Magnetotransport studies of the single and bilayer two dimensional electron gas in the quantum Hall regime

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2. Theoretical concepts

The quantum Hall effect (QHE) provides an exemplary study of quantum phase transitions (QPT). The most powerful theory to study (quantum) phase transitions available today is the renormalization group theory (RGT). Founding father of the RGT is K.G. Wilson [1-4], who in 1985 received the Nobel price for it. One of the strengths of the RGT is the capability of describing the system arbitrary close to the quantum critical point. This is done by renormalizing the system, i.e. redefining the concept of unity. Unity in the case of QPTs is defined by characteristic length scales, like the correlation length. When a system approaches the quantum critical point (QCP), a divergence of the characteristic length scales occurs. The RGT enables one to map a system asymptotically close to the QCP, thus having a certain length scale, onto the same system with a different length scale. None of the other existing theories, like mean field theory, and Landau-Lifshitz theory are capable of approaching the QCP as close as the RGT. The RGT predicts that the characteristic length scales of a system obey scaling. A system is said to scale if the equations of state, asymptotically close to the critical point, for different length scales are related to each other by so-called scaling factors (a_H and a_T). *Eq*. 2.1 gives a general expression of scaling behavior.

$$F_S(\lambda^{a_H} A, \lambda^{a_T} L) = \lambda F_S(A, L) \tag{2.1}$$

Critical exponents can always be expressed as a funtion of a_H and a_T. Systems with the same values of critical exponents and scaling functions belong to the same universality class. Essential to scaling is power-law behavior. If a system is proven to be renormalizable, this power-law behavior for the particular system is also observable by means of experiment. Pruisken has proven the renormalizability of the QHE for the non-interactive case in 1987 [5,6] and later for systems which include interactions [7]. The renormalizability of the QHE can be made visible in a so called renormalization flow diagram. An example is given in *Fig*. 2.1. Basically the diagram shows the evolution of a quantum Hall system in the σ_{xx}-σ_{xy} plane as a function of the perpendicular magnetic field. Ideal semi-circles represent a homogeneous electron gas at T = 0 K (relevant flow). The vertical arrows in *Fig*. 2.1 show the flow of the system from $T \neq 0$ to T = 0 K (irrelevant flow).

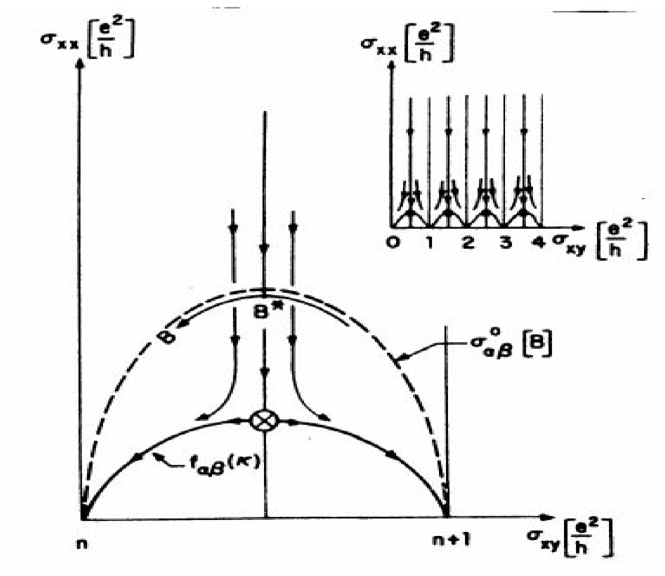

Figure 2.1 Renormalization flow diagram for the QHE. The top right corner shows four different transitions that obey the same scaling laws i.e. belong to the same universality class. Picture taken from *Ref.* [19].

The integer values i = 0, 1, 2 etc. on the σ_{xy} axis represent the stable fixed points. The values i + ½ represent the unstable fixed points. These are the QCPs at which the QPTs occur. At the QCP the components of the conductivity tensor attain the values [8]:

$$\sigma_{xx,c} = \frac{1}{2}, \sigma_{xy,c} = n + \frac{1}{2}; n = 0,1,2... \tag{2.2}$$

Using *Eq*. 2.2 we can define the scaling variables:

$$\sigma = \sigma_{xx} - \frac{1}{2}, \ \theta = \sigma_{xy} - n - \frac{1}{2} \tag{2.3}$$

These are the variables that are transformed upon renormalization. The QHE effect is the only system in which the scaling variables can be directly measured. The conductivity tensor follows from the measured resistivities as shown in *Eq*. 2.4.

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}, \quad \sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} \tag{2.4}$$

Eq. 2.4 also describes the unusual state of both ρ_{xx} and σ_{xx} being zero when ρ_{xy} forms a plateau.

From a pure theoretical point of view the scaling functions can be constructed for $T = 0$. Experimentally this is inaccessible, so the scaling functions need to be adapted to the experimental situation of $T \neq 0$. This is done by coupling the temperature to the effective size of the system (finite-size scaling). As effective size is taken the phase coherence length L_φ:

$$L_\varphi = T^{-p/2}, \tag{2.5}$$

where p is the dynamical critical exponent. Renormalizing the system changes the effective size as: $L \to bL$, which in turn changes the scaling variables according to *Eq*. 2.1: $\theta \to b^{y_\theta}\theta$ and $\sigma \to b^{-y_\sigma}\sigma$, where y_θ is the relevant and y_σ the irrelevant critical exponent. Another relevant length scale is the localization length L_ξ:

$$L_\xi = \left|E - E_{n,s}\right|^{\chi}, \tag{2.6}$$

where $E_{n,s}$ is the energy at the center of the Landau level. When $L_\xi > L_\varphi$ electrons in the 2DEG are delocalized. The renormalization flow functions [9] have the shape:

$$\frac{d\sigma_{ab}}{d\ln L} = \beta_{ab}(\sigma_{xx}, \sigma_{xy}, c), \tag{2.7}$$

where

$$\sigma_{ab}(L_\varphi, B) = f_{ab}(L_\phi^{1/\chi}(B - B_c)) + O(L_\phi^{1/\chi}(B - B_c)^2, L_\varphi^{-y_\sigma}) \tag{2.8}$$

is the conductivity tensor and *ab* stands for the arbitrary combination of *x* and *y*.
It has been shown that the relevant exponent y_θ is just the inverse of the localization length exponent χ. From *Eq*. 2.8 we can draw two important conclusions. The first one is that the irrelevant exponent y_σ starts to play a role only at higher temperatures. The second (by

substituting the measurable quantity of T, using *Eq*. 2.5) is that the relevant critical exponent κ obtained by experiment equals:

$$\kappa = \frac{p}{2\chi} \tag{2.9}$$

The value of χ has analytically been estimated to be 2.3 ± 0.4 [10] for the non-interacting case. This is in good agreement with numerical results [11-16]. There is a strong uncertainty concerning the value of p, which is estimated to be bounded between 1 for the Fermi-liquid approach and 2, where the main scattering mechanism is due to electron-electron interactions, both for zero magnetic field [17]. Using these values for p and χ, the value of κ according to *Eq*. 2.8 should lie between 0.22 and 0.43. Consequently the exact value of κ is not predicted by theory, but can only be determined by experiment. In the experimental data certain quantities obey power-law temperature dependence involving κ as exponent. The width of the peaks in the ρ_{xx}-curves ($\Delta\nu$) and the maximum slopes of the transitions in the ρ_H-curves both obey power-law T dependence:

$$\Delta\nu \propto \left(\frac{d\rho_{xy}}{d\nu}\right)_{\max} \propto T^{\kappa} \tag{2.10}$$

In the same way the PI-transition around the crossing point satisfies the following relation (Shahar *et al.* [18]):

$$\rho_{xx}(\nu,T) = \frac{h}{e^2}\exp\left(-\frac{\nu-\nu_c}{\nu_0(T)}\right), \tag{2.11}$$

with

$$\nu_0(T) = (T/T_0)^{\kappa}, \tag{2.12}$$

where T_0 is a phenomenological temperature. The term between brackets in *Eq*. 2.11 is in fact the scaling variable X:

$$X = \frac{\nu-\nu_c}{(T/T_0)^{\kappa}} \tag{2.13}$$

It has been shown that for the QHE the components of the conductivity tensor, σ_{xx} and σ_{xy}, which are functions of both the magnetic field and the temperature (see *Fig*. 2.1), can in fact be described as functions of the single scaling variable X [19].

The corrections to scaling are described by the second term of *Eq*. 2.8. Even though present in every quantum Hall transition, the experimentally observable behavior that is only controlled by the irrelevant exponent y_σ is the deviation of the Hall resistance from the quantized value at the PI-transition:

$$\rho_{xy} = 1 + \eta(T)e^{-X}, \ \eta(T) = (T/T_1)^{y_\sigma} \tag{2.14}$$

Here ρ_{xy} is given in units of h/e^2 and T_1 is a phenomenological temperature scale. The role of the critical exponents can be easily explained using the renormalization flow diagram (*Fig*. 2.1). Central in this diagram are the fixed points. Starting from the unstable fixed points at $\sigma_{xy,c} = n + ½$, with n being an integer, there are four directions to go: Two are along the semicircle ($T = 0$) towards the stable fixed points. The other two directions are toward the unstable fixed point originating outside the semicircle ($T \neq 0$). Simply put, the flow along the semicircle, in the neighborhood of the unstable fixed point is characterized by the relevant critical exponent κ, and the flow in the 'vertical direction' originating outside the semicircle is characterized by the irrelevant critical exponent y_σ.

The relevant and irrelevant critical behavior is combined in the universal scaling functions (to first order in X and η) [20]:

$$\sigma_0(X,\eta) = \frac{e^{-X}}{1 + 2\eta e^{-X} + e^{-2X}} \tag{2.15}$$

and

$$\sigma_H(X,\eta) = \frac{1 + \eta e^{-X}}{1 + 2\eta e^{-X} + e^{-2X}} \tag{2.16}$$

These functions describe every point in the renormalization flow diagram in the critical regime. Also they describe all quantum Hall transitions in the critical regime and for this reason are universal. So the relevant and irrelevant critical behavior is the evolution of a quantum state near the critical point, under influence of the magnetic field and temperature. *Eqs*. 2.15 and 2.16 include two important symmetries concerning scaling behavior in the QHE.

- Particle-hole symmetry:

$$\sigma_0(X,\eta) = \sigma_0(-X,\eta) \tag{2.17}$$

$$\sigma_H(X,\eta) = 1 - \sigma_H(-X,\eta) \tag{2.18}$$

- Periodicity in σ_H.

The second symmetry is another expression of the universality of the quantum Hall transitions.

The concepts discussed in this chapter will be used to analyze the experimental results presented in chapters 4 and 5 of this thesis.

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