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Attention! *Might* in Inquisitive Semantics

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Abstract

This paper points out that the notion of meaning propounded by inquisitive semantics is not only suited to capture both informative and inquisitive content, but also a sentence's potential to *draw attention* to certain possibilities. This gives rise to a novel analysis of *might*.

1 Introduction

In inquisitive semantics, a sentence is taken to express a *proposal* to enhance the common ground of a conversation. It has been argued in previous work that this notion of meaning is suited to capture both informative and inquisitive content (cf. Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2009a,b). In the present paper we argue that it can do *more* than that. Namely, it is also suitable to capture what we will call the *attentive* content of a sentence: its potential to *draw attention* to certain possibilities.

One empirical phenomenon that, in our view, calls for an account of attentive content, is the behavior of *might* sentences, like (1):

(1) John might be in London.

This sentence clearly differs from the assertion in (2) and the question in (3).

(2) John is in London.

(3) Is John in London?

(1) differs from (2) in that it does not *provide* any information about the state of the world, and it differs from (3) in that it does not *request* any information: one may respond to (1) simply by nodding, or saying "ok", whereas (3) requires an informative response.

In this sense, (1) is neither informative nor inquisitive. But it is certainly meaningful. Thus, a semantic account of (1) must distinguish a third meaning component, different from informative and inquisitive content. Intuitively, the semantic contribution of (1) lies in its potential to *draw attention* to the possibility that John is in London. It is this attentive meaning component that we wish to capture, and we will find that the notion of meaning propounded by inquisitive semantics is especially well-suited for this purpose.

2 Inquisitive Semantics

The central feature of inquisitive semantics is that sentences are taken to express *proposals* to enhance the common ground of a conversation. Such a proposal does not necessarily specify just one way of enhancing the common ground. It may suggest several alternative ways of doing so.

Technically, the proposition expressed by a sentence is taken to be a set of alternative *possibilities*. Each possibility is a set of possible worlds—or *indices* as we will call them—embodying a possible way to update the common ground. In this setting, a sentence may be *informative*, in the sense that certain indices may be eliminated from the common ground by any of the proposed updates, and it may also be *inquisitive*, in the sense that it proposes two or more alternative updates, and invites other participants to provide information such that at least one of these updates can be established.

Thus, the proposition that a sentence expresses in inquisitive semantics embodies both the information that it *provides* and the information that it *requests* from other conversational participants. If a sentence φ expresses a proposition $[\varphi]$, it provides the information that at least one of the possibilities in $[\varphi]$ obtains, and requests from other participants information that could be used to establish for at least one possibility that it indeed obtains.

Now suppose that $[\varphi]$ contains two possibilities, α and β (possibly among others), such that it would take strictly more information to establish α than it would take to establish β. Technically, this would mean that α is included in β (recall that both α and β are sets of indices). In this case, α does not really help in any way to represent the information that φ provides or requests. For, on the one hand, saying that at least one of α and β obtains is just as informative in this setting as saying that β obtains, and on the other hand, asking other participants to provide enough information so as to establish at least one of α or β is just the same as asking them to provide enough information so as to establish β. Thus, possibilities that are included in other possibilities do not really contribute to representing the informative and inquisitive content of a sentence. Therefore, it is common practice in inquisitive semantics to disregard these non-maximal possibilities. A proposition is always taken to be a set of *alternative* possibilities—a set of possibilities such that no element is contained in any other element.[1]

Below we define an inquisitive semantics for a propositional language, mostly drawing on (Groenendijk and Roelofsen, 2009). The language is based on a finite set of proposition letters, with $\bot$, $\wedge$, $\vee$, and $\rightarrow$ as its basic logical connectives. $\neg\varphi$ is defined as $\varphi \rightarrow \bot$; *non-inquisitive closure*, $!\varphi$, is defined as $\neg\neg\varphi$; and *non-informative closure*, $?\varphi$, is defined as $\varphi \vee \neg\varphi$.

2.1 Indices, Possibilities, and Propositions

The basic ingredients for the semantics are *indices* and *possibilities*. An index is a binary valuation for the atomic sentences in the language. We use ω to denote the set of all indices. A possibility is a set of indices. We will use α, β as variables ranging over possibilities, and $\mathcal{P}$ as a variable ranging over non-empty sets of possibilities. Propositions are defined as non-empty sets of maximal possibilities:

Definition 1 (Propositions)**.** A proposition is a non-empty set of possibilities $\mathcal{P}$ such that for no $\alpha, \beta \in \mathcal{P}$: $\alpha \subset \beta$.

[1] There is an important caveat to note here: strictly speaking, non-maximal possibilities may only be disregarded if they are included in a maximal possibility. For, suppose that $[\varphi]$ consists of an infinite sequence of ever increasing possibilities $\alpha_1 \subset \alpha_2 \subset \alpha_3 \subset \ldots$. Then there is no maximal possibility, which means that disregarding non-maximal possibilities amounts to disregarding *all* possibilities. As long as there are only finitely many distinct possibilities, which is indeed the case in the setting that we will consider below and that has been considered in most previous work, a proposition can of course not contain an infinite sequence of ever increasing possibilities, and non-maximal possibilities may safely be disregarded across the board. However, as observed and discussed in detail in (Ciardelli, 2009) (which is in many ways an important predecessor of the present paper), this is not the case in the first-order setting. There, a possibility may only be disregarded if it is stricly contained in a maximal one.

In order to give a recursive definition of the propositions that are expressed by the sentences in our language, we define two auxiliary notions. First, for any sentence φ, the *truth set* of φ, denoted by $|\varphi|$, is the set of indices where φ is classically true. Thus, $|\varphi|$ embodies the classical meaning of φ.

Second, we define a function $\textsc{Alt}$ which transforms any set of possibilities $\mathcal{P}$ into a proposition by removing all the non-maximal possibilities in $\mathcal{P}$.[2]

Definition 2 (Alternative Closure)**.**
$\textsc{Alt}\,\mathcal{P} = \{\alpha \in \mathcal{P} \mid \text{ there is no } \beta \in \mathcal{P} \text{ such that } \alpha \subset \beta\}$

The proposition expressed by a sentence φ is denoted by $[\varphi]$, and is recursively defined as follows.

Definition 3 (Inquisitive Semantics)**.**

1. $[p] = \{|p|\}$ if p is atomic
2. $[\bot] = \{\emptyset\}$
3. $[\varphi \vee \psi] = \textsc{Alt}\,\{\alpha \mid \alpha \in [\varphi] \text{ or } \alpha \in [\psi]\}$
4. $[\varphi \wedge \psi] = \textsc{Alt}\,\{\alpha \cap \beta \mid \alpha \in [\varphi] \text{ and } \beta \in [\psi]\}$
5. $[\varphi \to \psi] = \textsc{Alt}\,\{\Pi_f \mid f : [\varphi] \to [\psi]\}$, where $\Pi_f = \bigcap_{\alpha \in [\varphi]} (\alpha \Rightarrow f(\alpha))$

In the clause for implication, $\alpha \Rightarrow f(\alpha)$ denotes the *pseudo-complement of* α *relative to* $f(\alpha)$, which is defined as the union of the complement of α with $f(\alpha)$: $\overline{\alpha} \cup f(\alpha)$.[3] Notice that the definition assures that $[\varphi]$ is always a set of alternative possibilities. We call the possibilities in $[\varphi]$ the possibilities *for* φ.

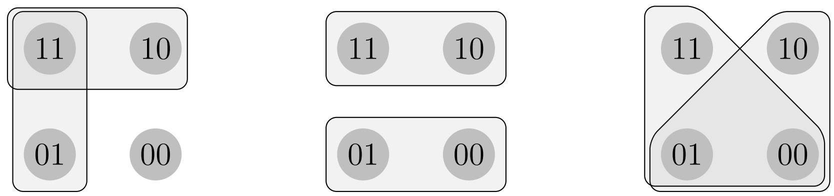
Let us briefly go through the clauses of the definition one by one. In doing so, it will be useful to make a distinction between *classical* sentences, whose proposition contains just one possibility, and *inquisitive* sentences, whose proposition contains at least two possibilities. Figure 1 provides some examples of inquisitive sentences.

[2] In the light of the remark made in footnote 1, $\textsc{Alt}$ must be defined in a slightly more involved way in order to carry over straightforwardly to the first-order setting:

- $\textsc{Alt}\,\mathcal{P} = \{\alpha \in \mathcal{P} \mid \text{ there is no } \textit{maximal } \beta \in \mathcal{P} \text{ such that } \alpha \subset \beta\}$

In the present setting, this definition is equivalent to the one given in definition 2.

[3] This terminology and notation is commonly used in the exposition of *Heyting algebra* (see, for instance, Partee *et al.*, 1990).



(a) $p \vee q$ (b) $?p$ (c) $p \to ?q$

Figure 1: Some examples of inquisitive sentences.

Atoms. The proposition expressed by an atomic sentence p always consists of just one possibility: $|p|$. So an atomic sentence is always classical.

$\bot$ and negation. The proposition expressed by $\bot$ consists of the empty possibility. This means that $\bot$ expresses the unacceptable proposal: if it were accepted, the common ground would become inconsistent.

Recall that $\neg\varphi$ is defined as $\varphi \to \bot$. So $[\neg\varphi] = \textsc{Alt}\,\{\Pi_f \mid f : [\varphi] \to \{\emptyset\}\}$. There is only one function f from $[\varphi]$ to $\{\emptyset\}$, namely the one that maps every element of $[\varphi]$ to $\emptyset$. The possibility Π_f associated with this function f is:

$$\Pi_f \quad = \quad \bigcap\nolimits_{\alpha\in[\varphi]} (\alpha \Rightarrow \emptyset) \quad = \quad \bigcap\nolimits_{\alpha\in[\varphi]} \overline{\alpha} \quad = \quad |\neg\varphi|$$

So $[\neg\varphi] = \{|\neg\varphi|\}$. This means, in particular, that $\neg\varphi$ is always classical.

Disjunction. Disjunctions are typically inquisitive. To determine the proposition expressed by a disjunction $\varphi \vee \psi$ we first collect all possibilities for φ and all possibilities for ψ, and then apply $\textsc{Alt}$ to obtain a proposition. Figure 1(a)–1(b) provide some examples: a simple disjunction of two atomic sentences $p \vee q$, and a polar question $?p$ (recall that $?p$ is defined as $p \vee \neg p$).

Conjunction. To determine the proposition expressed by a conjunction $\varphi \wedge \psi$ we take the pairwise intersection of all possibilities for φ and all possibilities for ψ, and then apply $\textsc{Alt}$ to obtain a proposition. Notice that if φ and ψ are both classical, then conjunction simply amounts to intersection, just as in the classical setting.

Implication. The clause for implication is the one that is most involved. Let us consider several cases separately. First, suppose that the consequent of the implication, ψ, is classical (note that this includes the case of negation, discussed above). As a concrete example, take $(p \vee q) \to r$. In this case, there exists only one function from $[\varphi] = \{|p|, |q|\}$ to $[\psi] = \{|r|\}$, namely the function that maps both $|p|$ and $|q|$ to $|r|$. Call this function f_*. Then the only possibility for $[\varphi \to \psi]$ is Π_{f_*}, which is defined as follows:

$$\bigcap_{\alpha \in [\varphi]} (\alpha \Rightarrow f_*(\alpha))$$

This amounts to $|(p \to r) \wedge (q \to r)|$, which can be further simplified to $|(p \vee q) \to r|$. Thus, $(p \vee q) \to r$ behaves classically. And this holds more generally: whenever the consequent ψ of a conditional $\varphi \to \psi$ is classical, the unique possibility for that conditional is $|\varphi \to \psi|$.

Now suppose that ψ is inquisitive, but that the antecedent, φ, is classical. Take as a concrete example the conditional question $p \to ?q$. In this case, there is one possibility for the antecedent, $|p|$, and two for the consequent, $|q|$ and $|\neg q|$. So there are two functions from $[\varphi]$ to $[\psi]$ in this case, one mapping $|p|$ to $|q|$, and one mapping $|p|$ to $|\neg q|$. Call the first f_q and the second $f_{\neg q}$. The corresponding possibilities are:

$$\begin{aligned} \Pi_{f_q} &= |p| \Rightarrow |q| &&= |p \to q| \\ \Pi_{f_{\neg q}} &= |p| \Rightarrow |\neg q| &&= |p \to \neg q| \end{aligned}$$

So the proposition expressed by $p \to ?q$ is $\{|p \to q|, |p \to \neg q|\}$, as depicted in figure 1(c). This proposition reflects the empirical observation that the expected answers to a conditional question like (4) are (5-a) and (5-b):

(4) If John goes to London, will he fly British Airways?

(5) a. Yes, if he goes to London, he will fly BA.
b. No, if he goes to London, he won't fly BA.

Finally, there are cases where both the antecedent φ and the consequent ψ are inquisitive. In this case, there are n^m functions from $[\varphi]$ to $[\psi]$, where $m \geq 2$ is the number of possibilities for φ and $n \geq 2$ is the number of possibilities for ψ. Each function delivers a potential possibility for $\varphi \to \psi$ (which may still be filtered out by ALT). To see how this works, let us take a concrete example: $(p \vee q) \to ?r$. There are $2^2 = 4$ functions from $[p \vee q] = \{|p|, |q|\}$ to $[?r] = \{|r|, |\neg r|\}$, and each of these functions yields a potential possibility for $(p \vee q) \to ?r$:

$$\begin{aligned}
\Pi_{f_{++}} &= |(p \to r) \wedge (q \to r)| = |(p \vee q) \to r| \\
\Pi_{f_{+-}} &= |(p \to r) \wedge (q \to \neg r)| \\
\Pi_{f_{-+}} &= |(p \to \neg r) \wedge (q \to r)| \\
\Pi_{f_{--}} &= |(p \to \neg r) \wedge (q \to \neg r)| = |(p \vee q) \to \neg r|
\end{aligned}$$

Here, f_{++} is the function that maps both $|p|$ and $|q|$ to $|r|$, f_{+-} is the function that maps $|p|$ to $|r|$ and $|q|$ to $|\neg r|$, etcetera. These are all alternative possibilities, so none of them will be filtered out by ALT.

As a natural language example, let us take a variant of (4), where the antecedent contains a disjunction:

(6) If John goes to London or to Paris, does he fly British Airways?

One could respond to this question in any of the following ways:

(7) a. Yes, if he goes to L or P, he flies BA.
b. If he goes to L, he flies BA, but if he goes to P, he doesn't.
c. If he goes to L, he doesn't fly BA, but if he goes to P, he does.
d. No, if he goes to L or P, he doesn't fly BA.

Each of these responses corresponds to one of the possibilities for $(p \vee q) \to ?r$.

2.2 Questions and Assertions

A proposition $[\varphi]$ is viewed as a proposal to enhance the common ground. If it contains more than one possibility, it embodies an *inquisitive* proposal: each possibility embodies a possible way to enhance the common ground, and other conversational participants are requested to provide information such that at least one of these possible enhancements can be established. If there are indices that are not included in any of the possibilities in $[\varphi]$, then φ is *informative*. For in this case certain indices will be eliminated by any of the possible enhancements proposed by φ.

Definition 4 (Inquisitiveness and informativeness)**.**

- φ is *inquisitive* if and only if $[\varphi]$ contains at least two possibilities;
- φ is *informative* if and only if $\bigcup[\varphi] \neq \omega$.

Assertions are defined as sentences whose only effect, if any, is to provide information, and questions as sentences whose only effect, if any, is to request information.

Definition 5 (Questions and assertions)**.**

- φ is a *question* if and only if it is not informative;
- φ is an *assertion* if and only if it is not inquisitive.

Notice that not every sentence is a question or an assertion. There are also *hybrid* sentences, which are both informative and inquisitive. A simple example of a hybrid sentence is the disjunction $p \vee q$ (see figure 1(a)).

Tautologies are defined as sentences that express a trivial proposal, and contradictions as sentences that express an unacceptable proposal.

Definition 6 (Contradictions and tautologies)**.**

- φ is a *contradiction* if and only if $[\varphi] = \{\emptyset\}$
- φ is a *tautology* if and only if $[\varphi] = \{\omega\}$

Note that contradictions are assertions, and that tautologies count both as questions and as assertions. It is easy to see that a formula is a contradiction iff it is a classical contradiction. This does not hold for tautologies. Classically, a formula is tautological iff it is not informative. In inquisitive semantics, a formula is tautological iff it is neither informative nor inquisitive. Classical tautologies may well be inquisitive, as exemplified by the question $?p$.

Definition 7 (Equivalence)**.** Two sentences φ and ψ are *equivalent*, $\varphi \sim \psi$, if and only if $[\varphi] = [\psi]$.

Proposition 8 (Alternative characterizations of questions)**.**
For any sentence φ, the following are equivalent:

1. φ is a question
2. φ is a classical tautology
3. $\neg\varphi$ is a contradiction
4. $\varphi \sim ?\varphi$

Proposition 9 (Alternative characterizations of assertions)**.**
For any sentence φ, the following are equivalent:

1. φ is an assertion
2. $[\varphi]$ contains exactly one possibility;
3. $[\varphi] = \{|\varphi|\}$;
4. $\varphi \sim\ !\varphi$.

Note that a sentence is an assertion if and only if the proposition it expresses consists of just one possibility, which corresponds with its classical meaning. In this sense, assertions behave classically. It can be shown that disjunction is the only source of non-classical, inquisitive behavior in our language: the disjunction-free fragment of the language behaves classically.

Proposition 10. Any disjunction-free sentence is an assertion.

Finally, the informative content of a sentence φ is embodied by $\bigcup[\varphi]$ (indices that are not in $\bigcup[\varphi]$ are proposed to be eliminated from the common ground). The following proposition guarantees that inquisitive semantics preserves the classical treatment of informative content.

Proposition 11. For any sentence φ: $\quad \bigcup[\varphi] = |\varphi|$.

3 Attention

We observed in the introduction that sentences like (8) can very well make a significant contribution to a conversation, even though they are neither informative nor inquisitive.

(8) John might be in London.

Intuitively, the semantic contribution of sentences like (8) lies in their potential to draw attention to certain possibilities, in this case the possibility that John is in London. The conception of a proposition as a set of possibilities is ideally suited to capture this intuition. If a sentence φ expresses a proposition $[\varphi]$ we can simply think of the elements of $[\varphi]$ as the possibilities that φ draws attention to; the possibilities that it proposes to take into consideration. At the same time, we can still think of φ as providing the information that at least one of the possibilities in $[\varphi]$ obtains, and as requesting information that could be used to establish for at least one of these possibilities that it indeed obtains. Thus, if a proposition is conceived of as a set of possibilities, it may in principle capture the informative, inquisitive, and attentive content of a sentence all at once.

Recall that in section 2 propositions were formally defined as sets of *alternative* possibilities. This was because non-maximal possibilities do not contribute in any way to the representation of informative and inquisitive content, and these were the only aspects of meaning that we were interested in. However, as soon as attentive content becomes of interest, non-maximal possibilities should be taken into account as well. In general, there is no reason why a sentence may not draw attention to two possibilities α and β such that $\alpha \subset \beta$. The only exception is that it seems unreasonable to think of any non-contradictory sentence as drawing attention to the *empty* possibility. Thus, we define propositions as arbitrary non-empty sets of possibilities, with the one exception that the empty possibility can only form a proposition on its own (the 'unacceptable' proposition, expressed by contradictions).

Definition 12 (Propositions)**.** A proposition is a non-empty set of possibilities $\mathcal{P}$ such that either $\emptyset \notin \mathcal{P}$ or $\mathcal{P} = \{\emptyset\}$.

In defining the semantics of our formal language, we will of course no longer make use of ALT (which turned any $\mathcal{P}$ into a set of *alternative* possibilities), but rather of a function PRO, which turns any $\mathcal{P}$ into a proposition in the sense of definition 12:

Definition 13 (Propositional closure)**.** $\textsc{Pro}\,\mathcal{P} = \begin{cases} \{\emptyset\} & \text{if } \mathcal{P} = \{\emptyset\} \\ \mathcal{P} - \{\emptyset\} & \text{otherwise} \end{cases}$

Definition 14 (Inquisitive semantics with non-maximal possibilities)**.**

1. $[\![p]\!] = \{|p|\}$ if p is atomic
2. $[\![\bot]\!] = \{\emptyset\}$
3. $[\![\varphi \vee \psi]\!] = \textsc{Pro}\,\{\alpha \mid \alpha \in [\![\varphi]\!] \text{ or } \alpha \in [\![\psi]\!]\}$
4. $[\![\varphi \wedge \psi]\!] = \textsc{Pro}\,\{\alpha \cap \beta \mid \alpha \in [\![\varphi]\!] \text{ and } \beta \in [\![\psi]\!]\}$
5. $[\![\varphi \to \psi]\!] = \textsc{Pro}\,\{\Pi_f \mid f : [\![\varphi]\!] \to [\![\psi]\!]\}$, where $\Pi_f = \bigcap_{\alpha \in [\![\varphi]\!]} (\alpha \Rightarrow f(\alpha))$

In comparing the system defined in section 2 with the one defined here, we will refer to the former as *restricted* inquisitive semantics, or $\mathsf{Inq}_{\not\subset}$ for short, and to the latter as *unrestricted* inquisitive semantics, or $\mathsf{Inq}_{\emptyset}$ for short.[4]

Notice also that in definition 14 we use the notation $[\![\varphi]\!]$ in order to avoid confusion with $[\varphi]$. Thus, $|\varphi|$ is the proposition that is classically expressed by φ, $[\varphi]$ is the proposition expressed by φ in $\mathsf{Inq}_{\not\subset}$, and $[\![\varphi]\!]$ is the proposition expressed by φ in $\mathsf{Inq}_{\emptyset}$. If no confusion arises, we will henceforth simply refer to $[\![\varphi]\!]$ as the proposition expressed by φ. The elements of $[\![\varphi]\!]$ will be called the possibilities *for* φ.

3.1 Informative, inquisitive, and attentive content

As in $\mathsf{Inq}_{\not\subset}$, the informative content of a sentence φ in $\mathsf{Inq}_{\emptyset}$ is determined by the union of all the possibilities for φ. Thus, $\mathsf{Inq}_{\emptyset}$ preserves the classical treatment of informative content, just as $\mathsf{Inq}_{\not\subset}$ did.

Proposition 15. For any sentence φ, $\quad \bigcup[\![\varphi]\!] = \bigcup[\varphi] = |\varphi|$

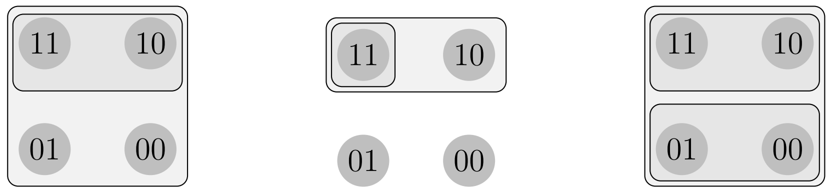
Also just as in $\mathsf{Inq}_{\not\subset}$, the inquisitive content of a formula φ in $\mathsf{Inq}_{\emptyset}$ is determined by the *maximal* possibilities for φ. For, these possibilities still completely determine which information is minimally required to establish at least one of the possibilities for φ. It is easy to check that a possibility α is a maximal element of $[\![\varphi]\!]$ if and only if it is an element of $[\varphi]$. Thus, as far as informative and inquisitive content are concerned, $\mathsf{Inq}_{\emptyset}$ and $\mathsf{Inq}_{\not\subset}$ coincide.

However, in $\mathsf{Inq}_{\not\subset}$, the meaning of a formula was completely exhausted by its informative and inquisitive content. In $\mathsf{Inq}_{\emptyset}$, these two components are still present, and still behave exactly the same, but they do no longer fully determine the meaning of a formula: a third, *attentive* meaning component has entered the stage.

Definition 16 (Informativeness, inquisitiveness, and attentiveness)**.**

- φ is informative iff $|\varphi| \neq \omega$;

[4] The restriction that the empty possibility can only form a proposition on its own should not be seen as a significant restriction. We could also simply have defined possibilities as non-empty sets of indices, and propositions as completely arbitrary (possibly empty) sets of possibilities, taking the empty proposition to be the semantic value of $\bot$. This is in fact the route taken in (Groenendijk and Roelofsen, 2009). The only reason we include empty possibilities here (and thus avoid empty propositions) is that it allows for a perspicuous formulation of the clause for implication.



(a) $\Diamond p = \top \vee p$ (b) $p \wedge \Diamond q$ (c) $\Diamond p \vee \Diamond \neg p$

Figure 2: Three examples of attentive formulas.

- φ is inquisitive iff $[\![\varphi]\!]$ contains at least two maximal possibilities;
- φ is attentive iff $[\![\varphi]\!]$ contains a non-maximal possibility.

Informativeness is defined just as in $\mathsf{Inq}_{\not\subset}$. The definition of inquisitiveness now explicitly requires the existence of two *maximal* possibilities—in $\mathsf{Inq}_{\not\subset}$ possibilities were always maximal, so there we could just require the existence of at least two possibilities. Finally, attentiveness requires the existence of a non-maximal possibility, something that could clearly never arise in $\mathsf{Inq}_{\not\subset}$.[5]

Perhaps it is worth emphasizing that *every* sentence draws attention to certain possibilities, not only attentive sentences. What is special about attentive sentences is that they draw attention to possibilities that do not contribute to representing their informative or inquisitive content. Attentive sentences do something *more* than providing or requesting information (if they provide or request any information at all).

3.2 Might

Let us consider some examples of attentive formulas. First consider the proposition depicted in figure 2(a). This proposition consists of two possibilities: the possibility that p, and the 'trivial possibility', ω. We take this to be

[5]In (Ciardelli, 2009) inquisitiveness and attentiveness are defined as follows:

- φ is *inquisitive* iff $|\varphi| \notin [\![\varphi]\!]$;
- φ is *attentive* iff there is a possibility for φ that is strictly included in a maximal possibility for φ.

In the propositional setting, these alternative definitions are equivalent to the ones given above. They may be slightly less transparent from our current perspective, but have the advantage of carrying over straightforwardly to the first-order setting.

the proposition expressed by '*might* p'. It draws attention to the possibility that p, but does not provide or request any information. This is indeed how *might* sentences typically behave in natural language.

We will add an operator $\Diamond$ to our formal language, representing *might*. But notice that our basic formal language already contains a formula that expresses the proposition in figure 2(a): the formula $\top \vee p$. This means that we can simply take $\Diamond p$ to be an abbreviation of $\top \vee p$. More generally, for any formula φ, we take $\Diamond\varphi$ to be an abbreviation of $\top \vee \varphi$. Thus, the effect of $\Diamond\varphi$ is to draw attention to the possibilities for φ without providing or requesting any information.

To see what this amounts to, let us consider two more concrete examples. First, take the proposition depicted in figure 2(b). This proposition is expressed by $p \wedge \Diamond q$. It consists of two possibilities: $|p|$ and $|p \wedge q|$. As such, it provides the information that p holds, and draws attention to the possibility that q may hold as well.

Finally, consider the proposition depicted in figure 2(c). This proposition is expressed by $\Diamond p \vee \Diamond\neg p$. It is especially instructive to consider how this formula differs from the polar question $?p$. The latter is inquisitive; it requires a choice between two alternative possibilities. $\Diamond p \vee \Diamond\neg p$ on the other hand, does not require any information: it highlights the possibility that p and the possibility that $\neg p$, and other participants may indeed confirm one of these possibilities in their response. But they are not required to do so; they may also just nod, or say "ok". These would not be compliant responses to $?p$.

3.3 Assertions, questions, and conjectures

As in $\mathsf{Inq}_{\not\subset}$, we define assertions as formulas whose only effect, if any, is to provide information, and questions as formulas whose only effect, if any, is to require information. As a third category, we now also distinguish formulas whose only effect, if any, is to draw attention to certain possibilities. We call such formulas *conjectures*.

Definition 17 (Assertions, questions, and conjectures)**.**

- φ is an *assertion* iff it is neither inquisitive nor attentive;
- φ is a *question* iff it is neither informative nor attentive;
- φ is a *conjecture* iff it is neither informative nor inquisitive.

The borderline cases, tautologies and contradictions, are defined just as in $\mathsf{Inq}_{\not\subset}$: tautologies are formulas that express the trivial proposition; contradictions are formulas that express the unacceptable proposition.

Definition 18 (Tautologies, contradictions)**.**

1. φ is a *tautology* if and only if $[\![\varphi]\!] = \{\omega\}$;
2. φ is a *contradiction* if and only if $[\![\varphi]\!] = \{\emptyset\}$.

Notice that, as in $\mathsf{Inq}_{\not\subset}$, contradictions are assertions, and tautologies now count not only as borderline cases of questions and assertions, but also of conjectures. The equality $\bigcup[\![\varphi]\!] = |\varphi|$ immediately entails that contradictions in $\mathsf{Inq}_{\emptyset}$ are precisely the classical contradictions; classical tautologies, however, may well express meaningful, non-trivial propositions in $\mathsf{Inq}_{\emptyset}$: they are never informative, but they may well be inquisitive and/or attentive. The meaning of a sentence in $\mathsf{Inq}_{\emptyset}$ is completely exhausted by its informative, inquisitive, and attentive content.

Proposition 19. If a sentence is neither informative, nor inquisitive, nor attentive, then it must be a tautology.

Equivalence is defined as expected:

Definition 20 (Equivalence)**.** Two formulas φ and ψ are *equivalent* in $\mathsf{Inq}_{\emptyset}$, $\varphi \approx \psi$, if and only if $[\![\varphi]\!] = [\![\psi]\!]$.

The characterization of assertions given in proposition 9 is preserved:

Proposition 21 (Alternative characterizations of assertions)**.**
For any sentence φ, the following are equivalent:

1. φ is an assertion;
2. $[\![\varphi]\!]$ contains exactly one possibility;
3. $[\![\varphi]\!] = \{|\varphi|\}$;
4. $\varphi \approx !\varphi$.

Also, all closure properties of assertions carry over from $\mathsf{Inq}_{\not\subset}$ to $\mathsf{Inq}_{\emptyset}$.

Proposition 22 (Closure properties of assertions)**.**
For any proposition letter p and any sentences φ and ψ,

1. p, $\neg\varphi$, and $!\varphi$ are assertions;
2. if both φ and ψ are assertions, then so is $\varphi \wedge \psi$;
3. if ψ is an assertion, then so is $\varphi \to \psi$.

In particular, any disjunction-free sentence is still an assertion. Thus, disjunction is the only source of non-classical behavior in $\mathsf{Inq}_{\emptyset}$, just as it was in $\mathsf{Inq}_{\not\subset}$.

Corollary 23. Disjunction-free sentences are assertions.

The characterization of questions in proposition 8 does no longer hold in $\mathsf{Inq}_{\emptyset}$. It remains a valid characterization of *non-informative* sentences; but only some of these count as questions in $\mathsf{Inq}_{\emptyset}$, namely the ones that are not attentive.

Proposition 24 (Questions and attentiveness)**.** $?\varphi$ is a question iff φ is not attentive.

Conjectures can be characterized very much in parallel with assertions.

Proposition 25 (Alternative characterizations of conjectures)**.**
For any sentence φ, the following are equivalent:

1. φ is a conjecture;
2. $[\![\varphi]\!]$ contains ω;
3. $\varphi \approx \Diamond\varphi$.

Notice that a sentence is a conjecture in $\mathsf{Inq}_{\emptyset}$ if and only if it is a tautology in $\mathsf{Inq}_{\not\subset}$. $\mathsf{Inq}_{\not\subset}$ refined the classical notion of meaning in such a way that some of the sentences that were tautological in classical semantics formed a new class of meaningful sentences, namely questions. $\mathsf{Inq}_{\emptyset}$ further refines the notion of meaning in such a way that some of the sentences that were tautological in $\mathsf{Inq}_{\not\subset}$ again form a new class of meaningful sentences, namely conjectures.

Proposition 26 (Closure properties of conjectures)**.**
For any formulas φ and ψ,

1. $\Diamond\varphi$ is a conjecture;

2. if φ and ψ are conjectures, then so is $\varphi \wedge \psi$;

3. if at least one of φ and ψ is a conjecture, so is $\varphi \vee \psi$;

4. if ψ is a conjecture, then so is $\varphi \to \psi$.

Thus, sentences like those in (9) are all conjectures.

(9) a. John might be in London. $\Diamond p$
b. John might be in London and Bill might be in Paris. $\Diamond p \wedge \Diamond q$
c. John is in London, or he might be in Paris. $p \vee \Diamond q$
d. If John is in London, Bill might be in Paris. $p \to \Diamond q$

4 *Might* meets the propositional connectives

It is well-known that *might* interacts with the propositional connectives in peculiar ways. In particular, it behaves differently in this respect from expressions like 'it is possible that' or 'it is consistent with my beliefs that', which is problematic for any account that analyzes *might* as an epistemic modal operator. The present analysis sheds new light on this issue.

Disjunction and conjunction. Zimmermann (2000, p.258–259) observed that (10), (11), and (12) are all equivalent.[6]

(10) John might be in Paris or in London. $\Diamond(p \vee q)$

(11) John might be in Paris or he might be in London. $\Diamond p \vee \Diamond q$

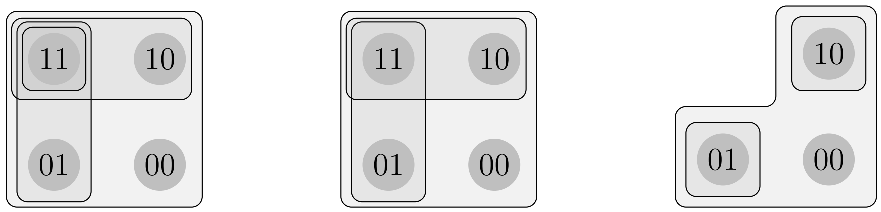
(12) John might be in Paris and he might be in London. $\Diamond p \wedge \Diamond q$

Notice that *might* behaves differently from clear-cut epistemic modalities here: (13) is not equivalent with (14).

(13) It is consistent with my beliefs that John is in London or it is consistent with my beliefs that he is in Paris.

(14) It is consistent with my beliefs that John is in London and it is consistent with my beliefs that he is in Paris.

[6]These type of examples have often been discussed in the recent literature in relation to the phenomenon of *free choice permission*, which involves deontic modals (cf. Geurts, 2005; Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007; Klinedinst, 2007).



(a) $\Diamond p \wedge \Diamond q$

(b) $\Diamond p \vee \Diamond q$
$\approx \Diamond(p \vee q)$

(c) $\Diamond p \wedge \Diamond q$
$\approx \Diamond p \vee \Diamond q$
$\approx \Diamond(p \vee q)$

Figure 3: *Might* interacting with conjunction and disjunction.

A further subtlety is that Zimmermann's observation seems to crucially rely on the fact that 'being in London' and 'being in Paris' are mutually exclusive. If they had not been chosen in this specific way, the equivalence between (10) and (11) on the one hand, and (12) on the other would not have obtained. To see this, consider the following examples:

(15) John might speak English or French. $\Diamond(p \vee q)$

(16) John might speak English or he might speak French. $\Diamond p \vee \Diamond q$

(17) John might speak English and he might speak French. $\Diamond p \wedge \Diamond q$

'Speaking English' and 'speaking French' are not mutually exclusive, unlike 'being in London' and 'being in Paris'. To see that (15) and (16) are not equivalent with (17) consider a situation, suggested to us by Anna Szabolcsi, in which someone is looking for an English-French translator, i.e., someone who speaks *both* English and French. In that context, (17) would be perceived as a useful recommendation, while (15) and (16) would not.

These patterns are quite straightforwardly accounted for in $\mathsf{Inq}_\emptyset$. The proposition expressed by $\Diamond p \wedge \Diamond q$ is depicted in figure 3(a), and the proposition expressed by $\Diamond(p \vee q)$ and $\Diamond p \vee \Diamond q$ (which are equivalent in $\mathsf{Inq}_\emptyset$) is depicted in figure 3(b). Notice that $\Diamond p \wedge \Diamond q$, unlike $\Diamond(p \vee q)$ and $\Diamond p \vee \Diamond q$, draws attention to the possibility that $p \wedge q$, that is, the possibility that John speaks both English and French. This explains the observation that (17) is perceived as a useful recommendation in the translator-situation, unlike (15) and (16).

In Zimmermann's example, p stands for 'John is in London' and q for 'John is in Paris'. It is impossible for John to be both in London and in Paris.

So indices where p and q are both true must be left out of consideration, and relative to this restricted common ground[7], $\Diamond(p \wedge q)$, $\Diamond p \vee \Diamond q$, and $\Diamond p \wedge \Diamond q$ express exactly the same proposition, which is depicted in figure 3(c).

Implication and negation. Let us first consider an example where *might* occurs in the consequent of an implication:

(18) If John is in London, he might be staying with Bill.

The corresponding expression in our formal language, $p \rightarrow \Diamond q$, is equivalent with $\Diamond(p \rightarrow q)$. It draws attention to the possibility that p implies q, without providing or requesting information. This seems a reasonable account of the semantic effect of (18). Indeed, one natural response to (18) is to confirm that John is staying with Bill if he is in London. But such an informative response is not required. Nodding, or saying "ok" would also be compliant responses.

Now let us consider an example where *might* occurs in the antecedent of an implication:

(19) If John might be in London, he is staying with Bill. $\Diamond p \rightarrow q$

This sentence is perceived as odd. In $\mathsf{Inq}_\emptyset$, this observation may be explained by the following general property of implication:

Proposition 27 (Implication and redundancy of non-informative content)**.** If ψ is an assertion, then for any φ: $\varphi \rightarrow \psi \quad \approx \quad !\varphi \rightarrow \psi$.

This proposition says that if the consequent of an implication is an assertion, then we could replace the antecedent φ by $!\varphi$ (which has exactly the same informative content as φ, but lacks any inquisitive or attentive content) without changing the meaning of the implication as a whole. In other words, the inquisitive and attentive content of the antecedent is redundant in such constructions: there is a simpler way to express exactly the same meaning. In particular, the meaning expressed by (19) could just as well be expressed by the simpler sentence "John is staying with Bill" (according to proposition 27, $\Diamond p \rightarrow q$ is equivalent to $\top \rightarrow q$, which reduces to q). This may be a

[7] For reasons of space, we do not explicitly define propositions relative to an arbitrary common ground here. But such a definition can be given in a straightforward way (see, for instance, Groenendijk and Roelofsen, 2009).

reason why constructions like (19) are generally not used, and are perceived as odd if they do occur.

Our general empirical prediction is that implications with an assertive consequent and an inquisitive or attentive antecedent are 'marked'.[8] This has particular consequences for negation, which is treated in our system as a special case of implication. In natural language, standard sentential negation cannot take wide scope over *might*: (20-a) cannot be interpreted as in (20-b).

(20) a. John might not be in London.
b. It is not consistent with my beliefs that John is in London.

Notice, again, that *might* behaves differently from clear-cut epistemic modalities here. This observation is explained in $\mathsf{Inq}_\emptyset$ by the fact that $\neg\Diamond\varphi$ is always a contradiction (recall that $\neg\Diamond\varphi$ is defined as $\Diamond\varphi \to \bot$; by proposition 27, this is equivalent with $\top \to \bot$, which reduces simply to $\bot$). Thus, $\neg\Diamond\varphi$ expresses a non-sensical, unacceptable proposal. $\Diamond\neg\varphi$ on the other hand, seems to have exactly the semantic effect of sentences like (20-a): it draws attention to the possibility that $\neg\varphi$.

5 The Bigger Picture

The idea that the core semantic contribution of *might* sentences lies in their potential to draw attention to certain possibilities has been entertained before. For instance, Groenendijk, Stokhof, and Veltman (1996) already wrote that "in many cases, a sentence of the form *might*-φ will have the effect that one becomes aware of the possibility of φ." However, it was thought that capturing this aspect of the meaning of *might* would require a more complex notion of possible worlds and information states, and a different way to think about growth of information. Thus, immediately following the above quotation, Groenendijk *et al.* (1996) write that their own framework "is one in which indices are total objects, and in which growth of information about

[8] In some cases, marked sentences may not be perceived as odd, but rather associated with a marked meaning, i.e., a meaning that differs from the one they are standardly associated with in our semantics. The association of marked forms with marked meanings (and unmarked forms with unmarked meanings) is widely assumed to play a significant role in interpretation (cf. Horn, 1984). In an extended version of this paper, which is currently in preparation, we will discuss the repercussions of this mechanism for the interpretation of *might* in more detail.

the world is explicated in terms of elimination of indices. Becoming aware of a possibility cannot be accounted for in a natural fashion in such an eliminative approach. It would amount to extending partial indices, rather than eliminating total ones. To account for that aspect of the meaning of *might* a constructive approach seems to be called for."[9]

The present paper has taken a different route. Indices are still total objects, and growth of information is still explicated in terms of eliminating indices. What has changed is the very *notion of meaning*. Our semantics does not specify what the truth conditions of sentences are, or what their update effect is, but rather what the proposal is that they express. And this shift in perspective immediately facilitates a simple and perspicuous way to capture attentive content.[10]

Let us end by briefly pointing out how our account relates to two basic observations about *might*, and the theoretical frameworks in which these observations are typically accounted for. Consider the following sentence:

(21) John might be in London.

The first observation, perhaps the most basic one, is that if someone utters (21) we typically conclude that he considers it *possible* that John is in London. Clearly, our semantics has nothing to say about this observation. The standard account is to analyze *might* as an epistemic modal operator.

The second observation is that if someone hears (21) and already knows that John is not in London, she will typically object, pointing out that (21) is inconsistent with her information state. In this sense, even though *might* sentences do not provide any information about the state of the world, they can be 'inconsistent' with a hearer's information state. The classical account of this observation is that of Veltman (1996). Veltman's update semantics specifies for any given information state σ and any given formula φ, what the information state $\sigma[\varphi]$ is that would result from updating σ with φ. The

[9] Groenendijk *et al.* originally used the terms 'possible world' and 'possibility' instead of 'index'. We have taken the freedom to adapt these terms in this quotation in order to avoid confusion with our own terminology.

[10] Brumwell (2009) and Franke and de Jager (2008) have recently developed ideas closely related to ours. For reasons of space, we do not discuss the differences and similarities between these approaches and ours here. We intend to do so, however, in an extended version of this paper.

update effect of $\Diamond\varphi$ is defined as follows:

$$\sigma[\Diamond\varphi] = \begin{cases} \emptyset & \text{if } \varphi \text{ is inconsistent with } \sigma \\ \sigma & \text{otherwise} \end{cases}$$

The idea is that, if φ is inconsistent with a hearer's information state, then updating with $\Diamond\varphi$ leads to the absurd state. To avoid this, the hearer must make a public announcement signalling the inconsistency of φ with her information state. As a result, whoever uttered $\Diamond\varphi$ in the first place may come to discard the possibility that φ as well. Again, our semantics clearly has nothing to say about this.

However, we believe that this is rightly so. In our view, both observations should be explained pragmatically. And they can be. It follows from the general conversational principles of inquisitive pragmatics, as described in (Groenendijk and Roelofsen, 2009), that the information state of a cooperative speaker who utters a sentence φ must be consistent with any possibility for φ. In particular, if a cooperative speaker utters (21) he must consider it possible that John is in London.

Moreover, it also follows from these general principles that if a hearer is confronted with a sentence φ, and one of the possibilities for φ is inconsistent with her information state, then she must signal this inconsistency, in order to prevent other participants from considering the possibility in question to be a 'live option'. Thus, both observations are accounted for.

And this pragmatic account, unlike the mentioned semantic analyses, extends straightforwardly to more involved cases. Consider for instance:

(22) John might be in London or in Paris.

This sentence is problematic for both semantic accounts just mentioned. The epistemic modality account predicts that the speaker considers it possible that John is in London or in Paris. But note that this is compatible with the speaker knowing perfectly well that John is not in London. What (22) implies is something stronger, namely that the speaker considers it possible that John is in London *and* that he considers it possible that John is in Paris. This follows straightforwardly on our pragmatic account.

Now consider a hearer who is confronted with (22) and who knows that John is not in London. We expect this hearer to object to (22). But Veltman's update semantics does not predict this: it predicts that an update with (22) has no effect on her information state. Our pragmatic account on the other hand, does urge the hearer to object.

Thus, the only task of our semantics is to specify which proposals can be expressed by means of which sentences. The pragmatics, then, specifies what a context—in particular, the common ground and the information state of the speaker—must be like in order for a certain proposal to be made, and how a hearer is supposed to react to a given proposal, depending on the common ground and her own information state. Together, these two components seem to provide a suitable account of the basic features of *might*.

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