Designing and implementing switch problems for mathematical discussion, reasoning and level raising

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Aims

Most of the current math methods in The Netherlands for secondary education are based on the fact that students should be able to learn autonomously. Teachers are used to this situation: they allow students to consult each other and they give explanations to students if they ask for it. Meanwhile, teachers often feel that students' learning is too little intrusive and that their ability to think mathematically, that is needed for level raising, is not enough developed. The term level raising refers to the levels of Van Hiele (1986), who distinguished levels of mathematical understanding. Crucial for student's level raising is reflection on one's own mathematical activity and learning process (Freudenthal, 1978). At the international level, the concept of level raising is connected to abstraction in context (Hershkowitz, Schwarz & Dreyfus, 2001).

In the present study we want to explore interventions that stimulate students to discuss and reason mathematically, aiming at level raising. Processes of mathematical level raising have been modeled by Dekker and Elshout-Mohr (1998), who created a framework to help collaborating students in the process of level raising. This model, the process model, distinguishes key activities and regulating activities. The key activities (showing, explaining, justifying and reconstructing one's work) help students to achieve a higher level of mathematical understanding. The regulating activities (for example student A criticizes student B's work) regulate the key activities of the other student and keep the interaction going.

One of the issues within this framework is how teachers can support students' mathematical level raising in collaborative learning settings. Research in The Netherlands does not show a clear picture so far. Dekker & Elshout-Mohr (2004) and Pijls, Dekker & Van Hout-Wolters (2007) investigated what kind of teacher intervention, aimed at helping 16-17 years old students in accomplishing collaborative tasks, is effective. Two kinds of teacher interventions were distinguished and compared: the process help where the teacher stimulates the students to perform...
the regulating and key activities for themselves; the product help where the teacher focuses on the mathematical content of the tasks and the products of the students.

In the study of Dekker & Elshout-Mohr occurred more level raising in classes where the teacher provided process help than in classes where the teacher provided product help. The study of Pijls, Dekker & Van Hout-Wolters did not show significant differences, although the product help did not work better than the process help.

In the study of Pijls, teachers told that they should like to have given more product help instead of only process help. This fact is also confirmed by the ILO/ELWIER research project of Dekker and Pijls (submitted).

Both studies stressed the importance of using suitable mathematical problems: (i) realistic and meaningful to the students; (ii) complex in order to allow several approaches; (iii) constructive character in a way that student’s thinking become visible to others; (iv) aiming at level raising in a way that the problem when approached at lower lever may not be accomplished. (Dekker, 1991).

The focus of the present study is to develop and use mathematical problems that fulfill these characteristics: switch problems. This type of problems will be used instead of the regular book on crucial moments regarding level raising. During these tasks students work in heterogeneous groups of three students and the teacher focuses on the progress of the mathematical discussion in the groups.

The research questions are:

1. Do the developed switch problems lead to mathematical discussions, reasoning and level raising?
2. Do the students in classes using switch problems reach more level raising than students in classes where the chapters are treated in the regular way?

Switch problems are strongly connected with the pedagogical and mathematical context where they are presented. Thus, the following mathematical problem may be a potential switch problem or just an regular problem depending on the context that it is used.

Given triangle \(ABC\) and equilateral triangles \(BCP\) and \(ACQ\). Proof that \(AP=BQ\).

This problem was taken from the school book and it is part of a chapter about proof in Geometry for 15/16 years old students at senior high school. We change a little bit the text of the question in order to underline the activity of ‘proofing’. At the moment that students started to
work on this topic they were not familiar with proof in Geometry. This topic is introduced in the book through familiar contexts as basic geometrical figures, angles and similarity of triangles.

In this research project we developed three worksheets with potential *switch problems* and we tried them out during 4 lessons spread through the chapter. The problems were selected from the book according to the four criteria already mentioned; the worksheets that we used were still in an initial developing stadium and they should be seen as draft versions. The example of switch problem presented above belongs to the first worksheet.

We aimed with these worksheets that students discuss mathematically with each other, experience the need of mathematical proof and, that they recognize and use mathematical arguments while reasoning and discussing. For example, an mathematical argument regarding the presented switch problem, can be: “triangles $\triangle ACP$ and $\triangle BCQ$ are congruent so, in particular $AP=BQ$”. On the other side arguments as “I have measured both segments and therefore they are equal” should not be accepted.

The *switch problem* that we present here, although a simple one, is not a standard problem for the students at the beginning of the book chapter. In order to solve this problem a student must:

1. understand the meaning of mathematical proof; use and recognize mathematical arguments and reasoning’s
2. get the idea of using congruence of triangles; identify $AP$ and $BQ$ as the respectively side of triangles $\triangle ACP$ and $\triangle BCQ$; justify that $AC=QC$, $CP=CB$ and $\triangle ACP=\triangle QCB$; justify that triangles $\triangle ACP$ and $\triangle BCQ$ are congruent.
3. write a mathematical proof using correct arguments, language and notation

In this way we consider it a key problem regarding this chapter. Further, it seems to fulfill the mentioned four criteria:

- it is realisitic since to proof that length equality of two segments is a meaningful problem within the specific mathematical context of this chapter;
- it is complex in the way that this was a non standard problem at the moment it was presented to students and to be able to solve it they should be able to perform the three steps described above, which demands several abilities.
- It has a constructive character in the way that students have to produce together a mathematical proof;
- It aims level raising in the way that when students start to solve the problem, they probably don’t possess enough knowledge to accomplish it. For example, it is new to students to use congruence as middle for geometrical proof; it is also new to recognize mathematical arguments and to produce a mathematical proof.

**Methodology**

In a beginning stage of the study, the focus is on the first research question and we will follow a design research approach. *Switch problems* will be developed by the researcher and teachers involved. The first step in this stage is to identify crucial moments in the subject matter where level raising should occur and to select and develop *switch problems* that may support this process.

The first learning materials are trailed in October 2008- May 2009 in mathematics lessons with 16/17 year old students at secondary level. The researcher specifically observes and analy-
ses the lessons where the students work with the developed materials to see if these ones lead to mathematical discussions, reasoning and level raising. Through pre- and post testing, it is checked if level raising has taken place.

In the second part of the study it will be examined if in classes with switch problems more raising level is reached than in the regular classes and the experiment will be extended to more than one school and class.

**Expected results**

In the first try-out held in November 2008 students worked on a geometry chapter and at certain moments they worked with worksheets with switch problems and the teacher experimented with process help. The researcher observed and analysed these lessons and that lead to some interesting results that we would like to present at CIEAEM 61. For example: the first and third worksheet worked good but the second one didn’t work at all. Why is that? What are the success or failing characteristics of switch problems involved? Another example: the researcher video taped and analysed two groups of students during these lessons: in one group it was found some evidence of level raising and in the other group not. Why is that? The switch problem was the same for both groups so…. What other factors play a role in the use of switch problems?

Further, it is expected that the present study will give more scientific insight into the way of teaching mathematics focused on mathematical discussion, reasoning and level raising, especially into the characteristics of the tasks, the composition of the small groups and the role of the teacher.

Practical outcome will be a series of switch problems for mathematical level raising and more information about the composition of the small groups and effective teacher interventions that may lead students to a higher level of mathematical understanding and competence in a setting that supports autonomous learning.
References


