Precision holography and its applications to black holes
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CHAPTER 2

THE FUZZBALL PROPOSAL FOR BLACK HOLES

(2.1) BLACK HOLE PUZZLES

Among the most challenging questions of black hole physics of the last 30 years are the origin of the Bekenstein-Hawking entropy, whether information is lost in black hole evaporation and how singularities are resolved in a full theory of quantum gravity.

According to the no-hair theorem of Einstein-Maxwell gravity, black holes are solely characterized by their mass, charge and angular momentum. Nevertheless to prevent the total entropy in the universe to decrease if matter falls in a black hole, which would be a violation of the second law of thermodynamics, it is necessary to assign black holes an intrinsic entropy. Following a formal analogy between the laws of thermodynamics and the laws of black hole mechanics, this entropy should be proportional to the horizon area of the black hole, the Bekenstein-Hawking entropy. The discovery of Hawking radiation by semiclassical quantization of the black hole geometry then showed that black holes indeed emit black body radiation according to their assigned temperature and furthermore fixed the precise prefactor of their entropy. Since then it has been a longstanding issue of gravitational physics to find the microscopic degrees of freedom corresponding to the Bekenstein-Hawking entropy and to explain why their number grows as $\exp(A/4G)$ with the horizon area $A$.

Hawking radiation also gave rise to the information paradox. Since the radiation is exactly thermal and the black hole finally evaporates, it seems as if information is lost in this process (see figure 2.1). In quantum-mechanical terms, conservation of information is equivalent to unitarity. If a final state in a quantum-mechanical process arises from unitary evolution,

$$|\psi\rangle_f = e^{-iHt}|\psi\rangle_i,$$

(2.1)
it means that the initial state can be reconstructed by inverting the evolution,

$$|ψ⟩_i = e^{iHt} |ψ⟩_f,$$

and hence information has been preserved. However, in Hawking’s calculation, entanglement between infalling and outgoing pair quanta at the horizon causes the state at $Σ_f$ to be mixed, since the infalling quanta have been destroyed. If the state at $Σ_i$ is pure, the evolution from $Σ_i$ to $Σ_f$ is necessarily non-unitary.

An alternative is to assume that information leaks out in subtle correlations of the Hawking radiation which are invisible in the semiclassical approximation. In this scenario the Hawking radiation is conceptually not very different from the black body radiation of a piece of burning coal. However, for a macroscopic black hole with mass well above the Planck mass, this requires that information must be non-locally transmitted from the infallen matter near the center of the black hole to the horizon. As a result, it seems that the information paradox requires either giving up unitarity or locality in a full quantum theory of gravity.

Assuming that AdS/CFT is valid gives an implicit solution to the information paradox: Since
the evolution in the dual QFT is unitary the evolution in the bulk gravity theory must be as well, so information must be conserved. Unfortunately however, this argument does not reveal how information escapes the black hole. It is not known how to calculate in the dual QFT correlations measured by an infalling observer.

The fuzzball proposal [27, 28], arising out of string theory, proposes to resolve the information paradox and to provide a microscopic description of the Bekenstein-Hawking entropy. The basic idea is to replace the black hole by a large number of horizonless solutions which asymptote to the black hole geometry but differ at the horizon scale. These horizonless solutions are thought to correspond to microscopic states in the black hole ensemble, and upon averaging over these geometries, the original black hole with its horizon is retrieved. The fuzzball proposal solves the information paradox because each individual microstate geometry does not possess a horizon which implies that information can escape the black hole after a very long time once one takes into account its pure state. After quantization of the solution phase space, it furthermore allows for a statistical explanation of the Bekenstein-Hawking entropy as the microstates are given by the (quantized) individual geometries.

So far, all candidate solutions which have been found only involve low energy supergravity fields. However there are only a few, atypical states which are believed to be well described by supergravity. Typical microstate geometries are expected to contain regions of string scale curvature in which higher string modes and higher modes arising in the compact space of a ten- or eleven-dimensional fuzzball solution become important.

Although the fuzzball proposal was originally formulated for black holes in asymptotically flat spacetimes, it can be analyzed using AdS/CFT if the near-horizon limit of the black hole has a known holographic dual at the boundary. In fact, as we will elaborate on below, AdS/CFT strongly supports the fuzzball program.

(2.2) **BLACK HOLE ENTROPY COUNTING BY STRING THEORY**

Already before the proposal of the fuzzball program, string theory has been able to count the entropy of black holes, mostly extremal BPS black holes. [29] As an example we review here the case of the five-dimensional black hole arising from the bound D1-D5-P system with respective charges \( Q_1, Q_5 \) and \( Q_p \) compactified on \( S^1 \times M_4 \), where \( M_4 \) is either \( T^4 \) or \( K3 \). Both D1 and D5 branes wrap the \( S^1 \) with radius \( R_z \gg \sqrt{\alpha'} \), while the volume of the compact space is taken of the order of the string length, \( vol(M_4) \sim \alpha'^2 \). The momentum \( P \) then denotes the momentum of excitations along the circle. This system preserves 1/8 of the supersymmetry. The solution in the decoupling limit is given by

\[
\frac{ds^2}{\sqrt{h_1 h_5}} = \left( -\left( dt^2 - dz^2 \right) + \frac{Q_p}{r^2} (dz - dt)^2 \right) \tag{2.3}
\]
\[ +\sqrt{h_1 h_5} dx^m dx^m + \sqrt{\frac{h_1}{h_5}} ds^2(M_4), \]

where \( h_i = 1 + Q_i/r^2 \), \( x^m \) denote the coordinates in the transverse direction and \( ds^2(M_4) \) denotes the metric on the compact space. The corresponding RR 2-form potential and dilaton is given by
\[ e^{-2\Phi} = \frac{h_5}{h_1}, \quad C_2 = (h_1^{-1} - 1) dt \wedge dz. \]

The charges \( Q_i \) can be expressed in terms of integral charges \( N_i \) via
\[ Q_1 = \frac{N_1 g_s \alpha'^3}{V}, \quad Q_5 = \frac{N_5 g_s \alpha'}{R^2 z}, \quad Q_p = \frac{N_p g_s^2 \alpha'^2}{R^2}, \]
with \( V = (2\pi)^{-4} \text{vol}(M_4) \). The Bekenstein-Hawking entropy of this black hole in the Einstein frame \( ds^2_E = e^{-\Phi/2} ds^2 \) can be calculated to be
\[ S_{BH} = \frac{A_{10}}{4G_{10}} = \frac{A_5}{4G_5} = 2\pi \sqrt{N_1 N_5 N_p}, \]
where \((A_{10}, G_{10})\) and \((A_5, G_5)\) are the horizon area and gravitational constant in ten and five dimensions respectively. The gravitational constants are given by
\[ G_{10} = 8\pi \kappa_{10} = 8\pi^6 g_s^2 \alpha'^4, \quad G_5 = \frac{G_{10}}{(2\pi)^5 R v}. \]

This supergravity description of the D1-D5-P system is valid if all \( Q_i \gg \sqrt{\alpha'} \).

The microscopic calculation of the entropy counts the excitations of the low-energy theory of the D-brane system. Since the volume of \( M_4 \) is of order of the string scale the system is described by an effective \((1+1)\)-dimensional theory living on the circle. This low-energy theory, which is conjectured to be the \( N = (4,4) \) sigma model on the symmetric orbifold \( (M_4)^{N_1 N_5}/S^{N_1 N_5} \), is also the AdS/CFT dual to IIB string theory on \( AdS_3 \times S^3 \times M_4 \), since the decoupling limit of the D1-D5-P solution above is \( BTZ \times S^3 \times M_4 \), and \( BTZ \) can be obtained by orbifolding \( AdS_3 \). The perturbative description of the dual theory is valid for all \( Q_i << \alpha' \).

However, unlike in [29] where the microscopic entropy was counted in the perturbative regime of the orbifold CFT and related to the supergravity regime by using non-renormalization theorems, we strictly speaking do not need the details of the CFT here. If we invoke AdS/CFT, the only necessary assumption is that there is a CFT dual to \( AdS_3 \times S^3 \times M_4 \) in the supergravity regime which is unitary. By either analyzing the asymptotic conformal symmetries [30] or by computing the conformal anomaly [13] one then finds that the central charge of this CFT dual is given by
\[ c = \frac{3l}{2G_3} = 6N_1 N_5. \]
In the supersymmetric case the momentum $P$ in the bound state correspond to the left-moving excitation level in the dual CFT, while the right-moving excitations are in their ground state. Due to the unitarity of the dual CFT we can calculate the degeneracy at high excitation number $N_p$ with Cardy’s formula \[31\],

$$d(c, N_p) \sim e^{2\pi \sqrt{N_p c/6}}.$$  \hspace{1cm} (2.9) 

As a result, the microscopic calculation of the entropy yields

$$S_{\text{mic}} = \ln d(c, N_p) = 2\pi \sqrt{N_1 N_5 N_p},$$  \hspace{1cm} (2.10) 

which precisely agrees with (2.6).

(2.3) **THE D1-D5 TOY MODEL**

Even though black hole entropy counting, which has been successfully performed for many more extremal and near-extremal black holes, offers an important glimpse of the microscopic origin of the Bekenstein-Hawking entropy, it only partly addresses the questions raised in the beginning of section (2.1). The counting is performed in the dual QFT and it is a priori not clear how the QFT states are related to gravitational states. Particularly it is not known how global properties of the gravitational description like horizons are encoded in the QFT. As a result, it does not give information on how the entropy is related to the horizon of the black hole and how the information paradox is resolved. In contrast, the fuzzball proposal goes further by suggesting an explicit representation of the microscopic states in terms of gravitational degrees of freedom. As we will see below, these gravitational degrees of freedom are related to the dual microscopic states by AdS/CFT.

The idea of the fuzzball proposal is to replace naive black hole solutions like (2.3) by an ensemble of solutions with the same asymptotic charges but a different geometry at the horizon scale. Unfortunately, the fuzzball solutions corresponding to the macroscopic 3-charge D1-D5-P black hole are quite intricate. An interesting toy model, which we will extensively explore in later chapters, is the 2-charge D1-D5 system. The naive solution is obtained by setting $N_p = 0$ in (2.3),

$$ds^2 = \frac{1}{\sqrt{h_1 h_5}}(-dt^2 + dz^2) + \sqrt{h_1 h_5} dx^m dx^m + \sqrt{\frac{h_1}{h_5}} ds^2(M_4),$$  \hspace{1cm} (2.11) 

and preserves 1/4 of the supersymmetry. The solution is only a toy model for a black hole, since its naive solution has no horizon but only a naked singularity. Only if one includes higher order corrections a small horizon appears, whose associated entropy agrees with the dual CFT calculation. The D1-D5 system can be related by U-duality to the F1-P chiral null model describing a fundamental string winding a compact direction with momentum, whose solution

\footnote{This is surprising since only a subset of higher order corrections are known and the curvature at the horizon of this small black hole is of order of the string scale. An explanation for black holes involving an $AdS_3$ factor in their (corrected) near-horizon geometry was given in \[32\].}
has been found in [33, 34]. We will exploit this in chapter 4 to find the most general fuzzball solution.

The D1-D5 fuzzball solutions corresponding to (2.11) are characterized by a curve \( F^I(v) = (F^I(v), F^\rho(v)) \) extending in the transverse and internal directions, on which D1 and D5 charge is distributed. If there are no internal excitations, \( F^\rho(v) = 0 \), the solution has a slightly simpler form (3.44) than in the general case (4.51). If \( F^I \equiv 0 \) the solution collapses to the naive solution; otherwise the size of the curve \( F^I(v) \) determines the scale at which the fuzzball geometry starts deviating. If the curve does not intersect with itself and if \( \frac{d}{dv} F^I(v) \neq 0 \) everywhere, the geometry close to the curve resembles a Kaluza-Klein monopole and remains smooth. Furthermore, since the Killing vector field \( \partial/\partial t \) is timelike everywhere, there is no horizon and the solution has the same Penrose diagram as Minkowski space.

Classically, there is an infinite number of solutions parametrized by the curve \( F^I(v) \). If one wishes to obtain a statistical entropy out of this ensemble of solutions one has to quantize the phase space by geometric quantization, which has been done for the D1-D5 system with only transverse excitations in [35]. Geometric quantization in this case yields commutation relations, in which the Fourier modes of \( F^I(v) \) behave as oscillators. Counting the appropriate subspace yields precisely the fraction of entropy expected for transverse excitations. However one should mention that the counting includes regions in the phase space where higher-derivative corrections to supergravity are non-negligible. It is not clear why in this case the (mostly unknown) higher-derivative corrections do not seem to influence the counting.

### (2.4) AdS/CFT Supports the Fuzzball Proposal

If according to the fuzzball proposal there is an explicit representation of the microscopic states of a black hole in terms of geometries, it should be possible to map these states back to the microscopic states in the dual theory which we counted in section 2.2. In fact, this is what most of our discussion in chapter 3 and 4 will be about.

In the first step, we have to bring the fuzzball solutions in a form where we can analyze them with AdS/CFT: We replace the asymptotically flat region by an asymptotically AdS region, which corresponds to the replacement \( h_{1,5} \to h_{1,5} - 1 \). While the naive solution becomes locally \( AdS_3 \times S^3 \times M_4 \), the fuzzball geometries will only asymptotically be AdS, differing by normalizable modes from the naive solution. This is due to the fact that the curve \( F^I(v) \) spreads out in the transverse directions, generating higher multipole moments in addition to the asymptotic charges. By the AdS/CFT dictionary developed in chapter 1 we can then relate the normalizable modes to the vevs of gauge-invariant operators in the dual CFT. Knowing all such vevs determines in principle the (pure) state of the CFT which corresponds to the fuzzball geometry.

In practice this procedure is however complicated by the fact that we have to use Kaluza-Klein holography to extract holographic data from a ten-dimensional (or in this case six-dimensional,
since the $M_4$ is taken very small) geometry. As discussed in section 1.5, the formula (1.28) for the vevs of the dual operators in terms of the six-dimensional geometry contains in general non-linear contributions which have the same radial falloff as the linear contribution. In our case that means that the vev of a dual operator of dimension $k$ does not only get contributions from the $k$-th multipole moment but also from the product of lower multipole moments. As we are doing perturbation theory in the order of the multipole moments it follows that for example at cubic order we can only extract the vevs of operators of lowest and second-lowest dimension.

In chapter 3 we will therefore conjecture a specific map between fuzzball geometries and CFT states. The extraction of the dual vevs of lowest and second-lowest dimension operators will serve as a way to perform kinematical and dynamical test of this map.

Nonetheless, on a more general level we can invert above arguments to show how AdS/CFT supports the fuzzball proposal for every black hole whose entropy we believe to be counted by a (strongly coupled) dual QFT. Also in this QFT we would be able to distinguish the dual states of a black hole ensemble by the vevs of gauge invariant operators. By AdS/CFT every such (pure) state maps to a geometry with different normalizable modes and hence with different subleading asymptotics. Replacing again the asymptotically AdS region by an asymptotically flat region we obtain an ensemble of fuzzball solutions which have the same asymptotic geometry as the black hole but differ in the interior.

This procedure however does not imply that the fuzzball geometries obtained in this way are resolvable in supergravity. In fact, as we will see for the D1-D5 system in section 3.11, many states in the dual CFT do not yield geometries which are distinguishable in supergravity.

## 2.5. ARE ASTROPHYSICAL BLACK HOLES FUZZBALLS?

Even though we restrict our attention for the rest of this thesis to supersymmetric fuzzballs, we would like to mention that eventually the fuzzball program should also resolve the information loss and entropy problem for astrophysical black holes. These black holes differ from the supersymmetric D1-D5-P black hole in that they are four-dimensional and non-extremal, with an electric charge much smaller than their mass. While counting entropy and constructing fuzzball solutions for four-dimensional black holes are in principle not any more difficult than for five-dimensional black holes, the non-extremality adds an additional challenge. Extremal black holes are particularly easier to handle if there are (sufficiently) supersymmetric. Supersymmetry can not only protect the microstates in the dual QFT as the coupling is changed from weak to strong coupling, a requirement for many black hole counting arguments, but BPS (ie. supersymmetric) supergravity solutions are often given by harmonic functions which can be linearly superposed. For supersymmetric solutions like the D1-D5 system, the coarse-graining of the fuzzball geometries to the naive solution can be be achieved by a simple linear superposition.
For non-extremal black holes however coarse-graining, and particularly how it leads to a horizon in the naive solution, is not yet understood. A few candidate geometries for non-extremal fuzzballs are known \cite{36,37,38}. All these geometries possess a superradiant instability which is thought to correspond to black hole evaporation, only that the decay time is much shorter than the evaporation time. This is not a contradiction if these states are atypical in the ensemble of the non-extremal black hole.

In principle, if we assume that also the microstates of a non-extremal black hole are given by a dual QFT, there should be corresponding fuzzball geometries. Following the general argument in the last section, we can invoke AdS/CFT to infer the existence of a large number of geometries with the same ADM charges as the black hole but differing in the interior. Since each of these geometries should correspond to a pure state, they should be horizonless. However the way in which the asymptotic data prevents the presence of a horizon in the interior, which then only appears after coarse-graining, still remains to be understood.

Finally we would like to mention that in all known fuzzball geometries corresponding to macroscopic black holes, typical geometries may contain regions of high curvatures or geometries may not be distinguishable in supergravity. A full gravitational description of an ensemble of fuzzball geometries most likely requires an understanding of these geometries as solutions of the full string theory. Overcoming the technical challenges associated with such a description would be a big progress in the fuzzball program.