$. In light of Assumption 3, this implies that there exists at least one other firm s that sets $p_s^o \geq p_j^o$ and therefore $p_s^o = p_j^o$. Note that, due to Assumption 1, all firms that price at p_j^o face positive demand. Yet, given that firm j does not produce up to capacity at p_j^o , reducing the price slightly to $p_j^o - \varepsilon$ yields a discrete increase in demand, which is profitable for ε sufficiently small. To show that firm j does not produce up to capacity at p_j^o we will assume the opposite and derive a contradiction. Suppose firm j does fully utilize its capacity at p_j^o . Then, by Assumption 1 (ii) we know that all firms that price at p_j^o produce up to capacity. This implies $D(p_j^o) - \sum_{i \in \Delta(p_j^o)} k_i \geq \sum_{i \in \Omega(p_j^o)} k_i \implies D(p_j^o) \geq \sum_{i \in N} k_i$, which violates Assumption 3. Hence, we have shown that all outsiders charge a price that is strictly higher than p^c with zero probability.

Now suppose that all outsiders price at p^c . Then, Assumption 3 in conjunction with Assumption 1 (i) implies that all firms face a positive demand and have excess capacity. Firm j therefore earns profits equal to $\pi_j^o = (p^c - c) D_j(p^c)$, with $D_j(p^c) < k_j$. However, by reducing its price to $p^c - \varepsilon$ it can earn $\pi_j^o = (p^c - \varepsilon - c) k_j$, which is larger for ε sufficiently small. Charging $p_i^o = p^c - \varepsilon$, $\forall i \in N \setminus \Gamma$, is therefore optimal if all outsiders produce up to capacity at $p^c - \varepsilon$, i.e., if $D(p^c - \varepsilon) \geq \sum_{i \in \Omega(p^c - \varepsilon)} k_i$. This must necessarily hold, because if $D(p^c - \varepsilon) < \sum_{i \in \Omega(p^c - \varepsilon)} k_i$ cartel demand would be zero, which means that the cartel is not profitable. Note that a further reduction in price by fringe members is not optimal, because all outsiders produce up to capacity at $p^c - \varepsilon$. ■

Lemma 3.1 implies that a cartel Γ is profitable only if $D(p^c) > \sum_{j \notin \Gamma} k_j$, which is the case when outsiders do not have sufficient capacity to meet total market demand. In other words, the only viable cartels are those that have sufficient control over industry capacity. Also, as $\varepsilon \rightarrow 0$, fringe firms optimally charge the cartel price. Note the close similarity with the approach taken in collusive price leadership models in which it is typically assumed that outsiders take the cartel price as given.

As a consequence, total cartel profits are given by,

$$\pi^c = (p^c - c) D^c = (p^c - c) \left[D(p^c) - \sum_{j \notin \Gamma} k_j \right]. \quad (3.2)$$

The cartel has to divide π^c among its members according to some profit sharing rule. Suppose therefore that the cartel establishes a profit allocation rule α . Given a capacity vector (k_1, k_2, \dots, k_n) , a cartel Γ and cartel price $p^c(\Gamma)$, α prescribes an allocation of cartel demand. Hence, every firm $i \in \Gamma$ receives a share $\alpha_i \in (0, 1)$ of total cartel profits, i.e.,

$$\pi_i^c = (p^c - c) \left[D(p^c) - \sum_{j \notin \Gamma} k_j \right] \alpha_i, \forall i \in \Gamma. \quad (3.3)$$

We require α to be efficient in the sense that cartel members allocate all cartel profits among themselves, i.e., $\sum_{i \in \Gamma} \alpha_i = 1$.

3.2.2 Infinitely Repeated Game

In the following, we will analyze the infinitely repeated version of this game. The collusive value of firm $i \in \Gamma$ is given by,

$$V_i^c(p^c, \Gamma) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i^c,$$

with t indicating the date of a single period and $\delta \in (0, 1)$ denoting the common discount factor. We assume that collusive arrangements are sustained through grim-trigger strategies. That is, every member of Γ adheres to the collusive strategy until one firm deviates. In the event of cheating, the coalition collapses with a one-period time

lag, i.e., all firms compete in all periods following a period of defection.¹⁰ Consequently, for collusion to be sustainable as a subgame perfect Nash equilibrium (SPNE) of the infinitely repeated game, the following incentive compatibility constraint must be satisfied for all participants,

$$V_i^c(p^c, \Gamma) \geq \pi_i^d, \quad (3.4)$$

with π_i^d denoting the maximum one-period gain from defection. The following result determines the maximum profit to be obtained by a chiseling firm during the period of defection.

Lemma 3.2 *As $\varepsilon \rightarrow 0$, $\pi_i^d = (p^c - c) k_i$.*

Proof. Consider a cartel that sets a price $p^c > c + \varepsilon$. By Lemma 3.1 we know that if the cartel is not all-inclusive outsiders set a price $p_j^o = p^c - \varepsilon, \forall j \in N \setminus \Gamma$. A chiseling firm i aims to maximize π_i^d . If it deviates by charging $p_i^d > p^c$, then its demand would drop to zero due to Assumption 3, which is clearly not optimal. Instead, suppose firm i reduces its price to $p_i^d = p^c - \varepsilon$. This defection strategy is optimal if firm i sells up to capacity at $p^c - \varepsilon$. In this case it would earn $\pi_i^d = (p^c - \varepsilon - c) k_i$. Note that $\varepsilon \rightarrow 0$ yields $\pi_i^d = (p^c - c) k_i$. Alternatively, firm i may have excess capacity at $p^c - \varepsilon$. Therefore, given that ε is sufficiently small, it could be profitable to reduce p_i^d further to $p_i^d = p^c - 2\varepsilon$, which would be the lowest price in the industry. Assumption 2 guarantees that firm i fully utilizes its capacity at this price. That is, $D(p^m) \geq k_i$ implies $D(p^c - 2\varepsilon) \geq k_i$, because $p^m \geq p^c$. Hence, a further reduction of p_i^d is never optimal. However, as $\varepsilon \rightarrow 0$, we have that $p^c - 2\varepsilon = p^c$. ■

Note that as $\varepsilon \rightarrow 0$, $\pi_i^d = \pi_i^o$. We therefore might say that fringe firms permanently and optimally cheat on the cartel, without being punished. In other words, from the start of the game, outsiders behave *as if* they optimally defect from the cartel agreement in every period. Furthermore, $\pi_i^d > \pi_i^c$ for all $i \in \Gamma$ whenever $\sum_{i \in N} k_i > D(p^c)$, which due to Assumption 3 always holds. Nevertheless, it is well-known that, despite the potential instability, collusion may be an equilibrium strategy if the interest rate that is used to discount future profit streams is sufficiently low.¹¹ Using Lemma 3.2, (3.4) can be rearranged to,

$$\delta \geq \delta_i^* = 1 - \alpha_i \frac{\pi_i^c}{\pi_i^d} = 1 - \frac{\alpha_i}{k_i} \left[D(p^c) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right]. \quad (3.5)$$

Observe that δ_i^* is decreasing in α_i and increasing in k_i , all else unchanged. Clearly, a member has a lower incentive to deviate the larger the share of total cartel profits it receives. Also, given some profit allocation rule, an increase in production capacity

¹⁰Note that the grim-trigger strategy is the most severe *credible* threat. Hence, whenever some level of collusion cannot be sustained by the threat of eternal competition it cannot be sustained by any other credible punishment strategy. For a detailed analysis of optimal penal codes in price-setting supergames the reader is referred to Lambson (1987, 1994).

¹¹The seminal work is due to Friedman (1971).

yields a stronger incentive to deviate, because larger capacity allows a firm to earn more during a single period of defection. In addition, (3.5) reveals that a cartel is only viable if it has control over at least $\sum_{i \in N} k_i - D(p^c)$ of total industry capacity. Finally, for sufficiently high δ , none of the firms can prevent its rivals from forming a cartel that is effective. This is due to Assumption 2, which ensures that none of the suppliers has sufficient capacity to meet $D(p^c)$.

Notice that the individual profits of conspirators increase with total cartel value. The objective of a coalition is therefore to choose a cartel price that maximizes total cartel profits, while satisfying (3.5). Hence, the optimal cartel price of a cartel Γ is defined by the following constraint optimization problem,

$$\max_{p^c} V^c(p^c, \Gamma) = \frac{1}{1-\delta} (p^c - c) \left[D(p^c) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right], \quad (3.6)$$

subject to,

$$\frac{\alpha_i}{k_i} \left[D(p^c) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right] - (1-\delta) \geq 0, \forall i \in \Gamma.$$

The next result shows that the solution to (3.6) is increasing in the combined capacity of colluders.

Lemma 3.3 *Fix $\frac{\alpha_i}{k_i}, \forall i \in \Gamma$. The optimal cartel price is strictly increasing in total cartel capacity.*

Proof. Consider a sustainable cartel Γ and for notational convenience, let $K^c = \sum_{j \in \Gamma} k_j$. The incentive compatibility constraint (3.5) may or may not be binding for one or more members. First, suppose it is not binding for any member. The optimal cartel price is then defined by,

$$D(p^c) - \sum_{i \in N} k_i + K^c + (p^c - c) D'(p^c) = 0.$$

Rearranging yields,

$$K^c = \sum_{i \in N} k_i - D(p^c) - (p^c - c) D'(p^c). \quad (3.7)$$

The first-order derivative of (3.7) with respect to p^c is,

$$\frac{\partial K^c}{\partial p^c} = -2D'(p^c) - (p^c - c) D''(p^c).$$

In order to evaluate how a change in total cartel capacity affects the optimal cartel price we take the inverse,

$$\frac{\partial p^c}{\partial K^c} = -\frac{1}{2D'(p^c) + (p^c - c) D''(p^c)} > 0.$$

Now suppose (3.5) is binding for at least one member, which means that the cartel price is chosen such that $\delta = \max \delta_i^*$, $i \in \Gamma$, i.e., the firm(s) for which the ratio $\frac{\alpha_i}{k_i}$ is smallest. Hence,

$$\delta = 1 - \frac{\alpha_i}{k_i} \left[D(p^c) - \sum_{i \in N} k_i + K^c \right].$$

Rearranging yields,

$$K^c = \frac{k_i}{\alpha_i} (1 - \delta) + \sum_{i \in N} k_i - D(p^c). \quad (3.8)$$

Taking the first-order derivative of (3.8) with respect to p^c yields,

$$\frac{\partial K^c}{\partial p^c} = -D'(p^c).$$

Taking the inverse gives,

$$\frac{\partial p^c}{\partial K^c} = -\frac{1}{D'(p^c)} > 0.$$

Notice that the function $p^c(K^c)$ is continuous, but might be non-differentiable. It can be concluded that the larger the share of industry capacity that is under the control of a cartel, the higher will be the optimal cartel price. ■

To what extent collusion yields higher industry prices therefore essentially depends on *total* cartel capacity relative to *total* fringe capacity, which, given industry size, are ‘communicating vessels’. Clearly, industry prices are highest when all firms take part in the cartel arrangement.

3.3 Optimal Cartel Size

In the previous section, it has been shown that any cartel that controls a large enough share of industry capacity is feasible provided that its members are sufficiently patient. In this section, we explore what is the optimal cartel size. We define optimal cartel size as follows.

Definition 3.1 *The size of a cartel is optimal when: (i) the cartel does not find it profitable to include additional firms, and (ii) no subset of participants finds it in their interest to exclude one or more members.*

At this point, it is important to emphasize that any result depends quite heavily on the profit allocation rule α . In particular, total cartel profits must be allocated in such a way that the cartel under consideration is sustainable. One way to think about this problem is by assuming that firms attempt to form a cartel with the support of a cartel manager (e.g., a ringleader). The prime task of the cartel manager is then to design an optimal cartel contract by deciding on a profit allocation rule.

In the following, we distinguish between a situation in which cartelizing is costless and a situation in which cartelizing is costly. By ‘costless’ we mean that, apart

from opportunity costs, membership of a price-fixing coalition is free. Hence, cartel members do not incur costs associated with the formation and management of the cartel. Furthermore, expected costs due to antitrust enforcement are assumed absent. By contrast, these type of costs are present when cartelizing is costly.

3.3.1 Costless Collusion

In this subsection, we investigate what cartel size is optimal under the assumption that cartelizing is costless. A cartel considered here is therefore *de facto* a tacit agreement among firms.

In a wide variety of settings, cartel profits are positively correlated with the size of the cartel.¹² Therefore, we might *a priori* expect the cartel manager to maximize the number of participants. However, fringe profits also tend to increase with cartel size. As a result, we may suspect that, *ceteris paribus*, it is increasingly difficult to expand the size of the cartel without rendering the cartel unstable. Yet, as the following result shows, larger cartels are feasible given that the total cartel value is allocated properly.

Theorem 3.1 *A cartel can always allocate its profits in such a way that it Pareto dominates every smaller cartel.*

Proof. Consider a stable cartel Γ consisting of x members that were willing to join given some profit sharing rule. Consequently, $n-x$ outsiders did not want to join given this profit sharing rule. For a firm $i \in N \setminus \Gamma$ to join the cartel it must be offered an amount that exceeds its current earnings. Following lemma 3.1, an outsider i earns,

$$\frac{1}{1-\delta} (p^c(\Gamma) - \varepsilon - c) k_i.$$

Suppose therefore that it is offered the following amount,

$$\frac{1}{1-\delta} (p^c(\Gamma) - c) k_i.$$

In what follows, let $\Psi \subseteq N \setminus \Gamma$ denote a (sub)set of outsiders and let $\Gamma_{+\Psi}$ indicate the cartel consisting of all members of Γ and Ψ . In order for all $j \in \Psi$ to join, they must be offered a total amount that is equal to,

$$\frac{1}{1-\delta} (p^c(\Gamma) - c) \sum_{j \in \Psi} k_j.$$

After paying these fringe members, there must be a sufficient amount of money left to cover, at a minimum, the total earnings of Γ , which amount to,

$$\frac{1}{1-\delta} (p^c(\Gamma) - c) \left[D(p^c(\Gamma)) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right].$$

¹²Note that there does not always exist a monotonic relationship between cartel size and cartel profitability. Consider, for example, a standard symmetric Cournot setting with linear demand and constant unit costs. In such a setting, the ‘empty cartel’ is more profitable than any coalition that spans less than 80% of the market. See Salant *et al.* (1983).

Hence, including outsiders is (weakly) profitable only if the following condition holds,

$$\begin{aligned} & \frac{1}{1-\delta} (p^c(\Gamma_{+\Psi}) - c) \left[D(p^c(\Gamma_{+\Psi})) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j + \sum_{j \in \Psi} k_j \right] \geq \\ & \frac{1}{1-\delta} (p^c(\Gamma) - c) \left[D(p^c(\Gamma)) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right] + \frac{1}{1-\delta} (p^c(\Gamma) - c) \sum_{j \in \Psi} k_j, \end{aligned}$$

which is equivalent to,

$$\begin{aligned} & (p^c(\Gamma_{+\Psi}) - c) \left[D(p^c(\Gamma_{+\Psi})) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j + \sum_{j \in \Psi} k_j \right] \geq \quad (3.9) \\ & (p^c(\Gamma) - c) \left[D(p^c(\Gamma)) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j + \sum_{j \in \Psi} k_j \right]. \end{aligned}$$

It is *a priori* unclear if this condition holds generally. Suppose however that the new cartel $\Gamma_{+\Psi}$ is not changing its price, i.e., $p^c(\Gamma_{+\Psi}) = p^c(\Gamma)$. Then, (3.9) holds with equality. It remains to be shown that such a cartel is indeed sustainable. Using (3.4), the incentive compatibility constraint of a newcomer is given by,

$$\frac{1}{1-\delta} (p^c(\Gamma) - c) k_i \geq (p^c(\Gamma) - c) k_i,$$

which is satisfied for all values of δ . After paying all new participants, the old cartel members have the following pie to share,

$$\frac{1}{1-\delta} (p^c(\Gamma) - c) \left[D(p^c(\Gamma)) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right].$$

This is equal to the value of the old cartel Γ , which was given some profit sharing rule sustainable.

To complete the proof, it is important to note that with $p^c(\Gamma_{+\Psi}) = p^c(\Gamma)$, (3.9) only holds with equality and that none of the members of the old cartel strictly benefits from the cartel expansion. However, unlike $p^c(\Gamma)$, $p^c(\Gamma_{+\Psi}) = p^c(\Gamma)$ is not the optimal cartel price for the new cartel $\Gamma_{+\Psi}$. In fact, following Lemma 3.3, a cartel $\Gamma_{+\Psi}$ would optimally set $p^c(\Gamma_{+\Psi}) > p^c(\Gamma)$, which implies that the left-hand side of (3.9) will be strictly larger than the right-hand side. ■

The result that a cartel Pareto dominates any smaller cartel is non-trivial and depends in part on the structure of the model. In particular, the result does not necessarily hold when firms compete in quantity (strategic substitutes) instead of price (strategic complements). To see this, consider the following example.

Example 3.1 Consider a standard Cournot market with n identical firms that produce a homogeneous product. For simplicity, production costs are normalized to zero and there are no fixed costs. Let the linear inverse demand function be given by,

$$P = 1 - Q,$$

where $Q = \sum_{i=1}^n q_i$ is total market output.

Now suppose there exists a cartel of x members that attempts to include t outsiders. In this setting, Pareto-improving cartel expansion requires,

$$\frac{1}{(n - x - t + 2)^2} \geq \frac{t + 1}{(n - x + 2)^2},$$

which does not always hold (e.g., this condition is violated for $n = 10$, $x = 8$ and $t = 1$).

Hence, when firms compete in quantity it does not always pay to expand cartel size. In Cournot competition without capacity constraints, outsiders benefit a lot from the collusive behavior of rivals. Consequently, firms have a strong incentive to free-ride on a cartel formed by rivals. The incentive to free-ride is prevalent within our model, but it is mainly caused by the ‘umbrella effect’ of cartel pricing. In the absence of capacity constraints, we may expect external suppliers not only to increase prices, but also to expand fringe production. In principle, therefore, fringe profits might be too high in that, even when colluding is costless, it would be too costly to include a subset of outsiders in the cartel.

The implication of the above result is less surprising. The unique Pareto efficient collusive equilibrium is the one in which all firms take part in the price-fixing agreement.

Corollary 3.1 *The optimal cartel size is all-inclusive.*

This result holds quite generally, because the profits of an all-inclusive cartel are typically higher than the combined profits of an incomplete cartel and fringe members. The above analysis suggests that the way in which total cartel profits are allocated plays a significant role in designing this optimal anticompetitive arrangement. Establishing a profit sharing rule acceptable to all parties is unlikely to evolve naturally and typically requires direct communication between firms. However, when collusion is explicit, it is more natural to assume that cartelizing is costly.

3.3.2 Costly Collusion

A coalition comprising the entire industry yields the highest possible gain for sellers under the assumption that cartelizing is costless. Yet, the mere fact that the total pie must be divided in a particular way to achieve this outcome reveals that this assumption is restrictive. Establishing a profit sharing rule that is acceptable to all

participants requires at least one, but typically multiple negotiation rounds.¹³ Also, once a cartel becomes effective, members often have to monitor each other in order to ensure compliance with the agreement.¹⁴ In addition, the content of these discussions makes participating firms subject to antitrust enforcement. All these factors make that cartelizing is a costly exercise. Arguably, firms will take these (expected) costs into account when deciding on whether or not to join a cartel.

Not much is known about the magnitude of the cost of cartelizing, which is in large part due to the secret nature of anticompetitive organizations. Moreover, these costs possibly vary widely among cartels. Be that as it may, based on the factors mentioned above it appears reasonable to assume that the costs of cartelizing are increasing in the number of parties involved. Discussions are, *ceteris paribus*, easier in small groups than in large groups, e.g., because lines of communication are shorter. Also, diversity among participants is likely to be higher in larger groups, which is generally believed to adversely affect the probability of reaching consensus. In addition, we may suspect that (expected) costs that are created by antitrust law enforcement will increase in the number of participants. In its simplest form, the costs created by antitrust enforcement equal the probability of conviction times the level of the fine. We may conjecture that the risk of detection increases with the size of the cartel, because larger cartels are presumably “more visible”.¹⁵ Furthermore, many antitrust agencies adopted a policy to grant amnesty to undertakings that confess and prove their involvement in a cartel. Obviously, such a leniency program is effective only if the first firm to self-report receives the highest benefits and the probability of being the first is, *ceteris paribus*, decreasing in the number of participants. In sum, there is a variety of reasons for why we may expect the cost of colluding to be increasing in cartel size.

We will take account of the cost of colluding in the simplest possible way. Let the per-period cost of cartelizing be given by some function $T(x)$, with x being the number of cartel participants. Note that we assume T to be independent of firm size. The reason for this is that most costs associated with collusion do not seem to depend on the market position of cartel members. For example, it is difficult to see how the risk of a cartel member applying for leniency and costs associated with the formation of a cartel would depend on the production capacity of a firm. In light of the above discussion, we naturally assume $T(\cdot)$ to be strictly increasing in the number of cartel members. No additional assumptions are made to further define the shape of $T(\cdot)$. For example, it may have a concave shape if we believe that the marginal cost of colluding is decreasing in x . Alternatively, if we believe that including more firms in

¹³In addition, firms often make agreements about other factors, e.g., what to do if market conditions change? The study of the U.S. sugar cartel conducted by Genesove and Mullin (2001) illustrates how frequent communication might help firms to collude.

¹⁴Levenstein and Suslow (2006) and Harrington (2006) report on cartels that have installed monitoring mechanisms (e.g., a joint sales agency). Although, these investments are potentially quite costly, it still is beneficial for the cartel if it prevents punishment phases caused by (perceived) cheating.

¹⁵One could argue, however, that a partial cartel may be more visible than an industry-wide cartel, because with the latter there is no outside competition that can be used as a benchmark. I thank Jeroen Hinloopen for pointing this out. In this case there indeed might be a discontinuity in $T(\cdot)$. See Chapter 4 of this thesis for a discussion on how outside competition can be used in detecting (incomplete) cartels.

the coalition adds more than proportionally to the cost of colluding, we may think of $T(\cdot)$ as a convex function. For instance, the latter may be the case when negotiations become increasingly complex and when the rate of detection significantly increases with the number of participants.

To see how the cost of cartelizing affects the internal stability of a cartel, consider a firm $i \in \Gamma$. This firm will adhere to the agreement if,

$$\delta \geq \delta_i^* = 1 - \alpha_i \frac{\pi^c}{\pi_i^d} = 1 - \alpha_i \frac{(p^c - c) \left[D(p^c) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right] - T(\cdot)}{\pi_i^d}. \quad (3.10)$$

As can be observed, taking into account the cost of colluding tightens the incentive compatibility constraint for all $i \in \Gamma$, because $T(\cdot) > 0$ for all cartel sizes, but the “empty cartel”. The set of collusive equilibria is therefore smaller compared to a situation in which colluding is costless, which is in line with expectations. Clearly, no cartel is viable for T sufficiently large and we therefore will restrict our attention to situations in which T is ‘not too high’.

The impact of the cost of colluding on optimal cartel size depends on a trade-off. On the one hand, both residual cartel demand as well as the optimal cartel price are increasing in the size of the cartel. On the other hand, every additional cartel member increases total cartel costs. What cartel size is optimal therefore depends on the magnitude of these two antagonistic forces. Due to the general structure of the current setting, the value of these effects cannot be determined explicitly. Nevertheless, as the following result indicates, if the smallest firm in the industry is sufficiently small, then the optimal cartel size is not all-inclusive.

Theorem 3.2 *If the smallest firm in the industry is sufficiently small, then the optimal cartel size is less than all-inclusive.*

Proof. We will compare the all-inclusive cartel, denoted Γ , with a cartel Γ_{-n} , which is the cartel consisting of the $n - 1$ (weakly) largest firms in the industry. First, note that whenever $\pi^c(\Gamma) > 0$, $\exists \delta$ for which Γ is sustainable. Clearly, the optimal cartel size is not all-inclusive if the full cartel is not sustainable. Suppose therefore that the following condition holds,

$$\delta \geq \delta_i^* = 1 - \alpha_i \frac{(p^m - c) D(p^m) - T(n)}{\pi_i^d}, \forall i \in \Gamma.$$

The total value of Γ_{-n} is given by,

$$\frac{1}{1 - \delta} [(p^c(\Gamma_{-n}) - c) [D(p^c(\Gamma_{-n})) - k_n] - T(n - 1)]. \quad (3.11)$$

This value exceeds the value of the full cartel if,

$$(p^c(\Gamma_{-n}) - c) [D(p^c(\Gamma_{-n})) - k_n] - T(n - 1) > (p^m - c) D(p^m) - T(n).$$

Rearranging yields,

$$T(n) - T(n-1) > (p^m - c) D(p^m) - (p^c(\Gamma_{-n}) - c) [D(p^c(\Gamma_{-n})) - k_n]. \quad (3.12)$$

It is *a priori* unclear if this inequality is satisfied or not, because the terms on both sides of the inequality sign are positive. However, as $k_n \rightarrow 0$, we have that $p^m = p^c(\Gamma_{-n})$ due to Lemma 3.3. Therefore, as $k_n \rightarrow 0$, the right-hand side of (3.12) equals zero, while the left-hand side of (3.12) is strictly positive because $T(x) > T(x-1)$ by assumption.

We have established that, for k_n sufficiently small, Γ_{-n} is more profitable than Γ . It remains to be shown that $\exists \delta$ for which Γ_{-n} is sustainable. Hence, we must show that the following condition holds for all $i \in \Gamma_{-n}$,

$$\delta \geq 1 - \alpha_i \frac{(p^c(\Gamma_{-n}) - c) [D(p^c(\Gamma_{-n})) - k_n] - T(n-1)}{\pi_i^d}.$$

This condition holds, because for $k_n \rightarrow 0$ it was shown that $\pi^c(\Gamma_{-n}) - \pi^c(\Gamma) > 0$, while π_i^d remains the same in both cases. Therefore, the maximum critical discount factor for Γ_{-n} is lower than for Γ , which was sustainable. ■

This result is intuitive and holds quite generally. In particular, the result does not depend on the size distribution of firms in the industry. For example, the optimal cartel size is also less than all-inclusive when firms are of equal size and sufficiently small. The intuition behind this result is that it does not pay to include a firm for which the marginal benefit to the cartel falls short of the marginal cost of colluding. Very small firms hardly contribute to cartel revenues, but at the same time are responsible for a nonnegligible part of the cost of colluding. As a result, the value of an incomplete cartel might exceed that of a full cartel when colluding is costly and the smallest firms are sufficiently small.

3.4 Incentives to Collude

It has been established that the optimal cartel size is not all-inclusive when colluding is costly and the smallest firms are sufficiently small. However, what cartel will emerge is a different matter. This typically depends on the incentives of firms to collude or to remain an independent outsider to a cartel formed by others instead. In this section, we explore what (type of) undertakings are likely to participate and what (type of) firms are likely to become fringe members.

Clearly, in competition all firms have an incentive to coordinate actions, because none of them is earning economic profits. At the same time, however, there exists a strong incentive for all sellers to wait in the hope that a cartel is formed by rivals. This incentive to free-ride is caused by the fact that cartel members do not fully utilize their capacity, while non-participants produce up to capacity and sell their products at approximately the same price. It has long been recognized that the incentive to

cheat *ex ante* makes it difficult to understand how incomplete coalitions emerge.¹⁶ This problem is particularly severe in symmetric settings, because it is difficult to see why identical firms would take non-identical decisions. However, in the current analysis firms are characterized by their capacity stock and we may conjecture this to cause a diversity in free-riding incentives.

In order to address this issue, we extend the model laid out in the previous sections by assuming that prior to the cartel there exists a period in which a cartel might be formed. More specifically, we assume that at $t = 0$ firms play an open membership game in which they simultaneously announce whether or not to join the coalition. In making this participation decision, a firm naturally will take into account how all sellers will adjust their behavior *ex post*, i.e., after it has made its choice.

Let π^c and π^o denote the profit of a cartel member and an outsider respectively. Formally, outsiders have no incentive to join a cartel Γ if the following ‘external stability condition’ is satisfied,

$$\pi_i^o(p^c(\Gamma)) \geq \pi_i^c(p^c(\Gamma_{+i})), \forall i \in N \setminus \Gamma, \quad (3.13)$$

with $p^c(\Gamma_{+i})$ being the price set by a coalition consisting of Γ and firm i . In a similar fashion, cartel members prefer to remain in the cartel if the following condition holds,

$$\pi_i^c(p^c(\Gamma)) \geq \pi_i^o(p^c(\Gamma_{-i})), \forall i \in \Gamma, \quad (3.14)$$

with $p^c(\Gamma_{-i})$ being the price set by a coalition consisting of Γ excluding firm i . We will refer to (3.13) in combination with (3.14) as the ‘participation constraint’. In addition, it is convenient to refer to the game analyzed so far as \mathcal{K} and to denote \mathcal{K}^E the extended version of \mathcal{K} , i.e., \mathcal{K}^E is \mathcal{K} including the participation stage. Hence, a cartel can be explained as an equilibrium outcome of \mathcal{K}^E only when (3.13), (3.14) and (3.10) are satisfied. As such, the participation constraint can be viewed a *refinement criterion*. Thus, for profitable coalitions, the incentive to participate implies sustainability, but not necessarily *vice versa*. That is to say, extending the game with a participation stage at $t = 0$ (weakly) narrows the set of subgame perfect equilibria of \mathcal{K} .

To make the participation decision, firm i must be able to determine $\pi_i^c(p^c(\Gamma_{+i}))$ *ex ante*, which is difficult because so far α was not defined explicitly. Hence, in order to analyze the incentives to collude we must specify how firms value the various coalitions. To that end, we suppose in the following that participants receive a proportional share of total cartel profits, i.e.,

$$\alpha_i = \frac{k_i}{\sum_{j \in \Gamma} k_j}, \forall i \in \Gamma. \quad (3.15)$$

There are at least two reasons for this assumption.¹⁷ First, capacity may be taken as a proxy for market share and there exists evidence that some cartels based their profit sharing rule on the market shares of members in years prior to the cartel.¹⁸

¹⁶See Stigler (1950).

¹⁷In Bos and Harrington (2008) it is shown that the proportional sharing rule can also be derived endogenously based on a Rawlsian notion of justice.

¹⁸See, for example, Harrington (2006).

Second, this allocation rule facilitates collusion in the sense that it minimizes the highest critical discount factor. In other words, if a cartel is not sustainable with a proportional allocation of profits, it cannot be sustained with any other profit sharing rule.

Typically, there exist multiple coalitions that can be sustained as subgame perfect equilibrium of \mathcal{K}^E and it is *a priori* unclear which of these will actually emerge. To begin, we might ask how the incentive to join a cartel relates to the production capacity of a firm. The following result shows that the incentive to collude is positively correlated with firm size.¹⁹

Theorem 3.3 *Assume $\delta > \frac{K-D(c)}{K_\Gamma}$ and consider $i, j \notin \Gamma$. If $k_j > k_i$ then: i) if firm i finds it optimal to join cartel Γ then so does firm j ; and ii) if firm j does not find it optimal to join cartel Γ then neither does firm i .*

Proof. Consider a cartel Γ comprising $x - 1$ firms and a firm that is not a member of Γ . If its capacity is k , it prefers to join cartel Γ iff

$$([p^c(K_\Gamma + k) - c][D(p^c(K_\Gamma + k)) - (K - K_\Gamma - k)] - T(x)) \left(\frac{k}{K_\Gamma + k} \right) \geq [p^c(K_\Gamma) - c]k,$$

where the left hand side expression is the stationary profit from joining the cartel and the right hand side is the stationary profit from remaining outside the cartel. This condition can be re-arranged to

$$\frac{[p^c(K_\Gamma + k) - c][D(p^c(K_\Gamma + k)) - (K - K_\Gamma - k)] - T(x)}{K_\Gamma + k} - [p^c(K_\Gamma) - c] \geq 0. \quad (3.16)$$

Take the derivative of the expression in (3.16) with respect to k ,

$$\begin{aligned} & p^{c'}(K_\Gamma + k) \left(\frac{D(p^c(K_\Gamma + k)) - (K - K_\Gamma - k)}{K_\Gamma + k} \right) + [p^c(K_\Gamma + k) - c] \left(\frac{1}{K_\Gamma + k} \right)^2 \times \\ & [D'(p^c(K_\Gamma + k))p^c(K_\Gamma + k)(K_\Gamma + k) + (K_\Gamma + k) - D(p^c(K_\Gamma + k)) + (K - K_\Gamma - k)] \\ & + \frac{T(x)}{(K_\Gamma + k)^2} \\ & = p^{c'}(K_\Gamma + k) \left(\frac{1}{K_\Gamma + k} \right) \times \quad (3.17) \\ & [D(p^c(K_\Gamma + k)) - (K - K_\Gamma - k) + (p^c(K_\Gamma + k) - c)D'(p^c(K_\Gamma + k))] \\ & + \frac{[p^c(K_\Gamma + k) - c](K - D(p^c(K_\Gamma + k))) + T(x)}{(K_\Gamma + k)^2}. \end{aligned}$$

The second term in (3.17) is positive because $p^c(K_\Gamma + k) - c > 0$, $K - D(p^c(K_\Gamma + k)) > 0$ and $T(x) > 0$. Since $p^{c'}(K_\Gamma + k) > 0$ was established in the proof of Lemma 3.3, the first term in (3.17) is non-negative iff:

$$D(p^c(K_\Gamma + k)) - (K - K_\Gamma - k) + (p^c(K_\Gamma + k) - c)D'(p^c(K_\Gamma + k)) \geq 0. \quad (3.18)$$

¹⁹A similar result without the cost of colluding can be found in Bos and Harrington (2008).

(3.18) holds with equality when the ICC is not binding and with inequality when the ICC is binding. Hence, (3.17) is positive.

Suppose $k_j > k_i$. Since the expression in (3.16) is increasing in k , if (3.16) holds for firm i then it holds for firm j ; and if (3.16) does not hold for firm j then it does not hold for firm i . ■

Thus, the larger the production capacity of a firm, the stronger its incentive to take part in a cartel, all else unchanged. We can make a stronger claim by showing that very small firms never have an incentive to join a cartel. The following result shows that the participation constraint is always violated for firms that are sufficiently small.

Lemma 3.4 *A sufficiently small firm has no incentive to join any cartel.*

Proof. Consider a firm $r \in N \setminus \Gamma$, which has to decide on whether or not to join a cartel Γ that is formed by x members. Following the external stability condition as defined by (3.13), firm r has no incentive to join coalition Γ if,

$$(p^c(\Gamma) - c)k_r \geq \left[(p^c(\Gamma_{+r}) - c) \left[D(p^c(\Gamma_{+r})) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j + k_r \right] - T(x+1) \right] \frac{k_r}{\sum_{j \in \Gamma} k_j + k_r},$$

with $p^c(\Gamma_{+r})$ being the solution of (3.6) for the new cartel consisting of members of Γ and firm r . Rearranging yields,

$$k_r \leq \frac{(p^c(\Gamma_{+r}) - c) (\sum_{i \in N} k_i - D(p^c(\Gamma_{+r}))) + T(x+1)}{p^c(\Gamma_{+r}) - p^c(\Gamma)} - \sum_{j \in \Gamma} k_j. \quad (3.19)$$

Following Lemma 3.3 we have that $p^c(\Gamma_{+r}) = p^c(\Gamma)$ as $k_r \rightarrow 0$. Therefore, the right-hand side of (3.19) tends to infinity for ever smaller values of k_r , while the left-hand side of (3.19) tends to zero. ■

The intuition behind these results is as follows. Both residual cartel demand and the extent to which a cartel can raise industry prices depend in part on the combined capacity of its members. As Lemma 3.3 indicates, the more industry capacity that is under the control of the cartel, *ceteris paribus*, the higher is the optimal cartel price. If a small outsider joins the coalition, then the change in both cartel and fringe capacity is only marginal. Consequently, the marginal contribution of small sellers to the cartel is limited. These small firms prefer to remain independent outsiders, because as a fringe member they do not have the commitment to reduce production. By contrast, large fringe members leave little residual demand for a cartel. As a result, a cartel facing a competitive fringe with significant production capacity has limited power to raise the cartel price above marginal costs. In turn, this limits the incentive to free-ride for a large firm, because even though it would produce up to capacity, it would also face a relatively small mark-up. Instead, if a large firm decides to join the cartel it will sell fewer products, but at the same time taking part in the cartel allows for a substantial

price increase. Therefore, a firm will find it in its interest to join a conspiracy when it is sufficiently large.

The previous results indicate that, *ceteris paribus*, there exists a negative correlation between the incentive of firms to free-ride on a cartel formed by others and firm size. The next proposition shows that a cartel comprising the largest firms in the industry is indeed a solution of \mathcal{K}^E .

Proposition 3.2 *If δ is sufficiently close to one, then there exists $m \in \{2, \dots, n\}$ such that $\{1, \dots, m\}$ is a SPNE of \mathcal{K}^E .*

Proof. Consider a cartel Γ in which the largest firms participate. Suppose that whenever the smallest participant leaves Γ this would render the cartel unstable. This implies that (3.14) is satisfied for all $i \in \Gamma$. It remains to be shown that (3.13) holds for all $i \in N \setminus \Gamma$. If so, the proof is complete. If not, then at least one outsider has an incentive to join Γ . By Theorem 3.3, whenever (3.13) is violated for some $i \in N \setminus \Gamma$, it must be violated for the largest fringe member. Let $r \in N \setminus \Gamma$ denote the largest outsider. If r joins Γ , it becomes the smallest member of the new cartel Γ_{+r} . Clearly, (3.14) is satisfied for firm r , because otherwise firm r would have had no incentive to join Γ . Note that Theorem 3.3 implies that all firms larger than firm r prefer to stay in Γ . Hence, the participation constraint is satisfied for all members of Γ_{+r} . Repeating this exercise leaves two possible outcomes. Either, at some point (3.13) is satisfied for the largest outsider. Then, by Lemma 3.4, all smaller fringe members have no incentive to join either. Alternatively, no outsiders are left, i.e., the cartel is all-inclusive. ■

Notice, however, that there typically exists a whole range of coalitions for which (3.13), (3.14) and (3.10) hold simultaneously. Cartels in which the smallest members are smaller than the largest outsiders might well be a solution of the game. More generally, whether or not moderate-sized firms have an incentive to take part in a cartel often depends on what type of ‘formation game’ is played.

To illustrate, suppose that firms do not take their participation decision simultaneously, but instead play a sequential formation game as in Bloch (1996) and Prokop (1999). Recall that both studies conclude that when players are identical and far-sighted, the last x firms in a sequence of proposers will join the cartel, leaving the first $n - x$ firms as outsiders.²⁰ This result is driven by the fact that in equilibrium outsiders earn more profits than insiders and, as a result, all firms have a strong incentive to free-ride on the cartel. The first firms to propose anticipate that announcing “no” yields a situation in which the remaining firms have an incentive to collude, because the alternative for the latter is competition.²¹

The predicted equilibrium cartel size is unique in these games, which is due to the assumption of symmetry. The symmetry assumption, however, also implies that the outcome is unique up to permutation. Applying such sequential formation rules to

²⁰Bloch (1996) considers a situation in which firms sequentially propose a coalition and applies this approach to a simultaneous symmetric Cournot game. Prokop (1999) allows firms only to say “yes” or “no” and applies this approach to the symmetric model of collusive price leadership as analyzed in d’Aspremont *et al.* (1983). See Chapter 2 of this thesis for a more detailed description.

²¹Note that this outcome requires all firms to commit to their choices.

the current heterogeneous setting typically yields a variety of equilibria. In particular, what coalition will form depends heavily on the position firms take in the sequence of proposers. For instance, a relatively large firm at the beginning of the sequence may well announce “no” realizing that a relatively small firm will have an incentive to say “yes” later on.

It has been shown in Lemma 3.4. and Theorem 3.3 that sufficiently small firms will always say “no” and that large firms are more inclined to say “yes” to a coalition. This result is *independent of the order of moves*. To what extent moderate-sized firms have an incentive to collude is sensitive to the order of moves and therefore strongly depends on the specific rules of the formation game that is played. The order in which firms take their decision is quite arbitrary and what cartel will actually form remains therefore difficult to predict.

3.5 The Most Profitable Cartel

In a wide array of settings, total cartel value is highest for a cartel that is all-inclusive. We have already established that when cartelizing is costly this might not be true. In this section, we explore if and under what conditions firms have an incentive to form the cartel agreement for which *total cartel profits* are highest. Note that the most profitable cartel is not necessarily all-inclusive as in the current setting colluding is costly. In the following, let Γ^* denote the cartel which generates the highest total cartel value. First, we examine what firms take part in Γ^* and then we explore whether or not Γ^* is a solution of \mathcal{K}^E .

The following result shows that, for a fixed number of cartel participants, the most profitable cartel consists of the largest firms in the industry.

Proposition 3.3 *For any given cartel size, the most profitable cartel comprises the largest firms in the industry.*

Proof. Consider a sustainable cartel Γ in which x members participate. Total cartel value amounts to,

$$V^c(p^c, \Gamma) = \frac{1}{1 - \delta} \left[(p^c - c) \left[D(p^c) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right] - T(x) \right].$$

Observe that, given x , $V^c(p^c, \Gamma)$ is increasing in total cartel capacity. With the proportional profit allocation rule as given by (3.15) all cartel members face the following critical discount factor,

$$\delta^* = \frac{(p^c - c) (\sum_{i \in N} k_i - D(p^c)) + T(x)}{(p^c - c) \sum_{j \in \Gamma} k_j},$$

which is decreasing in total cartel capacity. It can be concluded that for a given number of participants the most profitable cartel consists of the largest firms in the industry. ■

The above result implies that, given the number of cartel members, whenever some collusion is sustainable, than a cartel comprising the largest firms is sustainable too. Hence, we know that Γ^* is formed by the largest sellers in the market. In addition, we have already established that the most profitable cartel is less than all-inclusive when colluding is costly and the smallest firms are sufficiently small. As a result, the size of Γ^* is uniquely determined.²²

To see whether or not firms have an incentive to form the most profitable cartel Γ^* notice first that (3.10) trivially holds, because otherwise Γ^* would not be viable. Furthermore, it has been established that the smallest member of Γ^* is (weakly) larger than the largest outsider. By Proposition 3.2 we know that a cartel comprising the largest firms in the industry is indeed an equilibrium outcome of \mathcal{K}^E . However, this result is in itself not sufficient. For Γ^* to be a solution of \mathcal{K}^E it is also required that the right number of sellers has an incentive to participate.

The next result reveals that (3.13) is always satisfied for firms that are not included in Γ^* .

Lemma 3.5 $\pi_i^o(\Gamma^*) \geq \pi_i^c(\Gamma_{+i}^*)$ for all $i \in N \setminus \Gamma^*$.

Proof. Without loss of generality, assume Γ^* consists of x members and let firm $r \in N \setminus \Gamma^*$ denote some outsider. Firm r has no incentive to join Γ^* whenever the following condition holds,

$$\pi_r^o(\Gamma^*) \geq \pi_r^c(\Gamma_{+r}^*),$$

which is equivalent to,

$$(p^c(\Gamma^*) - c) \left(\sum_{j \in \Gamma^*} k_j + k_r \right) \geq \pi^c(\Gamma_{+r}^*). \quad (3.20)$$

Note that by definition $\pi^c(\Gamma_{+r}^*) \leq \pi^c(\Gamma^*)$, because otherwise Γ^* would not be the most profitable cartel arrangement. Hence, (3.20) holds whenever the following condition is satisfied,

$$(p^c(\Gamma^*) - c) \left(\sum_{j \in \Gamma^*} k_j + k_r \right) \geq \pi^c(\Gamma^*),$$

or,

$$(p^c(\Gamma^*) - c) \left(\sum_{j \in \Gamma^*} k_j + k_r \right) \geq (p^c(\Gamma^*) - c) \left[D(p^c(\Gamma^*)) - \sum_{j \in N} k_j + \sum_{j \in \Gamma^*} k_j \right] - T(x).$$

Rearranging yields,

$$(p^c(\Gamma^*) - c) k_r \geq (p^c(\Gamma^*) - c) \left[D(p^c(\Gamma^*)) - \sum_{j \in N} k_j \right] - T(x),$$

²²Note that when some firms have equally large capacity stocks the composition of the most profitable cartel arrangement might not be unique.

which always holds. It can be concluded that (3.13) is satisfied for all outsiders to Γ^* .

■

Hence, whenever Γ^* is less than all-inclusive, none of the fringe members has an incentive to take part in the most profitable cartel arrangement. For Γ^* to be a SPNE of \mathcal{K}^E it remains to be shown that none of the participants prefers to be a fringe member. We will show that this is not generally true. In fact, whether or not firms have an incentive to form Γ^* depends on the size of the smallest member.

Theorem 3.4 *If the smallest member of Γ^* is sufficiently large, then Γ^* is a SPNE of \mathcal{K}^E .*

Proof. Recall that for Γ^* (3.10) is naturally satisfied. Moreover, by Lemma 3.5 we know that (3.13) always holds. Hence, for Γ^* to be a SPNE of \mathcal{K}^E it remains to be shown that $\pi_i^c(\Gamma^*) \geq \pi_i^o(\Gamma_{-i}^*)$ for all $i \in \Gamma^*$. To that end, let r be the smallest member of Γ^* and suppose without loss of generality that the most profitable cartel consists of x firms. Firm r prefers to stay in Γ^* if,

$$\pi_r^c(\Gamma^*) \geq \pi_r^o(\Gamma_{-r}^*).$$

This is equivalent to,

$$\left((p^c(\Gamma^*) - c) \left[D(p^c(\Gamma^*)) - \sum_{i \in N} k_i + \sum_{j \in \Gamma^*} k_j \right] - T(x) \right) \frac{k_r}{\sum_{j \in \Gamma^*} k_j} \geq (p^c(\Gamma_{-r}^*) - c) k_r.$$

Rearranging yields,

$$k_r \geq \frac{(p^c(\Gamma^*) - c) (\sum_{i \in N} k_i - D(p^c(\Gamma^*))) + T(x)}{p^c(\Gamma^*) - p^c(\Gamma_{-r}^*)} - \sum_{\substack{j \in \Gamma^* \\ j \neq r}} k_j. \quad (3.21)$$

Observe that this inequality holds for k_r sufficiently large. Note that whenever (3.21) is satisfied for firm r it will be satisfied for all larger members too (by Theorem 3.3.), which completes the proof. ■

The above result shows that for the most profitable cartel to be a solution of \mathcal{K}^E it must be the case that the smallest member owns a sufficient part of total industry capacity. To illustrate, suppose that the cost of colluding is so low that Γ^* is all-inclusive. Whether or not firms have an incentive to form Γ^* then depends on the share of industry capacity that is under the control of the smallest market player. If this firm is very small it prefers to be a fringe member and the most profitable cartel arrangement will not emerge.

3.6 Incomplete Cartels and Mergers

The incentive to collude is positively related to the capacity stock available to sellers. If and what cartel is likely to emerge therefore partly depends on the size distribution of

firms in the industry. For instance, a cartel is unlikely to be all-inclusive in markets with one or more very small undertakings. A particular way in which the size distribution of firms can change is through a merger. In this section, mergers are discussed in relation to incomplete cartels. We address two issues: Merger incentives and the potential coordinated effects of a merger. In what follows, we assume that mergers do not create synergies, i.e., the level of production costs remains unchanged post-merger.²³ The effect of a merger is then a reduction in the number of firms and two or more firms become one larger firm with a capacity stock equal to the sum of capacities of the merging parties.

3.6.1 Merger Incentives

In this subsection, the incentives of firms to engage in a merger are analyzed. A distinction can be made between merger incentives in the presence and absence of collusion. Note that, under the assumptions made, the competitive equilibrium is unaffected by a merger. The reason is that the static Nash equilibrium is neutral to the number of firms as long as $n \geq 3$, which holds by assumption. As a result, none of the firms has an incentive to engage in a merger in absence of collusion. It must be noticed, however, that competing firms might find it in their interest to merge when collusion is anticipated at a later stage.

Alternatively, consider a situation in which a cartel is already present. To begin, notice that a cartel Γ which adopts a proportional profit allocation rule as given by (3.15) faces the following constrained optimization problem,

$$\max_{p^c} V^c(p^c, \Gamma) = \frac{1}{1-\delta} \left[(p^c - c) \left[D(p^c) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right] - T(\cdot) \right], \quad (3.22)$$

subject to,

$$\delta \geq \delta^* = \frac{\sum_{i \in N} k_i - D(p^c)}{\sum_{j \in \Gamma} k_j} + \frac{T(\cdot)}{(p^c - c) \sum_{j \in \Gamma} k_j}. \quad (3.23)$$

Observe that δ^* is the same for all members and decreasing in total cartel capacity, all else equal. Furthermore, (3.23) reveals that the sustainability of a cartel does not depend on the size distribution of participants, but on the total share of industry capacity that is under the control of the cartel. This does not mean that a merger between two or more cartel members has no effect on the incentive constraint. In fact, a merger between two or more cartel participants loosens the ICC. The reason is that there are fewer firms in the cartel post-merger, which leads to a reduction in $T(\cdot)$.

Hence, when the cartel is all-inclusive, all firms have an incentive to merge. If the cartel is not all-inclusive, there are two more cases to consider: (i) a merger between fringe members, and (iii) a merger between one or more cartel members with one or more fringe firms. In analyzing the latter, we assume that whenever the new larger firm does not take part in the cartel this will be perceived as cheating by cartel members

²³Clearly, it is beneficial for firms to merge whenever this leads to lower unit costs.

not involved in the merger. The following result shows that firms have an incentive to merge only when they are part of the cartel or become part of the cartel post-merger.

Proposition 3.4 *In the presence of a cartel,*

- (i) *Fringe firms have no incentive to merge,*
- (ii) *Cartel members do have an incentive to merge,*
- (iii) *Fringe firms and cartel members have an incentive to merge only if they are part of the cartel post-merger.*

Proof. Fix some collusive equilibrium in which there exists a cartel Γ consisting of x members. Three type of mergers may occur (i) a merger among two or more fringe members, (ii) a merger among two or more cartel members and (iii) a merger between one or more cartel members with one or more outsiders. We will discuss each case in turn.

(i) Note that a merger among two or more outsiders does not alter total cartel capacity. Following Lemma 3.3, we know that the optimal cartel price and therefore the mark-up of merging parties remains unaltered. The merger yields no additional profits, because total capacity of the merger equals the sum of production capacities of the merging firms.

(ii) The total cartel value of Γ equals,

$$\frac{1}{1-\delta} \left((p^c(\Gamma) - c) \left[D(p^c(\Gamma)) - \sum_{i \in N} k_i + \sum_{j \in \Gamma} k_j \right] - T(x) \right).$$

Observe that a merger between two or more cartel participants only leads to a decline in the number of cartel participants, i.e., the merger only negatively affects $T(x)$. Consequently, any merger between cartel members increases total cartel value. Notice further that the cartel is naturally sustainable post-merger, because $\frac{\partial \delta^*}{\partial T} > 0$. It can be concluded that cartel members always have an incentive to merge.

(iii) In the following, we show that a merger between cartel members and fringe members yields a larger firm that has no incentive to defect from the cartel agreement. To that end, let Ψ denote the coalition of merging parties. The total value that this coalition could earn by deviating optimally amounts to,

$$(p^c(\Gamma) - c) \sum_{j \in \Psi} k_j.$$

Such deviation is beneficial only if total deviating profits exceed the sum of profits that the merging parties earn separately, i.e.,

$$(p^c(\Gamma) - c) \sum_{j \in N \setminus \Gamma, \Psi} k_j + \sum_{j \in \Gamma, \Psi} \pi_j^d \geq \frac{1}{1-\delta} (p^c - c) \sum_{j \in N \setminus \Gamma, \Psi} k_j + \sum_{j \in \Gamma, \Psi} V_j^c.$$

This inequality is violated, because Γ is sustainable, i.e., $V_j^c \geq \pi_j^d$ for all $j \in \Gamma, \Psi$. Finally, note that (3.23) is decreasing in total cartel capacity and that the number of cartel firms will not increase. Hence, the cartel is always sustainable post-merger.

Fringe firms and cartel members therefore might have an incentive to merge and when they do they will be part of the cartel post-merger. ■

The above result is in contrast with ESCRIHUELA-VILLAR (2008b) which analyzes incomplete cartels in a symmetric infinitely repeated Cournot model. He shows that, in the presence of a cartel, mergers among outsiders and mergers among insiders lead to a price increase. In the current setting, the price level depends on total cartel capacity and therefore remains unaffected by a merger.²⁴ He also finds that a cartel reduces merger incentives, while in our case merger incentives are higher under a collusive regime. Furthermore, he shows that a cartel provides additional incentives for fringe firms to merge. In our model, an incomplete cartel provides no incentives for outsiders to organize their production capacity in a single company.²⁵ It is important to emphasize that the above result is largely driven by the fact that competitive profits do not depend on the number of sellers in the market. This is typical for homogeneous goods markets in which firms compete in price. If, instead, firms compete in quantity or when goods are differentiated, firms might find it beneficial to merge in absence of collusion.

3.6.2 Coordinated Effects of Mergers

In this subsection, we study potential coordinated effects that result from a merger. To provide some preliminary intuition, consider a merger between two very small sellers so that, following Lemma 3.4, both sellers have no incentive to join a cartel. Clearly, if the new firm is sufficiently small, merging parties still have no incentive to join a cartel post-merger. Hence, a merger between sufficiently small firms has no impact. In a related vein, a merger between sufficiently large firms, in that both would have joined a cartel pre-merger, has no impact either. That is, a merger between sufficiently large firms leaves total cartel capacity unaltered and therefore the merger has no impact on the equilibrium cartel price. The analysis presented below suggest that the composition of a stable cartel is most likely affected by a merger among moderate-sized firms.

Compte *et al.* (2002) and Vasconcelos (2005) are two important contributions that explore how the distribution of capacity stocks affects collusion. Assuming an all-inclusive cartel, both studies examine an infinitely repeated game in which firms differ in terms of production capacity. Compte *et al.* (2002) assumes that firms compete in price and find that the critical discount factor is increasing in the size of the largest firm(s). Vasconcelos (2005) considers a Cournot setting and finds that not only the size of the largest firms matter, but also that of the smallest. The smallest firm has the highest incentive to deviate from the cartel agreement, while the largest firm has the strongest incentive not to punish in the event of cheating. Consequently, redistributing capacity from the largest to the smallest seller facilitates collusion. Both papers show that the minimum critical discount factor is obtained with a symmetric capacity

²⁴This is not true for the case in which cartel members merge with fringe firms. However, in ESCRIHUELA-VILLAR (2008b) this possibility is not considered.

²⁵Likewise, fringe firms have no incentive to form a cartel. As a consequence, in the current setting it is never optimal to have multiple cartels.

distribution and it is in this sense that more symmetry facilitates collusion. Hence, both studies suggest that the strongest coordinated effects result from a redistribution of capacity involving either the largest firms or the smallest firms, or both.

By contrast, when cartel membership is endogenous and incomplete the strongest coordinated effects might well come from a redistribution of capacity among moderate-sized firms. For example, a merger between two firms of medium size results in a larger firm that might have an incentive to join a cartel post-merger. It is not immediately clear, however, if cartel capacity increases due to a merger, because a merger that joins the cartel may induce other members to leave the coalition. It is difficult to derive analytical results, because any outcome quite generally depends on the entire capacity distribution. In Bos and Harrington (2008), we perform simulations to explore this issue in more detail. These results are discussed below. First, however, we believe it is instructive to examine some numerical examples.

3.6.2.1 Numerical Examples

For the sake of simplicity, suppose that colluding is costless and that market demand $D(p)$ is linear,

$$D(p) = 1 - p.$$

Thus, monopoly demand is equal to $\frac{1}{2}$. By Assumption 2, we then must have that,

$$\frac{1}{2} > k_1 \geq k_2 \geq \dots \geq k_n > 0,$$

Furthermore, assume that $c = 0$ so that by Assumption 3,

$$\sum_{i=2}^n k_i \geq 1.$$

We further suppose that $\delta \simeq 1$, which implies that the ICC is not binding. Hence, in equilibrium the cartel price of a cartel Γ is either:

$$p^c(K_\Gamma) = \frac{1 - K + K_\Gamma}{2} \text{ if } K_\Gamma > K - 1,$$

or

$$p^c(K_\Gamma) = c = 0 \text{ when } K_\Gamma \leq K - 1.$$

For Γ to be stable, we must have that none of its members has an incentive to leave the coalition and all outsiders should prefer to stay part of the competitive fringe. A firm $i \in \Gamma$ has no incentive to leave a profitable cartel Γ if this renders the cartel unprofitable, i.e., when $p^c(K_\Gamma - k_i) = c = 0$. When $p^c(K_\Gamma - k_i) > c = 0$, it has no incentive to join the fringe if,

$$[p^c(K_\Gamma) - c] \left(\frac{D(p^c(K_\Gamma)) - K + K_\Gamma}{K_\Gamma} \right) k_i > [p^c(K_\Gamma - k_i) - c] k_i,$$

which is equivalent to,

$$k_i > \frac{K_\Gamma^2 - (K - 1)^2}{2K_\Gamma}, \forall i \in \Gamma \quad (3.24)$$

Likewise, a firm $i \notin \Gamma$ has no incentive to join Γ if,

$$[p^c(K_\Gamma) - c]k_i \geq [p^c(K_\Gamma + k_i) - c] \left(\frac{D(p^c(K_\Gamma + k_i)) - K + K_\Gamma + k_i}{K_\Gamma + k_i} \right) k_i,$$

which can be rearranged to,

$$k_i \leq \sqrt{K_\Gamma^2 - (K - 1)^2}, \forall i \notin \Gamma. \quad (3.25)$$

Hence, a cartel Γ is stable when both (3.24) and (3.25) hold.

We consider an industry of eight firms ($n = 8$) pre-merger. Pre-merger, firms can take three sizes:

- Large firms with capacity $\frac{4}{10}$
- Medium firms with capacity $\frac{2}{10}$
- Small firms with capacity $\frac{1}{10}$

In the following, we analyze three cases.

Case 3.5 *A merger among non-cartel members could result in the merged firm joining the cartel and expand cartel membership.*

Suppose that the capacities of the eight firms are distributed as follows,

$$\begin{aligned} k_1 &= k_2 = \frac{4}{10} \\ k_3 &= k_4 = k_5 = \frac{2}{10} \\ k_6 &= k_7 = k_8 = \frac{1}{10}. \end{aligned}$$

- Pre-merger: Suppose that the pre-merger cartel is formed by firm 1 and 2, i.e., $\Gamma = \{1, 2\}$. This cartel is profitable because $K_\Gamma = \frac{8}{10} > K - 1 = \frac{7}{10}$. Furthermore, if any of the two members leaves the cartel it renders unprofitable, which implies it is internally stable. Γ is also externally stable, because

$$k_i^2 \leq \frac{15}{100}, \forall i \notin \Gamma.$$

Hence, the pre-merger cartel $\Gamma = \{1, 2\}$ is stable.

- Post-merger: Now suppose firm 3 and 4 merge, so that $k_3 + k_4 = \frac{2}{10} + \frac{2}{10} = \frac{4}{10} = k_m$. Firm 3 and 4 have an incentive to join Γ post-merger if,

$$k_m > \sqrt{K_\Gamma^2 - (K - 1)^2} \Rightarrow \frac{16}{100} > \frac{15}{100}.$$

Clearly, the firms want to join post-merger. Also, the new cartel $\Gamma + \{3/4\}$ is stable. It is internally stable, because we have shown that firms of size $\frac{4}{10}$ have an incentive to join a cartel of size $\frac{8}{10}$. It is also externally stable because:

$$k_5^2 = \frac{4}{100} \leq \frac{95}{100},$$

Thus, a merger among non-cartel members could result in the merged firm joining the cartel and expand cartel membership.

Case 3.6 *A merger between a cartel and a non-cartel member may leave the composition of the cartel unaffected and result in more cartel capacity.*

Now suppose the capacity distribution is as follows,

$$\begin{aligned} k_1 &= \frac{4}{10}, \\ k_2 &= k_3 = k_4 = \frac{2}{10} \\ k_5 &= k_6 = k_7 = k_8 = \frac{1}{10}, \end{aligned}$$

so that $K - 1 = \frac{4}{10}$. Also, $\sum_{i=2}^n k_i = \frac{10}{10} \geq 1$.

- Pre-merger: again, assume that there is a cartel $\Gamma = \{1, 2\}$, which is profitable because $K_\Gamma = \frac{6}{10} > K - 1 = \frac{4}{10}$. This cartel is internally stable, because when any of the two members leaves we have $K_\Gamma \leq K - 1$, which implies zero profits for all firms. Moreover, Γ is externally stable because,

$$k_i^2 \leq K_\Gamma^2 - (K - 1)^2 \Rightarrow k_i^2 \leq \frac{20}{100}, \forall i \notin \Gamma.$$

Therefore, $\Gamma = \{1, 2\}$ is a stable cartel.

- Post-merger: Now suppose firm 2 and firm 3 merge, so that $k_2 + k_3 = \frac{2}{10} + \frac{2}{10} = \frac{4}{10} = k_m$, which will be part of the cartel. The new cartel $\Gamma^m = \Gamma - \{2\} + \{2/3\}$ is internally stable, because whenever one of the two firms leave we have that $K_\Gamma \leq K - 1$, which implies zero profits, while profits are positive pre-merger. To see that it is also externally stable note that,

$$k_i^2 \leq K_\Gamma^2 - (K - 1)^2 \Rightarrow k_i^2 \leq \frac{48}{100}, \forall i \notin \Gamma^m.$$

Hence, Γ^m is stable. We have shown that a merger between a cartel and a non-cartel member may leave the composition of the cartel unaffected and result in more cartel capacity.

Case 3.7 *Total cartel capacity can be lower post-merger.*

Consider the following capacity distribution:

$$\begin{aligned} k_1 &= \frac{4}{10} \\ k_2 &= k_3 = k_4 = k_5 = k_6 = \frac{2}{10} \\ k_7 &= k_8 = \frac{1}{10}, \end{aligned}$$

so that $K - 1 = \frac{6}{10}$ and $\sum_{i=2}^n k_i = \frac{12}{10} \geq 1$.

- Pre-merger: Suppose firm 1,2 and 3 are in the cartel, i.e., $\Gamma = \{1, 2, 3\}$. Total cartel capacity is then given by $K_\Gamma = \frac{8}{10}$. Thus, Γ is profitable because $K_\Gamma = \frac{8}{10} > K - 1 = \frac{6}{10}$ and internally stable, because whenever one of the participants leaves we have that $K_\Gamma \leq K - 1$. It is also externally stable because,

$$k_i^2 \leq K_\Gamma^2 - (K - 1)^2 \Rightarrow k_i^2 \leq \frac{28}{100}, \forall i \notin \Gamma.$$

As a result, $\Gamma = \{1, 2, 3\}$ is stable.

- Post-merger: Now suppose firm 2 merges with firm 7 so that $k_2 + k_7 = \frac{2}{10} + \frac{1}{10} = \frac{3}{10} = k_m$. Denote Γ^m the cartel post-merger. Note that both firm 1 and firm m must be part of Γ^m because total cartel capacity must be at least $\frac{7}{10}$ in order to be profitable. For instance, firm 1 and 3 have total capacity of $\frac{6}{10}$, which is not enough to make the cartel profitable. Yet, firm 3 potentially can leave the cartel post-merger without making it unprofitable because combined capacity of the merger and firm 1 equals $\frac{7}{10} > \frac{6}{10}$. Firm 3 has an incentive to leave the cartel $\Gamma^m = \Gamma - \{2\} + \{2/7\}$ when,

$$k_3 \leq \frac{K_{\Gamma^m}^2 - (K - 1)^2}{2K_{\Gamma^m}} = \frac{\frac{81}{100} - \frac{36}{100}}{\frac{180}{100}} \Rightarrow k_3 \leq \frac{1}{4},$$

and this holds because $k_3 = \frac{1}{5}$. Hence, $\Gamma^m = \Gamma - \{2\} + \{2/7\}$ is not stable and firm 3 will leave the cartel. Firm 1 and 2/7 will not leave because that would imply zero profits. Consequently, $\Gamma - \{2\} + \{2/7\} - \{3\}$ is internally stable. It is also externally stable, otherwise firm 3 would have had no incentive to leave (all outsiders are weakly smaller than firm 3). Finally, note that total cartel capacity post-merger is equal to $\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$, while pre-merger it was equal to $\frac{4}{10} + \frac{2}{10} + \frac{2}{10} = \frac{8}{10}$. As a result, total cartel capacity is lower post-merger.

The first two examples illustrate how a merger between moderate-sized firms can increase the impact of a cartel. The third example shows how a merger involving a medium firm can lower cartel capacity. These examples suggest that the strongest

coordinated effects might come from mergers involving moderate-sized firms. It is clear, however, that a more advanced analysis is required to say anything more substantial.

In Bos and Harrington (2008), we make an attempt to address this issue more rigorously by means of simulations. We assess the impact of a two-firm merger on a stable cartel comprising the largest firms. By Proposition 3.2 we know that a cartel agreement between the largest firms is an equilibrium outcome of the game. For simplicity, assume that colluding is costless. The next result shows that when demand is linear there is a unique stable cartel consisting of the largest undertakings.

Theorem 3.8 *If demand is linear, then there is a unique stable cartel comprising the largest firms.*

Proof. Denote the smallest stable cartel $\Gamma = \{1, \dots, m\}$ so that all cartels smaller than Γ are unstable. We will show that any larger cartel is unstable too. To that end, we need to consider three cases. First, suppose the ICC is not binding for Γ , then we know the ICC is not binding for any larger cartel, because

$$\delta^* = \frac{\sum_{i \in N} k_i - D(p^c)}{\sum_{j \in \Gamma} k_j},$$

is the same for all cartel members and decreasing in total cartel capacity. Second, for δ relatively low, the ICC may be binding for Γ and for all larger cartels. Thirdly, the ICC might be binding for Γ , but not for some larger cartel(s). We discuss the three cases in the following.

(a) If δ is sufficiently high, then the ICC is not binding for Γ and for any larger cartel. By assumption, Γ is stable, which implies firm $m + 1$ has no incentive to join, or,

$$k_{m+1} \leq \sqrt{(K_\Gamma)^2 - (K - D(c))^2}. \quad (*)$$

A similar condition holds for outsiders to larger cartels. Note that,

$$k_{h+1} \leq k_{m+1} \leq \sqrt{\left(\sum_{j=1}^m k_j\right)^2 - (K - D(c))^2} < \sqrt{\left(\sum_{j=1}^h k_j\right)^2 - (K - D(c))^2}, \text{ for all } h > m. \quad (**)$$

Hence, if $\{1, \dots, m^*\}$ is a stable cartel then $(*)$ holds. By $(**)$, $\{1, \dots, f\}$ is not a stable cartel, for $f > m^*$, since firm f does not want to join.

(b) Suppose the ICC is binding for Γ and for all larger cartels. Γ is stable, which means that firm $m + 1$ has no incentive to join, or:

$$k_{m+1} \leq \frac{D(c) - K + \delta \left(\sum_{j=1}^m k_j\right)}{1 - \delta}.$$

Next note that:

$$k_{h+1} \leq k_{m+1} \leq \frac{D(c) - K + \delta \left(\sum_{j=1}^m k_j\right)}{1 - \delta} < \frac{D(c) - K + \delta \left(\sum_{j=1}^h k_j\right)}{1 - \delta}, \text{ for all } h > m. \quad (**)$$

By the same argument as above, there is a unique value for m whereby $\{1, \dots, m\}$ is a stable cartel.

(c) Suppose the ICC is binding for Γ , but non-binding for some larger cartel(s). There are two cases to consider:

(i) The ICC is binding for Γ , but non-binding for any larger cartel. Thus, we have that,

$$p^*(K_\Gamma) = \frac{a - K + \delta K_\Gamma}{b}, p^*(K_\Gamma + k_{m+1}) = \frac{a + bc - K + K_\Gamma + k_{m+1}}{2b},$$

and

$$k_{m+1} \leq \frac{D(c) - K + \delta K_\Gamma}{1 - \delta}.$$

Note that because Γ is assumed stable, the cartel $\Gamma + \{m + 1\}$ is unstable, because firm $m + 1$ wants to be an outsider to Γ . Therefore, consider cartel $\Gamma + \{m + 1\} + \{m + 2\}$, which faces an ICC that is non-binding. Firm $m + 2$ wants to leave this cartel if,

$$k_{m+2} \leq \sqrt{(K_\Gamma + k_{m+1})^2 - (K - D(c))^2}.$$

Rearranging gives,

$$k_{m+2}^2 - k_{m+1}^2 \leq K_\Gamma^2 + 2K_\Gamma k_{m+1} - (K - D(c))^2.$$

Note that $k_{m+1} \geq k_{m+2}$ so that the LHS is weakly negative. It therefore suffices to show that the RHS is larger or equal to zero.

$$K_\Gamma^2 + 2K_\Gamma k_{m+1} - (K - D(c))^2 \geq 0,$$

which is equivalent to,

$$\sqrt{K_\Gamma^2 + 2K_\Gamma k_{m+1}} \geq K - D(c).$$

This condition holds because $K_\Gamma > K - D(c)$ otherwise Γ would not be stable. Hence, the cartel $\Gamma + \{m + 1\} + \{m + 2\}$ is unstable. By repeating the exercise it can be shown that all larger cartels are unstable too.

(ii) Suppose the ICC is binding for Γ and non-binding for one or more larger cartels, but not all. Note that this implies that the ICC is binding for $\Gamma + \{m + 1\}$. Let $\Gamma^* = \{1, \dots, t\}$ be the largest cartel for which the ICC is binding, which implies $t \geq m + 1$. Note that by a similar analysis as in (b) any larger cartel than Γ that faces a binding ICC is unstable. For example, for the smallest member of Γ^* it holds that,

$$k_t \leq k_{m+1} \leq \frac{D(c) - K + \delta \left(\sum_{j=1}^m k_j \right)}{1 - \delta} \leq \frac{D(c) - K + \delta \left(\sum_{j=1}^{t-1} k_j \right)}{1 - \delta}.$$

Hence, if there exists another stable cartel it will face a non-binding ICC. In the following, Let $\Gamma^{**} = \{1, \dots, t + 1\}$ be the smallest cartel for which the ICC is non-binding. We have to show that Γ^{**} is unstable. To that end, consider firm $t + 1$ which

compares being in Γ^{**} with being outside Γ^* . Γ^{**} is unstable if firm $t + 1$ wants to leave the cartel, which is the case if,

$$(p^*(K_{\Gamma^{**}}) - c) \left[\frac{a - bp^*(K_{\Gamma^{**}}) - K + K_{\Gamma^{**}}}{K_{\Gamma^{**}}} \right] - (p^*(K_{\Gamma^*}) - c) \leq 0,$$

which is equivalent to,

$$\delta \geq \frac{(D(c) - K + K_{\Gamma^{**}})^2 + 4K_{\Gamma^{**}}(K - D(c))}{4K_{\Gamma^{**}}K_{\Gamma^*}}. \quad (3.26)$$

In this particular situation it holds that,

$$\frac{K - D(c) + K_{\Gamma^*}}{2K_{\Gamma^*}} \geq \delta \geq \frac{K - D(c) + K_{\Gamma^*} + k_{t+1}}{2(K_{\Gamma^*} + k_{t+1})},$$

because the ICC is binding for Γ^* and non-binding for Γ^{**} . The condition (3.26) therefore holds if:

$$\frac{K - D(c) + K_{\Gamma^{**}}}{2K_{\Gamma^{**}}} \geq \frac{(D(c) - K + K_{\Gamma^{**}})^2 + 4K_{\Gamma^{**}}(K - D(c))}{4K_{\Gamma^{**}}K_{\Gamma^*}}.$$

Rearranging yields,

$$\begin{aligned} K_{\Gamma^*}^2 &\geq (K - D(c))^2 - 2D(c)k_{t+1} + 2Kk_{t+1} + k_{t+1}^2, \\ K_{\Gamma^*}^2 &\geq (K - D(c) + k_{t+1})^2, \\ k_{t+1} &\leq D(c) - K + K_{\Gamma^*}. \end{aligned}$$

This holds because $K_{\Gamma^*} - k_{t+1} \geq K_{\Gamma} > K - D(c)$ otherwise Γ would not have been stable. Hence, Γ^{**} is unstable. Following the analysis in (i) above it can be shown that firm $t + 2$ does not want to join Γ^{**} so that a cartel $\{1, \dots, t + 2\}$ is unstable. In a similar fashion, it can be shown that all larger cartels are unstable too.

Consequently, there is a unique stable cartel comprising the largest firms when demand is linear. ■

We perform simulations for $D(p) = 1 - p$ and $c = 0$. A single simulation involves the following five steps.

1. Fix the number of firms, n .
2. Randomly select a vector of capacities (k_1, \dots, k_n) according to a uniform distribution over $(0, \frac{1}{2})^n$. Requiring that each firm's capacity is less than $\frac{1}{2}$ ensures that Assumption 2 is satisfied. Next check that $\sum_{h \neq i, j} k_h \geq 1, \forall i, j$. If that condition holds then Assumption 3 is satisfied both in the pre-merger and any post-merger situation, in which case go to step 3. If instead $\sum_{h \neq i, j} k_h < 1$ for some i, j then redraw the vector of capacities until the condition is satisfied.
3. Given (k_1, \dots, k_n) , randomly select the discount factor δ according to a uniform distribution over $\left(\frac{K - (a - bc)}{K}, 1\right)$ where $K \equiv \sum_{i=1}^n k_i$. By drawing δ from this interval, some collusion is sustainable. (Otherwise, a merger has no effect.)

4. Given (k_1, \dots, k_n) and δ , derive the unique stable cartel involving the largest firms. Record the pre-merger price.
5. Consider every possible two-firm merger and, for each of them, derive the new stable cartel and post-merger price. Record the change in price due to the merger as well the rank of the firms (in terms of capacity) involved in the merger.

This procedure is repeated 100,000 times and we compute the average price change that results from a merger for all two-firm mergers. With n firms, the number of possible mergers is equal to $1 + 2 + \dots + (n - 1)$. The simulations are performed for $n \in \{5, 6, 7, 8, 9, 10\}$. The results for $n = 5$ are reported in Table 3.1.

Table 3.1: Average price change due to a merger, $n = 5$
Capacity Rank of
Merger Partners

Firm A	Firm B	Average Price Change
4	5	0,0297
3	5	0,0185
1	5	0,0114
2	5	0,0114
3	4	0,0099
1	4	0,0047
2	4	0,0047
1	2	0
1	3	0
2	3	0

With $n = 5$, there are ten possible mergers. In Table 3.1, the mergers are ordered in terms of the size of their average price effects. The highest average price change is equal to 0,0297 and obtained with a merger between the two smallest firms. A merger between the median firm (firm 3) and the smallest firm has the next biggest price effect. A merger between any of the three largest firms has no impact. Observe that the price effects tend to be larger the smaller are the firms involved in the merger. Yet, with only five firms, the number of moderate-sized firms is limited. We therefore performed the same exercise for $n = \{6, 7, 8, 9, 10\}$.

When $n > 5$, there are many types of merger and, as a result we obtain quite a few average price changes. To organize the data, we partition firms in three categories: large, medium, and small. When there are six or nine firms in the industry, there is a natural categorization; when $n = 6$ (9), the largest two (three) firms are labelled *large*, the firms with the two (three) smallest capacities are labelled *small*, and the remaining firms are labelled *medium*. When the number of firms is not divisible by three, we consider the three partitions that are closest to having $n/3$ in each category. For example, when $n = 7$, one partition has firms ranked 1st, 2nd, and 3rd (in terms of capacity) being large, those ranked 4th and 5th being medium, and those ranked 6th and 7th being small; a second partition has firms ranked 1st and 2nd being large, those ranked 3rd, 4th, and 5th being medium, and those ranked 6th and 7th being small; and a third partition has firms ranked 1st and 2nd being large, those ranked 3rd

and 4th being medium, and those ranked 5th, 6th, and 7th being small. The results are presented in Table 3.2 below.

Table 3.2: Average price change due to a merger, $n \in \{6, 7, 8, 9, 10\}$

Categorization of Firms*				Merger Type**					
n	Large (L)	Medium (M)	Small (S)	L/L	L/M	L/S	M/M	M/S	S/S
6	1,2	3,4	5,6	0	0,0036	0,0161	0,0153	0,0255	0,0240
7	1,2,3	4,5	6,7	0	0,0125	0,0139	0,0258	0,0202	0,0138
	1,2	3,4,5	6,7	0	0,0067	0,0131	0,0201	0,0186	0,0138
	1,2	3,4	5,6,7	0	0,0034	0,0132	0,0144	0,0193	0,0184
8	1,2,3	4,5,6	7,8	0	0,0110	0,0099	0,0212	0,0149	0,0087
	1,2,3	4,5	6,7,8	0	0,0096	0,0112	0,0200	0,0173	0,0125
	1,2	3,4,5	6,7,8	0	0,0054	0,0109	0,0151	0,0155	0,0125
9	1,2,3	4,5,6	7,8,9	0	0,0081	0,0090	0,0169	0,0136	0,0094
10	1,2,3,4	5,6,7	8,9,10	0,0011	0,0085	0,0081	0,0163	0,0118	0,0074
	1,2,3	4,5,6,7	8,9,10	0	0,0063	0,0077	0,0136	0,0111	0,0074
	1,2,3	4,5,6	7,8,9,10	0	0,0063	0,0083	0,0117	0,0120	0,0097

* "Categorization of Firms" allocates a firm to being large, medium or small based on its rank in terms of capacity.

** "Merger Type" refers to the size - large, medium or small - of the firms participating in the merger.

Table 3.2 reports the average price changes from various mergers. Observe that in all the cases considered the biggest impact comes from mergers involving either two medium sized firms or one medium and one small firm. Furthermore, a merger involving two large firms always has the lowest impact. Also, a merger between two small firms never yields the highest average price change. Contrary to previous research that assumed an all-inclusive cartel, we do not find that it is mergers between large firms and/or small firms that have the biggest impact. Intuitively, a merger involving a medium sized firm may lead to a significant increase of cartel output post-merger.

3.7 Concluding Remarks

This chapter provides a rationale for the existence of incomplete cartels. We show that the value of an incomplete cartel might exceed that of a full cartel if colluding is costly and the smallest firm is sufficiently small. Also, there is a positive correlation between firm size and the incentive to join a cartel. Moreover, we have established that the most profitable cartel comprises the largest firms in the industry and that firms have an incentive to form this cartel when its smallest member is sufficiently large. Finally, we have analyzed merger incentives and the potential coordinated effects that may result from a merger. We have shown that firms have an incentive to merge only when they are part of a cartel or become part of the cartel post-merger. Our analysis further suggests that the most severe coordinated effects might come from mergers involving firms of medium size.

Part of the theoretical predictions find considerable support in 'real-world' examples of incomplete cartels. In Chapter 2, some stylized facts were distilled from descriptive

cartel studies and antitrust cases. Many cartels were incomplete, but took a dominant position in the market. In the model developed in this chapter, it is shown that an incomplete cartel is viable only when it controls a sufficiently large part of industry capacity. Furthermore, incomplete cartels often consisted of the larger firms in the market. We have established theoretically that larger firms are indeed more inclined to join a cartel. Finally, incomplete cartels tend to lose market share over time. In the current model, fringe firms do have an incentive to increase production in response to an output reduction by cartel members. However, outsiders are bound by the available production capacity and as a result output expansion by fringe members is limited. A potentially interesting extension of the model would be to endogenize total industry capacity.

There exists various ways in which total industry productivity might change. Sellers investing in additional production capacity is all but one example.²⁶ Investments not only lead industry capacity to expand, but might also cause a change in the size distribution of firms. Note that, akin to the merger incentives discussed above, firms have no incentive to invest in absence of collusion. The reason is that, under the assumptions made, the competitive equilibrium is unaffected by a change in firm size. However, given that total cartel profits are allocated proportionally, cartel members might have an incentive to increase their production capacity. The reason is that it allows a participant to claim a bigger share of total cartel value, all else unchanged. Whether or not participants decide to invest in part depends on how this will affect the sustainability of the cartel. All members face an identical incentive compatibility constraint, which is strictly increasing in cartel investments. One can imagine that participants do not want to undermine the stability of the cartel. To that end, a cartel might attempt to control the investment plans of its members.²⁷ As to fringe members, investments lead to an increase of fringe profits as long as the optimal cartel price remains unchanged. However, additional fringe capacity directly reduces cartel demand and creates a downward pressure on the cartel price. In addition, an expansion of fringe capacity potentially undermines the stability of the cartel, because it leads to an increase of the critical discount factor. In sum, it is to be expected that both the cartel participants as well as the competitive fringe have an incentive to invest in additional production capacity, which potentially threatens the existence of the cartel arrangement. To make this claim, however, a formal analysis is warranted, which is left for future research.

Another potentially interesting extension is to model the actual cartel formation process. At this point, it is important to re-emphasize that the above analysis yields no (unique) prediction about what cartel will actually form. That is, we have focused exclusively on the incentive of firms to collude and did not model the cartel formation process. In the last decades, some progress has been made in the analysis of group formation with externalities.²⁸ Yet, the study of endogenous coalition

²⁶Market entry/exit is another prominent example.

²⁷Notice that with a different profit allocation rule investments in additional capacity do not threaten cartel stability as long as the members facing the highest critical discount factor abstain from investing.

²⁸For an overview of this strand of literature see Yi (1997).

formation with externalities is complex. In particular, the problem easily becomes analytically intractable if one attempts to introduce firm heterogeneity in coalition formation games.²⁹ Also, as discussed in Section 3.4, results tend to be highly sensitive to the precise structure of the game. Part of this problem can be overcome by assuming identical firms, but as we have seen, this assumption makes it difficult to give results a practical interpretation.

Firm heterogeneity is an important issue also because it is commonly believed to hamper collusion. Indeed, diversity among sellers complicates both the formation and management of a cartel. However, this should not lead one to conclude that no collusion will occur in industries in which firms differ substantially. Although, a cartel that controls the entire industry is unlikely to emerge, a subgroup of sufficiently identical sellers can potentially form a cartel that is effective. One of the peculiar observations in our model is that symmetry does not foster collusion. In fact, the asymmetry of firms has no effect at all on whether or not the cartel is sustainable. Clearly, this is due to the restrictions that we imposed on the absolute size of the largest firms. As the study of Compte *et al.* (2002) reveals, relaxing these assumptions yields a setting in which the size distribution of cartel participants affects the sustainability of a cartel.

Assuming an upper bound on firm size is convenient in that it greatly simplifies the analysis. It might be that allowing for firms that are very large, in a sense that their capacity stock is sufficient to serve the entire market, does not affect the main results of this study. When colluding is costly and the smallest firms are sufficiently small, then the optimal cartel size will still be less than all-inclusive. Clearly, any cartel would be viable only when very large firms participate. This is in line with the prediction that sufficiently large firms have an incentive to take part in any cartel. However, whether or not the analysis yields similar results without Assumption 2 and Assumption 3 cannot be determined without further inquiry. Furthermore, it may be interesting to conduct a similar analysis in a Cournot setting. It must be noted, however, that one is likely to encounter several problems. For example, optimal punishment schemes are typically more involved compared to the current model. A particular problem occurs if one wants to assess the coordinated effects of merger in a standard Cournot framework. The reason is that a two-firm merger is unprofitable, because their combined market share is typically less than 80%.

Finally, this chapter is a modest attempt to delineate tacit collusion and overt collusion. Traditionally, theoretical work on cartels focuses on conditions under which collusion can be sustained without communication between undertakings. Our analysis suggests that communication can be a powerful ‘tool’ in dealing with the incentive of firms to cheat on a cartel, either *ex ante* or *ex post*. A better understanding of what distinguishes tacit from explicit collusion is particularly important in view of antitrust enforcement, because only the latter is considered illegal. One can imagine that certain market structures are more conducive to overt collusion as opposed to tacit collusion. As such, a more narrow focus on explicit cartel arrangements in future theoretical research may prove to be particularly valuable in the detection of cartels.

²⁹Belleflamme (2000) is one of the few attempts to study group formation with externalities and firm heterogeneity.

