Rationalised panics: The consequences of strategic uncertainty during financial crises

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Chapter 2

Self-fulfilling Currency Crises: Redux

2.1. Introduction

Financial crises often involve an element of sudden panic. For instance, the proximate causes of vast currency crises typically appear to be “swings in sentiment” on either international capital or foreign exchange markets, or on both. Often, such swings seem to occur in response to an economic event of negligible importance,\(^{17}\) or—perhaps even more mysteriously from the perspective of economic theory—market participants seem to suddenly take note of information that must already have been available to them.\(^ {18}\) This should lead economic theorists to take interest in models that can explain how small changes in economic conditions or information about these conditions can produce the seemingly disproportionate disruptive behaviour of markets that is associated with currency crises.

In this chapter I introduce and discuss a stylised model of a speculative attack on a currency peg that can account for such features. This model is a bare-bones variant of the canonical speculative attack model of Morris and Shin [136], the first global game currency crisis model, and builds on the second generation approach to currency crises, pioneered by Obstfeld [145]. The purpose of the chapter is twofold. First of all, I simply wish to highlight the mechanics of second generation crisis models and those of self-fulfilling crises. As far as this aim is concerned, the purpose of the chapter is to provide an overview of ideas and techniques that will be recurrent in the rest of the dissertation. But its second aim is of more argumentative nature. A theoretical model is necessarily an abstraction of the many dimensions involved when addressing economic questions, and often heroic in its assumptions. I wish to persuade the reader that the mechanics of the second generation model provide a helpful abstraction of those of currency crises in reality, and indeed of financial crises when understood in a broader sense.

\(^{17}\)Allegedly, the 2001 currency crisis in Turkey was triggered when its premier stormed out of a national security council meeting claiming he had been insulted (BBC News, 22 February 2001).

\(^{18}\)The 1997 Asian crisis is partly attributed to structural distortions in the financial sector due to government policies [49]. It seems hard to defend the view that markets were completely ignorant of the presence of such distortions in an earlier stage. See chapter 1 for discussion.
CHAPTER 2. SELF-FULFILLING CURRENCY CRISIS: REDUX

From the modelling perspective, there are two important aspects to consider. On the one hand, there is the issue whether the model captures the essential features of the economic environment that agents are confronted with. I will argue that the approach taken in this chapter captures elemental aspects of the incentive structure that agents face during crises, and that these are crucial to understand their behaviour. On the other hand, there is the issue of explaining how agents will act in such an environment. The main contribution of this text is found in its emphasis on what might be called an “epistemic” game theoretical foundation of the actions of agents. The intent of epistemic game theory is to model how agents may reason about a situation of strategic choice. This approach is perhaps best characterised as providing explanations of the following kind: “if agents reason like this or that, then they will act in this or that way”. More concretely, in this chapter we will consider explicitly what the reasoning of rational economic agents may look like, and how this will determine their actions. My aim is to introduce this approach in a rigorous yet accessible way. There is, however, also a highly formal literature that relates epistemic considerations to the foundations of game theory (see e.g. Battigalli and Bonanno [17] for a survey and De Bruin [30] for a critical review). Implicitly, the discussion below draws on this literature, and more explicit links will appear in chapter 3.

With an emphasis on epistemic explanations, I aim to highlight the middle ground between the conventional literature on currency crises, which typically does not consider this aspect of reasoning at all but assumes agents have superhuman abilities for finding equilibria, and models of “boundedly rational” behaviour, which typically impose rather naive assumptions on the behaviour of agents. The heart of this chapter is formed by a theoretical argument that relates the predictions of the model to a reasoning process that is called infection. This process, that stems from the global games literature, formalises or mimics an intuitive kind of reasoning where doubts about what other economic agents might do “infects” the behaviour of agents. That the model can be founded on an intuitive reasoning process is by itself a fact of considerable merit. It means that the model can be micro-founded, not just in the traditional economic sense of grounding aggregate behaviour in decisions at the level of individual agents, but also in the less traditional sense that individual decisions are in turn grounded in rational reasoning. In general, even the simplest game theoretic models lack intuitive epistemic micro-foundations (Aumann and Brandenburger provide some rather unappealing foundations for the widely used Nash Equilibrium [15]). Moreover, as we will see, the infection process reproduces the disruptive “sudden stops” and “swings in sentiment” commonly associated with currency crises.

19 Here, I paraphrase De Bruin [30], p. 11.
20 In related literature this “infection” process is sometimes called contagion. However, in the context of international financial crises “contagion” has the specific economic connotation of a crisis passing contagiously from one country to another, so I prefer to use the term “infection” to describe the reasoning process. This follows terminology used by Morris et alii [135].
Apart from the epistemic considerations outlined above, in its description of the economic environment the model in this chapter highlights all the important elements commonly found in current theoretical literature on currency crises. The point of departure of this chapter, the second generation currency crisis model, was introduced by Obstfeld in [146] and [147] (though the model in Obstfeld [145] also has self-fulfilling features). The main ingredient of this model is a specific mathematical structure that emphasises “strategic complementarities”: a desire of agents to coordinate their actions. The most salient consequence of this is that it generates multiple rational expectations equilibria. In the context of currency crises, the presence of multiple equilibria means that for the same range of “economic fundamentals”, a crisis may occur depending solely on what might be called the “sentiment” of the market. Since currency crises often seem to be related to swings in sentiments on financial markets, it is unsurprising that this feature of the second generation model is one of its main selling points.

And yet the multiple equilibria approach has also attracted criticism. First, models with multiple equilibria provide no insight in which equilibrium may arise under what conditions of the economic fundamentals. Therefore, the use of models that have multiple equilibria has been criticised on the grounds that they lack predictive power and lack scope for policy recommendations. Moreover, the use of such models can be positioned diametrically opposite to the epistemic approach followed in this chapter, because models with multiple rational expectations equilibria clearly lack epistemic foundations: they do not explain how the market selects an equilibrium, let alone why one particular equilibrium over another. This constitutes a more fundamental source of critique.

Morris’s and Shin’s global game currency crisis model [136] can be used to reconcile the multiple equilibria approach on the one hand, and the epistemic approach, on the other. Global games are based upon theory that was pioneered by Carlsson and Van Damme [40], and provide a technique to derive exact and deterministic conditions under which each outcome (i.e. either a currency crisis or tranquility) will arise, thus resolving the issue of multiple equilibria. As Morris and Shin show, the only modification required to apply this technique to the currency crisis literature, compared to the archetypal second generation model, is the injection of a small amount of informational heterogeneity into the beliefs of agents. Arguably, this modification merely adds to the realism of the model. Global games will play a prominent role in this dissertation, as they facilitate the development of

21 An accessible introduction to second generation models that predates the global game era is Jeanne [105].
22 Rational expectations equilibria were introduced by Muth [143], and the concept is widely used in macroeconomic modelling.
23 The “second generation” type of reasoning has been applied by several scholars to explain the suddenness of recent currency crises: for instance the fall of the French franc during the European EMS crisis (Obstfeld [147]) and that of the peso that set off the Tequila crisis (Sachs et alii [160]).
24 Similar issues in macroeconomic modelling are discussed by Evans [74].
second generation style crises models that provide a (deterministic) understanding how economic conditions may affect the possibility of a crisis arising, and so ultimately convey policy implications.

In sum, the model in this chapter is essentially the “second generation” global game currency crisis model of Morris and Shin, but our analysis of it will be different, both in emphasis and in depth. Though the language used by Morris and Shin reveals that they are aware of the epistemic foundations of their model, they touch only briefly on epistemic considerations ([136, p. 594]). In contrast, in this chapter we will explicitly link the way the model is solved to a process of belief formation. This provides the epistemic justification that we are after. Moreover, the explicit epistemic treatment highlights the infection process mentioned, and provides some economic intuition for understanding swings in sentiment on financial markets.

Technically, the differences between the approach of Morris and Shin [136] and the one taken in this chapter may be summed up as follows. Morris and Shin use an elegant but non-constructive argument, which in particular does not show how one constructs the outcome of the model from more primitive aspects of agents’ reasoning. In contrast, in this chapter we use an approach known as “iterated elimination of dominated strategies” (IEDS) to determine the outcome of the model, which can be related explicitly to a process of belief formation. This approach for solving models with strategic complementarities is not new: it can already be found in Milgrom and Roberts [131] for non-Bayesian supermodular games, who in turn draw on heavily on well-known methods from lattice and order theory (see e.g. Davey and Priestley [55]), and in a number of subsequent papers that apply the same methods. The argument developed in this chapter is also similar to the one used by Carlsson and Van Damme’s original paper on two player global games [40]. However, the papers by Milgrom and Roberts and Carlsson and Van Damme focus on mathematical techniques and do not provide the economic context and justification that link these to crucial features of currency crises.

This chapter introduces three key interlinked economic ideas, and motivates their relevance for understanding currency crises and financial crisis more generally:

1. The presence of interaction between the behaviour of economic agents and policy makers’ objectives, which provides a channel for the existence of panic-induced crises that anticipate the possibility of sudden policy changes (sections 2.2 and 2.3);
2. Strategic complementarities, or the presence of mutually reinforcing behaviour of economic agents operating in the financial sector (sections 2.4 and 2.5);
3. Infection arguments (sections 2.6 and 2.7).

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25The method used in this chapter is also applied (but again not extensively discussed) by Morris and Shin in [139], which remains an important reference on global games, reviewing a great number of techniques.
The first two of these ideas are predominantly related to what we have called agents’ economic environment, while the last is predominantly related to founding the actions of agents in an epistemic fashion. The conclusion of this chapter contains some remarks that connect these ideas to key properties of global games in general, and a brief outlook on what this may mean for economic analysis.

2.2. Why Abandon a Currency Peg?

Assume a currency peg is in place. A currency crisis is a situation in which economic agents fear that policy makers might decide to abandon this currency peg. As we will see in more detail below, this fear will trigger agents to undertake actions that lead to pressure on the exchange rate and (further) undermine the current exchange rate regime. In this case, we say that the currency is “under attack”, or speak of a “speculative attack”. In virtually all circumstances policy makers can try to defend the currency against such an attack, and they have a number of options to do so. The most obvious option is to (sharply) increase interest rates: the central bank may set the interest rate in such a way that the returns on domestic deposits compensate for that of the possibility of the collapse of the currency, which will alleviate the pressure on the exchange rate. However, raising interest rates may lead to a liquidity squeeze that exposes fragility in the domestic financial sector, harms the domestic real sector, and depresses domestic demand. Hence, a policy maker who is confronted with mounting pressure on the exchange rate may decide that devaluation of the currency is a more attractive option than inflating interest rates. Other defence options, such as implementing capital controls, also have drawbacks. Thus, the real costs for the economy caused by implementing the defence may lead a policy maker to refrain from doing so.

The decision to abandon a currency peg or not must take into account the costs and benefits of these two alternative courses of action. Consequentially, a policy maker’s capitulation under speculative pressure should not be viewed as a deterministic consequence of the attack, but rather as a policy response. Ultimately, this response is a political decision. Of fundamental importance for understanding the onset of a crisis is an appreciation of how a policy maker exerts her discretion in such matters and how this decision is anticipated by markets. Indeed, the idea that a policy maker in principle could abandon the currency peg is exactly what instills fears in the minds of economic agents in the first place, thus precipitating the attack. We begin this chapter with an analysis of this policy decision.

The observation that a policy maker has to trade off policy objectives can be made mathematically precise, but in this case modelling choices have to be made. We start off with a parsimonious model of a policy maker pegging a currency that builds on the idea of the policy trade-off. When a currency peg is in place, then the policy maker—presumably—derives some benefit from maintaining the peg. Nevertheless, maintaining the peg is also
costly because from time to time it conflicts with other policy objectives. Both the cost and benefit of pegging the currency will depend on a wide variety of economic modalities. Mathematically we can express this idea by representing the costs and benefits using a net benefit function, \( B(\cdot) \). The net benefit is a function of the variables that the policy maker considers relevant for the decision whether or not to maintain the currency peg. When \( B(\cdot) > 0 \), the policy maker maintains the peg, when \( B(\cdot) < 0 \), she abandons it, and when \( B(\cdot) = 0 \) she is indifferent. The decision of a policy maker to defend or not is influenced by whatever range of variables she considers relevant and affect net benefit.

We will suppose that \( B(\cdot) \) depends on an economic variable \( u \). This is meant to capture the idea that worsening economic conditions may create a conflict between the current exchange rate regime and other policy objectives, and that this conflict could be relieved by a devaluation of the currency. In particular, \( B(\cdot) \) may reflect the net contribution of the currency peg to social welfare as perceived by the policy maker. Here, it is important to understand that a devaluation does not necessarily reduce welfare for the domestic economy: it may, for instance, boost employment (see Corsetti et alii [50] for a discussion of possible welfare effects). A picture that emerges from empirical literature is that currency pegs are more frequently abandoned when unemployment is high (see Eichengreen et alii [70]), or more generally, when the currency is substantially overvalued. In these cases, a devaluation of the currency may be more in line with the policy maker’s goals than adhering to the currency peg. For concreteness, we will suppose \( B \) is decreasing in \( u \), in line with the idea that a higher unemployment rate or greater overvaluation of the currency worsen the trade-off for the policy maker.\(^{26}\)

Obviously, there is no reason why the set of variables that the policy maker cares about should be confined to unemployment, real overvaluation, or any of the other “traditional” economic fundamentals. Quite likely the policy maker cares about a wide range of economic goals and intermediate variables, including what are sometimes called “intangibles”—such as reputation or political status or market sentiment. Any variable that bears relevance to the objectives of the policy maker contributes to her decision. Given the observation that currency crises are often related to swings in sentiments on financial markets, particularly interesting is how the policy maker’s trade-off is affected by—what is often called—“market sentiments”.

\(^{26}\)Perhaps the static formulation in this chapter leaves the impression that the policy maker cares about the economic variable \( u \) in a rather myopic way. After all, to evaluate the costs and benefits of the decision to abandon the peg, the policy maker should also take into account the future path of \( u \). Clearly, in a dynamic model we might have to make more sensitive modelling choices than we have done here. Still, if the path of \( u \) influences the decision of the policy maker, her decision can only be based on past observations of \( u \), and perhaps also her expectations on the future path of \( u \), given the trajectory already observed. Therefore, a consistent way to interpret the economic variable \( u \) in the present model is as a sufficient statistic for the expected future path of a relevant economic variable. We will consider a dynamic model in which this is true in chapter 4.
2.2. WHY ABANDON A CURRENCY PEG?

Admittedly, the term market sentiments is rather vague, and I will provide a more concrete interpretation further below. Intuitively, however, pessimistic market sentiments may lead to a costlier trade-off for the policy maker. Market sentiments generally influence a wide array of economic variables, such as interest rates on foreign debt or wage aspirations. Sentiments are thus a natural part of the economic environment the policy maker has to consider. For instance, if the private sector’s stance towards the peg is pessimistic, that is, the peg’s credibility is low, unions may demand increases in wages despite the peg’s effect of anchoring monetary policy. If wage demands drive up inflation, this forces the policy maker to pursue a more contractionary monetary policy if it wants to sustain the peg, and consequently adverse shocks hurt the economy more than when sentiments would have been positive. It is easy to find other channels that drive up the policy maker’s perception of the costs of the peg as a result of deteriorating market sentiment (Obstfeld [147, p. 1044-1046] discusses a substantial number of them). All other things equal, when sentiments worsen, the costs of maintaining the peg increase.

Formally, let $\lambda \in [0, 1]$ be a measure for the market sentiment, or “pessimism”, such that net benefit is decreasing in $\lambda$. We suppose that $B$ is a function of the measure for market sentiment, $\lambda$ as well as of the economic variable $u$. Figure 9 depicts how the policy maker’s decision is influenced by $u$ and $\lambda$. The position on the horizontal axis gives the magnitude of the fundamental variable $u$, and the curve labelled $B$ in the figure depicts the negative dependence of the net benefit of the peg on $u$ for $\lambda = 0$. If $u$ exceeds the point $h$, the costs of the peg exceed the benefits, and the policy maker will choose to abandon the currency peg. This should be interpreted as a rational, calculated, response to the economic circumstances the policy maker is confronted with, given the underlying preferences represented by the net benefit function $B$.

**Figure 9.** Graphical representation of the policy maker’s preferences
CHAPTER 2. SELF-FULFILLING CURRENCY CRISSES: REDUX

Net benefit derived from the peg decreases when market sentiments worsen. For example, the curve $\mathcal{B}'$ shows how net benefit depends on $u$ for $\lambda = 1$, reflecting very pessimistic market sentiment. This curve lies everywhere below the original $\mathcal{B}$ curve. The graphical analysis shows that the policy maker will now be inclined to abandon the currency peg already when $u$ exceeds the point $\ell$, so already at less deteriorated values of the fundamental $u$.

The relevant distinction between $u$ and $\lambda$ is that $u$ represents an exogenous economic variable, or the “fundamentals” of the economy, while $\lambda$ reflects economic agents’ assessment of the economic environment. The fact that the policy maker’s decision depends on agents’ assessment of the economic environment, an environment which is in turn crucially affected by policy decisions, leads to a curious channel of interaction between the policy maker and markets, which is at the heart of self-fulfilling crises. This will be the focus of the next section.

2.3. Self-Fulfilling Crises

Intuitively, pessimistic market sentiments reflect the fears of economic agents in the private sector or on financial markets that the policy maker will abandon the currency peg. But the presence of such sentiments will also influence the actual decision of the policy maker. To get a grip on the interaction that arises between the private sector, or markets, and the policy maker, the first step is to formally characterise the decision of the policy maker on the fate of the currency peg in terms of $\lambda$ and $u$.

Under mild conditions, it is possible to assure the existence of a unique and well-behaved function $u^*$ that traces out the points at which the policy maker is indifferent between abandoning the peg and maintaining it. That is, the policy maker will abandon the peg if and only if $u$ exceeds a certain threshold $u^*(\lambda)$. We pin down sufficient conditions for this type of analysis.\(^27\)

**LEMMA 1:** Let $\mathcal{B}$ be continuously differentiable, and let the derivatives $\mathcal{B}_u$ and $\mathcal{B}_\lambda$ be strictly negative. Further, suppose that there exist points $\ell$ and $h$ such that for all $\lambda \in [0, 1]$, $\mathcal{B}(\ell, \lambda) \geq 0$ and $\mathcal{B}(h, \lambda) \leq 0$. Then:

1. there exists a unique continuously differentiable function $u^*(\lambda) : [0, 1] \to \mathbb{R}$ such that $\mathcal{B}(u^*(\lambda), \lambda) = 0$ for all $\lambda \in [0, 1]$;
2. $u^*_\lambda = -\frac{\mathcal{B}_\lambda}{\mathcal{B}_u} < 0$ for all $\lambda \in [0, 1]$.\(^28\)

\(^27\)These conditions are not too restrictive: we demand that the dependence of $\mathcal{B}$ on $u$ and $\lambda$ is negative, to rule out perverse effects of $u$ and $\lambda$ on $\mathcal{B}$. Furthermore we require that for any sort of sentiment $\lambda$ it is possible to imagine values of the fundamental $u$ that are large or small enough to result in a devaluation or a continuation of the peg respectively. This assumption is implicit in figure 9.

\(^28\)In all chapters, proofs of formal statements are found in the accompanying appendices.
2.3. SELF-FULFILLING CRISES

Lemma 1 provides a very basic characterisation of the behaviour of the policy maker. Let us dwell a bit on its interpretation. For each given \( \lambda \in [0, 1] \), the point \( u^*(\lambda) \) is the point \( u \) such that \( B(\cdot) = 0 \). Hence, for given \( \lambda \), the set \( \{ u \in \mathbb{R} \mid u > u^*(\lambda) \} \) is the set of states, that is, the event, where the policy maker abandons the peg. The value of \( u^*(\lambda) \) at \( \lambda \) is therefore called the failure point for \( \lambda \). A convenient way to look at the function \( u^* \) is in \((u, \lambda)\)-space. The function \( u^* \) completely and uniquely describes the behaviour of the policy maker: it cuts the space in two regions, as in figure 10, so that the policy maker devalues the currency if and only if \( u \) lies to the right of \( u^* \). As the failure point depends on \( \lambda \), so does the behaviour of the policy maker. In fact, this dependence is negative, so that more pessimistic sentiments will lead the policy maker to abandon the peg for smaller (hence milder) values of the variable \( u \). This means that the function \( u^* \) is downward sloping, as is formally stated in the lemma.

This last observation has some striking implications. Suppose that market sentiment may suddenly shift for exogenous (for now) reasons. A sudden slide from optimism towards pessimism may be so costly for the policy maker that it will bring her to the point where she gives in to this pessimism. In figures 9 and 10, this may happen if \( u \) lies in the region between the points \( \ell \) and \( h \), and if the shift in \( \lambda \) is sufficiently large. The resulting policy response, a devaluation, will validate the change in sentiment after the event. Yet, while with hindsight the market’s pessimism turns out to be justified, such crises would not have occurred in the absence of a shift of sentiment. For obvious reasons, these types of crises have been dubbed self-fulfilling in the literature.

The possibility that crises might be self-fulfilling offers an interesting perspective on their swiftness and virulence. Opinions may change very abruptly, even without observable changes in fundamentals; perhaps driven by changes in information that affect the complex layer of beliefs and motivations that underlies the actions of financial agents. In this way, the interaction between private sector expectations and policy as outlined above can explain the abruptness of financial crises. However, this should lead us to probe further into the role that beliefs play in financial crises. The self-fulfilling perspective on financial crises shifts
the burden of explanation of abrupt financial crises onto the economic dynamics that shape beliefs of financial actors.

2.4. Strategic Uncertainty and Strategic Complementarities

Negative sentiments provoke actions that are costly to the policy maker. This interface between pessimism on markets and the policy maker’s trade-off enables us to provide a more substantive account of what we have called “market sentiments”.

In the private sector there is an array of parties that take special interest in the peg. Among these are speculators, domestic firms, unions, domestic banks, and foreign creditors. In times of financial distress, each of these parties increases the costs for the policy maker through certain actions: speculators go short hoping to profit from a possible devaluation, domestic firms hedge against shifts in the exchange rate to guard the health of their balance sheets, unions demand high nominal wages to protect workers against a deterioration of their real wages, foreign banks refuse to roll over debt or demand high interest rates, and so forth. While all these actions stem from different motives, from the viewpoint of the policy maker their effect is the same in the sense that they result in pressure on the exchange rate. For instance, if a large number of firms hedge against a possible devaluation, the effect on the exchange rate is not very different from that of speculation. Similarly, wage demands may drive up inflation, resulting in real currency overvaluation and—ultimately—pressure on the exchange rate. The cost of all the actions of all agents combined enters the policy maker’s evaluation of the net benefit of the peg, and each of these actions will thus affect her decision in a small way, which turns substantial at the aggregate level. “Negative market sentiment” may be identified a prevalence of such costly actions. For concreteness, in this dissertation I will usually simply speak of the actions of speculators instead of negative market sentiment or of all the different actions of all categories of economic agents that make up the private sector. Implicitly it is understood that speculative pressure stems from a wide variety of sources.

Costly actions are ultimately triggered by gloomy expectations of future policy decisions on the agents’ part, which create an interdependency of incentives. If agents have reason to believe that others will resort to actions that are costly for the policy maker, this belief will increase the incentives to do the same—because all other things being equal, the policy maker will more likely abandon the peg if costly actions are prevalent. Thus the risk of financial distress further increases. Because of this interdependency, decisions of individual agents must be based in part on their beliefs of what the actions of others will be. In situations like this, we say that agents have to reason strategically or that there is strategic interaction between agents. The most important aspect of strategic interaction during crises is that behaviour of agents is mutually reinforcing. When actions are mutually reinforcing
we say that they are strategic complements, and say that the situation exhibits strategic complementarities. Since agents prefer to act in similar ways in times of distress, pessimistic sentiments may snowball into a cumulative downward spiral, quickly shifting the economy from a situation where all is well into a crash. In this case, behaviour that is rational at the level of the individual results in massive disruption of the economy at the aggregate level. Strategic complementarities therefore expose financial institutions to inherent instability and panics, even if all agents are rational.

In an economic environment, agents who are weighing options are typically confronted with two kinds of uncertainty. On the one hand, there is uncertainty about certain economic quantities, such as the magnitude of $u$, that may conceivably be approached in a statistical manner. On the other hand, there is strategic uncertainty. If there is non-trivial strategic interaction (in the framework in this chapter whenever $u$ is in the region between $\ell$ and $h$), outcomes are affected by the behaviour of other agents, and thus become more uncertain whenever the behaviour of other agents becomes less certain. Strategic uncertainty is caused by the interdependence between agents and how agents deal with it can be addressed in a meaningful way only through an analysis of the intentions and beliefs that shape the actions of agents.

In modern economic systems, the relevance of strategic interdependence is substantial. Many prices of assets move primarily in response to decisions made and expectations held by economic agents. The stability of a financial system depends to a large extent on the agent’s belief and trust in the behaviour of other agents and in the stability of the financial system. It is not inconceivable that even well-established and stable financial systems as found in mature and developed economies would eventually fall apart if enough people were to lose their faith in it. The argument, then, is that the $[\ell, h]$-region is quite sizable and can be an important source of trouble that comes in the form of opinion slides. If we want to understand the onset of a financial crises, we should study how and why beliefs might shift when $u$ is in this region.

### 2.5. A Model of a Currency Crisis

We now turn to the question how beliefs may be determined endogenously in the framework discussed above. In this section, we develop a mathematical model of a speculative attack that brings together the key features discussed in sections 2.2, 2.3, and 2.4. As indicated, this model is fully based on the canonical work of Morris and Shin [136].

Note that the way the function $\theta$ depends on the economic fundamental, (which is denoted $\theta$ by Morris and Shin), is exactly reversed in our model. However, the notation used here is consistent with earlier second generation models, such as Obstfeld [147].
Let $N = [0, 1]$ be a continuum set of speculators. The assumption that $N$ is a continuum is made to capture the idea that each individual speculator is small so that individual decisions have no substantial impact at the aggregate level. We will identify the measure of market sentiment $\lambda$ with the fraction of speculators who attack the currency.

We assume there is an exogenous fixed, true value of $u$, unknown to the agents, which we treat as a parameter. Each individual speculator $i$ has a good idea of the true value of $u$, perhaps based on statistical information, but not an infinitely accurate idea. Formally, each speculator receives a noisy private signal $x_i$ on $u$, where we assume her signal is correct up to some small, uniformly distributed noise term, that is statistically independent across speculators. For each $i \in N$, we let $x_i \sim U[u - \epsilon, u + \epsilon]$. We call $2\epsilon$ the signal’s dispersion, and assume this dispersion is small, in particular that $0 < 2\epsilon < h - \ell$. For speculator $i$, the signal $x_i$ constitutes a piece of (private) information; based on the signal, speculator $i$ considers the set $X_i := [x_i - \epsilon, x_i + \epsilon]$ to be admissible descriptions of the true economic state $u$. Speculator $i$ considers each of the states $x \in X_i$ equally likely.\(^{30}\) Note that the signals of speculators are closely correlated, since they are all distributed around the true value of $u$.

Following Morris and Shin\(^{136}\), we consider a sequential set-up in which time is divided into two stages. In the first stage, individual speculators receive their signals and can opt to take a position against the currency or not, conditional on their signal. A speculator who takes position against the currency is said to “attack” it. Subsequently, the policy maker devalues conditional on $u$ and on the speculative pressure exerted by the speculators combined, according to her preferences which are shaped by $B$.\(^{31}\) The sequential structure allows us to solve the model backwards. First, we use the characterisation of the optimal policy given in lemma 1 to determine the response of policy maker for given $u$ and $\lambda$. Using this response, we can then solve for the situations in which speculators would like to attack.

Normalise a speculator’s revenues of a successful attack to unity, and assume a small, but fixed, transaction cost $q$ of attacking the peg, $0 < q < 1$.\(^{32}\) That is, writing $\pi(u, \lambda)$ for profits from attacking when the economic state variable is $u$ and a fraction $\lambda$ of speculators attacks, we have:

\[
\pi(u, \lambda) = \begin{cases} 
1 - q & \text{if } u \geq u^*(\lambda) \\
-q & \text{if } u < u^*(\lambda).
\end{cases}
\]

\(^{30}\)This means that an agent’s posterior (i.e., her belief after receiving the signal) is uniform on the set $X_i$. Many global game papers in the literature stress that there is also a relationship between an agent’s prior belief and her posterior belief. When signals are uniformly distributed, the uniformity of posteriors always holds if priors are uniform, and always holds approximately if the dispersion of the signals $\epsilon$ is sufficiently small and agents’ priors are continuous.

\(^{31}\)Although this sequential approach is widely adopted in the global games literature, it is not clear that it is the most realistic. Alternative approaches are discussed in chapters 4 and 5.

\(^{32}\)The assumptions that the costs and gains of attacking are fixed are relaxed in chapter 5.
2.6. Infection

In this section we will solve the model by considering what kind of reasoning we may expect of rational agents. We will consider a process that iteratively discards “unreasonable” behaviour, in order to determine under which circumstances agents will want to attack the currency. Our analysis will differ in style from the one given by Morris and Shin [136], and makes more explicit use of higher order beliefs. While this approach requires us to develop a somewhat technical perspective on the model of the previous section, in my view it is the most natural way to solve the model, and yields substantial insight in the outcome that the model generates. I shall make no vain attempts to formulate importance of higher order beliefs to the economic decision making process in a clearer and more compelling way than J.M. Keynes. In a famous passage of The General Theory [110], Keynes argued how financial decisions can be likened to:

...those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not the faces which he himself finds the prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view.

It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree when we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.34

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33Morris and Shin were well aware of the importance of higher order beliefs and provide some discussion in the article [136, p. 594]. However, they provide no explicit link between the way they solve model their model mathematically and the role of higher order beliefs.

34Keynes, in turn, appears to have been inspired by a passage in Littlewood’s “mathematical miscellany”, which details a variant of the well known “muddy children” puzzle. The link doesn't stop here: see Van Benthem [18] for a non-trivial analysis that relates the resolution of strategic interaction through rational reasoning to the muddy children puzzle.
CHAPTER 2. SELF-FULFILLING CURRENCY CRISES: REDUX

Keynes’s pageant metaphor suggests that the beliefs of financial actors possess dynamics of their own, that may operate out of touch with their actual opinion on the relevant matters of fact. At first sight, any attempt to untangle this process seems to lead into a psychological mess. Yet, strikingly, in the present case, we can exploit precisely this kind of interaction to determine the outcome of the model. The reason is that, due to the presence of those strategic complementarities, agents have an incentive to do “as everyone else does”. As we shall see, this allows certain extremal beliefs to “infect” the entire decision making process of each and every agent, which allows us to pin down the outcome of the analysis.

Consider the situation of a given speculator $i$, whose idiosyncratic signal is $x_i$. On the basis of her signal, the speculator has to decide whether to speculate against the peg or not. A strategy is a plan of action based on an agent’s observations and information about her environment—thus in the present case a strategy is a rule that transforms her idiosyncratic signal into a decision to attack or not.\(^{35}\) A joint strategy profile, denoted $\sigma$, is a sequence of strategies, specifying a strategy for each agent $i$ in $N$.

Recall that $\lambda$ is the fraction of speculators who attack the currency. The aggregate distribution of signals depends on the true state $u$, and so each joint strategy profile followed by speculators gives rise to a specific value of $\lambda$ for a fixed value of $u$. Therefore we may write speculative pressure as a function $\lambda(u, \sigma)$ when the true state is $u$ and the joint strategy profile followed by speculators is $\sigma$.\(^{36}\) The expectation value of the payoff obtained when attacking the currency, conditional on the piece of information $x_i$, is given by the function:

$$
\pi^e(x_i; \sigma) := \frac{1}{2\varepsilon} \int_{X_i} \pi(v, \lambda(v, \sigma)) \, dv;
$$

where $\pi$ is defined as in equation (1). Now denote by $D(\sigma) := \{u \in \mathbb{R} | u \geq u^*(\lambda(u, \sigma))\}$ the set of states, or “event”, in which the policy maker devalues at the second stage if the joint strategy profile followed by speculators is $\sigma$. Using equation (1), we can rewrite $\pi^e$ as the follows:

$$
\pi^e(x_i; \sigma) := \frac{1}{2\varepsilon} \int_{D(\sigma) \cap X_i} dv - q.
$$

Of course, determining this expectation value presupposes that we are able to specify $\lambda(u, \sigma)$; which is unobservable and must be deduced by the speculators. For this purpose,

---

35 This description corresponds exactly to the definition given by Von Neumann and Morgenstern [177], p. 79. Throughout this dissertation, we only consider “pure” strategies, that is, agents do not deliberately randomise over actions (with the exception of a general game-theoretic result in appendix 3.A).

36 This representation (which is similar to the one used by Morris and Shin [136, p. 591]) implicitly assumes that all speculators use the same strategy; that is, the joint strategy profile is symmetric. The analysis that follows below does not depend on this assumption, but all the crucial formal steps in the infection argument appeal only to symmetric joint strategy profiles. Therefore this simple representation suffices for our purposes.
some signals are a bit more informative than others. Consider the points $\ell := u^*(1)$ and $h := u^*(0)$ in figure 10. These are extremal points: if the true state $u$ falls short of $\ell$, then the policy maker will never devalue the currency, even if all speculators were to attack. On the other hand, if $u$ exceeds $h$, the policy maker will certainly abandon the peg. Now suppose, hypothetically, that a speculator’s signal is $\bar{x} := \ell - \epsilon$. In this case, the speculator can claim to know for sure that the true state must be below $\ell$, as every point in the set of possible states $X := [\bar{x} - \epsilon, \bar{x} + \epsilon]$ lies below $\ell$. In other words, speculator $i$ can deduce that the policy maker will not devalue, even if every other speculator attacks. It is easy to see that this argument holds for any speculator who receives a signal below $\bar{x}$. We may conclude that speculators who receive such signals will not decide to attack.

Now change the perspective slightly and consider a speculator $j$ who receives signal $x_j$ just slightly above $\bar{x}$ (i.e., $0 < x_j - \bar{x} < \epsilon$). This speculator will deduce that it is quite possible that the true state is below $\bar{x} + \epsilon$ (that is, below $\ell$). This observation makes attacking the currency less desirable than at higher signals, because when the true state is below $\bar{x} + \epsilon$, the policy maker will not devalue. The closer $j$’s signal $x_j$ is to $\bar{x}$, the more likely she thinks it is that the true state is below $\bar{x} + \epsilon$.

Moreover, $j$ already knows that speculators who receive signals below $\bar{x}$ do not attack. In other words, speculator $j$ believes there might be a group of other speculators who believe the policy maker will not devalue—and in the words of Keynes, we have reached the second degree of beliefs. The closer $j$’s signal $x_j$ is to $\bar{x}$, the larger she believes that this group of agents may be. From $j$’s perspective, a larger group of agents with signals below $\bar{x}$ also makes attacking the currency less desirable. Why? For any given true state $u$, it is more likely that the policy maker devalues when more agents attack. Hence, the size of the group of attackers affects the probability of a successful attack, and therefore the expected profit from attacking.

Now suppose $j$ believes that all speculators with signals above $\bar{x}$ attack the currency. Given that $j$ already knows that speculators who receive signals below $\bar{x}$ do not attack the currency, this behaviour would maximise the probability of a successful attack. Even under the belief that speculators with signals above $\bar{x}$ attack, if $x_j$ is sufficiently close to $\bar{x}$, $j$ will think that expected profits from attacking are negative. This is because speculation is costly, and the probability of success is small given that $x_j$ is close to $\bar{x}$. In other words, expected profits decline when $x_j$ falls. Let us write $\bar{x}$ for the signal that makes $j$ indifferent between attacking and not attacking, under the belief that all speculators with signals above $\bar{x}$ attack.

---

37 Note that since signals are distributed closely around $u$, and $u$ need not be close to $\bar{x}$, it may very well be the case that none of the speculators receives the signal $\bar{x}$. But the argument that follows does not depend on this. It depends solely on the fact that a situation in which some speculator receives the signal $\bar{x}$ is logically conceivable, and this fact is guaranteed by the assumptions made in lemma 1.
(below, we will show that this signal exists and is unique). If \( j \) receives a signal below \( x \), she will certainly not attack the currency. Note that we must have \( x < x' \).

Next, we may consider a speculator \( k \) who receives a signal just slightly above \( x \). Speculator \( k \) knows that agents who receive signals below \( x \) do not attack the currency. For which values of the signal \( x_k \) will \( k \) consider attacking the currency? We can apply the same kind of reasoning as we did with speculator \( j \). However, in order to answer the question in more generality, let us write \( \pi(x, y) \) for the expected profit of attacking the currency, for a speculator who receives signal \( x \), and who has deduced that every speculator who has a signal below \( y \) does not attack, under the “optimistic” belief that everyone who has a signal greater than \( y \) does not attack, under the “optimistic” belief that everyone who has a signal greater than \( y \) will join her in the attack if she chooses to attack the currency.

Note that the value of \( \pi(x, y) \) actually constitutes an upper bound on the expected payoff from attacking the currency, given that all speculators who receive signals below \( y \) do not attack. This is because the scenario where everyone with signals above \( y \) attacks is the most optimistic scenario that a speculator with signal \( x \) can think of—provided she decides to attack the currency—given her conviction that everyone with signals below \( y \) refrains from attacking. After all, if she deduces that all with signals below \( y \) refrains from attacking, the attack is most likely to succeed when all remaining speculators join in the attack. Moreover, note that this belief describes a belief about the joint strategy profile that is followed by others: every speculator with a signal below \( y \) does not attack the currency and every speculator with a signal greater than \( y \) attacks the currency. This joint strategy profile is called the joint threshold strategy around \( y \).

**Lemma 2:** \( \pi(x, y) \) is increasing in its first argument, \( x \), and decreasing in its second argument, \( y \). Moreover, there exists a unique, continuous function \( F : \mathbb{R} \to \mathbb{R} \) such that \( \pi(F(y), y) = 0 \).

The first part of this lemma tells us that a signal about weaker fundamentals (a higher \( x \)) increases the upper bound on expected profit of attacking; and also that a more optimistic belief about the behaviour of others (that is, a lower \( y \)) increases the upper bound on expected profit of attacking. The second part states that our assumptions so far suffice to guarantee that \( \pi \) is “well behaved”: given the belief that exactly those who have a signal greater than \( y \) attack, there is a unique signal \( x \) that makes the speculator receiving signal \( x \) indifferent.

Let us cast the argument so far in terms of the function \( \pi \). Clearly:

\[
\pi(x, y) = -q < 0 \text{ for all } y.
\]

This simply corresponds to our earlier observations that the signal \( x \) reveals the behaviour of the policy maker with certainty. Furthermore, note that \( x \) is given by the condition \( \pi(x, x) = 0 \), so that \( x = F(x) \).
Now turn back to speculator $k$ and consider the implications of lemma 2. By the lemma, $F(x)$ gives the unique point $x_*$ such that $\pi(x, x) = 0$. In other words, there is again a unique signal $x_*$ (with $x_* < x_0$) such the speculator who receives exactly this signal is indifferent between attacking the currency and refraining from it, conditional on the belief that everyone with a signal below $x_*$ refrains from attacking and that everyone with a signal above $x_*$ attacks. Indeed, it follows that no speculator who receives a signal below $x_0$ will want to attack, even under the optimistic belief that every other speculator receiving a signal greater than $x_*$ attacks. A fortiori, no speculator that receives a signal below $x_0$ will want to attack when speculators any other reasonable joint strategy profile (not necessarily a joint threshold strategy) is followed by other speculators. After all, the attack is most likely to succeed when every speculator with a signal above $x_*$ joins in the attack. Since $x_k < x_0 = F(x_0)$, speculator $k$ will not want to attack the currency, and we have now reached the third degree of reasoning, in which we have established that no speculator will attack if her signal is below $F(x_0)$.

This observation cannot be the end point of our analysis, as $x_0 < F(x_0)$, and since all agents with signals that are between $x_0$ and $F(x_0)$ prefer to refrain from attacking. So, speculators will be able to deduce that in fact no-one with a lower signal than $F(x_0)$ will attack, leaving our argument in a similar place as it started from. Whence we can iterate the argument, now taking $F(x_0)$ as a starting point, rather than $x_0$. And thereafter we repeat the argument starting at $F(F(x_0))$ (that is, $F(x_0)$), and so on.

If agents reason in this recursive fashion, the set of signals at which they refuse to attack expands with each level of the reasoning process. This iterative process is called infection, because at each stage observations on how to behave optimally at some given set of signals “infects” the decision that is to be made at neighbouring sets. The infection process must continue until we find a signal such that $F(x^*) = x^*$. If $F(x^*) = x^*$, we say that $x^*$ is a fixpoint of $F$. The significance of the fixpoints of $F$ is that these do not suffer from the “inconsistency” $F(x) > x$ that drives the iterative process further and further at each level of reasoning. Indeed, it follows straight from the definition of $F$ that if $x^*$ is a fixpoint, then $x^*$ is a signal such that speculators will want to attack if and only if their signal is greater than $x^*$, provided that all other speculators follow this strategy; so a fixpoint is a stable outcome of the strategic interaction. The following procedure “computes” the smallest fixpoint of $F$ in a way that mimics the iterative infection process outlined above. Define recursively: $L^0 = x_0$ and $L^{n+1} = F(L^n)$ and consider the (monotonic) sequence $\langle L^n | n \in \mathbb{N} \rangle$. We claim:

**LEMMA 3:** *The map $F(x)$ has a least fixpoint given by $L = \lim_{n \to \infty} L^n$.***

In sum, using the observation that speculators with signals smaller than or equal to $x_0$ must refuse to speculate against the currency, we can iterate the map $F$ starting from this
point \( x \), and (in the limit) obtain a joint threshold strategy that is stable in the sense that, if speculators follow the joint threshold strategy around \( \bar{L} \), every individual speculator will want to speculate if and only if her signal exceeds \( \bar{L} \).

We can develop a dual argument. A speculator who receives a signal at least equal to \( \bar{x} := h + \varepsilon \) will deduce the policy maker will abandon the peg with certainty, whatever the joint strategy profile of speculators. Such an agent will definitively opt to speculate against the currency. As it can be deduced that speculators with signals above \( \bar{x} \) have clear incentives to attack, it must be the case that speculators with signals slightly below \( \bar{x} \) will also attack. Now, a similar recursive exercise can be done starting from the point \( \bar{x} \). To formalise this dual argument, write \( \pi(x, y) \) for the expected profit of attacking for a speculator with signal \( x \), conditional on the belief that everyone who has a signal lower than \( y \) refrains from attacking the currency, and everyone who receives a signal above it attacks. It requires only a moment’s thought to see that \( \pi(x, y) = \pi(x, y) \) for all \( x, y \). To find a fixpoint define recursively: \( H^0 = \bar{x} \) and \( H^{n+1} = F(H^n) \). The next result then mirrors lemma 3:

**Lemma 4:** The map \( F(x) \) has a greatest fixpoint given by \( \bar{H} = \lim_{n \to \infty} H^n \).

Figure 11 shows a typical map \( F \) together with its least and greatest fixpoint.

### 2.7. Unique Equilibrium

Joint threshold strategies have played a special role in our analysis and we have shown that joint threshold strategies arise naturally as outcomes of the strategic interaction. Indeed, as hinted at above, the joint threshold strategies around \( \bar{L} \) and \( \bar{H} \), satisfy the following game-theoretic stability criterion:
**Definition:** A joint strategy profile $\sigma$ is a \textbf{(Bayes-Nash) equilibrium point} if no agent can improve her expected payoff by unilaterally resorting to a different strategy. □

A joint strategy profile is called an \textbf{equilibrium point in joint threshold strategies} if and only if it is both an equilibrium point and a joint threshold strategy. As a matter of fact, a joint threshold strategy around $x$ is an equilibrium point in joint threshold strategies if and only if $x$ is a fixpoint of $F$.

Figure 12 gives a graphical overview of the infection process that we have outlined in the previous section. Summing up, we first identified two regions where refrain and attack are the only “reasonable” choices. (In the literature, these regions are called \textbf{dominance regions}.) From this observation, an infection process starts that affects the behaviour of those agents with signals that are sufficiently close to the identified regions. Consequently, the sequence $\langle L^n \mid n \in \mathbb{N} \rangle$ iteratively expands the refrain region. The sequence is bounded by the least fixpoint of $F$, $L$. We have shown that for all signals to the left of $L$, refrain is the \textit{only} reasonable action to choose. Similarly, the sequence $\langle H^n \mid n \in \mathbb{N} \rangle$ iteratively expands the attack region. This sequence is bounded by the largest fixpoint of $F$, $H$. To the right of $R$, attack is the \textit{only} reasonable action to choose.

The main result of Morris and Shin [136], restated in terms of this framework, is that the “attack” and “refrain” regions eventually touch in a point $x^\ast$. This means that we can completely characterise the optimal behaviour of speculators for any signal they may receive: a speculator attacks if and only if her signal exceeds $x^\ast$. The key to this result is the observation that:

THEOREM 5: \textbf{There is a unique equilibrium point in joint threshold strategies.}

Since $L$ and $H$ are both equilibrium points in threshold strategies, to prove there is a unique equilibrium point in threshold strategies, $x^\ast$, it suffices to show that $L = H = x^\ast$. But \textit{a fortiori} this shows that no other strategies are compatible with the reasoning process. Indeed, in view of definition 2.7 we have proved there cannot be any other equilibrium points, by showing that any other joint strategy profile than the joint threshold strategy around $x^\ast$
must be suboptimal for some agent. Thus the equilibrium in threshold strategies $x^*$ is in an exact sense the only possible outcome of the model.

**Equilibrium Implications.** The implications of theorem 5 can be summed up as follows. As stated, there is a unique equilibrium point in threshold strategies, giving rise to a unique threshold $x^*$. Recall that signals are distributed around $u$ with small dispersion $2\epsilon$. The theorem implies that whenever $u$ is smaller than $x^* - \epsilon$, all speculators refrain from attacking the currency, and whenever $u$ is larger than $x^* + \epsilon$, all of them attack. In the region $[x^* - \epsilon, x^* + \epsilon]$, $\lambda$ increases rapidly (and linearly) from 0 to 1, and there is a uniquely induced state $v \in [x^* - \epsilon, x^* + \epsilon]$, such that for all $u > v$, speculative pressure exceeds the amount of pressure that is tolerated by the policy maker. Hence, if $u$ exceeds the point $v$, the policy maker abandons the peg. This analysis is summarised in figure 13. The figure shows that, given the equilibrium point, the net benefit for the policy maker is found by combining the two curves in figure 9 with a sudden shift in market sentiment around the point $x^*$, where $\lambda$ jumps from optimistic to pessimistic. At the point $v$, the policy maker then gives in to speculative pressure.

Although the devaluation at the point $v$ is triggered by a large shift in market sentiment, this shift occurs in response to only a small perturbation of the economic variable $u$. Nevertheless, such a shift does not come about as an arbitrary change of opinion. It is clearly a response to a change in an economically meaningful fundamental variable, and hence the unique equilibrium point ties the outcome of the model to the economic fundamental. However, since only a very small change in the underlying fundamental $u$ induces a sweeping change in sentiment, the relationship between fundamentals and market sentiment is highly
2.7. UNIQUE EQUILIBRIUM

non-linear. Moreover, crises typically have self-fulfilling features, since around the point \( v \), in principle the peg could have been sustained, if only all agents had chosen to “disarm”.

The Role of Higher Order Beliefs and Absence of Common Knowledge. Our style of proof has related the “threshold equilibrium” outcome to the analysis of agents’ higher order beliefs. More precisely, the infection process is driven by properties of the agents’ belief hierarchies. The infection result does not depend on the fact that some speculators actually receive the extreme signals \( \bar{x}, x \), but only on the idea that this situation is conceivable in terms of higher order beliefs.

An example will illustrate why these higher order beliefs matter. If the dispersion \( 2\varepsilon \) is small enough, a speculator who receives a signal \( x < \bar{x} \) may certainly rule out that someone has received a signal above \( \bar{x} \). What cannot be ruled out, however, is that “she believes that someone believes that someone believes . . . that someone received a signal above \( \bar{x} \). More generally, even though a speculator may claim to know or believe that the true state is close to some state \( x \), such statements always fail to be common knowledge: it is never true that

\[
(5) \quad \text{“everyone knows that everyone knows that . . . that the true state is close to } x \text{”}.
\]

And indeed, as suggested above, the opposite is true: for any \( x \in \mathbb{R} \) it is true that

\[
(5') \quad \text{“everyone believes that someone may believe that . . . the true state is close to } x \text{”}.
\]

This can be formalised completely (see Ref. [135]) but let us briefly sketch the argument supporting these claims. Recall that a speculator receiving signal \( x \) knows that the true state is in the set \( [x - \varepsilon, x + \varepsilon] \). She cannot rule out that she has received a signal at the low end of the distribution around the true fundamental \( u \), which would mean that the true state is in fact \( x + \varepsilon \)—this is the highest possible value of the fundamental \( u \), considering the signal \( x \). However, if the belief that \( u = x + \varepsilon \) turns out to be true, another speculator will have received a signal at the high end of the distribution \( (x + 2\varepsilon) \), yet also cannot rule out that his signal comes from the low end of the distribution. In fact this second speculator cannot rule out that \( u \) equals \( x + 3\varepsilon \). This means that, even though the first speculator (who receives signal \( x \)) rules out that \( u > x + \varepsilon \), she cannot rule out that someone else believes that \( u > x + \varepsilon \). By a similar argument, she cannot rule out that someone else believes that \( u < x - \varepsilon \). These are the speculator’s “second order” beliefs, since they are not statements about her own knowledge, but about what she believes that others might believe. Next, we may consider the speculator’s third order beliefs, which are the statements about what she “believes that others believe that others believe”, and so on.

Proceeding in this way, for any value \( \hat{u} \), there exists some higher order belief in the belief hierarchy (that is, perhaps a second order belief, or perhaps a third order belief, etc.) such that \( u > \hat{u} \) or \( u < \hat{u} \) may hold. These higher order beliefs are relevant for the speculator
because the beliefs of other speculators will determine their decisions, which—given the strategic interaction—are in turn relevant for her own. Ultimately, the behaviour of agents in extreme but conceivable states stays relevant. A speculator who receives a signal close to $x^*$ may certainly rule out that someone has received a signal above $\bar{x}$, but she also knows that higher order beliefs matter, and thus that the possibility of a sure attack on the currency in extreme states “infects” the behaviour of agents receiving signals close to $x^*$. Even though each agent’s behaviour is ultimately related to the information she receives about fundamentals, there are such pronounced strategic complementarities in the region $[\ell, h]$ that any choice must be driven predominantly by beliefs of what others will do. This allows infection to shape the outcome of the model in the stated, non-linear way. Therefore, in the equilibrium point, the model exhibits the features of an economy-wide panic when speculators’ signals exceed $x^*$.

**Equilibrium and Strategic Uncertainty.** What exactly changes in the minds of speculators at the point $x^*$? To conclude this section, let me illustrate how strategic uncertainty shapes the equilibrium outcome, and that this strategic uncertainty sways the decisions of agents towards attacking the peg at the point $x^*$.

Suppose a speculator receives the signal $x_i$. If she is completely unsure about what other speculators will do, speculative pressure $\lambda$ can be anything between 0 and 1. Now consider the function:

$$V(x_i) := \int_0^1 \pi(x_i, \lambda) \, d\lambda$$

$$= \begin{cases} 
-q & \text{if } x_i < \ell \\
1 - \lambda^*(x_i) - q & \text{if } \ell \leq x_i \leq h \\
1 - q & \text{if } h < x_i.
\end{cases}$$

This function approximates the expected payoff for the speculator who receives signal $x_i$, if we assume that $x_i$ reflects the true state fairly well (which is true when $\epsilon$ is small), under the belief that $\lambda$ may range over all possible values. Recall that the variable $\lambda$ captures how the devaluation outcome depends on the decisions of other speculators. We will say that the risk dominant action at $x_i$ is “attack” if $V(x_i) \geq 0$, and that it is “refrain” if $V(x_i) \leq 0$.\(^{38}\) If we let the noise in speculators’ signals become small, uncertainty about the fundamentals disappears from the model. However, strategic uncertainty remains, and indeed agents will act accordingly.

\(^{38}\)The notion of risk dominance defined here departs from the usual one, which is only clearly defined for strategic situations involving two agents and two actions. In such contexts, an agent’s action is called risk dominant if it is the optimal action under the belief that the other agent is equally likely to choose either of her two actions. This idea is so similar to the one discussed above that I prefer to stick to the same terminology. For the (axiomatic) introduction of the notion of risk dominance refer to Harsanyi and Selten [96].
THEOREM 6: As \( \epsilon \) becomes vanishingly small, in the unique equilibrium point of the model, all agents choose the risk dominant action.

Thus, for sufficiently small \( \epsilon \), speculators act as if they are completely unsure about the behaviour of others. Under this belief, the optimal decision changes exactly at \( x^* \).

The theorem characterises a close relationship between strategic uncertainty and the solution of the model. For vanishing dispersion of the noise, the function \( V(x_i) \) converges to the payoff of attacking the currency for an agent who faces strategic uncertainty, and treats strategic uncertainty “as if” she is facing a particular kind of probabilistic risk; concretely, as if she is facing situation where she is completely unsure about the behaviour of others. Being completely unsure can be represented by a particular probability distribution, which is “uniform” in \( \lambda \), to the outcomes that may obtain: in equation (6) \( \lambda \) is integrated out using a uniform measure, which is called a “Laplacian belief” by Morris and Shin in [139] (where theorem 6 and the underlying idea were perhaps first discussed in detail). There, the risk dominant action is called the “Laplacian” action. This terminology refers to the “principle of indifference”, widely attributed to Laplace [124], especially p. 7, which states that unless one has good reason to do otherwise, one should assign equal probability to all possible mutually exclusive outcomes. Thus, one interpretation of theorem 6 is that in an equilibrium point each agent will tend to this Laplacian principle as far as she is confronted with strategic uncertainty, choosing to attack if based on her signal \( x_i \) she thinks the value of \( V \) is positive, and choosing to refrain if \( V \) is negative.\(^{39}\) Under this interpretation, the function \( V(x_i) \) may be thought of as representing the “tolerance” for strategic uncertainty of an agent that receives signal \( x_i \).

2.8. Concluding Remarks

While there appears to be no hope that an all-conclusive theory about how people reason will be developed any time soon, this chapter discussed a minimalistic currency crisis model that can be solved by analysing a reasoning process which simply discards non-optimal strategies in an iterative fashion. In this way, the process has been grounded in epistemic considerations. The main ingredients of the model are the presence of strategic complementarities in the interaction structure, and the presence of a small amount of heterogeneity in the information available to each agent. I have suggested that strategic complementarities may arise quite naturally in economic decision environments, particularly during financial crises. Arguably, the same holds true for heterogeneity in information. Nevertheless, I conclude with some further remarks on how this heterogeneity functions in the model.

\(^{39}\)Frankel et alii [80] also discuss this result. They note a similarity between the characterisation offered by equation (6), the potential functions for games studied by Monderer and Shapley [134], and—more generally—the “robust” equilibria studied by Kanjii and Morris [107], Ui [174] and Morris and Ui [142].
The model in this chapter is an archetypal example of a *global game* model. Some form of informational heterogeneity is a crucial ingredient in such models (but the particular kind of information structure chosen in this chapter is not). In the currency crisis model, agents form higher order beliefs about who receives what kind of information on the fundamentals. In an abstract sense the information structure simply induces the existence of different kinds, or “types”, of agents. Specifically, an agent’s type is determined by the signal she receives, and the difference in agents’ types is induced by the noisy distribution of signals around the true state of the economy. Since all agents observe signals about a similar fundamental, they have reason to believe the types of other agents are quite similar to their own. However, every agent has a slightly different type, as the dispersion of signals, however small, induces some heterogeneity.

The model identifies types with beliefs about or knowledge of fundamentals, and the informational heterogeneity destroys agents’ *common* knowledge about fundamentals. The following may make the crucial relationship between informational heterogeneity and departure from common knowledge more concrete. While a person may claim to know that economic fundamentals are not so bad, she usually cannot claim to know that everyone knows that everyone knows that fundamentals are not so bad, let alone that there is common knowledge that the fundamentals are not so bad (let alone that she knows everyone’s actions). In fact, “Common knowledge” of some fact or belief (which coincides with statement (5) for the purpose of this chapter, but see Ref. [130] for a formal definition) is a highly theoretical concept, and it is not at all clear if and how it can obtain in reality: the existence of heterogeneity in beliefs, knowledge, the interpretation of information, etc. is an unquestionable fact of life. Thus, arguably, failure of common knowledge about other people’s beliefs about the fundamentals is the more realistic feature of real-life decision environments. Also in global game models—crucially—agents’ common knowledge of the similarity between each others’ types (beliefs) does not hold, even though agents know “individually” that other agents are of a type quite similar to their own. Slight uncertainty about the distribution of types drives the process of infection, which forces agents to take into account all possible beliefs, and thus all the different conditions that economic fundamentals may assume. Strikingly, this infection process eventually leads to a unique equilibrium point.40

Yet even if agents deduce the unique equilibrium strategy (so that they know exactly what each *individual* type will do, since the equilibrium strategy is a map from *type* to *action*), they still face strategic uncertainty about the behaviour of other agents which originates in the slight uncertainty about the *distribution* of types, as each type of agent may behave differently. Therefore agents’ decisions are predominantly driven by strategic

40A paper by Morris and Shin [141] provides a more in-depth analysis of the role of the type space and how it drives the uniqueness result in global games.
uncertainty rather than by their knowledge of the fundamentals. The strategic uncertainty, which is captured in equation (6), drives agents to choose the risk dominant action. The risk dominant choice takes into account that beliefs may vary slightly from agent to agent, and that—due to the high non-linearity of the threshold equilibrium—such small changes in beliefs may have a big impact on the actual outcome of the model, particularly around the threshold.

The current model admits generalisations and different interpretations as long as the feature of failure of common knowledge about types is preserved. For instance, while we assumed in this chapter that signals were uniformly distributed around the true state, the results do not crucially depend on this. Uniqueness of equilibrium holds for any continuous distribution of signals that has a compact (infinite) support and small enough dispersion (and also for the normal distribution with small enough variance). Perhaps more strikingly, also the characterisation given in equation (6) for vanishing dispersion remains valid for any compactly supported continuous distribution of signals. In fact, Frankel et alii [80] show for a much wider class of similar models that neither the existence of a unique outcome nor its particular form depend on the information structure, provided the dispersion is small enough.

Failure of common knowledge about types can also arise in other ways. Hellwig [101] develops a model where agents are uncertain about each other’s degree of risk aversion, rather than about an underlying economic fundamental. Burdzy et alii [34] discuss a dynamic model where heterogeneity of types stems from small frictions, so that every agent chooses actions at slightly different moments in time. Steiner and Stewart [165] consider a model where people are known to look for similarities in different situations and are slightly uncertain how the other agents will perceive the current one. The uniqueness results in all of these models depend on infection in the type space, and all of them are in a way related to the basic global game.

The outcome of the model takes the form of a (joint) threshold strategy. Let me conclude this chapter with three comments on the properties of equilibrium points in threshold strategies. First, there are some good reasons why one may regard the equilibrium point in threshold strategies to be a particularly robust outcome of the analysis. If we want to approximate heuristics practised in reality, we are led to consider some natural restrictions on the class of strategies that we consider to be reasonable approximations of human behaviour. A plan of action must be based on a rule that results in clear cut behaviour and that is easy to implement. Threshold strategies, that identify cut-off points for observed variables so that if the variable remains below the cut-off point, one decision is taken, and if the variable

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41Noise independence of equilibrium does not hold for all global game models. A counter-example is the model in Corsetti et alii [47], which has a unique, yet noise dependent equilibrium.
CHAPTER 2. SELF-FULFILLING CURRENCY CRISES: REDUX

exceeds it, another, satisfy this requirement. They are natural decision rules that are easy to implement and economically intuitive. Indeed, from a behavioural perspective, it seems justified to restrict one’s attention to these types of strategies (we have not done so in this chapter, but in an application considered in chapter 6 of the dissertation we will restrict ourselves to finding threshold strategies).

Second, although threshold strategies give rise to self-fulfilling crises, clear scope for policy analysis is introduced. The analysis in this chapter shows that self-fulfilling crises take place when \( u \) enters the region \([v, h]\). This begs the question what policy measures can be taken that influence the position of the point \( v \) relative to \( \ell \) and \( h \). In the model discussed in this text, \( v \) is primarily influenced by the size of the transaction cost, \( q \), but in richer models, a wide variety of factors can play a role. This observation can be connected to the question of how to properly design robust financial institutions. Suppose one is able to pin down a probability distribution on the values that the economic fundamental \( u \) may take. The \textit{ex ante} probability of a crisis varies with the size of the region \([v, h]\), and hence the global game modelling approach provides insight into the likelihood of the economic system breaking down, and into what kind of measures may bolster it.

A third and final remark concerns the static nature of the model presented, which appears somewhat unsatisfactory. In chapter 4, we will show that the main implication of the model, the existence of a unique equilibrium point in threshold strategies, also emerges when a more dynamic context is considered.

2.A. Appendix: Proofs

Proof of Lemma 1. The conditions outlined in the lemma are sufficient to allow the application of a global implicit function theorem; see e.g. Zhang and Ge [182] for discussion. From the implicit function theorem, the result immediately follows.

Proof of Lemma 2. The first step in the proof is to derive an explicit expression for \( \pi(x_i, y) \). If everyone who receives a signal above \( y \) attacks, then we can write speculative pressure as a continuous and piecewise differentiable function of the true state \( u \):

\[
\lambda(u, y) = \begin{cases} 
0 & \text{if } u \in (-\infty, y-\varepsilon) \\
\frac{u-(y-\varepsilon)}{2\varepsilon} & \text{if } u \in [y-\varepsilon, y+\varepsilon] \\
1 & \text{if } u \in (y+\varepsilon, +\infty)
\end{cases}
\]

By lemma 1, the inverse of \( \lambda^* \), \( \lambda^* \), is defined on a compact convex subset \([\ell, h]\) of the real line, is continuous, and has slope \(-\frac{\partial \lambda}{\partial u} < 0\) and so is strictly decreasing. Fix \( y \) momentarily—for fixed \( y \), \( \lambda(u, y) : \mathbb{R} \to [0, 1] \) can be seen as an increasing continuous
We build upon two simple auxiliary claims. We derive the second part using a simple application of the implicit function theorem. Note where again $\sigma(u)$ and $\lambda(u, y)$ have a unique intersection point, say $a^*(y)$. To the left of the point $a^*(y)$, speculative pressure falls short of bringing down the peg, while to its right, speculative pressure exceeds the amount of pressure needed to bring down the peg. Whence, setting $D(\sigma) = \{u \in \mathbb{R} | u \geq a^*(y)\}$, equation (3) is the appropriate function to evaluate expected payoff for a speculator receiving signal $x_i$ given that everyone who receives a signal above $y$ attacks, i.e., is sought expression for $\pi$.

From the fundamental theorem of calculus, it is immediate that $\pi$ is continuous, since its integrand is bounded. Moreover, $\pi$ is increasing in its first argument—after all increasing $x_i$ while holding $y$ fixed broadens the support of the integral in (3). On the other hand, $\pi$ is decreasing in $y$, because whenever $y' \leq y$, $\lambda(u, y')$ lies everywhere below $\lambda(u, y)$. Since $\lambda^*(u)$ is decreasing, $a^*(y)$ must then increase. This settles the first part of the lemma.

We derive the second part using a simple application of the implicit function theorem. Note that equation (4) gives a point $x$ such that $\pi(x, y) < 0$, and similarly there is a point $\bar{x}$ such that $0 < \pi(\bar{x}, y)$. Since $\pi$ is continuous in $x$, there is a point $\hat{x}$ such that $\pi(\hat{x}, y) = 0$. By equation (3), at the point $\hat{x}$ we must have:

$$
\frac{1}{2\epsilon} \int_{\hat{x} \cap D(\sigma)} dv = q,
$$

where again $D(\sigma) = \{v \in \mathbb{R} | v \geq a^*(y)\}$, and it follows that $a^*(y)$ must lie in the interior of $\hat{x}$. Differentiating the left hand side of (7) with respect to $x$ gives $\pi_x = 1/2\epsilon > 0$ on a sufficiently small open interval about the point $\hat{x}$. Since $\pi$ is continuous and increasing in $x$, this shows that the zero point $\hat{x}$ is unique. Hence, setting $F(y) = \hat{x}$, we find a unique function $F : \mathbb{R} \to \mathbb{R}$ such that $\pi(F(y), y) = 0$.

Finally, we wish to show $F$ is continuous. Since $\pi_x = 1/2\epsilon$, $\pi$ is locally one-one for fixed $y$. Applying the implicit function theorem in (for instance) Kumagai [121] for locally one-one functions, we see that $F(y)$ is locally continuous. Since $y$ was arbitrary, it follows $F$ is continuous in every point $y \in \mathbb{R}$, hence, continuous.

* * *

Proof of Lemma 3. The proof of this result mirrors a well-known theorem from order theory. We build upon two simple auxiliary claims.

Claim 1: $F$ is monotonic.

Proof of Claim. We will prove this by contradiction. Suppose that $F$ is not monotonic. Then there exists points $x$ and $x'$ such that $x < x'$ and $F(x) > F(x')$. By definition, $\pi(F(x), x) = 0$ and similarly $\pi(F(x'), x') = 0$. By lemma 2, for each $x'$, $\pi$ has a unique
zero, and is increasing in its first argument. So $\pi(F(x), x') > \pi(F(x'), x')$. Again by lemma 2, $\pi$ is decreasing in its second argument. So $\pi(F(x), x) \geq \pi(F(x), x')$. By transitivity, $0 = \pi(F(x), x) > \pi(F(x'), x') = 0$, which is absurd. 

**Claim 2:** $F$ is bounded from above.

**Proof of Claim.** We have $\pi(x, y) = 1 - q$ for all $y$. As $\pi$ is increasing in its second argument, and bounded by $1 - q$, it must be that $\pi(x, y) = 1 - q$ for all $x \geq x$ (and all $y$). It follows immediately from the definition of $F$ that $F(y) < x$ for all $y$. 

We will exploit the continuity of $F$ to show a fixpoint can be obtained as the limit of the sequence $\langle L^n \mid n \in \mathbb{N} \rangle$. As $L^0 = x \leq F(x) = L^1$ and as $F$ is monotonic by claim 1, it follows this is an increasing sequence of reals. By claim 2, the sequence is bounded above. An infinite increasing sequence of reals that is bounded above must converge to a point $\mu$, by the completeness property of the reals. As continuous maps preserve limits, we find:

$$L = \lim_{n \in \mathbb{N}} L^n = \lim_{n \in \mathbb{N}} F(L^n) = F(\lim_{n \in \mathbb{N}} L^n) = F(L).$$

So $L$ is a fixpoint.

Finally, let $x^*$ be any fixpoint of $F$. It is obvious that $x \leq x^*$. As $x^*$ is a fixpoint, $F(x^*) = x^*$, while $F(x) = L^1$. As $F$ is monotonic, $L^1 \leq x^*$. By induction $L^n \leq x^*$ for all $n$. It follows $L = \lim_{n \in \mathbb{N}} L^n \leq x^*$. So $L$ is the least fixpoint. This completes the proof. 

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**Proof of Theorem 5.** To prove the result we will derive necessary conditions for a point $x^*$ to be an equilibrium in threshold strategies; and show these are satisfied by at most one point.$^{42}$

Suppose speculators implement a threshold strategy around $x^*$. First, note that when speculative pressure is $\lambda$ by lemma 1 the failure point for $\lambda$ is $u^*(\lambda)$, and that the function $u^*(\lambda)$ is invertible (recall we denote its inverse by $\lambda^*(u)$).

Second, given the strategy around $x^*$, let $v$ be a corresponding failure point. If all speculators follow a strategy around $x^*$, then, whenever the true state is $v$, it must be that $\lambda$ is just the proportion of agents whose signal is above $x^*$. From the uniform distribution we have:

$$\lambda = 1 - \frac{x^* - (v - \epsilon)}{2\epsilon}$$

with the obvious qualification that $\lambda \in [0, 1]$.

As $\lambda^*(v) = \lambda$, we find an implicit expression for the failure point:

$$\lambda^*(v) = 1 - \frac{x^* - (v - \epsilon)}{2\epsilon}.$$  

$^{42}$This proof was given by Professor Shin during a summer school on global games in 2004.
Third, if \( x^* \) is an equilibrium in threshold strategies, the agent receiving signal \( x^* \) must be indifferent:

(9) \[ P(u > v|x^*) - q = 0 \iff 1 - \frac{v - (x^* - \epsilon)}{2\epsilon} = q. \]

Solving equation (9) for \( x^* \) and substitution for \( x^* \) in equation (8) gives:

\[ \lambda^*(v) = 1 - q; \]

which a unique solution, since \( \lambda^*(v) \) is a strictly decreasing continuous function. Consequently, from equation (9) we obtain the following expression for the threshold:

(10) \[ x^* = \epsilon(2q - 1) + u^*(1 - q). \]

It follows that there is exactly one threshold strategy to be considered as a candidate for an equilibrium in threshold strategies. This observation completes the proof.

\[
\text{Proof of Theorem 6.} \text{ By the proof of theorem 5, speculative pressure at the failure point } v \text{ is given by } \lambda^*(v) = 1 - q. \text{ Consider the speculator who receives the threshold signal } x^*. \text{ From her perspective, the true state } u \text{ is in the set } [x^* - \epsilon, x^* + \epsilon], \text{ and—since } x^* \text{ is the signal above which all speculators attack—} \lambda \text{ increases linearly from 0 to 1 on this set. Hence, from her perspective, the probability that } \lambda \text{ exceeds } 1 - q \text{ is just } q, \text{ which equals } 1 - \lambda^*(v). \text{ If we let } \epsilon \to 0, \text{ then by equation (10) the corresponding sequence of thresholds } x^*_n \text{ converges to } v, \text{ so that (by the continuity of } \lambda^* \text{) the expression } 1 - \lambda^*(x^*_n) - q \text{ converges to } 1 - \lambda^*(v) - q—\text{which is just the expected payoff for the speculator receiving the threshold signal. Since the speculator who the signal } x^* \text{ is indifferent in equilibrium it must be that—for vanishing } \epsilon—V(x_i) = 1 - \lambda^*(x_i) - q = 0 \text{ when } x_i = x^*. \text{ Since } \lambda^*(x_i) \text{ is a decreasing function, } V(x_i) < 0 \text{ when } x_i < x^* \text{ and } V(x_i) > 0 \text{ when } x_i > x^*. \]

\* * *