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**A supersymmetric model for lattice fermions**

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# Summary

In the past decades the study of solid state materials with strongly interacting electrons has become a major area of research. These materials exhibit exotic effects such as high temperature superconductivity and giant magnetoresistance. While extensively studied in the lab, these systems challenge our theoretical understanding, since traditional condensed matter techniques fail when analyzing strongly interacting electrons. Due to the strong interactions, these systems are truly governed by many-body effects, rendering descriptions based on perturbation theory unreliable. It is well known that for fermionic particles, such as electrons, the analysis of many-body physics is plagued by sign problems.

This thesis describes a lattice model for itinerant spin-less fermions with strong repulsive interactions. A judicious tuning of the kinetics and the interactions of the fermions leads to models possessing supersymmetry. Quite remarkably, the notion of supersymmetry, which was developed in the context of high energy physics, turns out to be a powerful tool in the analysis of strongly correlated itinerant fermions. In the model discussed here, the supersymmetry induces a subtle competition between the kinetics, on the one hand, and the strong repulsive interactions, on the other hand. As a result the system realizes its lowest energy states in a regime where both the kinetics as well as the interactions play an important role. It is precisely this regime that is inaccessible via traditional techniques. Supersymmetry, however, provides us with a rich mathematical structure that can be employed to derive rigorous results for some of the key features of the model.

The supersymmetric model was first introduced by P. Fendley, K. Schoutens and J. de Boer in 2003 [22]. A variety of results for this model on one and two dimensional lattices was known at the time the research for this thesis started. In 1D it was shown that the supersymmetric model on the chain is quantum critical, with the low energy continuum limit described by an  $\mathcal{N} = (2, 2)$  superconformal field theory. On 2D lattices one generically finds superfrustration, a strong form of quantum charge frustration leading to an extensive ground state entropy. A heuristic understanding of superfrustration was given by the "3-rule": to minimize the energy, fermions prefer to be mostly 3 sites apart. For generic two dimensional lattices the 3-rule can be satisfied in an exponential number of ways. This picture was supported by a remarkable relation found by J. Jonsson between quantum ground states of the supersymmetric model and tilings of the lattice.

The main subject of this thesis is the supersymmetric model on the square lattice. First of all, inspired by the work of J. Jonsson, we establish a rigorous mathematical result which relates quantum ground states to certain tiling configurations on the square lattice. Due to supersymmetry, the quantum ground states of the model are in one-to-one correspondence with cohomology elements. To compute the cohomology is in general very difficult, however, using a spectral sequencing technique this problem can be cut into several smaller problems. For the square lattice with periodic boundary conditions all these sub-problems

turn out to be non-trivial, rendering the proof of the ground state-tiling theorem rather involved. From this theorem, however, we obtain a closed expression for the total number of ground states on the square lattice and the number of particles in each ground state. For periodic boundary conditions we find that the number of ground states grows exponentially with the linear dimensions of the system, leading to a sub-extensive ground state entropy. We furthermore find that the ground states occur for densities between  $1/5$  and  $1/4$ .

Second of all, we find substantial analytic and numerical evidence that for the square lattice with open boundary conditions the system has gapless edge modes. For this result we built on the understanding of the supersymmetric model on the 1D chain. Numerical studies of this system provide a detailed relation between the lattice model on the one hand, and its continuum description, a superconformal field theory, on the other hand. In particular, we relate a boundary twist in the lattice model to a spectral flow in the superconformal field theory. This relation is not exclusive to the chain. Consequently, the analysis of a boundary twist, or spectral flow analysis, is an important new technique to investigate the supersymmetric model. By applying this spectral flow analysis to ladder realizations of the square lattice, we find compelling evidence for quantum criticality in these systems. Furthermore, by combining the ground state-tiling relation with our findings for the ladder models, we arrive at a picture in which the tilings not only count the number of ground states, but actually dominate the ground state wavefunctions. This picture allows us to propose the existence of critical edge modes in the supersymmetric model on the two dimensional square lattice with a boundary.

Finally, combining these two results, the ground state-tiling relation on the one hand, and the results for the ladders on the other hand, we conclude that the supersymmetric model on the square lattice exhibits a novel phase. This phase is characterized by a sub-extensive ground state entropy and edge criticality. The work in this thesis underlines the intricate relation between supersymmetry on the one hand, and criticality and quantum charge frustration on the other hand, thereby making the supersymmetric model a promising system for future research.