Magnetotransport studies of the single and bilayer two dimensional electron gas in the quantum Hall regime

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Publication date
2010

3. Experimental aspects

3.1 Introduction

Traditionally magneto-transport measurements on two-dimensional electron-gasses (2-DEG's) in order to probe the quantum Hall effect have been done using the lock-in technique. The reason for this is that the measurements take place at temperatures ranging from a few Kelvin to sub-Kelvin temperatures and a very small current has to be used (varying from a few nano-amperes to even less than 1nA) in order to prevent Joule heating. Measuring with a small current becomes even more important when the 2DEG reaches the insulating regime and the resistance of the sample diverges exponentially, since heat is dissipated as the product of the resistance with the square of the current. Under these experimental conditions measuring the electrical resistance using the lock-in technique will give the best signal-to-noise ratio. The sinusoidal signal also causes thermal voltages present in the circuit to be averaged out, without any additional manipulation of the data required. The Hall and longitudinal resistances are measured using a four point configuration, such that the voltage and current leads in the circuit do not play a role.

Nevertheless, in spite of this great advantage of the lock-in technique, it turned out to be inadequate for our magnetotransport experiments when probing the 'irrelevant' critical behavior of the PI-transition of the quantum Hall effect. The main reason for this is that when the 2DEG enters the insulating state, its resistance increases drastically and the experimental circuit (wiring, cables etc.), due to the AC-nature of the measurement, gives rise to an additional capacitive coupling. Consequently not all the current flows through the Hall bar and the voltages measured over the Hall bar contacts become unreliable.

In this chapter the above mentioned method will be discussed together with its inadequacy to work under the extreme experimental conditions needed to observe the critical behaviour of the PI transition. Following this we will present an alternative method that has been used. Even though this DC-method cannot compete with the signal-to-noise ratio offered by the lock-in technique, its application has overcome the main difficulty of capacitive coupling, connected to the ac-measuring method, mentioned before. Then a comparison of both methods will be made. Finally we will mention some adjustments that we made on the size of the Hall-bar in order to reduce the total resistance of the 2DEG once the system enters the insulating state.

3.2 Measuring techniques

3.2.1 AC-measuring

The alternating current (ac) - method is widely used to measure electrical resistance, by means of the lock-in amplification technique. The basic working principle of the lock-in technique is phase sensitive detection, i.e. the ability to extract a sinusoidal signal of known frequency and phase which is immersed in a ‘background’. The total input signal measured with the lock-in amplifiers (V_{in}) can be represented within the complex plane in the following way:

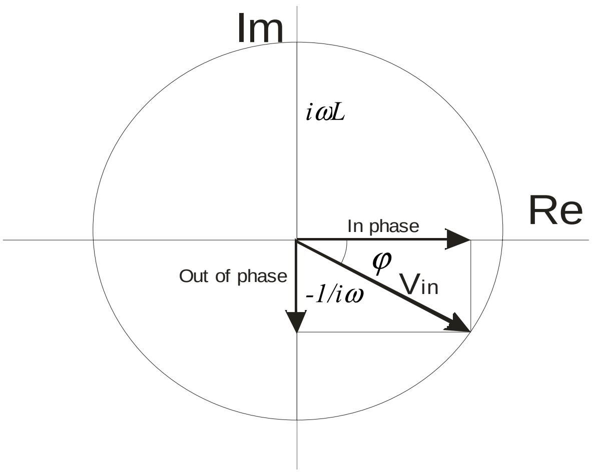

Figure 3.1 Representation of V_{in} in the complex plane. The input signal is shown here decomposed in an in-phase and an out-of-phase component. Note that a capacitor gives a phase shift of -90° with respect to the in-phase component and an inductance gives a phase shift of +90° with respect to the in-phase component.

A capacitor causes a phase shift of -90° and an inductance a phase shift of + 90°. The out-of-phase signal in *Fig*. 3.1 is the net vector of the capacitive effect and the contribution of a possible inductance. A measurement scheme representing our experimental case is given in *Fig*. 3.2 [1].

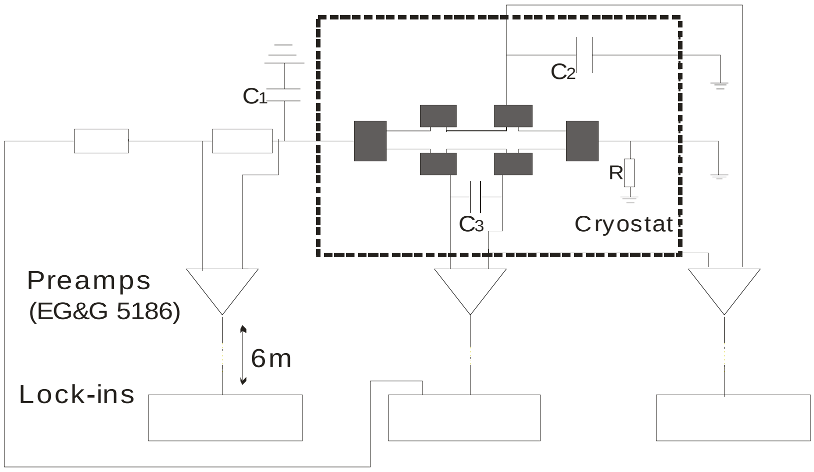

Figure 3.2 Scheme of the lock in amplification 4-point resistance measurement technique on a Hall bar. C_1, C_2, C_3 and R show unwanted capacitors and resistors an ac-current can flow through.

As *Fig. 3.2* shows, a big part of the measurement scheme consists out of wiring:

- The coax-cables connecting the lock-in amplifiers to the pre-amplifiers and the current cable connecting one lock-in amplifier to the top of the cryostat (~6 m)
- The coax-cables connecting the pre-amplifiers to the connector at the top of the cryostat
- The wiring connecting the connector at the top of the cryostat to the sample (~2 m)

At this point we should remark that the Hall bar needs to be completely isolated from its environment (sample-holder, cryostat) in order to prevent current flowing to ground, so '*R*' in *Fig.* 3.2 should be much larger than the resistance of the Hall bar. Basically there are three main constructions within the experimental setup that can contribute to the effect of capacitive coupling. These are shown in *Fig.* 3.2 as C_1, C_2 and C_3.

- C_1 is the capacitor formed by the core and the shield of a coax cable.
- C_2 is the capacitor formed by a wire and its environment, e.g. the cryostat.
- C_3 is the capacitor formed by two neighboring wires.

The impedance of a capacitor is given by:

$$Z = 1/(i \cdot \omega \cdot C) \tag{3.1}$$

where ω is the angular frequency and C is the capacitance. Capacitive coupling becomes a significant problem when the resistance of the sample becomes comparable to the impedances caused by C_1, C_2 and C_3 and is the main reason for the ac-method to become inadequate. From *Fig.* 3.2 the problem becomes clear immediately. Not all the current flows through the Hall bar. This means that the voltages measured by the lock-in amplifiers over the different Hall bar contacts are based on a current that is not exactly known to us. To give a quantitative example: Coax cables have a capacitance of 50 pF/m.
In the past in another similar cryostat the typical capacitance between wires for one contact pair has been shown to be around 700 pF [1]. Using the typical frequency of 13 Hz will give an impedance of $1/(\omega C) = 1/(2\pi fC) \approx 17$ MΩ. This becomes a problem in the insulating regime where the total resistance of the sample can reach values > 1 MΩ, meaning that not all the current flows from one current contact to the other. Measuring with a very low frequency is not a practical solution to the problem. In that case we would have to integrate over a much larger time scale (at least over ten periods of the input-signal [1]) to get an accurate averaging, which will cause a considerable delay in the measurement. This delay could be overcome by increasing the magnetic field at a lower sweep-rate, a process that would become too much time and helium consuming.

One could think of measuring the values of C_1, C_2 and C_3 and correct for the 'current-loss' due to them. This is also not a very practical solution. The reason is obvious. The reality is far more complicated than shown in *Fig.* 3.2. In fact current can leak away between every pair of wires and between every wire and its environment. To correct for this is a very difficult task at least. In the past a criterion has been set up based on the magnitude of the out-of phase component in order to determine whether the measured signal is reliable. It has been shown that if the out-of-phase component stays below 10 % of the in phase component, than the (systematic) error in the measured R (B) stays below one percent [1]. This follows easily from *Fig*. 3.1, where if the out of phase component is 10 % of the in-phase component, then $\varphi \approx 5$ deg and the in-phase component is still > 99 % of the real value. This criterion, regardless how helpful it has been in the previous measurements, has turned out to be not strict enough for the purpose of measurement presented in the following chapters.

It turned out that even if the 10 %-criterion was satisfied, the effect of capacitive coupling was still to high to give reliable data. Considering this limitation of the lock-in technique we decided to switch to a different method, based on a commutating DC- signal. This method will be discussed in the next paragraph.

3.2.2 DC-measuring

To measure DC we used a 6220 DC current source and a 2182A nanovoltmeter, both from Keithley Instruments Inc. The 6220 current source can generate currents from 0.1 pA to 105 mA. The 2182A voltmeter can measure voltages from 1 nV to 120 V. The input resistance of the voltmeter lies above 10 GΩ for all the measuring ranges. Measuring DC will give additional problems. Since there is no phase- or frequency dependent detection, we will have to deal with a higher noise level. Also a way has to be found to deal with thermoelectric voltages throughout the circuit. To deal with thermoelectric voltages we applied a measuring technique that makes use of commutating DC-current. A simple measurement scheme is shown in *Fig.* 3.3.

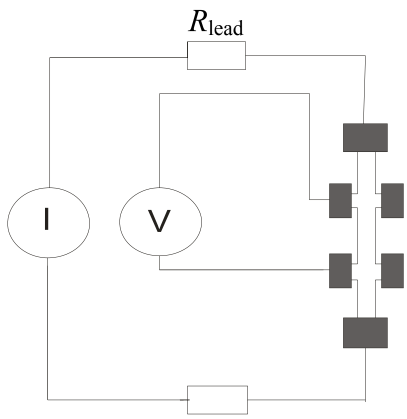

Figure 3.3 Schematic representation of 4-points DC measuring:
$V_{measured} = V_{Hallbar} + V_{thermoelectric} + V_{noise}$

Thermoelectric voltages can be dealt with by commutating the current. By averaging the measured voltages for $+I$ and $-I$ we can eliminate this contribution. In formula form this looks like:

$$V_{I+} = V_{Hallbar} + V_{Thermal}$$
$$V_{I-} = -V_{Hallbar} + V_{Thermal} \tag{3.2}$$

Thermoelectric voltages may vary in time (drift). In order to deal with the above mentioned problems we used the 3-step delta-method [2], [3] described next.

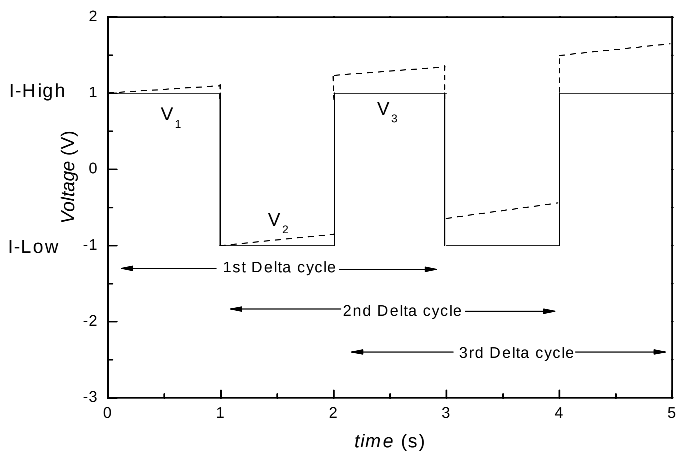

Figure 3.4 three-step delta method. The dashed line shows the situation, of a linear dependence of the thermoelectric voltages with time. Figure taken from [2], [3].

The final value V_f of the first delta cycle is calculated in the following way [2], [3]:

$$V_a = \frac{V_1 - V_2}{2}; V_b = \frac{V_3 - V_2}{2};\quad , \qquad V_f = \frac{V_a + V_b}{2} \tag{3.3}$$

where V_1, V_2 and V_3 are the values of three successive steps in the delta method. The advantages of this method are twofold. It deals better with thermoelectric variation in the system and it does additional averaging on the data which helps improve the accuracy of the measurement. Every step in this procedure can be divided in two parts. The *delay time* and the *integration time*. The delay is needed to take into account the response time of the system. By adjusting this we can make sure that all the current goes through the circuit, before we start measuring the voltage during the integration time. The delay time should increase as the resistance we want to measure increases. The length of this integration-time is expressed in PLC (power line cycle). One PLC is 0.02 s for 50 Hz. After each step the voltmeter gives one value, which is the average over the number of PLC's (NPLC) in that step. The higher the NPLC the more accurate the measurement of the voltage will be, but the longer it takes (see *Fig*. 3.5).

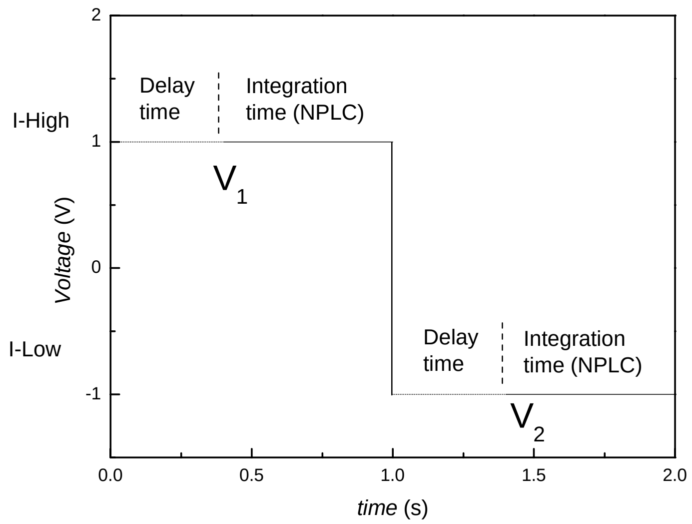

Figure 3.5 For one current polarity the measuring time is divided in a 'delay' and a 'time of integration', expressed in number of PLC's. Figure taken from [2].

A delay of 400 ms and a NPLC of 30 will give total period time of 2 s, which can be considered as 0.5 Hz (*Fig*. 3.5). Expressing this commutating behavior in Hertz can create the impression that this method is fundamentally similar to the AC method. This is actually not the case and the next point we make is a very important one. In the DC method a current source with adjustable output value is being used. The delay time in the voltage measurement makes sure that all the set-current goes through the circuit before the voltage is measured. Since the current during one step is completely DC in nature, this means that, in theory, all the set-current has to go through the Hall bar before the voltage measuring starts. The only period in time in which the current changes is when its polarity changes. Considering that during this period the voltage is not measured (if the delay time (*Fig.* 3.5) is bigger than the response time of the system), what is left is a way of measuring that is effectively 100% DC and the previous mentioned problem of capacitive coupling does not play a role anymore. Important now is to find a compromise between the accuracy of the measurement and the time it takes, since we are dealing with a signal that varies in time. To reduce the noise level further, a so called 'moving filter' has been added, which averages over a subsequent number '*n*' of values of V_f (*Eq*. 3.3), moves one value further, repeats the averaging, and so on. The question that now rises is: How does this DC-method compare to the traditional AC-method? This will be discussed in the next paragraph.

3.2.3 AC versus DC

The easiest way to compare these methods is to look at the insulating regime of the quantum Hall effect. Since the resistance of the 2DEG in this regime diverges exponentially as function of the magnetic field, we can expect that the method of measuring sooner or later will reach its limitation. In *Fig.* 3.6 the insulating regime measured both AC and DC is presented. The settings during both measurements where such that the time-constants (i.e. the time over which is averaged before the equipment gives a resistance value) were similar (~ 5 s). *Fig.* 3.7 shows that the out of phase component reaches about 6 % of the maximal resistance value. This is within the previous mentioned 10 % criterion. There is a clear difference between the ac and dc measured curve. This difference, however, is not only explained by the ac method losing accuracy due to capacitive coupling. In *Fig*. 3.8 the Hall resistivity is shown for the same density and temperature. A difference between the measured plateau values and the expected plateau values is obvious. The measured plateau values are about 1.5 % below the expected values. This difference can be attributed to an inaccuracy in the multiplication factor of the pre-amplifiers (*Fig.* 3.2). So every curve measured ac should be corrected for this. Multiplying the ac measured curve in *Fig.* 3.6 with a factor of 1.015 allows us to make a better comparison of both methods (see *Fig*. 3.9). The difference between the AC- and DC-curves in *Fig*. 3.9 is still considerable. At the critical value 'B_c' the difference amounts 7.5 %. The total sample resistance at this point is about 182 kΩ. The DC measured values are at each point higher than the AC measured values. Even though the above mentioned is a simple approach to the problem, it certainly speaks in favor of the DC-method. In section 3.3 we will look at the differences observed in the measurements of the Hall resistance.

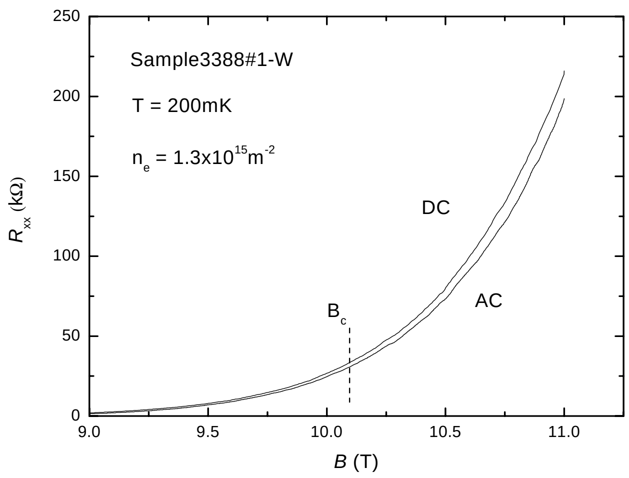

Figure 3.6 Insulating regime measured both AC and DC. AC: f = 2.6 Hz, I = 1 nA, TC = 5 s; DC: Del = 0.2 s; $NPLC$ = 15; MF = 2; I= 1 nA.

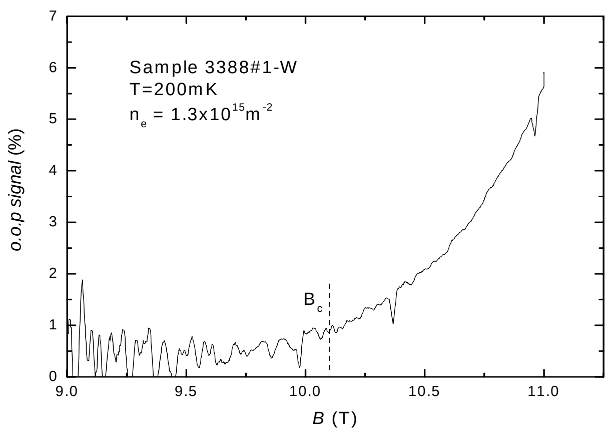

Figure 3.7: Out of phase component in percent of R_{xx}.

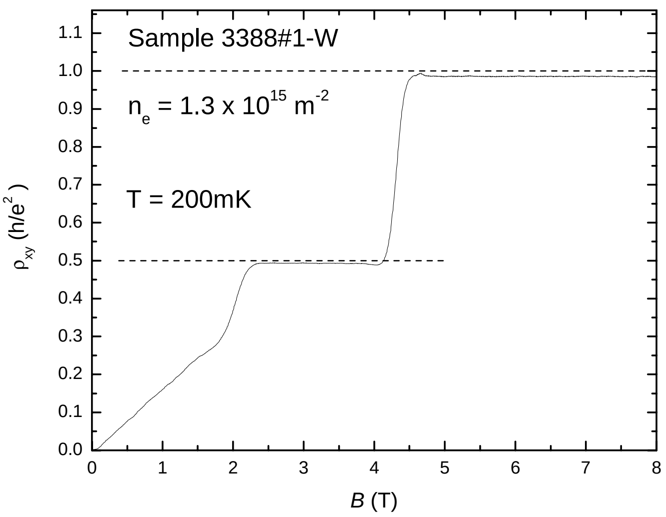

Figure 3.8 Hall resistance measured AC for $n_e = 1.3 \times 10^{15}$ m^{-2}. Dashed lines indicate the expected plateau values.

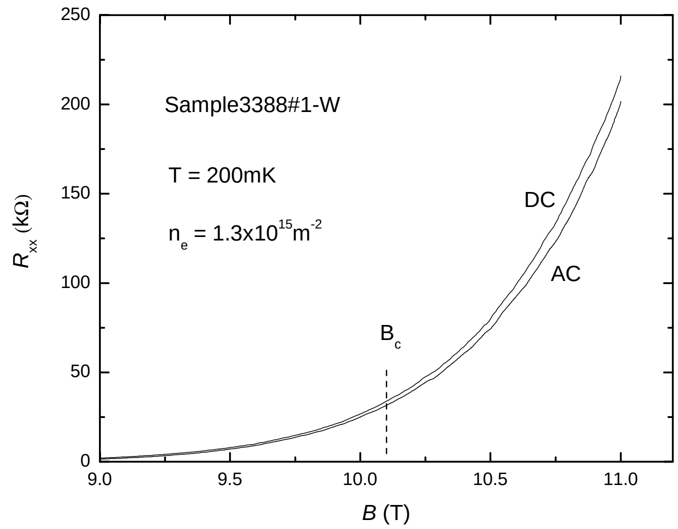

Figure 3.9 Same curves as in *Fig.* 3.6 only with the ac curve multiplied with a factor 1.015.

3.3 Capacitive coupling: The subtle difference

In the previous paragraph it has been shown that the effect of capacitive coupling strongly influences the resistance measurement in the insulating regime. We can assume that the influence of this capacitive coupling will be even more drastic while measuring the deviation from quantization in the insulating regime, since the effect we are after is very small (~ 100 Ω), compared to the actual measured signal (> 25 kΩ) and the total resistance of the sample. In *Fig.* 3.10 the ρ_{xy} -data measured for two field polarities at T = 0.8, 1.0 and 1.2 K are presented. From previous experiments we expect the averaged curves to show a positive deviation from the quantized value ρ_{xy} = 1 in units h/e^2 [1],[4]. This is clearly not the case for T = 0.8 K. In *Fig*. 3.11 we show the averaged curves both for the ac- and the dc-method. There is a big difference between the ac and dc measured data. Notably is that for the dc-method all the curves show a positive deviation. In *Fig*. 3.12 we show the out-of-phase component for the measured Hall resistances.

Indeed something peculiar happens: The increase of the out of phase signal is asymmetric for both field polarities. The effect of capacitive coupling is larger when measuring with positive field polarity. This explains why the ac-measured curves are always lower in magnitude than the dc-measured ones.

One might wonder where this asymmetry comes from. By changing the field polarity, we also change the polarity of the Hall voltage. Since every contact is wired differently in terms of length of cables, position of wires with respect to each other and with respect to the cryostat, it can be assumed that the effects of C_1, C_2 and C_3 (*Fig.* 3.2) are different for different field polarities.

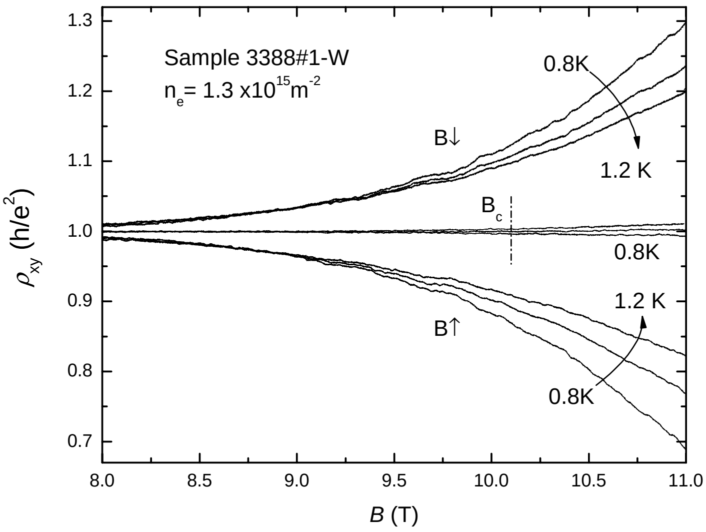

Figure 3.10: Hall resistance measured AC for both positive and negative field near the PI-transition at T = 0.8, 1.0 and 1.2 K plus the averaged curves for both field polarities.

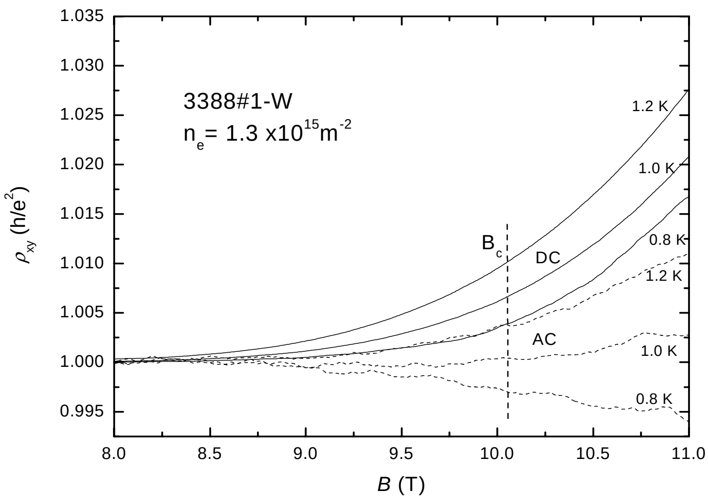

Figure 3.11 Hall resistivity for T = 0.8, 1.0 and 1.2 K measured with both the ac- and dc-method. Dashed lines: AC; solid lines: DC.

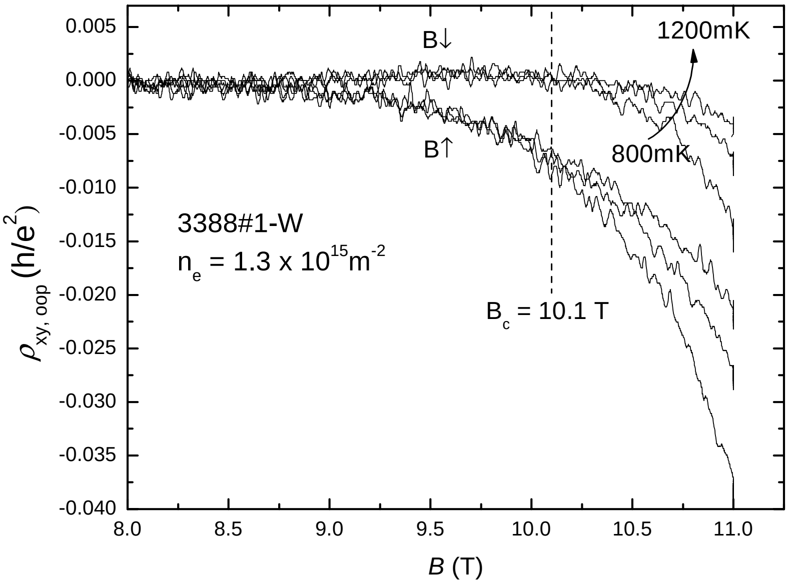

Figure 3.12 Out op phase component of the R_{xy} measurement for positive and negative magnetic field.

3.4 Shape of Hall-bar

Since we want to keep the resistance of the 2DEG as low as possible we have designed a Hall-bar with different dimensions than the one used in previous measurements [1]. The newly designed Hall bars have a width of 200 μm against the 75 μm of the Hall bar described in [1]. The total length of the Hall bar has been adapted to assure that the current between the voltage-contacts flows parallel to the Hall bar. Our decision has been supported by numerical calculations [1]). The dimensions of the Hall bar that we used are displayed in *Fig.* 3.13.

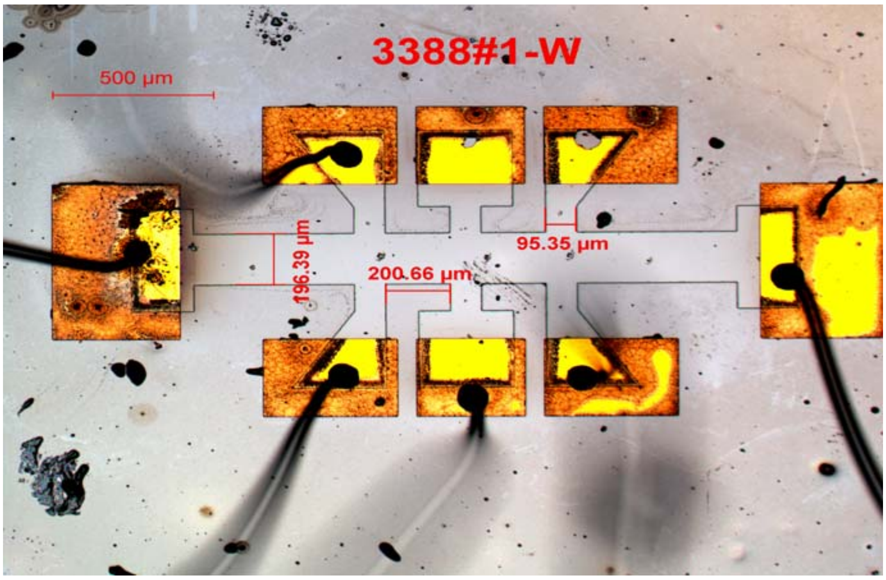

Figure 3.13 Sample 3388#1-W with respective dimensions.

Notice that the geometrical factor (Length/Width) now becomes 1.5, with respect to the previous 5.2 [1], keeping the longitudinal resistance more than three times as low. In *Table* 3.1 a comparison is made between both Hall bars at the PI- transition. Both samples were made from the same wafer, which is an $In_{0.2}Ga_{0.8}As/GaAs$ quantum well grown by MBE. The quantum well is a 12 nm thick $In_{0.2}Ga_{0.8}As$ layer, separated by the doping layer by a 20 nm thick spacer. There is no caplayer. The Hall bars were etched by photolithography.

Table 3.1 Comparison of R_{xx} and $R_{Hallbar}$ at the PI-transition for 3388#1 and 3388#1-W

Hallbar	(L/W)	R_{xx} (kΩ)	$R_{Hallbar}$ (kΩ)
3388#1	(5.2)	134.2	498.5
3388#1-W	(1.5)	38.7	219.3

It is clear that Hall bar 3388#1-W is the most suited of the two for our aim of keeping the total resistance of the Hall bar as low as possible.

3.5 Conclusions

In this chapter two different methods of measuring the magnetotransport properties of a Hall bar made of a 2DEG in the quantum Hall regime have been discussed.

- The traditional low frequency ac-method using the lock-in technique
- The dc-method which through signal averaging and commutation has been adapted to make it suitable for measurements using very low excitation currents.

Due to the nature of the dc-method the problem of capacitive coupling, inherent to the ac-method, can be strongly reduced or avoided completely, depending on the measurement parameters ‘delay time’ and ‘NPLC’ (*Fig.* 3.5). Thus the dc method compares favorable when measuring Hall bars in the high-ohmic regime of the plateau-insulator transition.

3.6 References

[1] L.A. Ponomarenko, *Ph.D Thesis* (University of Amsterdam, 2005), unpublished.

[2] Keithley Instruments Inc., Model 6220 DC Current Source User’s manual, *622X-900-01 Rev. B*/June 2005. www.keithley.com/products/locurrhiresist/?path=6220/Documents#6

[3] Keithley Instruments Inc., Model 2182/2182A Nanovoltmeter, User’s manual, *2182A-903-01 Rev. A*/ June 2004. www.keithley.com/products/lovoltloresist/nanovoltmeters/?path=2182A/Documents

[4] A.M.M. Pruisken, D.T.N. de Lang, L.A. Ponomarenko and A. de Visser, *Solid State Commun.* **137** (2006) 540.