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Intergenerational risk sharing within funded pension schemes*

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Abstract

We study risk sharing between generations for a variety of realistic collective funded pension schemes, where pension benefits and contributions may depend on the funding ratio and the asset returns. The collective pension schemes organizing intergenerational risk sharing are optimized with respect to the generosity of pension benefits, the asset allocation and the risk allocation rules. We perform a welfare comparison between the collective plans with intergenerational risk sharing and the optimal individual benchmark. We show that well-structured intergenerational risk sharing can be welfare-enhancing vis-a-vis the benchmark. Furthermore, from an ex ante perspective the expected welfare gain of the current entry cohort is not at the cost of the older and future cohorts.

Keywords: value-based generational accounting, intergenerational risk sharing, pension funds. JEL codes: G13, H55

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1 Introduction

Systematic risks such as labor and capital market shocks can cause dramatic and long-lasting swings in consumption.¹ It is well known from the literature that optimally designed intergenerational smoothing of such risks, or intergenerational risk sharing (IRS), is potentially welfare improving. However, the private market fails to provide insurance products based on IRS because human capital is non-tradable and current generations cannot sign contracts with future generations. Allen and Gale (1997) argue that there are two strategies which can arrange intergenerational risk sharing, namely public programs and asset accumulation via financial intermediaries. In this paper, we examine whether collective funded pension schemes could be used as dedicated financial institutions to facilitate intertemporal smoothing of systematic risks. Due to their long run nature and mandatory participation rules, pension funds seem natural entities to organize IRS.² The questions we address are whether IRS is desirable in funded pension schemes and if so, what the optimal design regarding risk sharing rules is.

This paper focuses on intergenerational sharing of systematic investment risk in realistic funded pension schemes. The schemes analyzed are stylized versions of collective (public sector, industry-wide, or multi-employer) pension funds operating in countries like the US, UK, Canada and the Netherlands. In these collective schemes, assets are pooled and owned collectively by young and old generations. Mandatory participation is required by legislation. The schemes invest in the financial markets, typically partly in risky assets (stocks) to capture the equity premium. The schemes have no sponsor and investment risk is borne by the overlapping generations of pension plan members. Surpluses or deficits in the funding process are shared among young, old and future generations by adjusting either contributions, benefit levels or both, which leads to intergenerational transfers. In this setup, we carry out a welfare comparison of various collective schemes with IRS, and use the optimal individual scheme as the benchmark.

More specifically, this paper specifies a number of realistic collective pension schemes consisting of 55 overlapping generations (from age 25 to 80). During the first 40 years, workers contribute to the pension fund and the last 15 years retirees receive benefits. Partic-

¹The credit crunch since 2007, the internet bubble in the beginning of this century, the lost decade in Japanese market, the oil crisis in 1970s, and the great depression in 1930s are typical examples of such systematic risks.

²See also Gollier (2008) and Teulings and De Vries (2006).

ipants are short-sales and borrowing constrained, and their only savings are for retirement purposes. The pension funds invest the savings of all cohorts in one asset pool, and part of it may be invested in the stock market. The return on the risky asset is exogenous and unaffected by the scheme's design or portfolio composition. Fluctuations in asset returns lead to mismatch risk between the pension fund assets and liabilities. To absorb funding surplus, contributions and/or benefits are adjusted as a function of surplus. As a result, the three schemes under study differ in their risk allocation rules, which specify who of the stakeholders, when, and to what extent is taking part in risk-bearing.³ The three schemes are optimized with respect to the target contribution rates, the risk allocation rule and the portfolio weights, both from the point of view of a newly entering cohort, as well as from the point of view of a social planner who also takes the utility of future generations into account. We also examine the market value of contributions and benefits derived from the optimal schemes, to check whether the schemes are actuarially fair.

Our main findings are the following. (i) From the entry cohort point of view, the well designed realistic collective pension schemes can be welfare improving over the optimal individual life-cycle benchmark. In a stylized example, the hybrid collective scheme leads to a 2.3% increase in certainty equivalent consumption per annum vis-à-vis the optimal individual pension scheme. Despite the fact that risk allocation rules and asset allocation are fixed, the hybrid scheme still outperforms the optimal individual benchmark with full flexibility regarding asset mix and saving rate. (ii) It is possible for well-structured pension schemes to absorb funding deficits up to 10% to 20% by intergenerational risk sharing and still enhance the welfare for her participants. (iii) Hybrid pension plan with flexibility in adjusting both contribution and benefit levels to absorb funding residue are the most preferred in welfare terms. It outperforms plans that only allow for adjustments in either contributions or benefit levels. (iv) Due to IRS, participants are more capable to exploit the positive equity premiums by taking more risk. (v) The welfare gains for the new entry cohort are not at the cost of older and future cohorts. From an ex-ante point of view, the expected welfare of the future cohorts is even higher compared to the currently entering cohort. The key intuition is that the long term nature of the collective pension funds allows for smoothing of risks across many generations. Risk pooling or forming a mutual insurance across generations is welfare enhancing for all from ex ante perspective. (vi) Although optimization occurs by the entering generation, the fund is expected to build up a buffer, and is, in expectation,

³In a defined benefit scheme with contribution adjustment (DB_{CA}), the workers bear the funding risks, whereas in a defined benefit scheme with benefit adjustment (DB_{BA}), the retirees bear the funding risks. In the hybrid defined benefit (DB_H) scheme, both workers and retirees bear the funding risks.

overfunded in the long run. Therefore, both intertemporal risk smoothing strategies (namely IRS and the asset accumulations) can be used within combination in collective pension funds to improve intertemporal smoothing. When we design the fund so as to maximize the utility of all generations, the steady state buffer increases.

There are a few differences between our paper and the existing literature. Allen and Gale (1997) and Gollier (2008) study the role of shareholder-sponsored financial intermediary (or corporate-sponsored pension funds) in facilitating intertemporal risk smoothing. They find large welfare improvements if shareholders are able to contribute or accumulate sufficiently large financial buffer as means for intertemporal smoothing. Importantly, they point out that, without mandatory participation, the system will break down and go back to the market solution once the buffer is exhausted or become insufficient. In addition to the potential threat from underfunding, Van Bommel (2007) argues that intergenerational risk sharing will not be sustainable when the fund is overfunded, as living generations might renegotiate and raid the surplus. These studies suggest that without compulsory entry and/or with the opportunity to renegotiating risk sharing rules, funded IRS-pension schemes cannot improve upon the market equilibrium in which agents act individually. However, their models are much more abstract, with two or three overlapping generations respectively. Our OLG model is much more realistic, with 55 generations. In such a world, it is realistic to assume that participations can be enforced, and asset-raids can be avoided.

Furthermore, we extend the IRS literature with applications to funded pension systems. Traditionally, intergenerational transfers are implemented by government through fiscal policy and public debt management, monetary policy and Pay-As-You-Go social security programs. It is well known from public finance literature that well designed intergenerational risk sharing under mandatory participation could be welfare improving, especially when dealing with systematic risks. For a general exposition of these issues see Gordon and Varian (1988) and Shiller (1999)). Some specific papers in this area are Fisher (1983), Gale (1990), and Bohn (2003) about fiscal policy and public debt management; Weiss (1979) about monetary policy; Merton (1983), Enders and Lapan (1982), and Krueger and Kubler (2006) about social security programs. In the pension fund context, Teulings and De Vries (2006) argue that it is welfare improving if individuals can borrow against their future labour income and invest before entering the labour market, and expect that pension funds with IRS has similar effect. Instead of assuming borrowing, we model IRS within funded pension schemes with risks borne by multiple overlapping cohorts.

The recognition of the welfare aspects of risk sharing within pension funds is important.

An analysis in terms of only market value may easily lead to the spurious conclusion that pension funds are irrelevant, as argued for example by Exley (2004). Indeed, the market value of contributions equals the market value of benefits in our collective pension schemes, and hence a zero-sum game in value-terms; But the schemes with IRS are potentially welfare enhancing, and thus a positive-sum game in welfare terms. These results have important implications related to the social security reforms worldwide. Many countries are gradually reducing (or planning to reduce) the PAYG social security and promoting the individual defined contribution saving schemes (see for instance recent discussions of Feldstein (2005) and Diamond and Orszag (2005)). Our results show that the individual defined contribution scheme is not the optimal funded scheme, even in a frictionless world with rational and sophisticated individuals. Collective funded schemes with well organized intergenerational risk sharing mechanisms are better choices from a welfare perspective. Furthermore, related to the impact of current global financial turmoil since 2007 on retirement provisions, our analysis implies that funded pension schemes with IRS could facilitate intertemporal smoothing of systematic risks and limit welfare loss for participants.

The paper is organized as follows. Section 2 describes the structure of the economy, the preferences of the pension scheme participants and the structure of the pension arrangements. Section 3 evaluates the welfare outcomes of the various pension schemes. Section 4 discusses the robustness of the results and Section 5 looks at the implications for future cohorts. Section 6 concludes.

2 Model

2.1 Economy, overlapping generations and preferences

Two asset classes are traded in the financial market: risky stock index and risk-free assets. For simplicity, we assume that the interest rate, r , is non-stochastic. Stock prices follow a geometric Brownian motion with a constant drift μ . The investment portfolio is a mix of stocks and risk-free assets, with portfolio weight ω_t for stocks. The dynamics of this asset portfolio are given by

$$dW_t/W_t = [r + \omega_t(\mu - r)]dt + \omega_t\sigma dZ_t \quad (1)$$

where σ is the volatility of stock returns. Both the expected return and the volatility increase linearly with the fraction ω_t invested in equities. The Sharpe ratio of the investment portfolio

equals

$$\lambda = \frac{\mu - r}{\sigma} \quad (2)$$

The stochastic discount factor M_t in this economy is the deflator for risky cash flows, which in our model evolves according to

$$dM_t/M_t = -r dt - \lambda dZ_t \quad (3)$$

This deflator can be used to calculate the market value of stochastic pension contributions and benefits.

The default values for the model parameters are in line with the values used in the recent literature. We assume the (real) interest rate is constant and equals $r = 2\%$. The expected (real) return on stocks is assumed to be $\mu = 6\%$, implying an equity premium of 4% , which is in line with the long run estimates in Fama and French (2002). The volatility of stock returns is $\sigma = 15\%$, which is close to the value (15.7%) estimated by Cocco, Gomes and Meanhout (2005).

We consider an economy populated by 55 overlapping generations (from age 25 to 80). We assume that all individuals start working at age 25 ($t = 0$), retire at age 65 ($t = R = 40$), and die at age 80 ($t = 55 = T$). During the first 40 years people work and earn a flat real labor income which is normalized to 1.⁴ The population (as well as the collective pension fund) has stationary age composition. During the retirement period ($R \leq t < T$), the individuals receive no income but consume their accumulated pension wealth, denoted by W_t . Apart from the pension wealth, there are no savings. The 55 homogeneous cohorts have the same population size and share the same preferences. Individuals have constant relative risk aversion (CRRA) utility function defined over a single nondurable consumption good. Let c_t denote the consumption level at time t , then the individual's preferences are defined by

$$U = E_0 \left[\int_0^T e^{-\delta t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right] \quad (4)$$

where γ is the risk aversion parameter and δ is the subjective discount rate. In the baseline setup, we set the subjective discount rate equal to $\delta = 4\%$, and the risk aversion parameter $\gamma = 5$ for a typical individual. In the robustness checks we allow for a lower equity premium and different values of δ and γ .

⁴All variables throughout this paper are expressed in real terms, i.e. scaled by the price level. It is assumed that wage inflation is identical to price inflation.

2.2 Cohort specific pension schemes

We consider two individual schemes, where the individual bears all the non-diversifiable investment risks and there is no intergenerational risk sharing. The first scheme is the optimal individual scheme, where the individual can optimally choose his consumption level and portfolio composition (under short sales and borrowing constraints) at any time throughout his life. This scheme serves as the benchmark for the collective schemes with IRS to be presented in Section 2.3. The second scheme is a defined contribution scheme with a fixed contribution rate which is chosen optimally at the beginning of the working life, but the consumer is still able to adjust the portfolio weight throughout his life. The later scheme captures common practice in the real world and is more realistic than the optimal individual scheme; Poterba, Rauh, Venti and Wise (2005) use a similar setup.

The optimal individual scheme (*OI*) implements the optimal life cycle consumption and portfolio choice of an individual investor under borrowing and short-selling constraints. During working period, the investor receives his labor income, which is normalized to 1. The individual chooses his consumption stream c_t optimally at every point in time, and invest the rest in financial markets. After retirement, the individual optimally consumes down his wealth. The individual is free to choose the portfolio weight of stocks ω_t continuously, within the short-selling and borrowing constraints, i.e. $0 \leq \omega_t \leq 1$. Formally, the individual's consumption and portfolio choice problem is characterized by the following objective function

$$U = \max_{\{0 \leq \omega_t \leq 1, c_t\}} E_0 \left[\int_0^T e^{-\delta t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right] \quad (5)$$

and subject to the pension wealth dynamics

$$dW_t^{OI} = [W_t^{OI}(r + \omega_t(\mu - r)) + 1 - c_t]dt + \sigma\omega_t W_t^{OI}dZ_t, \quad \text{for } 0 < t < R \quad (6)$$

$$dW_t^{OI} = [W_t^{OI}(r + \omega_t(\mu - r)) - c_t]dt + \sigma\omega_t W_t^{OI}dZ_t, \quad \text{for } R \leq t < T \quad (7)$$

$$W_0^{OI} = 0 \quad (8)$$

where W_t^{OI} is the accumulated financial wealth in the *OI* scheme.

Because of the short-selling and borrowing constraints, no analytical solution is available. We use dynamic programming to solve for the consumption and investment in the optimal individual scheme. We use the numerical procedures presented by Carroll (2006) to solve for the optimal consumption and portfolio policy before retirement, under the borrowing and short-selling constraints, $0 \leq \omega_t \leq 1$. Details of the solution method are given in the Appendix.

The defined contribution (*DC*) scheme differs from the optimal individual scheme in that the individual contributes a fixed fraction of his labor income into the *DC* scheme and consumes the rest of his income. At the entry date, the contribution rate m is fixed for the entire working period ($0 < m < 1$ during the working period and $m = 0$ after retirement). The individual's consumption before retirement is therefore $c_t = (1 - m)$, and after retirement the consumption is optimally chosen. The individual adjusts his portfolio (ω_t) throughout life. Formally, the individual's consumption and portfolio choice problem is characterized by the following preferences

$$U = \max_{\{m, 0 \leq \omega_t \leq 1, c_t\}} E_0 \left[\int_0^R e^{-\delta t} \frac{(1 - m)^{1-\gamma}}{1 - \gamma} dt + \int_R^T e^{-\delta t} \frac{c_t^{1-\gamma}}{1 - \gamma} dt \right] \quad (9)$$

subject to the pension wealth dynamics

$$dW_t^{DC} = [W_t^{DC}(r + \omega_t(\mu - r)) + m]dt + \sigma\omega_t W_t^{DC} dZ_t, \quad \text{for } 0 < t < R \quad (10)$$

$$dW_t^{DC} = [W_t^{DC}(r + \omega_t(\mu - r)) - c_t]dt + \sigma\omega_t W_t^{DC} dZ_t, \quad \text{for } R \leq t < T \quad (11)$$

$$W_0^{DC} = 0 \quad (12)$$

The optimal contribution rate m is found by numerical grid search, detailed in the Appendix. Since m and p both refer to a fixed contribution rate, the notation p is used in the tables for both collective and individual schemes.

Figure 1 shows the life cycle quantile distributions (at 5%, 50% and 95% quantile levels) of portfolio weight, consumption and wealth accumulation of the two individual schemes for a benchmark investor ($\gamma = 5$, $\delta = 4\%$). The left panel shows the results for the *OI* scheme and the right panel shows the *DC* scheme. The distribution of portfolio choices (ω_t) is shown in the top panel of Figure 1. On average, the optimal portfolio weights are decreasing with age. As explained by Campbell and Viceira (2002), this is due to the leverage effect of human capital (or to be precise, the present value of future pension contributions). The desired equity exposure in total wealth, including financial wealth and human capital, is constant over the life cycle. Individuals achieve the desired equity exposure of their total wealth by adjusting their financial portfolio.

The distribution of consumption (normalized by income) for the baseline parameters is shown in the middle panel of Figure 1. The median consumption level is relatively flat over the life cycle, but uncertainty in old-age consumption is increasing with time. In the *OI* scheme, the consumption is higher (hence new saving is lower) when the financial wealth is boosted by good returns. Finally, the accumulated assets over life cycle are presented in the lower panel of Figure 1. These asset values show wide dispersion. They peak at the

retirement date with a median amount standing around 11 times of the annual labor income for *OI* scheme or 13 times for *DC* scheme.

2.3 Pension schemes with intergenerational risk sharing

In this section, we model collective pension schemes that allow for intergenerational risk sharing. Mandatory participation is required by law. We first define the targeted pension benefit and contribution policies for such funds, and then specify the risk allocation rules which adjust the benefits and/or contributions as functions of finding surplus. Finally the design parameters are optimized with respect to a chosen objective.

2.3.1 Pension liability

Since the collective pension schemes considered in this paper are certain variants of defined benefit (DB) schemes, we shall start with traditional DB schemes to introduce the target liability, target benefit and contribution rates. In a traditional average salary defined benefit scheme, the retirement benefit is a fixed fraction (the so-called replacement rate) of labor income during the working period. These pension benefits are funded by the contributions plus investment proceeds. The higher the pension ambition, the more contributions are required. The actuarially fair contribution principle requires that *ex-ante* each generation finances its own pension. That is, the market value of the contributions equals the market value of the benefits. The actuarially fair contribution rate can be solved from the following present value equivalence:

$$\int_0^R e^{-rs} p ds = \int_R^T e^{-rs} b ds. \quad (13)$$

where b denotes the target benefit and p the target contribution (recall that we have normalized the flat real labor income to 1). For each age cohort, denoted by x , the target DB liability equals the difference between the present value of risk-free benefits and the present value of yet-to-be-paid risk-free contributions.

$$\begin{aligned} L_x &= \int_R^T e^{-r(t-x)} b dt - \int_x^R e^{-r(t-x)} p dt, \quad \text{for } x < R \\ &= \int_x^T e^{-r(t-x)} b dt, \quad \text{for } R < x < T \end{aligned}$$

At the aggregate level, the target liabilities of the fund can be calculated simply as the sum of the liabilities for each age cohort:

$$L = \int_0^R \left(\int_x^T e^{-r(t-x)} b dt - \int_x^R e^{-r(t-x)} p dt \right) dx + \int_R^T \left(\int_x^T e^{-r(t-x)} b dt \right) dx \quad (14)$$

Given the stationary age composition of the fund and the fixed target benefit level, the target liability L is time-invariant. If the fund invests full in risk free asset, then the actual liability follows exactly as the target liability L . If the fund accepts mismatch risk, for example by investing in stocks, there may be funding surpluses or deficits with respect to the target liability. Let A_t denote the value of the pension fund's assets, which starts off with an initial asset value of $A_0 = FR_0 \cdot L$, where FR_0 is the initial funding ratio. Let ω denote the fraction of assets invested in risky assets. The portfolio weight ω is time-invariant and the same for each cohort in the fund.⁵ Then, the fund's assets follow the dynamics

$$dA_t = [A_t (r + \omega (\mu - r)) + 40p_t - 15b_t] dt + \omega \sigma A_t dZ_t \quad (15)$$

where p_t and b_t denote the actual contribution and benefit levels which will be specified shortly. In most of the experiments we assume $FR_0 = 1$, but we shall show some results with initial over- or under-funding. The fund surplus is defined as the difference between assets and the target liability level:

$$S_t = A_t - L \quad (16)$$

2.3.2 Risk allocation rules

If the investments of the fund were completely risk free, there would never be any overfunding or underfunding. However, with a risky investment policy, or other sources of systematic risks, the mismatch risk has to be allocated in some way. This is done by adjusting contributions p_t and benefits b_t as a function of the fund surplus S_t and the target contribution (p) and benefit (b). This adjustment scheme is motivated by real-world examples.⁶ We now discuss three stylized schemes, that differ in their risk allocation rules. These risk allocation rules specify who of the stakeholders, when, and to what extent is taking part in risk-bearing.

In the defined benefit with contribution adjustments (DB_{CA}) scheme, benefits are fixed at $b_t = b$, but contributions are adjustable. So, the working cohorts bear all the funding

⁵This assumption is motivated by the observed common practice in collective pension funds, where the same investment policy is implemented for all cohorts.

⁶We refer to Ponds and Van Riel (2007) for a more detailed description.

risk. We specify a simple contribution policy, where contributions per cohort are a function of the target contribution level p and the funding residual per cohort S_t/R :

$$p_t = p - \alpha S_t/R \quad (17)$$

The slope coefficient α determines the speed of absorbing the funding imbalances. The choice $\alpha = 1$ implies that a funding imbalance is immediately and fully absorbed. A lower value of α implies that part of the funding residual is shifted to the future, and shared across generations.

In the defined benefit with benefit adjustments (DB_{BA}) scheme, contributions are fixed at $p_t = p$, but benefits are adjustable in order to absorb the funding surplus. In this scheme, retired cohorts bear the funding risk, where benefit per retired cohort are a function of the target benefit level b and the funding residual per cohort $S_t/(T - R)$:

$$b_t = b + \beta S_t/(T - R) \quad (18)$$

Again, the lower the value of β , the higher the degree of intergenerational risk sharing.

The hybrid defined benefit (DB_H) scheme adjusts both contributions and benefits simultaneously to absorb the funding residual. The contribution and benefit are linearly related to the funding residual S_t . More specifically, a fraction α of the funding residual is shared among employees and a fraction β of the funding residual is shared among retirees:

$$p_t = p - \alpha S_t/R \quad (19)$$

$$b_t = b + \beta S_t/(T - R) \quad (20)$$

Under this rule, consumption before and after retirement is related to the funding surplus. This resembles the optimal consumption rule in the classical consumption and portfolio choice problem in the spirit of Merton (1969) where the optimal consumption is linearly related to wealth.

The following table summarizes the differences among the collective schemes:

Defined benefit with contribution adjustments (DB_{CA})	$\alpha > 0 \quad \beta = 0$
Defined benefit with benefit adjustments (DB_{BA})	$\alpha = 0 \quad \beta > 0$
Hybrid defined benefit (DB_H)	$\alpha > 0 \quad \beta > 0$

When $\alpha + \beta$ approaches the risk free rate r , each generation in each period absorbs only the 'interest' accrued on their funding residual and passes the principal into the infinite future. To get stability over time (i.e., non-explosive values for the surplus), we need to restrict $\alpha + \beta > r$.

2.3.3 Pension policies optimization

We optimally chooses the target contribution rate (p) together with the risk allocation rule (α, β) and investment policy (ω), based on the expected life-time utility of the 25-year-old entering participants. That is, the policy parameters $\{p, \alpha, \beta, \omega\}$ of the three collective plans are optimized from the perspective of the cohort entering at time $t = 0$. Therefore, the specification of the utility function of the pension fund is the same as in the individual case (5). The optimization problem of the pension fund then becomes

$$\begin{aligned} U &= \max_{\{p, \alpha, \beta, \omega\}} E_0 \left[\int_0^T e^{-\delta t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right] \\ \text{with } c_t &= 1 - (p - \alpha S_t / R), \quad t < R \\ c_t &= b + \beta S_t / (T - R), \quad R \leq t \leq T \end{aligned} \tag{21}$$

subject to the wealth dynamics (15). Since the pension fund is short-sale constrained, the portfolio choice is constrained by $0 \leq \omega \leq 1$. The actual contributions and benefits (p_t, b_t) follow directly from the risk allocation rules (19, 20) in different schemes. The decisions $\{p, \alpha, \beta, \omega\}$ are time invariant. The optimization problem therefore is static and can be solved using Monte Carlo simulations combined with a standard grid search in four dimensions. The Appendix gives more details about the optimization procedure.

3 Evaluation of pension schemes

In this section we evaluate the performance of the optimal cohort specific and collective schemes. First, in subsection 3.1, we compare the investment policies and consumption outcomes. Subsection 3.2 then provides a welfare comparison of the optimal collective schemes with the individual benchmark. Subsection 3.3 studies the intergenerational transfers in the collective pension schemes.

3.1 Optimal pension schemes

Table 1 reports the optimal design parameters $\{p, \alpha, \beta, \omega\}$ for the pension schemes under the default parameter values ($r = 2\%$, $\delta = 4\%$, $\mu - r = 4\%$, $\gamma = 5$). All schemes are initially fully funded (i.e. $A_0 = L$).

We find that institutional settings have a significant impact on the optimal contribution

rates and investment policies. The target level of contribution p and portfolio choice ω vary significantly across schemes. The optimal contribution rate in DC scheme is the lowest of all, which is 11%. The optimal target contribution rate in DB_{CA} scheme is the highest, requiring 16.6% of annual salary. This leads to the highest risk-free replacement rate of 78.4% after retirement. The desired contribution rate in DB_{BA} scheme is the lowest among the collective schemes, requiring 13.1% of salary, which is close to the DC scheme. Consequently, its target replacement rate b is also the lowest at 53.6%. The hybrid scheme DB_H requires 14% of salary as contribution rate and leads to a 66% target replacement rate. The actual contributions and benefits are different from these risk-free levels, as the pension fund can run surpluses or deficits.

The portfolio choice of DB_{BA} scheme is less aggressive than the other collective schemes. The risky portfolio weight of DB_{BA} scheme is 60%, whereas the risky share of DB_{CA} scheme reaches 96%. The hybrid scheme DB_H accommodates the most aggressive investment portfolio, namely 100% in stocks. Comparing with the portfolio choices in the optimal individual schemes (Figure 1), where the stock portfolio share declines to 35% over the life cycle, the stock allocations of collective schemes are much higher. This shows that intergenerational risk sharing makes an aggressive investment policy more attractive. Due to intergenerational risk sharing, participants are more capable to exploit the positive equity premiums. This finding confirms the results of Gollier (2008) that intergenerational risk sharing increases the demand for risky investment. The values of the contribution and benefit adjustment parameters α and β are typically small, indicating that funding mismatches are absorbed gradually over time.

Figure 2 provides some graphical insight in the distribution of consumption over time. The figure plots the 5%, 50% and 95% quantiles of the normalized consumption, c_t , of the optimal collective plans, with the scheme parameters set corresponding to Table 1 with $\gamma = 5$. In general, the consumption profiles are upward sloping with age. The distributions of consumption indicate that contribution reductions are more frequent, and also larger, than contribution increases. This is due to the positive equity premium. The figure shows that the hybrid scheme (DB_H) is much better in distributing shocks over the full lifetime of the individual than the other collective schemes, where there are large fluctuations in consumption either just before retirement (DB_{CA}) or during the retirement period (DB_{BA}). This demonstrates that more efficient risk sharing can be achieved by using more risk absorbers. Through adjusting *both* contribution and benefit policies, the DB_H scheme spreads the funding residual over both workers and retirees, i.e. all cohorts are involved in the risk

sharing. From the risk allocation perspective, DB_{CA} and DB_{BA} represent two extremes, with one group of people bearing all risks, while the other group is fully insured. The hybrid scheme allocates the risks more efficiently among all generations, hence reducing the costs of risk taking and resulting in welfare improvements.⁷

3.2 Welfare evaluation

We use certainty equivalent consumption (CEC) as the welfare measure to gauge the performance of collective and individual pension schemes. The CEC can be backed out easily from the following equality

$$U = \int_0^T e^{-\delta t} \frac{(CEC)^{1-\gamma}}{1-\gamma} dt \quad (22)$$

The most important result in Table 1 is the welfare levels of the optimized pension schemes from the entry cohort perspective. Welfare level is reported as the normalized certainty equivalent consumption in units of annual salary, CEC . Let CEC^{OI} denote the welfare level achieved by the optimal individual scheme (OI). The ratio CEC/CEC^{OI} then shows the welfare of the collective schemes relative to this optimal individual benchmark. For participants with a default risk aversion ($\gamma = 5$), the hybrid defined benefit (DB_H) scheme provides a welfare gain of 2.3% per annum vis-a-vis the OI scheme in terms of certainty equivalent consumption. Over the full life-cycle, this amounts to more than one annual salary gain. The results indicate that well-structured intergenerational risk sharing is welfare enhancing compared to the optimal individual scheme. Despite the fact that adjustment speed coefficients and asset allocation are fixed, the hybrid collective scheme still outperforms the individual benchmark. The hybrid pension plan with flexibility in adjusting both contribution and benefit levels to absorb funding residue are the most preferred in welfare terms. It outperforms other collective plans that only allow for adjustments in either contributions or benefit levels. Particularly, the DB_{CA} scheme has a welfare loss of 0.3% relative to the optimal individual scheme, whereas the DB_{BA} scheme shows a welfare loss of 2.7%. Notice that the welfare of the DB_{BA} scheme is very close to the welfare of its individual counterpart, the DC plan. As the DC scheme induces a 2.8% welfare loss vis-a-vis the OI scheme, the hybrid defined benefit scheme outperforms the more realistic DC scheme by 3–6% per annum.

⁷This finding confirms the results of Van Hemert (2005), who shows that the optimal intergenerational risk sharing in social security program is neither a pure defined benefit type nor a defined contribution type, but a state contingent hybrid scheme.

3.3 Market valuation of the pension schemes

In this section, we focus on the market values of contributions and benefits in the optimal collective schemes. For an individual member, it may be difficult to trade his pension claims once he stepped into a pension contract, but a market valuation is useful ex-ante to evaluate a given pension deal. Furthermore, it is important to investigate whether the target contributions and benefits set according to Equation (13) and the risk sharing rules in Equations (19) and (20) are actuarially fair. If so, then it means the market value of contributions equals the market value of benefits, and ex ante, the starting generation does not borrow from future generations and therefore there is no "chocolate paradox" of the type described by Shell (1971).

From the perspective of a new pension fund member, the market value of the pension deal is the value of the actual benefits minus the contributions. We take all the (stochastic) cash flows (p_t, b_t) generated by the optimal pension schemes and value them using the deflator method.⁸ For the typical member with retirement date R and life expectancy T , the net present value (NPV) of the pension deal is the difference between the present value of benefits (PVB) and the present value of contributions (PVP)

$$NPV = PVB - PVP = E_0 \left[\int_R^T M_t b_t dt \right] - E_0 \left[\int_0^R M_t p_t dt \right] \quad (23)$$

where p_t and b_t are the (stochastic) actual contributions paid and benefits received, and M_t is defined as eq(3). From an ex-ante point of view, the pension scheme is a fair deal if the NPV is zero.

The collective schemes may shift the funding mismatches beyond one's life time, leaving surplus or deficit in the notional cohort account. An alternative way of calculating the present value of pension deals therefore is to look at the remaining balance left in the notional cohort account at the end of life of a cohort. A positive remaining balance indicates positive transfers from this generation to other cohorts. Similarly, a negative transfer means the cohort receives cash flows from other cohorts. In a collective pension scheme, any cohort writes a call option to share a funding surplus with the other cohorts, and holds a protective put option to receive protections from the other cohorts. When the value of the call equals the value of the put, the pension deal is a fair deal in value terms ex ante. Formally, let a_t denote the accumulated asset in the notional cohort account through out their life time

⁸This analysis builds on Bader and Gold (2002), Blake (1998), Chapman et al. (2001) and De Jong (2007), and results in the method of value-based generational accounting of Ponds (2003).

($0 \leq t \leq T$). Each cohort starts its notional account with no surplus or deficit, i.e. $a_0 = 0$. The dynamics of the cohort account are

$$da_t = [a_t(r + \omega(\mu - r)) + p_t - b_t]dt + \omega\sigma a_t dZ_t \quad (24)$$

Integrating out this dynamics and applying Equation (23), the NPV of the pension deal can be decomposed as the sum of the value of the call and the put option, valuing the positive and negative transfers to other cohorts:

$$NPV = -E_0[M_T a_T] = -E_0[M_T(a_T)^+] + E_0[M_T(-a_T)^+] = -call + put \quad (25)$$

Table 2 summarizes the market valuation of the intergenerational transfers and options for initially fully funded pension schemes ($FR_0 = 1$). As required by the assumption of actuarial fairness, the NPV is indeed zero for all collective schemes, i.e. market value of contributions equals the market value of benefits. Ex ante, the starting generation does not borrow from future generations and therefore there is no "chocolate paradox" of the type described by Shell (1971). However, the market values of positive and negative transfers (the implicit call and put options) are potentially large, between 0.2 to 1.25 annual salaries, indicating a substantial amount of ex post transfers. The magnitude of the transfers depends on the chosen asset mix, the level of contributions and the risk sharing rules. The DB_{CA} scheme results in the highest value of transfers whereas the DB_{BA} scheme results in the lowest value of transfers. This makes the DB_{CA} system less sustainable than the DB_{BA} system, as future generations can be confronted with larger deficits in the pension fund.

4 Robustness checks

4.1 Risk aversion, time preference and equity premium

We now perform some robustness checks to see if our results still hold under different risk and time preferences, and with a reduced equity premium. The degree of risk aversion has several effects on the optimal pension design, as shown in Table 1. Comparing the results as the coefficient of risk aversion increases from $\gamma = 3$ to $\gamma = 8$, one observes three changes: (i) the portfolios become less risky, (ii) the contribution rates increase, and (iii) the schemes rely more on intergenerational risk sharing by choosing lower values for α and/or β . Furthermore, the degree of risk aversion affects the welfare gain from intergenerational risk sharing. The welfare gain for a less risk averse investor ($\gamma = 3$) is 3.9% in the DB_H scheme, whereas it

is reduced to 0.9% for a more risk averse investor ($\gamma = 8$). The result that more risk-averse agent obtains less welfare gain may seem counter-intuitive at first sight, but observe that the less risk-averse agent is willing to accept a more risky portfolio and thus benefits more from intergenerational risk sharing. The less risk averse participants are also willing to accept larger adjustments in contributions and benefits in order to absorb funding mismatches, which is reflected in higher values for α and β .

The degree of risk aversion has a major impact on the portfolio choice. Figure 3 displays the average life cycle portfolio profiles for investors with different risk aversion $\gamma = 3, 5, 8$. Less risk averse investor allocates more assets in equities. The degree of risk aversion has only a minor impact on consumption and asset accumulation profiles, via the different portfolio choices. High risk averse individuals save more and consume less compared with the less risk averse individuals. Figure 3 (c,d) also shows the average life cycle consumption profiles for investors with different subjective discount rate $\delta = 4\%, 2\%$, and 0% . The subjective discount rate has a strong impact on consumption profiles. In general, from the first order condition for optimality (Equation 32 in the appendix) it follows that the expected consumption is increasing when the expected portfolio return is larger than the subjective discount rate. The figures show that for $\delta = 4\%$, the median consumption profile is fairly flat. For lower values of δ the median consumption is increasing with age. The subjective discount rate has only a minor impact on the portfolio choice, via the changes in total-wealth to financial wealth ratio.

With a lower equity premium, the results are in line with our baseline results reported in Table 1, supporting our claim that well-organized IRS by collective pension funds is welfare improving. However the welfare levels are lower and the relative welfare gains are smaller. For instance, when the equity premium is reduced from 4% to 3%, the welfare gain for participants in DB_H scheme vis-à-vis the OI scheme ranges now from 0% to 2%, depending on the degree of risk aversion, and 2% to 4% vis-à-vis the DC scheme. A lower subjective discount factor results in higher contribution rates, similar to the individual schemes. However, other design parameters are not significantly different from the baseline cases.⁹

4.2 Initially underfunded and overfunded schemes

The distributions of the funding status over time are critical for the welfare of the future cohorts and the sustainability of the schemes. Seriously underfunded situations are difficult

⁹Detailed results are available on request.

for the sustainability of the fund, as in these situations future generations may want to step out. Figure 4 shows quantile distributions of the normalized funding ratio's, $FR_t = S_t/L$, of the three optimal collective schemes. Due to the positive equity premium, funding surpluses are more frequent (and larger in size) than funding deficits.

Table 3 shows the welfare gains and losses of entering the initially over- and underfunded schemes (with funding ratios being $FR_0 = 1.1$, $FR_0 = 0.9$ and $FR_0 = 0.8$ respectively). These schemes are set at their optimal designs as in Table 1 (middle panel). The new cohort joining an underfunded collective scheme is not necessarily worse off in welfare terms, comparing with the optimal individual benchmark. For instance, when FR_0 falls to 0.9 (0.8) initially, the welfare of DB_H is 100.7% (98.8%) of that of the optimal individual scheme. Hence it is possible for well-structured collective pension schemes to absorb funding deficits up to 10% to 20% by intergenerational risk sharing and still enhance the welfare for her participants.

Table 3 also shows the market valuation of the intergenerational transfers when the collective schemes are initially underfunded. The net transfers ex ante are non-zero. The net transfers are large and positive, meaning the entry cohort makes up a large part of the initial deficits, by either higher contributions (DB_{CA} , DB_H) or lower benefits (DB_{BA} , DB_H). Furthermore, the net transfers are proportional to the degree of underfunding. For instance, the value of net transfers doubles when FR_0 is reduced from 0.9 to 0.8. Initially overfunded schemes are also reported in Table 3. The market values of the net transfers starting with $FR_0 = 1.1$ are mirror images of $FR_0 = 0.9$.

5 Welfare of future generations

Having shown that IRS in collective pension plans can be welfare improving for the new entry cohort, we investigate in this section whether or not the welfare gain of the new-entry cohort comes at the expense of the future cohorts. To do this, we compute the ex ante expected welfare of the future cohorts, who will join the optimal collective schemes described as in Table 1, with the initial condition that the pension funds are fully funded at $t = 0$. We calculate the certainty equivalent consumption, CEC^f , of the future cohorts (the future 25-year-old's) who will enter into the pension fund in f years' time, with $f = 0, 1, 2, \dots, 1000$ years. The expected welfare of cohort f obtained from participating a given pension scheme, during his life time from year f till $f + T$, is measured in terms of certainty equivalent

consumption as follows:

$$E_0 \left[\int_0^T e^{-\delta x} u \left(c_{f+x}^f \right) dx \right] \equiv \int_0^T e^{-\delta x} u \left(CEC^f \right) dx \quad (26)$$

Figure 5 presents the CEC of the future cohorts (up to cohort $f = 1000$) relative to that of the optimal individual scheme, CEC^f/CEC^{OI} , which can be seen as the relative welfare gain or loss over the individual benchmark. The expected welfare of all future cohorts are higher than the time 0 entry cohort. The welfare is steadily increasing for future cohorts and converging to a level much above the level for the entry cohort. For instance, the welfare of DB_H scheme stabilizes at 130% of that of the OI scheme after 100 years. From this result we can conclude that the expected welfare gain of the current entry cohort is not at the cost of the future cohorts from an ex ante perspective. Although optimization occurs by the entering generation, the fund is expected to build up a buffer, and is, in expectation, overfunded in the long run. For DB_H scheme, the median size of surplus becomes twice as large as the size of liability after 200 years. Due to these transfers, future cohorts are able to benefit from the positive equity premium before they are born. Intergenerational risk sharing together with asset accumulations greatly enhance the intertemporal risk smoothing capability of the economy with collective pension schemes.

An alternative way to design the collective schemes is to take into account the welfare of future generations by taking a social planner's view. Suppose the social planner's objective is to maximize a weighted sum of entry and future cohorts' life time expected utility by optimizing the pension scheme $\{p, \alpha, \beta, \omega\}$ at time zero:

$$U^{Social} = \max_{\{0 \leq \omega \leq 1, p, \alpha, \beta\}} E_0 \left[\sum_{f=0}^{\infty} \left(\Delta^f \int_0^T e^{-\delta x} u \left(c_{f+x}^f \right) dx \right) \right] \quad (27)$$

where $0 < \Delta < 1$ is the social planner's weighting factor for the future cohorts. There is no clear guidance as to how Δ should be chosen. Gollier (2008) imposes that the assets of the pension fund follow a martingale; this assumption implies $\Delta = 0.937$, given his other parameter values. We assume the weighting factor of the social planner is the same as individuals' subjective discount factor, hence $\Delta = e^{-\delta} = 0.96$.¹⁰ We calculate the social planner's designs for individuals with risk aversion of $\gamma = 5$.

Table 4 shows the results for the social planner's optimal pension schemes. Comparing the socially optimal schemes with the schemes optimized for the $f = 0$ entry cohort in Table 1, we see that the values of α and β are slightly lower and the portfolio choice becomes

¹⁰Feldstein (2005) provides an argument why a 4% discount rate is a reasonable value.

more risky. These results indicate that shocks in funding surplus are shared by more generations under the socially optimal schemes. Table 4 also shows the market value of the intergenerational transfers under the socially optimal schemes. Indeed the market value of transfers (calls and puts) are larger comparing with that of Table 2. CEC^{opt} shows the certainty equivalent consumption of the time 0 entry cohort under the scheme optimized by the social planner. From comparing CEC^{opt} with CEC in Table 1, we find that the welfare of the time 0 entry generation is slightly reduced under the social planner's schemes. These results do not materially change for different risk and time preferences or a lower equity premium; the long run welfare gains for the future cohorts are still substantial.

6 Conclusion

We have used the institutional setting of funded pension schemes to study welfare aspects of intergenerational risk sharing. Typical for such collective pension plans is that pension benefits and/or pension contributions may depend on the funding status. From the perspective of a newly entering cohort, we optimize the explicit asset allocation and risk allocation rules, which specify who of the stakeholders, when, and to what extent is taking part in risk bearing.

We show that well-designed funded schemes with intergenerational risk sharing are welfare improving over and above the fully optimal individual scheme by up to 2-4% in terms of certainty equivalent consumption. The hybrid defined benefit scheme (DB_H), where risks are shared between working and retired cohorts, performs better than schemes where risks are only borne by workers (DB_{CA}) or retirees (DB_{BA}). The initially fully funded collective pension schemes with intergenerational risk sharing are a zero-sum game in value terms, however they are potentially a positive-sum game in welfare terms. Furthermore, the expected welfare gain of the current entry cohort is not at the cost of the older and future cohorts, from an ex ante perspective. This result has important implications for the current trend of shifting from defined benefit to (individual) defined contribution schemes. In such individual schemes, the risks are concentrated mainly in the retirement phase, giving up the intergenerational risk sharing potential and the greater intertemporal risk smoothing capacity of collective plans.

Introducing more sources of systematic risks in addition to the modeled investment risks, like labor income risk and real interest rate risk, might further strengthen the welfare-

enhancing potential of intergenerational risk sharing via collective schemes compared to the optimal individual scheme.

A Solution methods

A.1 Optimal individual scheme

This appendix explains the solution procedure solving the optimal individual (OI) consumption and portfolio choice described in Section 2.2. We first rewrite the optimization problem (5) in discrete time form (at annual frequency), and rewrite the original objective function in the recursive form as follows

$$U(W_t) = u(c_t) + E_t [e^{-\delta} U(W_{t+1})] \quad (28)$$

$$s.t. \ W_{t+1} = (W_t - c_t) \left[R^f + \omega_t (\tilde{R}_{t+1}^s - R^f) \right] + 1 \quad \text{for } 1 \leq t < R \quad (29)$$

$$W_{t+1} = (W_t - c_t) \left[R^f + \omega_t (\tilde{R}_{t+1}^s - R^f) \right] \quad \text{for } R \leq t < T \quad (30)$$

where $W_1 = 1$, $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$, $\tilde{R}_{t+1}^s = \exp\left(\mu + \sigma \int_t^{t+1} dZ_s\right)$, and $R^f = \exp(r)$. The problem is solved using dynamic programming principle, solving backwards from the last to the first period.

For the final period, the optimal consumption $c_T^* = W_T$, and the functional form of indirect utility is known, as $U(W_T) = u(c_T^*) = \frac{(c_T^*)^{1-\gamma}}{1-\gamma}$. Then, we proceed backward in time to $t = T - 1$. The procedure for $T - 1$ starts by defining a new variable $a_t = W_t - c_t$, as the after-consumption-wealth. Following Carroll (2006), we construct a grid of $a_t = \{a_j\}_{j=1}^J$. Now we solve for the optimal consumption and portfolio policies for each given a_j . The first order conditions with respect to ω_t and c_t are

$$0 = E_t \left[e^{-\delta} U'(W_{t+1}) (\tilde{R}_{t+1}^s - R^f) \right] \quad (31)$$

$$u'(c_t) = E_t \left[e^{-\delta} U'(W_{t+1}) \left(R^f + \omega_t (\tilde{R}_{t+1}^s - R^f) \right) \right] \quad (32)$$

The envelope theorem implies that $u'(c_t) = U'(W_t)$, since

$$U'(W_t) = E_t \left[e^{-\delta} U'(W_{t+1}) \left(R^f + \omega_t (\tilde{R}_{t+1}^s - R^f) \right) \right] \quad (33)$$

Replace $U'(W_{t+1})$ by $u'(c_{t+1})$ in the two first order conditions, we have

$$0 = e^{-\delta} E_t \left[u'(c_{t+1}^*[W_{t+1}]) (\tilde{R}_{t+1}^s - R^f) \right] \quad (34)$$

$$c_t^*(a_t) = I_u \left(e^{-\delta} E_t \left[u'(c_{t+1}^*[W_{t+1}]) \left(R^f + \omega_t^*(a_t) (\tilde{R}_{t+1}^s - R^f) \right) \right] \right) \quad (35)$$

where $c_{t+1}^*[W_{t+1}]$ as the optimal consumption policy at time $t + 1$, and $I_u(\cdot)$ denotes the inverse function of $u'(c_t)$. Using any numerical solver, Equation (34) will give the optimal portfolio weight $\omega_t^*(a_t)$ for any given amount of investment a_t . Because of the borrowing and short-selling constraints, we then impose the restriction $0 \leq \omega_t^* \leq 1$. Then, Equation (35) gives the corresponding consumption $c_t^*(a_t)$ for any given amount of investment a_t . Finally, the optimal wealth process is endogenously determined by $W_t^* = c_t^*(a_t) + a_t$. The advantage of this method is that the numerical search is only needed once in solving $\omega_t^*(a_t)$, while $c_t^*(a_t)$ can be directly obtained from Equation (35).

Following the same procedure illustrated above, we can solve the model backward in time to $t = 1$. To generate the average pattern of life cycle portfolio holding we simulate the model from time 1 to T for 10,000 scenario's, and take the average over all simulated scenario's.

A.2 DC scheme

The optimal consumption and portfolio choice after retirement in the *DC* scheme is identical to the one in the *OI* scheme. Before retirement, the optimal portfolio weight ω_t is stochastic, depending on the relative size of financial capital and human capital. Campbell and Viceira (2002) show that the optimal individual portfolio choice before retirement in the *DC* plan without constraints is the leveraged myopic portfolio:

$$\omega_t^* = \frac{1}{\gamma} \frac{\mu - r}{\sigma^2} \frac{W_t^{DC} + PVC_t}{W_t^{DC}} \quad (36)$$

where PVC_t denotes the present value of the future pension contributions, as a fraction of the investor's human capital,

$$PVC_t = m \int_t^R e^{-rs} y ds \quad (37)$$

Since the investor is borrowing and short sales constrained, the optimal portfolio is then given by: $\omega_t = \min(\max(0, \omega_t^*), 1)$. The optimal contribution rate m is found by numerical search, given the optimal portfolio strategy.

A.3 Collective schemes

The optimization problem of collective schemes, Equation (21), is a static problem, since all the decision parameters $\{p, \alpha, \beta, \omega\}$ are constants. The collective schemes are solved using

Monte Carlo simulations and grid search. We first construct a four dimensional grid for the decision parameters. For each combination of the parameters, we simulate the wealth dynamics (15) at each point in time, evaluate the resulting funding surplus and determine the state contingent contributions and benefits p_t and b_t according to the risk sharing rules in Equations (19) to (20). When p_t and b_t are determined, we evaluate the objective function (21) for the new entry cohort. A numerical grid search identifies the global maximum of the welfare function (21) and its corresponding parameter values $\{p, \alpha, \beta, \omega\}$.

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Table 1: **Optimal pension schemes**

The table shows the optimal pension scheme parameters under the default values for the time preference ($\delta = 4\%$) and equity premium ($\mu - r = 4\%$). The optimal contribution rate and the corresponding replacement ratio are given by (p, b) . The optimal speed of risk absorbing by adjusting contributions or benefits is determined by (α, β) respectively. The optimal portfolio weight in equities is given by (ω) . The welfare levels achieved under these optimal collective schemes are indicated by CEC (in units of annual salary). The ratio CEC/CEC^{OI} shows the relative welfare gain or loss of the collective schemes relative to the optimal individual scheme (OI).

γ		OI	DC	DB_{CA}	DB_{BA}	DB_H
3	p	-	11.0%	16.6%	11.4%	14.0%
	b	-	-	78.4%	53.6%	66%
	α	-	-	0.07	-	0.06
	β	-	-	-	0.03	0.02
	ω	-	-	100%	94%	100%
	CEC	0.908	0.886	0.926	0.886	0.943
	CEC/CEC^{OI}	100%	97.6%	102.0%	97.6%	103.9%
5	p	-	12.8%	16.6%	13.1%	14.0%
	b	-	-	78.4%	53.6%	66%
	α	-	-	0.05	-	0.045
	β	-	-	-	0.03	0.02
	ω	-	-	96%	60%	100%
	CEC	0.892	0.867	0.889	0.867	0.912
	CEC/CEC^{OI}	100%	97.2%	99.7%	97.2%	102.3%
8	p	-	14.1%	17.5%	14.0%	15.7%
	b	-	-	82.5%	66%	74.4%
	α	-	-	0.04	-	0.04
	β	-	-	-	0.02	0.02
	ω	-	-	67%	55%	76%
	CEC	0.876	0.854	0.865	0.853	0.888
	CEC/CEC^{OI}	100%	97.7%	98.7%	97.3%	100.9%

Table 2: **Market value of intergenerational transfers**

This table reports the market value of the actual contributions (PVP) and benefits (PVB) for a new entry cohort and an initially fully funded pension scheme ($FR_0 = 1$). The market value of the intergenerational transfers are shown in columns 'call' (for positive transfers) and 'put' (for negative transfers), as defined in equation (25). The market values are expressed in terms of multiples of annual salary. The welfare levels achieved under these optimal collective schemes are indicated by CEC (in units of annual salary). The ratio CEC/CEC^{OI} shows the relative welfare gain or loss of the collective schemes relative to the optimal individual scheme (OI).

	DB_{CA}	DB_{BA}	DB_H
PVP	4.62	3.64	3.89
PVB	4.61	3.64	3.88
<i>put</i>	1.26	0.22	0.77
<i>call</i>	1.25	0.22	0.77

Table 3: **Initially overfunded and underfunded schemes**

FR_0 denotes the initial funding ratio. The optimal scheme designs are as shown in Table 1 for $\gamma = 5$. CEC/CEC^{OI} shows the relative welfare gain or loss to the optimal individual scheme.

	DB_{CA}	DB_{BA}	DB_H
$FR_0 = 1.1$			
PVP	4.05	3.64	3.55
PVB	4.61	3.79	3.91
call	1.02	0.15	0.62
put	1.58	0.30	0.98
NPV	0.56	0.15	0.36
CEC	0.909	0.871	0.928
CEC/CEC^{OI}	101.9%	97.5%	104.0%
$FR_0 = 0.9$			
PVP	5.16	3.64	3.50
PVB	4.61	3.50	3.86
call	1.53	0.31	0.94
put	0.98	0.16	0.60
NPV	-0.55	-0.15	-0.35
CEC	0.867	0.862	0.896
CEC/CEC^{OI}	97.4%	96.8%	100.7%
$FR_0 = 0.8$			
PVP	5.71	3.64	4.53
PVB	4.61	3.35	3.84
call	1.75	0.42	1.14
put	0.65	0.12	0.45
NPV	-1.10	-0.30	-0.70
CEC	0.845	0.856	0.881
CEC/CEC^{OI}	94.8%	96%	98.8%

Table 4: **Socially optimal collective schemes**

This table shows the social planner's optimal designs with weighting factor $\Delta = \delta = 0.96$ and risk aversion $\gamma = 5$. CEC^{opt} is the certainty equivalent consumption of the time 0 entry cohort under the optimal schemes chosen by the social planner.

	DB_{CA}	DB_{BA}	DB_H
p	19.2%	14%	15.7%
b	90.7%	66%	74.3%
α	0.04	-	0.03
β	-	0.025	0.01
ω	100%	65%	100%
PVP	5.35	3.87	4.36
PVB	5.34	3.87	4.35
call	1.46	0.30	0.98
put	1.45	0.30	0.97
CEC^{opt}/CEC^{OI}	98.7%	96.4	101.8%

Figure 1: **Life cycle profiles of the cohort specific schemes (OI and DC)**

Taking a benchmark investor (with $\gamma = 5$, $\delta = 4\%$), the following charts show the life cycle quantile distributions of portfolio weights in equities for the *OI* scheme (a) and the *DC* scheme (b); the life cycle quantile distributions of consumptions for the *OI* scheme (c) and the *DC* scheme (d); and the life cycle quantile distributions of wealth accumulation for the *OI* scheme (e) and the *DC* scheme (f).

Figure 2: **Consumption profiles obtained from the collective schemes**

The charts below show the (5%, 50%, 95%) quantile distribution of consumptions in the optimal collective schemes (DB_{CA} , DB_{BA} , and DB_H). The scheme parameters are fixed at their optimal values as given in Table 1 for $\gamma = 5$. The consumption levels are normalized by annual salary, i.e. c_t/y .

Figure 3: The impacts of different risk aversion and subjective discount rates on individual schemes.

Varying the degree of risk aversion among $\gamma = 3, 5$, and 8 , while keeping other parameters at their default values, (a) shows the median of portfolio weight distribution in the *OI* scheme over ones life cycle, and (b) shows the median of portfolio weights distribution in the *DC* scheme. Varying the subjective discount rate among $\delta = 4\%$, 2% , and 0% , while keeping other parameters at their default values, (c) shows the median of consumption distribution in the *OI* scheme, and (d) shows the median of consumption distribution in the *DC* scheme.

Figure 4: **Funding status of the collective schemes.**

The charts below show the (5%, 50%, 95%) quantile distribution of the surplus ratio, S_t/L , of the optimal collective schemes (DB_{CA} , DB_{BA} , and DB_H). The scheme parameters are fixed at their optimal values as given in Table 1 for $\gamma = 5$.

Figure 5: **Future generations under the generation-0 optimal schemes.**

Consider a future generation f who enters a given collective scheme in f years' time, and the scheme is optimized for the time-0 generation as characterized in Table 1 ($\gamma = 5$). The upper panel shows the welfare improvements obtained from entering the given collective schemes (DB_{CA} , DB_{BA} , and DB_H) above the optimal individual benchmark (OI), CEC^f/CEC^{OI} , for these 1000 future generations ($f = 1, 2, \dots, 1000$).