"Nonmonotonic" does not mean "probabilistic"

Stenning, K.; van Lambalgen, M.

Published in:
Behavioral and Brain Sciences

DOI:
10.1017/S0140525X0900048X

Citation for published version (APA):
Baye–way to deal formally with the pervasive nonmonotonicity of common issues, because this denies the environmental relativity of adaptation and a true rational analysis accordingly takes account of this another. Adaptation is defined with respect to a specific environment, so that adaptation is defined with respect to a specific environment. For example, the definition of adaptation in the rational analysis approach of Anderson (1990) is to view cognitive processes as approximating some normative standard of correct performance. This goes beyond the point at which a rational analysis becomes applicable, which is when the approximation is sufficiently close to the normative standard.

O&C argue that their probabilistic model – which represents the conditional “if p, then q” as a conditional probability P(q|p) – can account for suppression effects. To explain the model and our criticism, we must first state the main Bayesian mechanism for updating probabilities, “Bayesian conditionalization”: (BaCo). The absolute subjective probability of event D given that evidence E has been observed is equal to the conditional probability P(D|E). Here E should encompass available evidence. In this context it is important to note that Bayesian conditionalization is a nonmonotonic principle: An extension of the evidence p may invalidate a previous posterior probability for q derived by (BaCo) applied to P(q|p).

Informally, the O&C model works like this: One is given the conditional probability P(q|p) with value, say, 1 – e for some positive but small e. If few exceptions are salient, e is small, and, given p, (BaCo) yields that q can be concluded with high probability (namely, 1 – e). The second conditional highlights a possible exception (the library being closed), which leads to an increase in e, and hence to a decrease in the a posteriori probability of q. But appealing though this picture is, it is not Bayesian (Stenning & van Lambalgen 2008b). Consider two putative Bayesian processes that can change the value of P(q|p) when new possible exceptions – say, not-r with probability 1 – P(r) – become salient.

(1) The probability space is enlarged from {p, q} to {p, q, r} (where r stands for, say, “the library is open”), and this leads to a new representation of P(q|p). One may write

\[
P(q | p) = \frac{P(p \land q)}{P(p)} = \frac{P(p \land q \land r) + P(p \land q \land \neg r)}{P(p)}
\]

\[
= \frac{P(p \land q \land r)}{P(p)} \frac{P(p \land q \land r)}{P(p \land r)} + \frac{P(p \land q \land \neg r)}{P(p \land r)} \frac{P(p \land q \land \neg r)}{P(p)} = P(q | p \land r) P(r)
\]

Figure 1.

where the last equality follows under the plausible assumption that p and r are independent. O&C assume that the subject assigns a lower probability to P(q|p) in the enlarged representation P(q|p & r)P(r), suggesting that this is because the subject lowers P(r) from 1 to a value smaller than 1 when becoming aware of possible exceptions. In probability theory, P(q|p & r) P(r) would simply be a different representation of P(q|p), and the values of these expressions must be the same. There are no rationality principles governing changes in probabilities when enlarging the probability space, or rather there is one such
probability distribution first assigns probability 0 to \( q \) and \( r \), and, upon becoming aware of the second conditional \( \text{“if } r \text{ then } q \text{”} \), a nonzero probability. The trouble with such a suggestion is that from a Bayesian point of view, the transition from the a priori probability \( P(\text{not-}r) = 0 \) to the a posteriori \( P(\text{not-}r) > 0 \) is not allowed, because this cannot be achieved via (BaCo): conditiona-
izing on more evidence cannot make a null probability positive. One thus needs an additional rationality principle (beyond [BaCo]) governing such transitions. In the absence of such a principle, one has to assume that the probabilities of all non-salient exceptions (such as \( \text{not-}r \)) are initially very small but nonzero. This increases the computational complexity of probabilistic reasoning enormously: One requires massive storage and intricate computations to maintain consistency of the probability assignment.

These considerations show that in order to account for the data on the suppression task any probabilistic model needs to be sup-
plemented with a theory about nonmonotonic and non-Bayesian, but still somehow rational, changes in degrees of belief. One may then question whether a probabilistic model is necessary at all; Stenning and van Lambalgen (2005; 2008a) provide a model cast entirely in terms of nonmonotonic logic.

(2) An orthodox Bayesian alternative would be a construction in which the probability spaces remain the same (namely, the universal space based on all possible propositions), but the probability distributions change. In our toy world, the probability space is in both cases \([p, q, r]\), but one could assume that the probability distribution first assigns probability 0 to \( \text{not-}r \), and, upon becoming aware of the second conditional \( \text{“if } r \text{ then } q \text{”} \), a nonzero probability. The trouble with such a suggestion is that from a Bayesian point of view, the transition from the a priori probability \( P(\text{not-}r) = 0 \) to the a posteriori \( P(\text{not-}r) > 0 \) is not allowed, because this cannot be achieved via (BaCo): conditiona-
izing on more evidence cannot make a null probability positive. One thus needs an additional rationality principle (beyond [BaCo]) governing such transitions. In the absence of such a principle, one has to assume that the probabilities of all non-salient exceptions (such as \( \text{not-}r \)) are initially very small but nonzero. This increases the computational complexity of probabilistic reasoning enormously: One requires massive storage and intricate computations to maintain consistency of the probability assignment.

These considerations show that in order to account for the data on the suppression task any probabilistic model needs to be sup-
plemented with a theory about nonmonotonic and non-Bayesian, but still somehow rational, changes in degrees of belief. One may then question whether a probabilistic model is necessary at all; Stenning and van Lambalgen (2005; 2008a) provide a model cast entirely in terms of nonmonotonic logic.