Incomplete cartels and antitrust policy: incidence and detection
Bos, A.M.

Citation for published version (APA):
Bos, A. M. (2009). Incomplete cartels and antitrust policy: incidence and detection Amsterdam: Tinbergen Institute

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: http://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
Appendix A

BaseLocator®

We have operationalized the cartel screen that has been developed in Chapter 5 in a simple topographic detection routine in Delphi®.¹ Here we illustrate the logic of the algorithmic steps taken in the software. Below we give the main program with the key subroutines TracingBP, which traces the base using function SumOfSquares, and EstimateLStatistic, which calculates the value of LoC.²

A.1 Steps to Trace the Base

Input is a structured data file (in Notepad) that has a column of individual mill locations, and a column of individual customer project locations with the volume of trade and the transaction price per project. Base tracing consists of three main steps: data sorting, base tracing, and calculation of the LoC-measure.

In the first step, the data are sorted. All transaction data \((P_i, q_i)\) are grouped by base group, using the information on project site locations. Those combinations of project locations that are aligned are disregarded as not independent—but this is a rare occasion. All base groups with less independent observations than the number of unknowns, which is four in the system developed in the text, are ignored—this small sample problem would normally not need to appear. What remains is \(N\) sets of independent observations, \(N \leq J\).

¹Excellent programming assistance has been provided by Eelko Ubels.
²The other subroutines called in the program are less insightful and lengthy. They are available upon request from the author.
In the second step, each constructed set of observations \( l = 1, \ldots, N \) is used to recover \( \tilde{c}, \tilde{F} \) and base location used for the base group considered, \( (\tilde{a}_l, \tilde{b}_l) \). For this, the specification of \( T(q_i, d_{li}) \) is crucial. The software would in principle allow for a variety of specifications of \( T(q_i, d_{li}) \)—for the user to choose from, or for the program to find the best fit amongst. In the present version, transportation costs are linear in distance and volume, and including a fixed component, as in equation (5.15) in the text.

In a bounded area—that we determine as the size of the convex hull area of customer projects locations, extended with twice that size in all directions—the program steps-in a grid for the specification of \( (\tilde{c}, \tilde{F}, (\tilde{a}_l, \tilde{b}_l)) \) that returns the lowest \( S \) value. To be computationally efficient, we first use the information of the partial first-order conditions to problem (5.19):

\[
\frac{\partial S}{\partial \tilde{c}} = -2 \sum_{i=1}^{I_G} q_i \left( P_i - \tilde{c} q_i - \tilde{F} - \sqrt{(a_i - a_{x_i})^2 + (b_i - b_{x_i})^2} \right) = 0, \quad \text{and} \\
\frac{\partial S}{\partial \tilde{F}} = -2 \sum_{i=1}^{I_G} \left( P_i - \tilde{c} q_i - \tilde{F} - \sqrt{(a_i - a_{x_i})^2 + (b_i - b_{x_i})^2} \right) = 0,
\]

to obtain

\[
\tilde{c} = \frac{\sum_{i=1}^{I_G} q_i \left( P_i - \tilde{F} - d_{li} \right)}{\sum_{i=1}^{I_G} q_i^2}, \quad \text{and} \\
\tilde{F} = \frac{\sum_{i=1}^{I_G} \left( P_i - \tilde{c} q_i - d_{li} \right)}{I_G},
\]

where \( d_{li} = \sqrt{(a_i - a_{x_i})^2 + (b_i - b_{x_i})^2} \).

Using the averages \( \bar{P} = \frac{\sum_{i=1}^{I_G} P_i}{I_G}, \quad \bar{q} = \frac{\sum_{i=1}^{I_G} q_i}{I_G}, \quad \text{and} \quad \bar{d}_l = \frac{\sum_{i=1}^{I_G} d_{li}}{I_G} \), some manipulation yields

\[
\tilde{c} = \frac{\frac{1}{I_G} \sum_{i=1}^{I_G} q_i \left( P_i - d_i \right) - \bar{q} (\bar{P} - \bar{d}_l)}{\frac{1}{I_G} \sum_{i=1}^{I_G} q_i^2 - \bar{q}^2}, \quad \text{and} \\
\tilde{F} = \bar{P} - \left( \frac{\frac{1}{I_G} \sum_{i=1}^{I_G} q_i \left( P_i - d_i \right) - \bar{q} (\bar{P} - \bar{d}_l)}{\frac{1}{I_G} \sum_{i=1}^{I_G} q_i^2 - \bar{q}^2} \right) \bar{q} - \bar{d}_l.
\]

Plugging these expressions for \( \tilde{c} \) and \( \tilde{F} \) into the criterion function (5.19), we obtain

\[
S = \sum_{k=1}^{I_G} \left( P_k - \bar{P} + \bar{d}_l - d_{lk} + \bar{q} - q_k \left( \frac{\frac{1}{I_G} \sum_{i=1}^{I_G} q_i \left( P_i - d_i \right) - \bar{q} (\bar{P} - \bar{d}_l)}{\frac{1}{I_G} \sum_{i=1}^{I_G} q_i^2 - \bar{q}^2} \right) \right)^2,
\]
for which we are to find the value(s) for \((\hat{a}, \hat{b})\) that return(s) the lowest \(S\)-value.

In the search area, the value of \(S\) is determined for each combination of \((a, b)\). The program stores the \(S\)-value and overwrites it when further grid-point yields a lower value. This is our candidate basing point.

In the third step, the base locations found are translated into the \(LoC\)-measure. The program determines the convex hull of firm locations and its surface. It determines \(\lambda\) on the basis of the theoretical competitive mean base point and variance, using project and mill locations only. Since the data are sorted per base group, in competition each mill would be found only once. The theoretical competitive mean base point therefore is the unweighted mean of the mill locations. The program subsequently calculates the mean recovered base location and the ‘distance spread-circle’ around it. This returns the surface \(S^C\). The intersection of these two areas gives the \(LoC\)-measure, a number between zero and one.

As output, the program returns the name of data set used, the location \(l^*\), which is referred to as the center of the convex hull for reference, the value of \(\lambda\), the sample mean base, the sample mean variance, the parameters of the bid structure estimated (normalized on \(t_a = 1\)), and the value of the \(LoC\)-measure. High values of \(LoC\) are indicative of collusion, in particular when supported by a small sample variance.

A.2 Kernel of the Software

```plaintext
function SumOfSquares(const p,q:Vector; const x:Matrix; const ag,bg:integer):Vector;

{uses criterion function to calculate sum of squares, marginal cost and fixed cost

begin
for given basepoint candidate
var s_i,s_j,s_jt,s_jn,pm,qm, dm:extended;
i,j,len:integer;
d,bp,s:Vector;

begin
len:=Length(p);
s_i:=0;

pm:=Mean(p);
qm:=Mean(q);
SetLength(d,len);
SetLength(bp,2);
bp[0]:=ag; bp[1]:=bg;
for i:=0 to len-1 do begin
h:=0;
for j:=0 to 1 do
h:=h+Sqr(bp[j]-x[i][j]);
d[i]:=Sqrt(h);
end;
dm:=Mean(d);
```
\begin{verbatim}
s_jt:=0; s_jn:=0;
for j:=0 to len-1 do begin
    s_jt:=s_jt+q[j]*((p[j]-d[j])*(pm-dm));
    s_jn:=s_jn+Sqr(q[j])-Sqr(qm);
end;

s_j:=s_jt/s_jn;
for i:=0 to len-1 do begin
    s_i:=s_i+Sqr(p[i]-pm+dm-d[i]+(qm-q[i])*s_j);
end;

SetLength(s,3);
s[0]:=s_i;
s[1]:=s_j;
s[2]:=pm-(s[1]*qm)-dm;
SumOfSquares:=s;
end; {SumOfSquares}

procedure TracingBP(const base:integer; const x:Matrix; const p,q:Vector; var BP:Matrix);
    const 10,j1,i1,j0:integer);
{determines location with lowest value of sum of squares}
var step,len_BP,Intm,a,b,a0,a1,b0,b1:integer;
sos:Vector;

begin {TracingBP}
step:=50;
len_BP:=Length(BP);
SetLength(BP,len_BP+1);
SetLength(BP[len_BP],4);
Intm:=Max(j1-j0,i1-i0);
a0:=i0-0*Intm;
a1:=i1-0*Intm;
b0:=j0-0*Intm;
b1:=j1-0*Intm;
sos:=SumOfSquares(p,q,x,a0,b0-1)[0];
c:=SumOfSquares(p,q,x,a0,b0-1)[1];
f0:=SumOfSquares(p,q,x,a0,b0-1)[2];
SetLength(sos,3);
for a:=a0 to a1 do begin
    if a mod step=0 then begin
        for b:=b0 to b1 do begin
            if b mod step=0 then begin
                sos:=SumOfSquares(p,q,x,a,b);
                if sos[0]<sum then begin
                    sum:=sos[0];
                    BP:=Matrix(a,b,Intm,Intm);
                    Intm:=Intm+1;
                end;
            end;
        end;
    end;
end;
end {TracingBP}
\end{verbatim}
BP[len_BP][0]:=a;
BP[len_BP][1]:=b;
c:=sos[1];
fc:=sos[2];
BP[len_BP][2]:=c;
BP[len_BP][3]:=fc;
end;
end;  {TracingBP}

procedure EstimateLstatistic(const h:Matrix; const sq,v:extended; const m:Vector;
out Ls:extended);
{determines overlap of circle in hull}
var i,j,i0,i1,j0,ot,tot:integer;
p:Vector;
begin
SetLength(p,2);
MinimalRectangle(h,i0,i1,j1,j0);
ot:=0;
tot:=0;
for i:=0 to 100 do begin
for j:=0 to 100 do begin
p[0]:=i0+i*(i1-i0)/100;
p[1]:=j0+j*(j1-j0)/100;
if PointInHull(h,p,sq) then begin
tot:=tot+1;
end;
if PointInCircle(m,p,r) then begin
ot:=ot+1;
end;
end;
end;
Ls:=1-ot/tot;
end;  {EstimateLstatistic}

begin  {main}
LoadFileName(s);
s_out:='LoCw1.txt';
AssignFile(f,s_out);
Rewrite(f);
SetLength(res,2);
for i:=0 to 1 do begin
SetLength(res[i],201);
end;
for j:=0 to 200 do begin //read 201 files with standard errors on price
for i:=0 to 1 do begin
SetLength(res[i][j],10);
end;
for k:=0 to 9 do begin //averageing over 10 files with same error
SetLength(arr_BP,0);
Writeln(k,Chr(9),j);
s1:=s+'-'+IntToStr(k)+'-'+IntToStr(j)+'-comp_n.txt';

LoadData(s1,BP,sd,int_c,int_F,int_I,int_J,arr_x,arr_y,arr_p,arr_q,all_x,all_p,all_q);

thMean:=MeanVector(arr_y);
ConvexHull(arr_y,arr_h,sqHull);
len_x:=Length(arr_x);
MinimalRectangle(all_x,west,east,north,south);
for i:=0 to len_x-1 do begin
if Length(arr_x[i])>3 then begin
TracingBP(i,arr_x[i],arr_p[i],arr_q[i],arr_BP,west,east,north,south);
end;
end;
end;
endSE:=SigBar(arr_y);
ConvexHull(arr_y,arr_h,sqHull);
lambda:=LambdRadius(thMean,arr_h,thSE);
r:=lambda*SigBar(arr_BP);
SetLength(mc,2);
mC:=MeanVector(arr_BP);
EstimateLstatistic(arr_h,sqHull,r,mC,LStat);
res[i][j][k]:=LStat;

SetLength(arr_BP,0);
s1:=s+'-'+IntToStr(k)+'-'+IntToStr(j)+'-col_n.txt';

LoadData(s1,BP,sd,int_c,int_F,int_I,int_J,arr_x,arr_y,arr_p,arr_q,all_x,all_p,all_q);

len_x:=Length(arr_x);
MinimalRectangle(all_x,west,east,north,south);
for i:=0 to len_x-1 do begin
if Length(arr_x[i])>3 then begin
TracingBP(i,arr_x[i],arr_p[i],arr_q[i],arr_BP,west,east,north,south);
end;
end;
r:=\lambda \text{SigmaBar}(arr\_BP);
SetLength(mC,2);
mC:=\text{MeanVector}(arr\_BP);
EstimateStatistic(arr\_h,sqHull,r,mC,LStat);
res[1][j][k]:=LStat;
end;\text{Writeln}(f,Mean(res[0][j]):4:2,Chr(9),Mean(res[1][j]):4:2,Chr(9),sd:4:0);

end;
\text{Writeln('the end');}

GetProfits(all\_x,all\_y,BP,\text{int}\_I,\text{int}\_J,all\_p,all\_q,\text{int}\_c,\text{int}\_F,pi\_col,pi\_comp);

for j:=0 to \text{int}\_J-1 do begin
\text{Writeln}(f,'firm ',j+1,Chr(9),pi\_comp[j]:4:2,Chr(9),Chr(9),pi\_col[j]:4:2);
end;
\text{CloseFile}(f);
\text{Readln;}
end. \{\text{main}\}