Abstract
What does it mean to say that language is a *potential*-infinite object, as opposed to an *actual*-infinite one? I want to inquiry into the picture of language as an infinite object in such a way that we move closer to an inquiry into language-understanding. Philosophers have attributed to language the property of infinity. This has motivated the study of compositional theories of meaning. Compositionality is also supposed to solve the problem of productivity of language, which allegedly is created by the infinity of language. I will argue that the assumption that languages are infinite objects leads to insurmountable obstacles to putting the notion of understanding back into the picture—and therefore, that compositionality doesn't do the job it was hired for. I will follow instead the idea that it is fruitful to analyze the problem of productivity in a parallel way to Wittgenstein’s discussion of the infinity of natural numbers.

1. Introduction

What does it mean to say that language is a *potential*-infinite object, as opposed to an *actual*-infinite one? What is at stake here? It is not only that our conceptualization of language is different from one choice of infinite to the other. In fact, it is our very understanding in language that changes. Let me elaborate. For the sake of the argument, if one accepts the metaphor that thought takes place in language, the ascription to language of either kind of infinite has consequences for the notion of thought. For thinking as a *process* is closer to a notion of a *potential*-infinite language, whereas thinking as a *way of perception* relates better to the idea of an *actual*-infinite language. But this is only a metaphor, to be sure, and a misleading one. Its only purpose here is to direct our attention to how we conceptualize language, because this might have consequences on how we put thought back into the picture.

I am interested in the understanding that takes place when one understands language. So I must distinguish the subject of my inquiry from a study of language as an object, as it occurs in, e.g., typology. It might be convenient to use an expression such as “language-understanding” to refer unambiguously to the understanding that is characteristic of what goes on when we read a book, conduct a conversation, give a speech, write a letter, etc. This understanding is clearly dependent on how one defines its object, namely language. But we must be clear that the definition of language must depend on our pre-theoretic notion of language-understanding.
Explaining language-understanding is too big a task. For the purposes of this paper, however, I need to show what it does not consist in. To bring the point home, let me resort to an analogy with the case of perception. In this area it is clear, I believe, that it is one thing to investigate into our experience of colors and shapes, and quite another to provide an algorithm for mapping two-dimensional arrays of intensity vectors into three-dimensional matrices. For one thing, a three-dimensional matrix is as much in need of interpretation as the original two-dimensional array. Therefore, an explanation of the mapping does not count as an explanation of the perceptual experience. In the case of language, too, there seems to be a difference between our language-understanding and the non-introspectable mechanisms which are supposed to constitute the language faculty.

I am not in a position to explain in every detail what our language-understanding consists in. My purpose in this paper is rather limited. I will approach the problem by inquiring into the conceptual roots that gave rise to the picture of language as an infinite object. In the first part of this paper I show that both this picture and its purported solution are utterly puzzling, if not wrong, with respect to how language-understanding occurs. Although my argument concludes that the notion of infinity must be dissociated from a conceptualization of language-understanding, I still want to argue in the second part of this paper that the argument for the infinity of language can be “re-analyzed” so as to still throw light on the problem of “language productivity”, but without the negative effects mentioned. To this effect, I will discuss our understanding of natural numbers and their infinity in the light of Wittgenstein’s late philosophy of mathematics, so as to draw positive consequences for a different picture of language-understanding.

2. Languages and Recursive Syntax

The claim to be discussed here is that language is an infinite object –i.e., an infinite set of sentences. This claim is both surprising and unsurprising. It is unsurprising when it comes to formal languages. The recursion of the syntax with which most formal languages are defined is on a par with the recursion of the successor function on natural numbers, so the same type of infinity is associated to both cases –traditionally, the actual infinite. However, the claim that language is an infinite object is surprising in the case of natural languages. What would support such ontological claim?

Language is infinite, so the received view goes, because it is generated by a recursive syntax. This would explain why language is infinite, in a parallel way to that of natural numbers and formal languages. That a natural
language such as English is infinite, so the argument goes, is a “fact” that follows from examples such as rule (1):

(1) If \( S \) is a sentence of English, then I believe that \( S \) is a sentence of English

However, it must be clear that we are dealing here with two different kinds of entities: rules and languages. Since it is languages that we are interested in, the idea that we analyze them by means of rules raises the question of the adequacy of rules: how do we know that these rules are the rules of this language? This question can only be addressed if one has an independent specification of the language—and one that shows that it is infinite—to which the rules have to conform. But since it is such specification that we are after, an analysis of language in terms of rules only pushes the problem one step back. That is, the argument of recursive rules purporting to show that language is infinite depends on the assumption that language is infinite.¹

A move here could be to abandon languages altogether in favor of rules. But this is not a viable move if what we are investigating is language-understanding. For we should ask ourselves what comes first in language-understanding: sentences or “tacitly known” rules? (Compare here my early remark about an investigation of perception.) Furthermore, “tacitly known” rules are non-starters. This is because to understand a rule is to have the ability to distinguish between what ought to be done and what ought not. But a “tacit knowledge” of rules is supposed to make a causal connection between such “knowledge” and performance. This is the wrong level of explanation, because how can a language user be wrong about the rules he “tacitly know”? Such rules cannot give us a reconstruction of language-understanding.

Thus, we are not compelled to accept this argument for the infinity of language. For even if certain recursive rules can generate an unbounded supply of sentences, nothing guarantees that these sentences are sentences of English, let alone that they will give us any insight into language-understanding. Moreover, it is the analysis of language by means of rules that depends on this picture of language as an infinite object.

The adequacy of rules is not the only problem for the argument that language is infinite. Another problem is the far reaching constrains that we need to

¹ A quote can help me substantiate my claim: “The language make[s] available [to a speaker] an infinity of sentences from which the speaker can select appropriate and novel ones to use as the need arises … A synchronic description of a natural language seeks to determine what a fluent speaker knows about the structure of his language that enables him to use and understand any sentence drawn from the INFINITE set of sentences of his language, and since, at any time he has only encountered a FINITE set of sentences, it follows that the speaker’s knowledge of his language takes the form of rules …” J.J. Katz and J.A. Fodor (1963, p. 171). [B&H Sense and nonsense p. 272]
apply to the notion of a sentence if the argument is to make sense. The problem unfolds in several points. First, only if we have a theory-independent notion of a sentence can we say that (1) is a fact of language. Second, the notion of a sentence should also be independent from what people actually utter and write. Otherwise the idea of infinitely many sentences is meaningless. But what could be a notion of a sentence that is both theory- and use-independent? Only the notion of a sentence either as a material or as a platonic object will do. However, there are not infinitely many material objects, so sentences must be platonic objects. But if sentences are platonic objects, how do we understand them? How do we know there are infinitely many of them? What would an argument to this effect look like? At the very least, the argument could not be an empirical one, and therefore (1) cannot be a fact of English.

Despite of this, philosophers do have attributed language the property of infinity. This has also provided motivation to come up with a compositional theory of meaning. In particular, one of the main issues in (formal) semantics is to “explain” how the meanings of sentences depend on the meanings of words and the way they are put together. This “feature” of language is supposed to explain how an infinite ability is accomplished with finite means and thus also how an infinite object can be learned. Moreover, compositionality is supposed to solve the following related problem. Along with the observation that people develop mastery of a language, consisting in their ability to understand its sentences, the presumed infinity of language gives rise to the “observation” that people can understand and use infinitely many sentences, in particular, sentences they have never heard before.

However, the problem of productivity –i.e., how to explain that people can understand and use sentences they have never heard before– is independent from the claim of the infinity of language. This becomes clear from the fact that productivity as such cannot be an argument for the infinity of language. Actually, “productivity” is an ill-formulated claim. It is classified as a claim about language, whereas it is a claim about language users (Groenendijk and Stokhof 2005). This distinction is usually overlooked because of the assumption that language as such is an individual possession; that there is no distinction between what an individual understands and uses and language as such. This, however, is an assumption we need not make. Productivity says that language users are able to understand and use sentences they have not heard before. But does this mean that no-one uttered or wrote these sentences? Does it mean that there are infinitely many sentences? No, it does not: Productivity is compatible with there being finitely many sentences.
Separating productivity and learnability from the picture of language as an infinite object has a far-reaching consequence: compositionality is no longer well-motivated. The reason is that it was summoned in order to explain ill-formulated “properties of language” which brought under its sleeve the picture of language as an infinite object. But instead of being an argument in favor of this picture of language, the notion of compositionality depends on it.2

The upshot of our discussion is this. The picture of language as an infinite object is a motivation, and not a consequence, for the reduction of language to rules and the principle of compositionality of meaning. But neither the picture itself, nor its purported explanations, give us any insight into the notion of language-understanding. The reason is that, on the one hand, it is far from obvious how one can understand an infinite object; on the other hand, reducing language to a set of rules that we “tacitly know” hardly takes us any further in solving this misleading picture of language.

In the next section I will argue that the picture of language as an infinite object that gave rise to the idea of language as rules and compositionality can be studied in a quite illuminating way. It will be illuminating because it will throw light into the notion of productivity, and a fortiori, into the notion of language-understanding.

3. The Infinity of Natural Numbers

The methodological strategy suggested is not to use mathematics as an uncritical source of understanding, but as a place where the kind of understanding that we want to conceptualize can be fruitfully discussed. The aim is to conceptualize language in such a way that it becomes perspicuous how we understand it. In particular, we need an account of the fact that we are able to understand sentences we have never heard before. To this effect, we will ask how natural numbers should be conceptualized so that it becomes perspicuous how we understand them. To be sure, the cases of numbers and language are not prima facie on a par. But the analogy might be interesting since it might suggest an improved methodology for the study of language-understanding.

2 A quote can help me substantiate my claim: “These matters appear to be connected in the following informal way with the possibility of learning a language. When we can regard the meaning of each sentence as a function of a finite number of features of the sentence, we have an insight not only into what there is to be learned; we also understand how an infinite aptitude can be encompassed by finite accomplishments.” D. Davidson (1984, p. 9).
We start our conceptualization of natural numbers in terms of the ability to write down numerals. The technique is easier to explain with strokes as numerals. Once a stroke for 1 is agreed upon, say \( | \), we define it as the numeral for the number one. The numeral for the successor of a number represented by a given numeral can be obtained by putting another stroke to the right of this numeral. In this way we can construct all the numerals, each of them corresponding to a natural number.

It is worth noting that this conceptualization does not commit us to actual-infinite entities, such as the set of all natural numbers. A technical reason can be found in the existence of strictly finitistic approaches to mathematics (for example van Bendegem 1987). Another reason is manifest in the intelligibility of the distinction between the actual and the potential infinite, which dates back to Aristotle (cf. Aristotle, Physics, book 3, chapter 6; cf. Moore 1991 for discussion.).

The conceptualization of the natural numbers as explained above can be analyzed in the following way (cf. Wittgenstein 1976, p. 31). One may ask how many numerals one has learned to write down. The answer could not be other than \( \omega \). For clearly, any technique for writing down numerals that only yields a limited number of numerals is different from our own technique, which is unbounded. Our experience of the technique is that it does not get exhausted—numbers are infinite precisely because of this! This shows that numbers are not epistemically accessible a priori. Dealing with numerals is a non-exhaustible technique that produces results that are not epistemically accessible a priori. If the natural numbers were conceptualized as an actual-infinite set in some platonic realm, our chances for explaining how we know them grow thinner. For how do we grasp them? How do we find which properties they have? But, even more importantly, the actual-infinite is not the way in which we experience them. The fact that we can not actually finish the process is what gives us the experience of there being infinitely many of them. We do not survey the totality of the natural numbers in our minds. We have a technique for constructing more and more, but each time this technique has to be applied. This is not an irrelevant remark about our understanding, which usually gets lost because of the constant fluctuation between a potential and an actual infinity with which the infinite as such is dealt with in the literature. But in order to gain a better idea of our understanding of numbers, we had better keep these two pictures apart.

Now, any explanation of our understanding of natural numbers requires, besides showing how to write down numerals, also showing that one can operate with them, that we can find relations between pairs or tuples of them—e.g., being lesser or equal than—, etc. But it is clear that the bigger the
numbers – i.e., the more strokes their numerals have —, the lesser the possibility of doing operations with them (and this is so even for machines, but that is beside the point). Big numbers do not appear to us in the same way as small ones do, and this has to do with how the techniques to handle numbers get more and more involved as the numbers get bigger. This also shows that positing a rule of understanding which is parallel to the successor function is not a fruitful strategy. For one thing, the rule would predict that we understand very big natural numbers in the same way as smaller ones. In fact, the rule would predict that we understand all numbers in the same way. But this just runs against our previous observation that such similarity breaks down at some point.

The way to bring these observations concerning numbers back to an observation of language is clear (cf. Baker and Hacker 1984; Groenendijk and Stokhof 2005). We do not need to take at face value the idea that the unboundedness of language resides in its being an infinite object — not even a potential one. (Think of ideographic languages such as Chinese, Hieroglyphic or sign languages to help resisting this temptation.) Rather, as masters of language we have a technique for constructing and using sentences. But this technique does not give us a way to survey the totality of sentences in an a priori manner. Dealing with sentences, too, is a technique that produces results that are not epistemically accessible a priori. In each case, when a sentence is presented to a hearer/reader, he can apply his ability without already having understood the sentence. There is no need for a rule of understanding to cope with this sentence, let alone one that applies to the construction of infinitely many of them. The reason is similar to the case of the natural numbers. To understand a sentence requires, among other things, the ability to operate with it, for example, drawing inferential relations. As it was the case with numbers, the bigger the sentence, the smaller the possibility of operating with it. Accordingly, our language-understanding is not uniform across sentences. Compositionality delivers a wrong “explanation” of our language-understanding, for it asserts that we have an a priori understanding of all language. As in the case of numbers, this is a wrong prediction.\

Literature


\footnote{With thanks to Martin Stokhof for his useful guidance in the preparation of this paper.}


