The sound of sediments: acoustic sensing in uncertain environments
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This chapter provides a brief introduction in inverse acoustic sensing. Just like basic acoustic sensing, inverse theory deals with a sound source, a propagation medium and receiving sensors. The scientific literature will be surveyed with regard to sound sources other than sonar transmissions, modeling of the shallow water sound propagation, sparse configurations of receiving sensors, and inversion schemes that are based on optimization with metaheuristics.

3.1 Introduction

Introductions to underwater acoustics are given in various handbooks. The first handbook to be mentioned is undoubtedly *Principles of underwater sound* by Urick [128]. The book is among the oldest of its kind (first edition 1967), with chapters that can be traced back to declassified U.S. Navy studies from World War II, of which each addresses a single sonar parameter. For many years, the book has been used to educate naval officers at the Royal Netherlands Naval College [68] and the work can still serve as a thorough introduction. Among the contemporary introductions are works of Lurton [82] and Medwin *et al.* [87]. These authors address a wide range of military and modern civilian applications (Lurton) and present-day scientific studies on bio-acoustics, ocean dynamics and the ocean bottom (Medwin *et al.*). With a minimum of technical jargon, Richardson *et al.* [110] concisely document underwater acoustic principles and effects of man-made noise on marine mammals. Other works focus more on fundamental mathematical and physical principles, such as Burdic [19], Kinsler [61], and Brekhovskikh [17]. A practical handbook for engineers is Urban [127], or with less stature that of Waite [131], with many practical figures and rules of thumb. And finally, various concise booklets have been compiled, such as a pocket handbook by Bradley [16] or the compendium of definitions by Ainslie [3].
3.1.1 Active and passive sonar

For military applications, basic acoustic sensing comes in two flavors: active and passive sonar.

Active sonar is based on the transmission of a signal that is meant to reflect from a target. The echo is to be discriminated from either ambient noise or from reverberant receptions, most often due to bottom reflections. Active sonar has most in common with radar. The principal difference is that radar is based on radio waves (that are quickly attenuated in water), while sonar depends on sound pressure waves. Nevertheless, the radar performance equations [114] and the active sonar equations [128] are essentially the same [79].

Passive sonar is a receiving-only system and concerns the analysis of sounds that are received from ships and submarines. Every platform has an acoustic signature, which is a composition of mainly machinery noise, propeller noise, and flow noise [111]. When the reception is strong enough, a sonar operator can separate signals from ambient noise aurally or by frequency analysis.

The performance of a sonar system can be expressed by two measures, these are the (maximum) detection range and the probability of detection. When the detection range is given, the sonar equations can be used to work out the probability of detection. And vice versa, when a minimum detection probability is specified, the equations are used to provide the maximum detection range. For a given frequency, the sonar equations contains the relevant knowledge of signal and noise. The passive sonar equation [128] specifies the performance of detection on shipping sounds, and can be formulated as

\[ SL - TL = NL - DI + DT \]  \hspace{1cm} (3.1)

where all parameters are expressed in decibels.

The left hand side of the sonar equation is the received signal strength. This part consists of the source level (SL) of the shipping sound, which is reduced by a transmission loss (TL). The source level of a point source in free space is defined as the sound pressure level measured at 1 m from the source [111]. Shipping noise is a combination of many sound sources, which are not radiating in free space but merely act as dipoles. For practical use, shipping noise can be treated in the far field as a point source with a complex radiating pattern. The transmission loss expresses the loss (in dB) during the propagation of the signal towards the receiver. A simple example to calculate TL in free space is with the mechanism of spherical spreading, in which case \[ TL = 20 \log_{10} R \] with \( R \) the range in m.

The right hand side of the equation is the effective noise level, and this part is determined by the received (ambient) noise level (NL), the directivity index (DI), and the detection threshold (DT). The ambient noise is due to factors such as wind, rain and distant shipping [129]. Ambient noise can either be measured at sea or
estimated from factors as wind speed and sea state, e.g. using the Wenz-Knudsen
curves [133, 63] that are printed in virtually any of the above mentioned acoustic
handbooks, and also in appendix A. Updates on Wenz and Knudsen are found
in an APL report from 1994 [8]. The directivity index is based on the directional
sensitivity of the receiving sensor, and this property effectively reduces the noise
level. A receiver array with a large aperture results in a large DI and much noise
suppression. The DT finally is a function of probability of detection and false
alarm, and expresses the signal to noise ratio, which is valid at the threshold of
the specified detection probabilities.

To determine the performance of a sonar system, the sonar parameters need to
be gathered from intelligence (SL), direct measurement (NL), and calculation (DI
and DT). In deep water the calculation of TL depends on spreading loss, attenua-
tion and refraction in accordance with Snell’s law. As such the only environmental
information that is needed is the sound speed profile in the water column. In shal-
low water more information is needed, as sound propagates in complex interaction
with the seabottom. Therefore, to determine the TL in shallow waters, this thesis
aims for means that complete the environmental model with acoustic bottom prop-
erties. Such an environmental model is input to a propagation code, which in turn
calculates the transmission loss and completes the prediction of sonar performance
using the sonar equations.

3.1.2 The environmental model

For both active and passive sonar, acoustic sensing is a battle of the decibel.
Detection by sonar depends on the balance between received signals strength and
the effective levels of noise or reverberation. The sonar equations express this
balance in terms of the precise and individual contribution of the sensor system,
the medium and the target. Figure 3.1 illustrates these components for a passive
sonar scenario. In order to listen to underwater sound from HNLMS Snellius, a
receiver array is deployed from a small boat. Signals of various acoustic frequencies
propagate through the water column and the sea bottom. In the water column
the propagation is determined by the sound speed profile and Snell’s law, as will
be explained in section 3.5. Sound speed in water can be derived from water
temperature, salinity and water pressure at the given water depth, some empirical
equations are discussed in appendix B. In this thesis we focus on propagation
loss due to the properties of the sea bottom. The propagation loss in Figure 3.1
displays a complex pattern, which is due to mechanisms such as spreading loss,
absorption and reflection on the layer of sediment and the subbottom.

Various properties of the underwater environment have an influence on the
propagation of sound [128]. An obvious parameter is the speed of sound \( c \) (in
m/s), which is a property of the medium. The sound speed defines the relation
Chapter 3. Acoustic inversion

Figure 3.1: Scenario of passive acoustic sensing with a sparse vertical receiver array in a shallow water environment. The listed environmental parameters are an example for a layered seabottom. The Transmission Loss is plotted as a function of depth and range.

between the acoustic frequency $f$ (in Hz) and the acoustic wavelength $\lambda$ (in m) by $c = f\lambda$. Another important acoustic property of marine sediments is density $\rho$ (in g/cm$^3$), that determines the acoustic impedance $\rho c$ and reflective properties of marine sediments (the physical principles of reflection and impedance according to Rayleigh will be discussed in section 3.5 using equations 3.3 and 3.4). Attenuation $\alpha$ (in dB/λ) is the damping of amplitude due to mechanisms such as absorption. Sound propagation in water concerns compressional waves. For solid media, sound also propagates in shear waves. In this work sound speed $c$ and attenuation $\alpha$ relate to compressional waves; for shear waves the sound speed and attenuation are noted as $c_s$ and $\alpha_s$. As an example, some realistic values for these parameters are given in Figure 3.1. The shown environmental model describes a shallow water environment (30 m depth) with a sediment layer of 20 m that covers a subbottom.

An environmental model that characterizes the local medium is highly beneficial to optimize a sonar system for operation in a given oceanic situation [31]. When the environmental model is used to calculate propagation losses, the sonar equations provide tactical information (such as probabilities of detection and counter detection) to plan strength and frequency of sonar signals and optimal sensor positioning.

The environment can also be exploited in the design of advanced sonar con-
cepts. Active and passive sonar are direct methods that rely on direct observation of underwater sound. An advanced concept like inverse acoustic sensing is different in being an indirect method that aims to find the plausible physical conditions, like a source position, that explain a certain observation of underwater sound.

### 3.2 Inverse acoustic sensing

A brief introduction to inverse acoustic sensing will be given next. Principal reference books and tutorial texts are written by Tolstoy [126], Munk et al.[94], and Sen and Stoffa [112].

The problem of inverse acoustic sensing is to infer from precise measurement of a received underwater sound, the state of the source or the acoustic medium. Etter [31] subdivides inverse techniques that concern propagation into three categories. *Matched field processing* [126] is a technique that aims to find a source position or to characterize the marine environment. For *ocean acoustic tomography* [94] the intention is to describe the state of the ocean with a focus on changes in density fields (eddies, currents) or temperature (as for climate monitoring). In case of *geoacoustic inversion* [112, 37, 48] the result is a geoacoustic model [44] of sediment parameters and sea-floor scattering characteristics.

The starting point of model-based inversion is that the propagation of underwater sound can be modeled with a forward propagation model. In this way, assumptions about the source position and properties of the environment can be evaluated by correlating modeled propagation loss with an actual observation of underwater sound. Inversion is then a search process for those acoustic parameters that bring about the best correlation. A schematic overview of the inverse process is given in Fig. 3.2.

Inversion begins with observations at sea where underwater sound is recorded, e.g., like in Figure 3.1. Experiments involve planning and extensive logging of the experimental geometry, like GPS positions and transmitting schedules, and the environmental conditions, like sound speed profiles in the water column and the bathymetry of the area. The experiment is followed by a quick-look analysis. Experiments with high potential are then selected for further analysis. Various selection criteria exist, such as a good signal to noise ratio, the quality of the logging, or operational needs to assess a certain area. During the processing phase, data logged such as sonar frequencies, sound speed profiles, array configurations, and – most important – the candidate parameters that describe the source position or the environment, are input to a replica model of the experiment. The virtual experiment calculates the propagation loss by means of a forward propagation code. The resulting replica data are correlated to the observed data by an objective function. With iterative process, an inversion scheme guides the search
for candidate solutions that are subject to evaluation. After a number of iterations a solution emerges that describes the source position (matched field processing) or the environment (geoacoustic inversion).

3.3 Sound sources of opportunity

Systems that acoustically assess the seabed depend on transmissions of continuous wave (CW) or frequency modulated (FM) signals of various frequencies [35]. To make sure that signals penetrate into the sea bottom, experiments with geoacoustic inversion mostly rely on transmissions of low frequency.

An known issue with underwater sound, and one that has drawn much interest from the media, concerns underwater noise pollution and possible dangers for human divers [4] and marine mammals. A broad introduction on the topic of marine mammals and underwater sound is given by Richardson [110]. A selection from the many studies and reports are reports by the National Research Council [99] and [95], which functions under the U. S. National Academy of Sciences (NAS), and a report by the National Resources Defense Council [57], an international environmental advocacy group. But even if sea life has been taken into consideration, e.g., with appropriate mitigation measures [11], the use of sonar transmissions is
not always an appropriate option. High power transmissions could give away the position or intentions of military units, or the required energy or equipment may simply not be available. These limitations are a motivation to study inversion with sound sources of opportunity.

In an attempt to exploit sound sources of uncontrolled nature, a variety of low and mid-frequency (0.1 kHz – 6 kHz) sources of opportunity has been investigated for geoacoustic inversion. Examples of opportunity sources are ambient noise [46], different kinds of sea life [28, 124], aircraft propellers [18], and land vehicles [29]. All these sources have successfully been exploited in experiments of environmental assessment, but with these sources it is hard to plan a survey campaign. Finding the positions of sea life, aircraft and land vehicles potentially pose another problem for practical application.

An alternative line of research focuses on the analysis of bottom reflected shipping sound. In the absence of sonar transmitters the self noise of the tow-ship can be used, albeit a source of uncontrolled nature. An example of this through-the-sensor approach is an experiment in the Mediterranean Sea in 2000, when the Alliance towed a horizontal array, while the self noise of this NATO research vessel was used for geoacoustic inversion [10]. In this case the Alliance was a cooperative source in sailing a requested course and provision of GPS logging. The concept is a variation on the through-the-sensor approach as studied in the RUMBLE project [113], where sonar transmission with a low frequency active sonar (such as LFAS) are the source.

When arbitrary ships are recorded, the shipping noise is a true opportunity source. A typical signature of a ship includes numerous tones with frequencies between 20 Hz and 2 kHz, which ensures good penetration of sound into the sea bottom. Geoacoustic inversion with opportunity ships has been reported [104, 64, 21] for long range receptions, of more than 10 km away, and the use of dense horizontal receiver arrays.

For many opportunity sources the position is not known beforehand. When this is the case, the range and depth of the source are often added to the search space. For short-range receptions of shipping sound, it is possible to estimate the position from Doppler analysis. Particularly when a vessel passes the receiver, the geometry can be worked out. In some cases the Doppler-solution is accurate enough for inversion, and otherwise the inversion can include a search interval for a modest correction to the Doppler range.

### 3.4 Receiving sensors

Conventional geoacoustic inversion involves the coherent reception of underwater sound on many hydrophones, typically 32 or more. For inversion with transmitted
sonar signals, the receiving sensors are often placed in a vertical line array [125, 47]. Opportunity sources are more often recorded with horizontal arrays, that count as many as eighteen [10] or more [104] hydrophones. A large number of hydrophones reduces the influence of noise, but the disadvantage is in the large volume of acoustic data that need to be recorded, handled and processed in what ultimately needs to be a real time application.

A class of flexible deployable receiving systems are drifters or a field of acoustic-oceanographic buoys (AOBs) [83] that transfer the acoustic data by a wireless communication channel. To limit the bandwidth, it makes sense to limit the number of phones in an array. The physical design and further development of AOBs has been studied in a Joint Research Project (JRP) of the NATO Undersea Research Centre (NURC), the University of Algarve (UAlg), the Netherlands Defence Academy (NLDA) and the Université Libre de Bruxelles (ULB). During the BP’07 sea trails, the University of Algarve participated with two AOBs. The buoys pictured in Figure 3.3 count 8 and 16 hydrophones.

![Figure 3.3: Acoustic-oceanographic buoys of the University of Algarve, deployed during BP’07. The buoys are equipped with sparse receiver arrays and means of wireless communication to transfer acoustic data.](image)

The next chapters examine geoacoustic inversion with data from sparse vertical arrays [47] that were deployed from a small boat. The arrays that were used count four or five phones, which makes the equipment highly portable. During the Saba bank experiments the equipment had been flown in with a commercial airliner. Sparse sampling of the water column also makes that the data require far less bandwidth than would have been the case with a dense arrays. When signals below 2.5 kHz are multiplexed together, just two radio channels suffice to transmit a continuous recording of underwater sound.

The downside of a sparse array is that such a configuration involves less noise cancellation than processing with data from a dense array. For a constant ex-
perimential geometry the receptions are coherent over receiver depth and time. Therefore the effect of ambient noise in the observations can be reduced by repeating the sonar transmissions many times. In case of inversion with close-range shipping sound, the source is moving and there is no longer a benefit of coherency over time.

In addition to the sparse arrays, the use of vector sensors [101] has been considered. These sensors measure the acoustic particle velocity [33]. Vector sensors can be individually beam formed in cardioid and non-linear hippoid shapes [117]. In theory the directivity index increases by 3 dB (cardioid) or more (hippioid), but in practice the processing gain strongly depends on the directivity of the ambient noise. With null-steering of the cardioid beam, loud interfering sources can effectively be removed from the received signal. In the absence of real acoustic recordings from vector sensors, this thesis does not further study acoustic particle velocity.

To summarize: at the cost of increased noise levels, this thesis studies the use of sparse vertical receiver arrays and uncontrolled shipping sound for geoacoustic inversion purposes.

### 3.5 The medium

This section briefly introduces some basic physics of underwater and sedimentary sound propagation. Apart from the sound wave speed, the most important mechanisms in shallow water are those of refraction (Snell) and reflection (Rayleigh). Even though the environment is not controlled by man, favorable conditions occur that can be exploited for geoacoustic inversion.

#### 3.5.1 Water column

The direction in which a wave front propagates can be visualized as a ray. Such a ray can be traced with Snell’s law of refraction [128, 17]

\[
\frac{\cos \theta_1}{c_1} = \frac{\cos \theta_2}{c_2},
\]

(3.2)

where $\theta_1$ and $\theta_2$ are grazing angles and $c_1$ and $c_2$ are the sound speeds (in m/s) for medium 1 and 2 (e.g. two layers in the water column).

For a constant sound speed, Snell’s law points out that the ray is a straight line. When the lower layer has a smaller sound speed ($c_2 < c_1$), it follows that $\theta_2 > \theta_1$ and the ray is refracted downwards.

A well-known phenomenon in underwater acoustics is the afternoon effect [128]. At day time, the top layer of the water column is warmed by the sun. As a
result, the sound speed of the upper water column increases and in the afternoon sound rays tend to refract downwards, which results in shorter detection ranges for receivers at the water surface. (The effect of weaker or no detection in the afternoon was discovered after World War I. Some time before World War II it was found that the sonar operators were not to blame, and that the receptions were actually weaker in the afternoon.)

For geoacoustic inversion purposes, the afternoon effect can be exploited. When the receiver array is positioned beyond reach of the direct propagated path, only bottom reflected signals are received. The omission of the strong direct path receptions means that more bottom information can be extracted with inverse techniques.

### 3.5.2 Sea bottom

When underwater sound encounters the sea bottom, a part is reflected back into the water column and a part is refracted upon entry of the sea bottom. The reflected signal has a grazing angle identical to the angle of the incident ray $\theta_1$. The refracted signal follows Snell’s law as expressed in equation 3.2.

The amplitudes of reflected and refracted waves depend on the acoustic impedance of the media. In the case of a plane wave, the acoustic impedance equals to the product of $\rho$ and $c$. According to Rayleigh [17, 61], the complex pressure amplitude of a reflected wave $P_{\text{reflected}}$ is related to that of the incident wave $P_{\text{incident}}$, by

$$\frac{P_{\text{reflected}}}{P_{\text{incident}}} = \frac{\rho_2 c_2 \sin \theta_1 - \rho_1 c_1 \sin \theta_2}{\rho_2 c_2 \sin \theta_1 + \rho_1 c_1 \sin \theta_2}.$$  \hspace{1cm} (3.3)

For the refracted wave that enters the bottom, the complex pressure amplitude $P_{\text{refracted}}$ is related to that of the incident wave $P_{\text{incident}}$, by

$$\frac{P_{\text{refracted}}}{P_{\text{incident}}} = \frac{2 \rho_2 c_2 \sin \theta_1}{\rho_2 c_2 \sin \theta_1 + \rho_1 c_1 \sin \theta_2}.$$  \hspace{1cm} (3.4)

Just as in water, the propagation of sound in sediments is further subject to mechanisms of spreading loss and attenuation.

Typical sound speeds, densities and attenuation values for marine sediments [58] are listed in Table 3.1. A first indication of the sediment type can be based on historical data or on (recent) side scan sonar imagery. For geoacoustic inversion, tables like 3.1, e.g. those compiled by Hamilton [44, 45] or by the U. S. Applied Physics Laboratory [8], can be used to set up initial search intervals for parameters that are to be inverted. As a closer inspection of listed bottom densities $\rho_b$ and sound speeds $c_s$ may suggest, there exists a correlation between $\rho_b$ and $c_s$ [113]. This feature can be used to further limit the search space in a geoacoustic inversion process.
3.5. The medium

<table>
<thead>
<tr>
<th>Bottom type</th>
<th>$\rho_b/\rho_w$</th>
<th>$c_p$ (m/s)</th>
<th>$c_s$ (m/s)</th>
<th>$\alpha_p$ (dB/(\lambda_p))</th>
<th>$\alpha_s$ (dB/(\lambda_s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>1.5</td>
<td>1500</td>
<td>&lt;100</td>
<td>0.2</td>
<td>1.0</td>
</tr>
<tr>
<td>Silt</td>
<td>1.7</td>
<td>1575</td>
<td>1.0</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td>1.9</td>
<td>1650</td>
<td>0.8</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Gravel</td>
<td>2.0</td>
<td>1800</td>
<td>0.6</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Moraine</td>
<td>2.1</td>
<td>1950</td>
<td>600</td>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td>Chalk</td>
<td>2.2</td>
<td>2400</td>
<td>1000</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>Limestone</td>
<td>2.4</td>
<td>3000</td>
<td>1500</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Basalt</td>
<td>2.7</td>
<td>5250</td>
<td>2500</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

$\rho_w = 1000 \text{ kg/m}^3$

The experiments that are documented in this thesis took place in areas where the bottom could be modeled as a stratified medium. In case of the Saba bank, the nautical charts indicated a flat bottom. For the BP’07 experiments in the Mediterranean Sea, the local bathymetry has been obtained with multibeam echo sounders (MBES) from HNLMS Snellius and with the single beam echo sounder (SBES) of the REMUS autonomous underwater vehicle. The experiments were then set up with the ship and receiving array on a contour line of constant depth.

3.5.3 Ambient noise

The performance of acoustic sensing depends on the ratio between signal and noise, as expressed by equation 3.1. Therefore less ambient noise means better reception of bottom reflected signals, and as such an increased sensitivity of the inversion process with regard to acoustic sea bottom parameters. Favorable ambient noise conditions occur during night time, but this is of limited or no use at all, due to limited visibility during a measurement campaign, and the absence of opportunity shipping and afternoon effects. But ambient noise has other properties that can be exploited.

Ambient noise is highly directional, both in elevation and bearing. This property can be exploited with beamforming, e.g., with vector sensors as described in the previous section about receiving sensors.

Ambient noise also strongly depends on frequency. This property is exploited in chapters 4 and 5, where the narrow banded signals to be inverted are selected at frequency bands of less ambient noise.
3.6 Forward propagation modeling

The general physics of underwater sound propagation are well understood [33], and there are various theoretical approaches to propagation modeling. At the basis is the evaluation of the wave equation

\[ \nabla^2 \Phi = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} \quad (3.5) \]

where \( \nabla^2 = (\partial^2 / \partial x^2) + (\partial^2 / \partial y^2) + (\partial^2 / \partial z^2) \) is the Laplacian operator, \( \Phi \) the potential function, \( c \) the speed of sound and \( t \) the time.

For harmonic solutions (single-frequency, continuous wave), this equation can be reduced to a simpler form known as the Helmholtz equation. Harmonic solutions can be written as

\[ \Phi = \phi e^{-i\omega t} \quad (3.6) \]

with \( \phi \) the time-independent potential function, \( \omega = 2\pi f \) the source frequency and \( f \) the acoustic frequency. The wave equation (3.5) then reduces to the Helmholtz equation:

\[ \nabla^2 \Phi + k^2 \Phi = 0 \quad (3.7) \]

where \( k = \omega / c = 2\pi / \lambda \) is the wave number and \( \lambda \) the wavelength.

The potential function \( \phi \) is of particular interest as the transmission loss (TL) follows from

\[ \text{TL} = 10 \log_{10}[\phi^2]^{-1} = -20 \log_{10} |\phi| . \quad (3.8) \]

In order to model underwater sound propagation, and take into account the geoacoustic properties that this thesis concerns, various theoretical approaches can be distinguished for solving equation 3.7. The most common propagation models are based on ray theory, normal mode theory or the parabolic equation [33, 31]. The differences between these techniques are found in the geometrical assumptions made for the environment, and the type of solution chosen for \( \phi \).

3.6.1 Ray theory

Models that are based on ray theory [100, 31] calculate the propagation loss by evaluation of all the occurring rays that connect a source with a receiver. Solutions are written as

\[ \phi = A(x, y, z) e^{-iP(x,y,z)} \quad (3.9) \]

where \( A \) is a pressure amplitude function, and \( P \) a phase function that is common referred to as the eikonal, a Greek word for “image”. By tracing each ray through the different media, \( A \) can be calculated by the application of mechanisms for spreading loss, attenuation, reflection and refraction. In addition, \( P \) is found from the length over which the ray was traced and the acoustic wave length.
3.6.2 Normal mode theory

Normal mode theory assumes a stratified medium (an environment that changes as a function of depth only). In this case the solution of the potential function $\phi$ can be written in cylindrical coordinates as

$$
\phi = F(z)S(r).
$$

(3.10)

When the Helmholtz equation is written in cylindrical coordinates, a separation of variables can be performed, using $\xi^2$ as separation constant. The resulting equations are

$$
\frac{d^2 F}{dz^2} + (k^2 - \xi^2)F = 0
$$

(3.11)

and

$$
\frac{d^2 S}{dr^2} + \frac{1}{r} \frac{dS}{dr} + \xi^2 S = 0.
$$

(3.12)

Equation 3.11 is the depth equation and describes the standing wave portion of the solution. This part is known as the normal-mode equation. The range equation 3.12 describes the traveling wave portion of the solution. As such, each normal mode can be described as a traveling wave in the $r$ direction and a standing wave in the $z$ direction. (The full solution for $\phi$ can be obtained using numerical mathematics [33, 31], and involves the Green’s function as a solution for equation 3.11 and a zero-order Hankel function of the first kind for equation 3.12.)

3.6.3 Parabolic equation models

The parabolic equation (or parabolic approximation) approach assumes solutions of the form

$$
\phi = \Psi(r, \vartheta, z)S(r),
$$

(3.13)

When compared with normal mode models, PE models are capable of variations in range, bearing ($\vartheta$) and depth. This thesis does not expand on inversion with PE models. Therefore the background of this model type is not further discussed here, details can be found in [33] and [31].

3.6.4 Criteria for model selection

To select a model type for inversion, two essential criteria are the applicable frequency range and whether or not the environmental model is range dependent.
Applicable frequency range

The acoustic frequency determines the physical applicability of a particular model type, and also whether or not it is computationally practical to do so.

- Models based on ray theory become less reliable at lower frequencies, due to the assumptions made about the geometry. A rule of thumb [31] that identifies a safe frequency range for ray based models is
  \[ f > 10 \frac{c}{H} \]
  where \( f \) is the applicable frequency, \( c \) is the speed of sound and \( H \) is the depth of the acoustic ducting layer. According to literature, the applicability of the method might require attention for frequencies below 150 Hz (Urick [128]) or 500 Hz (Etter [31]). Another limiting factor is the occurrence of sudden changes of sound speed within the medium.

- For models based on the parabolic equation the frequency also corresponds to calculation time. For practical use these models should be used only with frequencies below 500 Hz (Etter [31]).

- Normal mode models are applicable to a broad frequency range. Shipping sounds that are of interest to this thesis are found between 20 Hz and 2 kHz, and therefore this dissertation mainly addresses inversion with a normal mode model.

Range dependency

An environmental model is called range-dependent (or 3-dimensional) when environmental properties can vary over range. A horizontally stratified medium is called range-independent (or 2-dimensional). This dissertation is limited to range-independent modeling, of which all models are capable. In case of a large and range-dependent environment, such an area can be segmented into range-independent patches [103].

3.6.5 Available implementations

Etter [30, 31] has compiled an impressive list of over a hundred propagation models. Some models are freely available, like from the Ocean Acoustics Library\(^1\) that is supported by the U.S. Office of Naval Research. Other models are commercial products that are part of a tactical decision aid for sonar performance. The following three models have a connection with this thesis:

\(^1\)http://oalib.hlsresearch.com/, last visited June 19th 2009
3.7. Objective functions

- KRAKEN is normal mode program by M. B. Porter [105] and was developed at the (U. S.) Naval Ocean Systems Center and the (U. S.) Naval Research Laboratory. In time, the model has been improved to support research in matched field processing. Unlike many other normal mode models, KRAKEN handles multilayered environments and also deals with elastic media so that shear sound speed is included in the calculations. The model is well documented and has a large scientific user community, e.g., the NATO Undersea Research Centre has incorporated KRAKEN into its SNAP model. Different from standard normal mode models, KRAKEN is also capable of range-dependent geometries.

This thesis involves model based acoustic inversion with many evaluations of a propagation model. KRAKEN is regarded to be an accurate, stable and efficient code for the range-independent geometries and frequency ranges that are of interest. For these reasons, the model has been used for most inversions that are described in this dissertation.

- MMPE [115] is a parabolic equation code, that is unique in also being able to calculate acoustic particle velocity.

- The Royal Netherlands Navy uses the TNO model Almost. This ray based model is capable of range dependant environments [106] and also provides evaluations of the sonar equations.

3.7 Objective functions

When a propagation code is used to calculate replica transmission losses, the inversion process requires an objective function to correlate the results with observations of underwater sound.

3.7.1 The Bartlett processor

When a single frequency \( f \) needs to be observed, continuous wave signals can be analyzed in the frequency domain by means of Fourier transformation. A time sequence \( d_n(t_k) \) of sampled data from the \( n^{th} \) phone at time \( t_k \) \((0 \leq k \leq K)\) can be converted to the frequency domain using a discrete Fourier transformation

\[
D_{n,f} = \sum_{k=0}^{K} d_n(t_k)w(t_k)e^{-i2\pi ft_k}
\]

where \( t_k = k\Delta t, \Delta t = \tau/K, \tau \) the integration time, and \( w(t) \) the weighting or window function, e.g. Hamming or rectangular.
The result is a vector $\mathbf{D}_f$, that has complex valued entries for observations from $N_S$ separate hydrophones for the various depths. The cross-spectral data matrix is constructed as

$$
\hat{\mathbf{R}}_f = \mathbf{D}_f \mathbf{D}_f^* \quad (3.16)
$$

where $\mathbf{D}^*$ denotes the conjugate transpose operator. For fixed geometries and prolonged sonar transmissions, $\hat{\mathbf{R}}_f$ is based on the ensemble average of estimated cross-spectral matrices for various time intervals [38]. Taking the ensemble average reduces the influence of noise.

Replica data are constructed, with the propagation models discussed before, as a complex valued vector $\mathbf{w}_f$ with entries for the various hydrophones. The Bartlett processor [126, 38] then correlates replica data with the covariance matrix of the observations and is defined as

$$
B_f = \left[ \mathbf{w}_f^* \hat{\mathbf{R}}_f \mathbf{w}_f \right] \frac{\text{tr}[\hat{\mathbf{R}}_f]}{\|\mathbf{w}_f\|^2} . \quad (3.17)
$$

By definition $0 \leq B_f \leq 1$ and a perfect match between observed data and replica data is found when $B_f \equiv 1$.

**Normalization**

When a ship is used as an uncontrolled sound source, the magnitude and phase of radiated tones are usually not known. For this reason the vectors $\mathbf{D}_f$ and $\mathbf{w}_f$ are normalized to unity [85] (that is, all elements in a vector are divided by the largest occurring magnitude). The normalization makes that the Bartlett processor compares received signals by differences in phase and relative amplitude only.

**Moving source**

When an inversion experiment deals with a moving ship or sound source, the receptions are no longer coherent in time. Therefore the covariance matrix cannot be constructed as an ensemble average within a fixed geometry. In this case, noise in an objective function can be reduced by incoherent summation of Bartlett processors for various acoustic frequencies and source-receiver ranges. When $\mathbf{f}$ is a vector of $N_F$ frequencies (in Hz), a cost function to be minimized can be defined as

$$
E = \frac{1}{N_F} \sum_{i=1}^{N_F} \left[ 1 - B_{R(i)} \right] . \quad (3.18)
$$

And when $\mathbf{r}$ is a list of $N_R$ receiving ranges (in m), cost function 3.18 becomes

$$
E = \frac{1}{N_F N_R} \sum_{i=1}^{N_F} \sum_{j=1}^{N_R} \left[ 1 - B_{R(i), r(j)} \right] . \quad (3.19)
$$
Notice that in literature and following chapters, it is also custom to use symbol $\Phi$ to denote the objective function. This is in contrast with use in equations 3.5 and 3.7, where $\Phi$ is the symbol for the potential function.

### 3.7.2 Other processors

To study the potential of acoustic particle velocity for inversion [75], simulated data have been constructed with the MMPE propagation model [115] and objective functions were constructed that correlate particle velocity. But in the absence of experimental data from vector sensors, this technology is not further studied within this thesis. Another and alternative type of cost function that is not further studied here, is correlation in the time domain [104].

### 3.8 Optimization

When the objective function provides a measure for correlation between model parameters and observed underwater sound, the inversion scheme guides the search for ‘good’ model parameters.

For a simple search space, like with a few discrete values for a few parameters, all possible combinations can be evaluated with an exhaustive search. An example is matched field source localization with a vertical receiver array where the only unknowns are source range and depth. In this case an ambiguity surface can be calculated very fast, like with Fig. 2.2 in the previous chapter. For a single receiver depth, normal mode and PE models are very fast in the calculation of transmission losses over an range interval. But as a localization problem deals with a single source depth and many receiver depths, the trick is to use the acoustic reciprocity of source and receiver [58]. Transmission losses can then be obtained by modeling many sources and only one receiver depth.

Geoacoustic inverse problems typically deal with a far more complex search space, which is multi dimensional (e.g. 4 parameters or more) and encompasses many local optima. Therefore these problems are in need of proficient search strategies. The conventional metaheuristic optimization methods for inversion are Simulated Annealing [126, 62] and the Genetic Algorithm [37]. For Simulated Annealing the efficiency quickly decreases when the complexity of the search space increases. In these cases a Genetic Algorithm is much more capable, but this method needs tuning of its performance parameters, which is often done by trial and error. Other relatively new methods to guide the search are Differential Evolution [120] and Ant Colony Optimization [74, 73]. Discriptions of the four metaheuristics are given in chapter 7. Literature has shown that all these methods can provide solutions that go beyond local optimality. The tuning aspect of
these methods for inverse problems was found to be ill-documented.

Metaheuristics are a class of optimizers that are able to quickly identify solutions that are better than average solutions. But these methods are usually not able to quantify how close to optimality such a solution is [108]. For practical applications, the time to invert a solution is limited. As such, a strict time constraint makes that different runs of metaheuristic methods result in different solutions.

This thesis addresses the question how to select and configure a metaheuristic optimizer for the best performance of the inverse process. Conceptual comparisons of metaheuristics have been made before [108, 15]. But here the focus is on the performance for a specific application: acoustic inverse problems with a computational demanding objective function.

### 3.9 Inversion toolbox

The previous sections, and more particular figure 3.2, illustrate that geoacoustic inversion involves a range of models and algorithms. To carry out inverse processing, it makes sense to use a software toolbox. For some time now, there has been a free and open source\(^2\) inversion software package by Gerstoft [38] that goes under the name SAGA. One of the data sets from SAGA is used in chapter 7 to compare metaheuristic performance.

For the line of research documented in this thesis, it was decided to work on a toolbox that received the name LOBSTER [71] (the Low-frequency Observation Based Sonar Toolbox for Environmental Reconstruction). This object-oriented Matlab code interfaces with variants of the KRAKEN [105] and MMPE [115] (third party) propagation models and offers a number of objective functions. Like SAGA, the toolbox contains the conventional metaheuristics such as Simulated Annealing and the Genetic Algorithm, but LOBSTER also has implementations of Differential Evolution and Ant Colony Optimization.

### 3.10 Conclusions

This chapter gave a brief introduction in conventional matched field processing and geoacoustic inversion. Various general lines of research were identified that aim to bring these techniques to a higher technology readiness level. Among these are the use of sound sources of opportunity, reduction of data volumes by deployment of sparse receiver arrays, and inversion schemes that are based on optimization with metaheuristics.

\(^2\)http://www.mpl.ucsd.edu/people/gerstoft/saga/saga.html, last visited August 5th 2009