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**Abstract:** Oaksford & Chater (O&C) advocate Bayesian probability as a way to deal formally with the pervasive nonmonotonicity of common sense reasoning. We show that some forms of nonmonotonicity cannot be treated by Bayesian methods.

One argument that Oaksford & Chater (O&C) proffer in *Bayesian Rationality* (Oaksford & Chater 2007, henceforth BR) for a probabilistic approach to cognition is the pervasiveness of nonmonotonicity in reasoning: almost any conclusion can be overturned, if additional information is acquired. They claim that nonmonotonic inferences fall outside the scope of logical methods, and that probability theory must be preferred.

This judgment of the authors does not reflect the present state of play (see, e.g., Antoniou 1997, especially Part V). There are several good logical formalisms for nonmonotonic reasoning in existence, some of which even have a computational complexity that is vastly less than that of classical logic. More importantly, even in those cases where probabilistic modeling is in principle useful, one sometimes needs to supplement the model with a nonmonotonic logic to account for all the data. We illustrate this here by using an example from Chapter 5 of BR, not mentioned in the present Précis, but relevant throughout: the suppression task (Byrne 1989). When subjects are presented with the *modus ponens* material "If she has an essay she studies late in the library. She has an essay," they almost universally draw the conclusion: "She studies late in the library." When instead they are presented with the same two premises *plus* the premise: "If the library is open she studies late in the library," about half of them withdraw the inference. Logically speaking, we here have an instance of nonmonotonicity: addition of a premise leads to withdrawal of the original conclusion.

O&C argue that their probabilistic model – which represents the conditional "if p, then q" as a conditional probability \( P(q|p) \) – can account for suppression effects. To explain the model and our criticism, we must first state the main Bayesian mechanism for updating probabilities, "Bayesian conditionalization": (BaCo). The absolute subjective probability of event \( D \) given that evidence \( E \) has been observed is equal to the conditional probability \( P(D|E) \). Here \( E \) should encompass available evidence. In this context it is important to note that Bayesian conditionalization is a nonmonotonic principle: An extension of the evidence \( p \) may invalidate a previous posterior probability for \( q \) derived by (BaCo) applied to \( P(q|p) \).

Informally, the O&C model works like this: One is given the conditional probability \( P(q|p) \) with value, say, 1 – for some positive but small \( e \). If few exceptions are salient, \( e \) is small, and, given \( p \), (BaCo) yields that \( q \) can be concluded with high probability (namely, \( 1 – e \)). The second conditional highlights a possible exception (the library being closed), which leads to an increase in \( e \), and hence to a decrease in the \( a \) posteriori probability of \( q \). But appealing though this picture is, it is not Bayesian (Stenning & van Lambalgen 2008b). Consider two putative Bayesian processes that can change the value of \( P(q|p) \) when new possible exceptions – say, \( not-r \) with probability 1 – become salient:

1. The probability space is enlarged from \( \{p, q\} \) to \( \{p, q, r\} \) (where \( r \) stands for, say, "the library is open"), and this leads to a new representation of \( P(q|p) \). One may write

\[
\frac{P(q | p)}{P(p)} = \frac{\frac{P(p \land q)}{P(p)} - \frac{P(p \land q \land r)}{P(p)}}{P(p)} = \frac{\frac{P(p \land q \land r)}{P(p)}}{P(p)} = \frac{P(q | p \land r)}{P(r)}
\]

(Figure 1)

where the last equality follows under the plausible assumption that \( p \) and \( r \) are independent. O&C assume that the subject assigns a lower probability to \( P(q|p) \) in the enlarged representation \( P(q|p \land r)P(r) \), suggesting that this is because the subject lowers \( P(r) \) from 1 to a value smaller than 1 when becoming aware of possible exceptions. In probability theory, \( P(q|p \land r) \) would simply be a different representation of \( P(q|p) \), and the values of these expressions must be the same. There are no rationality principles governing changes in probabilities when enlarging the probability space, or rather there is one such
probability distribution first assigns probability from a Bayesian point of view, the transition from the a priori nonzero probability. The trouble with such a suggestion is that salient exceptions (such as principle, one has to assume that the probabilities of all non-BaCo) governing such transitions. In the absence of such a allowing on more evidence cannot make a null probability positive. This increases the computational complexity of probabilistic reasoning enormously: One requires massive storage and intricate computations to maintain consistency of the probability assignment.

These considerations show that in order to account for the data on the suppression task any probabilistic model needs to be supplemented with a theory about nonmonotonic and non-Bayesian, but still somehow rational, changes in degrees of belief. One may then question whether a probabilistic model is necessary at all; Stenning and van Lambalgen (2005; 2008a) provide a model cast entirely in terms of nonmonotonic logic.