The sound of sediments: acoustic sensing in uncertain environments
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Chapter 6

Inversion and Uncertainty Analysis with Ant Colony Optimization

Abstract

Inverse problems in ocean acoustics are generally solved by means of matched field processing in combination with metaheuristic global search algorithms. Solutions that describe acoustical properties of the bed and subbottom in a shallow water environment are typically approximations that require uncertainty analysis. In order to compare various metaheuristics for geoacoustic inversion in a later chapter, this chapter describes the use of Ant Colony Optimization. It is demonstrated that a MAX-MIN Ant System can find good estimates and provide uncertainty analysis. In addition, the algorithm can easily be tuned, but proper tuning does not guarantee that every run will converge given a limited processing time. Another concern is that a single optimization run may find a solution while there is no clear indication on the accuracy. Both issues can be solved when probability distributions are based on parallel MAX − MIN Ant System runs.

6.1 Introduction

Coastal waters allow for a high concentration of human activities and an expanding exploration of the underwater environment takes place mostly by acoustic techniques. The propagation of sound through the ocean medium is well understood and since no other energy propagates as efficiently, sonar has been the most effective sensor for many years. In a shallow water environment [20] sound interacts

with the bottom in complex ways and propagation modeling requires a detailed knowledge of the bottom acoustic properties in addition to the sound speed structure in the water column and the sea surface scattering conditions.

Geoacoustic inversion is a technique that aims to describe a shallow water environment by acoustical parameters which in turn are related to other geophysical parameters. An introduction to geoacoustic inversion is given in Section 2. The concept of matched field processing is explained together with the optimization problem that is part of an inversion process. Section 3 describes a benchmark of geoacoustic inversion based on the Yellow Shark experiments in Mediterranean shallow waters. The section also provides a brief introduction to metaheuristics that, according to literature, have been applied to inversion before. Ant Colony Optimization (ACO) \cite{26} is demonstrated to be feasible as well, when the world of the ants is regarded as an analogy for the geoacoustic environment. A MAX-MIN Ant System implementation is tuned for the benchmark and it is shown how accurate results are found. Tuning is important but does not guarantee that every inversion will converge to the same solution. In addition there is no clear indication on the accuracy of solutions. Section 4 addresses these issues with uncertainty analysis.

\section{Introduction to Geoacoustic Inversion}

Geoacoustic inversion is principally based on matched field processing (MFP) described in this section. The focus is on a benchmark that comprises a representative problem of geoacoustic inversion, based on real data from the 1994 Yellow Shark experiments.

\subsection{Inversion based on Matched Field Processing}

\textit{Inversion} is a technique that estimates the parameters of a physical model from a measurable quantity like a sound pressure field. Sound can be measured for a particular frequency and at a certain distance from the sound source. A physical model of a medium can be used to make predictions of pressure fields at a number of positions and frequencies. These predictions are called forward calculations. When the predictions match the measured fields it is fair to say that the model is \textit{acoustic equivalent} with the real world. In other words: when sound propagates through a medium, the sound pressure field upon reception will be the same as it would have been when the sound had propagated through the physical model. The whole process of matching reception with prediction, in order to obtain a physical model, is called \textit{matched field inversion}. An objective function defines mismatch and needs to be minimized to find the best acoustic equivalent model.
6.2.2 Inversion for Bottom Geoacoustic Parameters

In a typical shallow water environment\(^2\), the medium consists of several layers. The top layer is the water column where most acoustic quantities can be measured or otherwise accurately be predicted. But high-frequency echo sounders used for bathymetric survey do not deeply penetrate into the sedimentary layers and do not reach the underlying hard rock basement. Low-frequency sound however does penetrate these layers and in many cases inversion helps to find acoustic characteristics like sound speed and absorption. A profound understanding of the medium enhances acoustic sensing capabilities and permits accurate predictions of sonar detection ranges. A fast inversion scheme that obtains reliable geoacoustic parameters is a prerequisite for Rapid Environmental Assessment (REA) for sonar.

For an arbitrary environment, geoacoustic inversion encompasses four steps [37]: discretization of the environment, efficient and accurate forward modeling, efficient optimization procedures and finally uncertainty analysis. The first step is commented in the next paragraph for the Yellow Shark experiments. The forward modeling of sound propagation was presented in chapter 3. Steps 3 and 4 are addressed with ACO in sections that follow.

6.2.3 The Yellow Shark Experiments

One of the interesting features of the Yellow Shark experiments is that cores of the bottom material have been taken and the analysis of these samples provide ground truth for the solutions of geoacoustic inversion. The transmitted narrow-band tones, with frequencies between 200 Hz and 800 Hz, were propagated over a range of 9 km with an average bottom depth of 112 m. Further details on the experiments can be found in Hermand and Gerstoft [48]. For our preliminary study we will use a benchmark with geometric and geoacoustic parameters and an objective function based on phase-coherent processing of pressure time series (wave-forms) received on a vertical array using a model-based matched filter (MBMF), details can be found in [47]. The benchmark is in the form of a precomputed objective function [49] based on measured sound speed profiles in the water column, averaged over range, and a geoacoustic model of eight parameters that describe a single sediment layer over a half-space sub-bottom.

The objective function \( f : S \rightarrow \mathbb{R}^+ \) needs to be minimized. In YS94, \( S \) is an 8-dimensional search space, where every real parameter \( x_i \) is subject to \( a_i \leq x_i \leq b_i \), with constants \( a_i \) and \( b_i \) based on general \textit{a priori} information. If each parameter is

\(^2\)Shallow waters are usually found at the continental shelf and bound by a depth-contour line of 200 m. In these waters, sound typically propagates with multiple interaction with the sea bottom and surface.
sampled by just 10 samples, there are already $10^8$ possible combinations. Considering that each forward call to the objective function depends on a non-linear propagation model that is computationally demanding, geoacoustic inversion clearly benefits from a metaheuristic approach.

6.3 Ant Colony Optimization for Inversion

Various metaheuristics have been applied to the optimization part of inversion. Ant Colony Optimization is unique in being a population based method with a short term memory. A $\text{MAX} - \text{MIN}$ Ant System has been applied on the Yellow Shark benchmark and a rule of thumb for the performance parameters has been derived. Results of a tuned run are shown to match with reference solutions for the benchmark.

6.3.1 ACO and Other Metaheuristics for Inversion

Early environmental inverse problems were solved with an exhaustive search on a limited search space [126]. The introduction of Simulated Annealing and the Genetic Algorithm (GA) [37], [39] made it possible to invert more parameters on a wider range of samples. Only recently, other methods as Tabu Search and Differential Evolution entered the field. A careful comparison between the various metaheuristics will follow in the next chapter.

Ant Colony Optimization has most in common with Genetic Algorithms. Both types are population based algorithms that search a discrete search space and that are capable of providing uncertainty analysis. The main difference between the methods are the mechanisms that handle and re-combine components of better candidate solutions (pheromones trails versus genetic operators). ACO is further different in having a form of memory (the pheromone trails), while GAs are without memory. When pheromones evaporate, identifiers of paths with above average quality are fading out. High rates of evaporation mean that only recent information can be retrieved, as is typical for a short term memory. Low evaporation rates allow recollection of much older information and correspond to a long term memory.

6.3.2 Application of $\text{MAX} - \text{MIN}$ Ant System

When ACO is applied to inversion, the world of the ants acts as an analogy for the geoacoustic environment. Acoustic parameters that describe a sea bottom or water column are objects that block the path between the nest and a food source. Paths that bridge such objects are the sampled values each parameter may take, as illustrated in Fig. 6.1. An ant that leaves the nest, and explores the
6.3. Ant Colony Optimization for Inversion

world to reach the food source, represents the evaluation of an candidate solution. Real ants communicate by depositing pheromones on their paths, and succeeding ants tend to follow such markings. With ACO the markings are recorded in an \( n \times q \) pheromone matrix \( M \), where \( n \) is the number of acoustic parameters, and \( q \) the number of samples per parameter. For a candidate solution the pheromone marking is added to each of the \( n \) evaluated samples, like those connected by bold purple trace in Fig. 6.1. The amount of pheromones is determined by the mismatch between predicted and actually measured sound pressure fields.

\[
\begin{array}{cccccccc}
\rho_2 & \alpha_2 & c_2 & g_2 & d_2 & \rho_3 & \alpha_3 & c_3 \\
1.02 & 0.004 & 1442 & 1.16 & 0.22 & 1.03 & 0.057 & 1481 \\
1.04 & 0.007 & 1444 & 1.32 & 0.44 & 1.06 & 0.063 & 1484 \\
1.06 & 0.114 & 1446 & 1.48 & 0.66 & 1.09 & 0.070 & 1489 \\
1.08 & 0.168 & 1448 & 1.65 & 0.88 & 1.12 & 0.075 & 1500 \\
1.10 & 0.212 & 1450 & 1.84 & 1.10 & 1.15 & 0.082 & 1511 \\
1.12 & 0.275 & 1452 & 2.00 & 1.32 & 1.18 & 0.089 & 1522 \\
1.14 & 0.375 & 1454 & 2.16 & 1.55 & 1.21 & 0.096 & 1533 \\
1.16 & 0.666 & 1456 & 2.32 & 1.78 & 1.24 & 0.103 & 1544 \\
1.18 & 0.999 & 1458 & 2.48 & 2.00 & 1.27 & 0.110 & 1555 \\
1.20 & 1.333 & 1460 & 2.64 & 2.23 & 1.30 & 0.117 & 1566 \\
\end{array}
\]

Figure 6.1: The ACO analogy: acoustic model parameters as blocking objects and parametric values as bridges.

ACO is an iterative method, and each iteration concerns a group of exploring ants. For each of the \( N \) ants in the colony, a separate candidate solution is constructed. Parametric values of the candidate solutions are statistically selected in accordance with the distributions in the pheromone matrix. Every candidate solution is evaluated by the objective function and new pheromone markings are added to \( M \). Just as real pheromones evaporate in time, the whole matrix is subject to evaporation. The evaporation is applied after each iteration as \( M = (1 - \rho)M \), where evaporation rate \( \rho \) satisfies \( 0 \leq \rho \leq 1 \). Upon termination of the algorithm, the best solution so-far is returned as an obtained result.

In previous work [74] we have argued that ACO is a feasible optimizer for the geoacoustic inversion problem and this was demonstrated with a MAX−MIN Ant System (MMAS) as introduced by Stützle and Hoos [123]. MMAS has four characteristics [26]. First, only the best-so-far or iteration-best ant is allowed to deposit pheromones. Secondly, pheromone trail values are restricted to the \( [\tau_{\min}, \tau_{\max}] \) interval. All paths are initialized with \( \tau_{\max} \). And finally, in case of stagnation the system is reinitiated.

The MMAS implementation focused on here is called Geoacoustic Inversion with ANTs (GIANT). Some technical issues:

- Initially, pheromones are equally distributed. Each entry of the pheromone matrix is set up with the same value \( \tau_0 = 1/f_{\min} \), with \( f_{\min} \) the minimal
mismatch from the first iteration.

- Only the best-so-far ant $x^*$ is allowed to deposit pheromones.

- Apart from a maximum number of evaluations of $f$, a stopping criterion is defined as $f(x^*) \leq \epsilon$, for some small $\epsilon \geq 0$.

- Pheromone update $\Delta \tau = \frac{1}{f(x^*)}$ is deposited on each of the $n$ acoustic parameters that is covered by $x^*$. (Notice that division by zero does not occur for $\Delta \tau$, as $f = 0$ satisfies a stopping criterion.

- After each iteration, the entries in the pheromone matrix are scaled to fit an upper bound $\tau_{\text{max}}$ and a lower bound $\tau_{\text{min}}$. The boundaries are defined as

$$\tau_{\text{max}} = \frac{n}{f(x_{\text{iteration}} - \text{best})}$$ (6.1)

and

$$\tau_{\text{min}} = \frac{c}{\phi n^2},$$ (6.2)

where $c > 0$ is a constant and $\phi$ the average $f$ of the iteration [123]. For $c = 1$ it follows that $\tau_{\text{min}} > 0$ and therefore ants have access to the complete search space at any iteration.

### 6.3.3 Tuning of MAX – MIN Ant System

The efficiency of an average optimization run strongly depends on some algorithm-specific parameters, unique for a particular problem. A metaheuristic runs on a pair $(S, f)$, generally called an instance [12]. The solution space $S$ is determined by problem specific parameters $x_i$ with upper and lower boundaries $a_i$ and $b_i$ that are based on general a priori knowledge. For combinatorial search methods $S$ is usually discretized by sampling the $(a_i, b_i)$ intervals. The sampled geoacoustic parameters strongly influence the search space and the instance. Therefore, sampling has not been regarded as part of the tuning.

For geoacoustic inversion problems, the objective function $f$ is usually computationally demanding as for each forward call a mathematical propagation model needs to be calculated. The ideal solution with $f = 0$ is not always contained in $S$ due to simplifications in the models, the presence of ocean noise or limitations in the sampling of the search space. Parameters that have been tuned for GIANT are colony size $N$ and pheromone evaporation factor $\rho$.

In a typical application of geoacoustic inversion, many signals are transmitted and many instances are to be considered. As the objective functions are computationally time consuming, rapid assessments do not permit extensive tuning for separate instances. Tuning results and their sensitivity in general cannot directly be transferred to other instances.
6.3. Ant Colony Optimization for Inversion

Average fitness

<table>
<thead>
<tr>
<th>Evaporation factor $\rho$</th>
<th>Colony size $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>1500</td>
</tr>
</tbody>
</table>

Average number of calls

<table>
<thead>
<tr>
<th>Evaporation factor $\rho$</th>
<th>Colony size $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>1500</td>
</tr>
</tbody>
</table>

Figure 6.2: Tuning results for GIANT on the YS94 benchmark. Left: final mismatch over 30 runs. Right: average number of forward calls that were needed to find these minimal mismatches. In both plots the best settings are found in the (dark) blue areas.

6.3.4 Results for Yellow Shark

Good procedures exist for tuning metaheuristics [13], [12]. For the benchmark however, an exhaustive search was carried out to find a simple rule of thumb for optimal values for $N$ and $\rho$. This approach is time consuming and not recommended for practical use, but does provide a good understanding of the interaction between performance parameters and the speed of convergence. The method further indicates how sensitive tuning results are and just how bad non-optimal settings work out. Results are shown in Fig. 6.2 for a maximum number of $10^5$ forward calls. Averages are taken over 30 runs for various combinations of $N$ and $\rho$. A benchmark specific convergence threshold of $\epsilon = 1.743$ has been adapted.

The left diagram shows the average final mismatch. In most runs GIANT did not converge to $\epsilon$, plotted in dark blue. The change of color points out that $N$ and $\rho$ should carefully be chosen to find the best solution with the available forward calls. For $10^5$ available calls, there exists a range of good settings, briefly characterized as $N$ large and $\rho < 0.5$.

The diagram on the right shows how many forward calls were needed to get the minimal mismatch in the left diagram. The number of calls corresponds with computing time for the best-so-far solution. For a minimal number of forward calls, $N$ must be as small as the minimal mismatch permits, $\rho$ is less bounded.

Combined, the two plots reveal a small area of interest: $\rho < 0.4$ and $N \approx 250$. The low evaporation rates make $MAAX - MAX$ Ant System act like it has a long term memory. With small $\rho$, ants start to choose the same solution components and are reducing the intensity of exploration. For the benchmark a rule of thumb for colony size is $N \approx \sqrt{k}$, with $k$ the available number of forward calls [74].

Table 6.1 lists solutions for a run with $\rho = 0.1$ and $N = 250$. The results
Table 6.1: Geoacoustic parameters and results for tuned GIANT on YS94 ($\rho = 0.1$ and $N = 250$). Listed are parameters $x_i$ with their physical meaning, $a_i$ and $b_i$ are lower and upper bounds, $s^\ast$ is the reference solution and $s$ the solution found by GIANT.

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>Physical meaning</th>
<th>$a_i$</th>
<th>$b_i$</th>
<th>$s^\ast$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_2$</td>
<td>sediment density in g/cm$^3$</td>
<td>1</td>
<td>2</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>sediment absorption in dB/\lambda</td>
<td>0</td>
<td>0.175</td>
<td>0.03</td>
<td>0.0315</td>
</tr>
<tr>
<td>$c_2$</td>
<td>sediment sound speed in m/s</td>
<td>1440</td>
<td>1540</td>
<td>1470</td>
<td>1470</td>
</tr>
<tr>
<td>$g_2$</td>
<td>sediment sound speed gradient in s$^{-1}$</td>
<td>1</td>
<td>9</td>
<td>2</td>
<td>1.96</td>
</tr>
<tr>
<td>$d_2$</td>
<td>thickness of sediment layer in m</td>
<td>0</td>
<td>11</td>
<td>7.5</td>
<td>7.48</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>bottom density in g/cm$^3$</td>
<td>1</td>
<td>2.5</td>
<td>1.8</td>
<td>1.81</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>bottom absorption in dB/\lambda</td>
<td>0.05</td>
<td>0.375</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>$c_3$</td>
<td>bottom sound speed in m/s</td>
<td>1450</td>
<td>1600</td>
<td>1530</td>
<td>1531</td>
</tr>
</tbody>
</table>

are almost identical to the reference solutions [48]. Since the search space was linear discretized in 50 samples, $S$ does not contain all the values of the reference solution. This can be noticed for $g_2$, where neighboring samples are 1.96 and 2.12: the exact solution $g_2 = 2$ s$^{-1}$ is out of reach.

Good settings have been found after tuning, Fig. 6.2 reveals that on average, still some 40,000 forward calls are needed to get the desired solution. Another issue is that not every run of a metaheuristic is guaranteed to converge to the same solution. An alternative approach is to have less forward calls for each run, stops before final convergence and then let uncertainty analysis build up a bigger picture from intermediate results.

### 6.4 Uncertainty Analysis

Metaheuristics find solutions of above-average quality and tuning will speed up the average inversion process. Still, there is no guarantee that for a single run the convergences are fast enough and the final solution lacks a clear indication on the degree of confidence in the results. Both issues are solved when probability distributions provide uncertainty analysis.

#### 6.4.1 The Bayesian Framework for Genetic Algorithms

In geoacoustic inversion codes that use genetic algorithms, it has become common use to base posterior probability density (PPD) on the average of several parallel inversions [37]. In the absence of detailed a priori information, uniform a priori distributions are assumed [39]. Another assumption is that data errors are independent and identically Gaussian distributed. Gerstoft did show how the average
gene distributions and the marginal probability distribution became similar over 20 runs [37].

6.4.2 Uncertainty Analysis with MAX−MIN Ant System

Without a Bayesian argumentation but in line with the genetic approach, the probability distributions in Fig. 6.3 are based on the averages over 10 GIANT runs. For this multi-start procedure equal settings for performance were used: $N = 50$, $\rho = 0.05$ and a maximum of 200 iterations. In total, the cost of acquiring the distributions did not exceed $10^5$ forward calls. Arrows mark the reference solutions and are found at central positions within the distributions. Figure 6.3 not only gives correct central values, it adds an indication on the uncertainty. Intervals of likely values narrow down when more processing is allowed.

6.5 Conclusions

With geoacoustic inversion, a marine application has been presented for Ant Colony Optimization. A basic $\text{MAX}−\text{MIN}$ Ant System was implemented to search for geoacoustic properties of a shallow water environment. The algorithm has been tuned on a benchmark based on real world data from the 1994 Yellow Shark experiments. Accurate solutions were found within the given processing time. Runs that do not reach full convergence are shown to be useful in a multi-start approach, when $\text{MMAS}$ provides uncertainty analysis by combining final results of parallel runs into probability distributions. For practical use, one can
improve convergence for geoacoustic inversion by customizing $\mathcal{MMAS}$ with *a priori* knowledge on the search space. In the next chapter a comparison is made with other metaheuristic approaches and real geoacoustic inverse problems.