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# A progression ring for interfaces of instruction sequences, threads, and services \*

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## Abstract

We define focus-method interfaces and some connections between such interfaces and instruction sequences, giving rise to instruction sequence components. We provide a flexible and practical notation for interfaces using an abstract datatype specification comparable to that of basic process algebra with deadlock. The structures thus defined are called progression rings. We also define thread and service components. Two types of composition of instruction sequences or threads and services (called ‘use’ and ‘apply’) are lifted to the level of components.

## 1 Introduction

We can not simply say that this instruction sequence (*inseq*) has that interface because there are different ways to obtain an interface from an *inseq*, which is one of the main questions that we deal with in this paper. Instead we will do this:

- (i) we define and formalize so-called focus-method interfaces, or briefly, FMIs,
- (ii) we specify some relations between FMIs and *inseq*'s, and
- (iii) we define an *inseq component* as a pair  $(i, P)$  of an FMI  $i$  and an *inseq*  $P$  where  $i$  and  $P$  need to be related in the sense meant in (ii) above.

Focus method interfaces will also serve as thread-component interfaces, whereas MIs (method interfaces) will be used as service-component interfaces.

Like with an instruction sequence, a *thread component* results by pairing a thread and an FMI, and a service component consists of a pair of a service and an MI.

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Using *focus-method notation* (FMN) a *basic instruction* has the form

`f.m`

where `f` is a *focus* and `m` is a *method*. Furthermore, we have the following test instructions:

`+f.m` (a *positive test instruction*)

`-f.m` (a *negative test instruction*)

Some examples of instructions in FMN are:

`b17.set:true` and `+b18a.get`

where `b17` and `b18a` are foci addressing certain boolean registers, and `set:true` and `get` are methods defined for such boolean registers.

The idea of a positive test instruction is that upon execution it yields a reply `true` or `false`, and that upon the reply `true` execution continues with the next instruction, while upon reply `false` the next instruction is skipped and execution continues with the instruction thereafter. The execution of a negative test instruction displays the reversed behavior with respect to the reply values. Note that the reply values `true` and `false` have nothing to do with the example method `set:true` mentioned above.

In [BL02] the programming notation PGLB is defined: Next to a given set  $A$  of basic instructions and the test instructions generated from  $A$ , PGLB contains forward jumps  $\#k$  and backward jumps  $\backslash\#k$  for  $k \in \mathbb{N}$  as primitive instructions, and a termination instruction which is written as `!`. The instructions mentioned here are PGLB's so-called *primitive* instructions. The inseqs that we call PGLB programs are formed from primitive instructions using concatenation, notation `;`. The following three inseqs are examples of PGLB programs:

```
b17.set:true
+b18a.get;#2;b17.set:true
-b1.get;#3;b4.set:false;#3;b4.set:true;!
```

We consider PGLB as an inseq notation, and we assume that adaptations to other inseq notations can easily be made.

In [BL02] it is defined in what way a PGLB program defines a thread. Here we now discuss the interface of such inseqs. Consider

$P = -b1.get; \#3; b4.set:false; \#3; b4.set:true; !$

It is reasonable to view

`{b1.get, b4.set:false, b4.set:true}`

as the interface of  $P$ . We will call this a focus-method interface (FMI) for  $P$  as it consists of focus-method pairs.

For an interface it is important that its description is “simple”. If an interface of `inseq P` is automatically derived from `P` then storage of that interface as a part of the code is a matter of pre-computation.

These pre-computed datastructures should be easy to store and easy to use. This “use” can be a matter of application of static checks which prevent the occurrence of dynamic errors. It follows that one needs a notation for interfaces which allows notational simplification.

Writing `+` for union and omitting brackets for single elements one may write

$$\begin{aligned}
 \{\mathbf{b1.get}, \mathbf{b4.set: false}, \mathbf{b4.set: true}\} &= \mathbf{b1.get} + \mathbf{b4.set: false} + \mathbf{b4.set: true} \\
 &= \mathbf{b1.get} + \mathbf{b4.set: (false + true)} \\
 &= \mathbf{b(1.get} + \mathbf{4.set: (fals} + \mathbf{tru)e)} \tag{1}
 \end{aligned}$$

We find that this notation allows useful simplifications. The main contents of this paper is to specify the details of this particular interface notation.

We notice that if `i` is a plausible interface for `inseq P`, it may be the case that some `i'` extending `i` can be denoted with a significantly more simple FMI-expression, say `ie`. Then `ie` is also a plausible interface for `P`. We will provide technical definitions of various forms of matching `inseq`'s with interfaces in such a way that this intuition can be made formal.

From a mathematical point of view the task to provide the details of the interface notation as suggested in (1) is not at all challenging. From the point of view of abstract data type specification it requires the explicit resolution of several design alternatives, each of which have advantages and disadvantages. In particular, insisting that interface elements are in FMN, one needs to resolve some language/meta-language issues in order to provide an unambiguous semantics of interface expressions.

We hold that from some stage onwards the theory of instruction sequences needs to make use of a pragmatic theory of interfaces. It is quite hard to point out exactly when and where having full details of an interface notation available matters. For that reason we have chosen to view the design of an interface notation for instruction sequences as an independent problem preferably not to be discussed with a specific application in mind.

Summing up, this is the question posed and answered below:

Assuming that instruction sequence interfaces are important per se, provide a flexible and practical notation for interfaces by means of an appropriate abstract datatype specification.

The paper is structured as follows: In the next section we introduce progression rings and show how they underly an abstract datatype specification for interfaces. In Section 3 we define `inseq` components and thread components, and in Section 4 we discuss composition of such components with service components. In Section 5 we briefly discuss interfaces that are not minimal and make a remark about related work.

## 2 Progression Rings and Interfaces

We first formally define the *letters*, *lower case* and *upper case* in BNF-notation (enclosing terminals in double quotes):

$$\begin{aligned} V_{LLC} &::= \text{"a"} \mid \text{"b"} \mid \text{"c"} \mid \dots \mid \text{"z"} && (26 \text{ elements}) \\ V_{LUC} &::= \text{"A"} \mid \text{"B"} \mid \text{"C"} \mid \dots \mid \text{"Z"} && (26 \text{ elements}) \\ V_L &::= V_{LLC} \mid V_{LUC} \end{aligned}$$

Furthermore, we shall use digits ( $V_D$ ), the colon and the period as terminals for identifiers:

$$\begin{aligned} V_D &::= \text{"0"} \mid \dots \mid \text{"9"} \\ V_C &::= \text{"."} \\ V_{LDC} &::= V_L \mid V_D \mid V_C \\ V_P &::= \text{"."} \\ V_{LDPC} &::= V_{LDC} \mid V_P \end{aligned}$$

Let  $I$  be the set of interfaces. Elements of  $V_{LDC}$  and  $V_P$  (thus of  $V_{LDPC}$ ) are considered constants for  $I$ , and

$$\delta$$

is a special constant denoting the empty interface. Furthermore,  $X, Y, Z, \dots$  are variables ranging over  $I$ .

On  $I$  we define  $+$  and  $\cdot$  as alternative and sequential composition, where  $\cdot$  binds stronger than  $+$  and will be omitted whenever possible. As usual, we use the notation  $V^+$  for the set of finite strings over alphabet  $V$ .

Consider a string, say  $\beta = \text{a7:b.b25:8.c}$  in  $(V_{LDPC})^+$ . The string  $\beta$  is understood as a selector path to be read from left to right. Stated in different terms,  $\beta$  contains a progression of information separated by periods (" $\cdot$ ") to be understood progressively from left to right. For this reason we will call  $\beta$  a progression. The notion of a progression thus emerging combines that of a string or word and that of a process. A process in a bisimulation model can be considered a branching progression where progression is thought in terms of time or less abstract in terms of causality.

Having recognized interfaces as progressions it is a straightforward decision to take an existing algebraic structure of progressions as the basis of their formal specification. To this end we provide the axioms of Basic Process Algebra with  $\delta$ , or briefly,  $BPA_\delta$  (see [BW90, Fok00]) in Table 1.

**Definition 1.** *A right progression ring is a structure that satisfies the equations of  $BPA_\delta$  (see Table 1).*

Thus sequential composition  $\cdot$  is right distributive over  $+$  in a right progression ring and  $\delta$  is its additive identity. Furthermore,  $\delta$  is a left zero element for sequential composition.

For the sake of completeness we also define left progression rings.

Table 1: The axioms of  $BPA_\delta$

---

$X + Y = Y + X$	$(X + Y) \cdot Z = X \cdot Y + X \cdot Z$	$X + \delta = X$
$(X + Y) + Z = X + (Y + Z)$	$(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$	$\delta \cdot X = \delta$
$X + X = X$		

---

**Definition 2.** A *left progression ring* is a non-commutative ring with respect to sequential composition, in which  $+$  is commutative, associative and idempotent, and sequential composition  $\cdot$  is left distributive over  $+$ . Furthermore there is a constant  $\delta$  that is both the additive identity and a right zero element for sequential composition.

So a left progression ring satisfies the axioms

$$\begin{aligned} X \cdot (Y + Z) = X \cdot Y + X \cdot Z & \text{ instead of } (X + Y) \cdot Z = X \cdot Y + X \cdot Z, \\ X \cdot \delta = \delta & \text{ instead of } \delta \cdot X = \delta. \end{aligned}$$

A right or left progression ring is *distributive* if  $\cdot$  is distributive over  $+$ .

$\mathbb{I}_{LD\mathcal{C}P}$  is the initial distributive right progression ring with constants taken from  $V_{LD\mathcal{C}P}$ . In this paper we will only consider  $\mathbb{I}_{LD\mathcal{C}P}$  as a structure for interfaces. We will omit the symbol  $\cdot$  in a sequential composition whenever reasonable. In the case that we consider closed terms that are built with sequential composition only, we will certainly omit this symbol, and for example write

`b114.get` instead of for example `b1 · 14 · .get`

although these two expressions of course represent the same closed term. Observe that when adopting this convention, each  $\beta \in (V_{LD\mathcal{C}P})^+$  can be seen as an element (an interface, a closed term) in  $\mathbb{I}_{LD\mathcal{C}P}$ .

We notice that most process algebras as found in [BK84, BW90, Fok00] are (non-distributive) right progression rings, some unital (i.e., containing a unit for sequential composition).

In  $\mathbb{I}_{LD\mathcal{C}P}$  we write

$$X \sqsubseteq Y \quad \text{if} \quad X + Y = Y.$$

For each  $\beta \in (V_{LD\mathcal{C}P})^+$  we need two additional operators on  $\mathbb{I}_{LD\mathcal{C}P}$ :

$$\begin{aligned} \frac{\partial}{\partial \beta}(X) & \text{ the } \beta\text{-derivative of } X: \\ & \text{yields the largest } Y \text{ such that either } Y = \delta \text{ or } X + \beta \cdot Y \sqsubseteq X, \end{aligned}$$

$$\begin{aligned} \overline{\frac{\partial}{\partial \beta}}(X) & \text{ the } \beta\text{-filter-complement of } X: \\ & \text{removes all progressions from } X \text{ which have } \beta \text{ as an initial segment.} \end{aligned}$$

For example,

$$\frac{\partial}{\partial \mathbf{a} \cdot \mathbf{b}}(\mathbf{a} \cdot + \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{b} \mathbf{c}) = \mathbf{c} = \frac{\partial}{\partial \mathbf{a} \cdot \mathbf{b}}(\mathbf{a} \cdot \mathbf{b} \mathbf{c})$$

and

$$\frac{\bar{\partial}}{\partial \mathbf{a} \cdot \mathbf{b}}(\mathbf{a} \cdot + \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{b} \mathbf{c}) = \mathbf{a} \cdot = \frac{\bar{\partial}}{\partial \mathbf{a} \cdot \mathbf{b}}(\mathbf{a} \cdot).$$

Both these operators can easily be defined. Let  $u, v \in V_{LD\mathcal{C}P}$  and  $\beta \in (V_{LD\mathcal{C}P})^+$ , then the  $\beta$ -derivative  $\frac{\partial}{\partial \beta}(-)$  is defined as follows:

$$\begin{aligned} \frac{\partial}{\partial u}(\delta) &= \delta, & \frac{\partial}{\partial u \beta}(X) &= \frac{\partial}{\partial \beta}\left(\frac{\partial}{\partial u}(X)\right), \\ \frac{\partial}{\partial u}(X + Y) &= \frac{\partial}{\partial u}(X) + \frac{\partial}{\partial u}(Y), \\ \frac{\partial}{\partial u}(v) &= \delta, \\ \frac{\partial}{\partial u}(vX) &= \begin{cases} \delta & \text{if } v \neq u, \\ X & \text{otherwise,} \end{cases} \end{aligned}$$

and the  $\beta$ -filter-complement  $\frac{\bar{\partial}}{\partial \beta}(-)$  is defined by:

$$\begin{aligned} \frac{\bar{\partial}}{\partial u}(\delta) &= \delta, & \frac{\bar{\partial}}{\partial u \beta}(\delta) &= \delta, \\ \frac{\bar{\partial}}{\partial u}(X + Y) &= \frac{\bar{\partial}}{\partial u}(X) + \frac{\bar{\partial}}{\partial u}(Y), & \frac{\bar{\partial}}{\partial u \beta}(X + Y) &= \frac{\bar{\partial}}{\partial u \beta}(X) + \frac{\bar{\partial}}{\partial u \beta}(Y), \\ \frac{\bar{\partial}}{\partial u}(v) &= \begin{cases} v & \text{if } v \neq u, \\ \delta & \text{otherwise,} \end{cases} & \frac{\bar{\partial}}{\partial u \beta}(v) &= v, \\ \frac{\bar{\partial}}{\partial u}(vX) &= \begin{cases} vX & \text{if } v \neq u, \\ \delta & \text{otherwise,} \end{cases} & \frac{\bar{\partial}}{\partial u \beta}(vX) &= \begin{cases} vX & \text{if either } v \neq u, \text{ or } v = u \\ & \text{and } \frac{\bar{\partial}}{\partial \beta}(X) \neq \delta, \\ \delta & \text{if } v = u \text{ and } \frac{\bar{\partial}}{\partial \beta}(X) = \delta. \end{cases} \end{aligned}$$

### 3 Instruction Sequence and Thread Components

A focus-method interface (FMI) is an interface which is provably equal to either  $\delta$  or a term of the form

$$+_{i=1}^k \beta_i \cdot +_{j=1}^{k_i} \beta_{i,j}$$

with  $\beta_i, \beta_{i,j} \in V_L(V_{LDC})^+$  and  $k, k_{i,j} > 0$ . Informally stated this means that after flattening (bringing  $+$  to the outside), we have words with exactly one period each and no words ending in  $\delta$ .

Let  $i \in \mathbb{I}_{LDCP}$  be an expression denoting some FMI (thus  $i \neq \delta$ ). Then  $\text{PGLB}_i$  is the instruction sequence notation with basic instructions taken from  $i$ . The number of possible basic instructions may grow exponentially with the size of the expression  $i$ .

**Example 1.** Consider the interface  $i$  defined by

$$i = \mathbf{f} \cdot (\mathbf{get} + (\mathbf{set} : + \mathbf{testeq}) (0 + 1 + 2) (0 + 1 + 2) (0 + 1 + 2)) + \mathbf{g} \cdot (\mathbf{get} + (\mathbf{set} + \mathbf{testeq}) (:)(\mathbf{true} + \mathbf{false} + \mathbf{error})) \quad (2)$$

which abbreviates  $1 + 2 \cdot 3^3 + 7 = 62$  basic instructions (and the availability of twice as much test instructions in  $\text{PGLB}_i$ ).

Here we notice a first reward of our formalization: a flexible specification format for a wide variety of different instruction sequence notations. Instead of  $\text{PGLB}$  one might of course use other other program notations such as  $\text{PGA}$ ,  $\text{PGLC}$  or  $\text{PGLD}$  from [BL02],  $\text{PGLA}$  from [BP08], or  $C$  from [BP09].

In [BP02] a ‘program component’  $[i, P]$  is defined as a pair of an interface and a program with the requirement that  $i$  contains at least all instruction names that occur in program  $P$ . We stick to this idea and hold that an *instruction sequence component* is a pair

$$(i, P)$$

of a focus-method interface  $i$  and an instruction sequence  $P$ , where some form of match between  $i$  and  $P$  needs to be assumed. We have a number of options which we will formally distinguish:

**Definition 3.** Let  $P$  be some inseq in  $\text{FMN}$ . Then  $P$  **requires** interface  $i$  if each element  $\mathbf{f} \cdot \mathbf{m}$  of  $i$  occurs in  $P$  and conversely, all basic instructions occurring in  $P$  are elements of  $i$ , where  $\mathbf{f} \cdot \mathbf{m}$  occurs in  $P$  if at least one of  $\mathbf{f} \cdot \mathbf{m}$ ,  $+\mathbf{f} \cdot \mathbf{m}$ ,  $-\mathbf{f} \cdot \mathbf{m}$  is an instruction of  $P$ .

Furthermore,  $P$  **subrequires** interface  $i$  if for some  $j$ ,  $j \sqsubseteq i$  and  $P$  requires  $j$ , and  $P$  **properly subrequires**  $i$  if for some  $j$ ,  $j \sqsubseteq i$ ,  $j \neq i$  and  $P$  requires  $j$ .

For example,

$$\#3; \mathbf{b2.set:false}; !; ! \quad \begin{cases} \text{requires} & \mathbf{b2.set:false}, \\ \text{subrequires} & \mathbf{b2.set:false} + i \text{ for any interface } i. \end{cases}$$

For threads we can consider thread components as pairs  $(i, T)$ . The definitions of  $T$  *requires*  $i$  and  $T$  *subrequires*  $i$  are obvious:

**Definition 4.** *Let  $T$  be some thread with actions in FMN. Then  $T$  **requires** interface  $i$  if each element of  $i$  occurs in  $T$  and conversely, all actions occurring in  $T$  are elements of  $i$ .*

*Furthermore,  $T$  **subrequires** interface  $i$  if for some  $j$ ,  $j \sqsubseteq i$  and  $T$  requires  $j$ , and  $T$  **properly subrequires**  $i$  if for some  $j$ ,  $j \sqsubseteq i$ ,  $j \neq i$  and  $T$  requires  $j$ .*

As an example, `|#3; b2.set:false; !; !|` (sub)requires  $\delta$ . Using these last two definitions, we can refine the (sub)-requirement notions as follows.

**Definition 5.** *Let  $i$  be an interface and  $P$  an inseq. Then  $P$  **1-requires**  $i$  if  $|P|$  requires  $i$ ,  $P$  **sub-1-requires**  $i$  if  $|P|$  subrequires  $i$ ,  $P$  **(sub-)n-requires**  $i$  if  $|#n; P|$  (sub)requires  $i$ , and  $P$  **(sub-)(1, n)-requires**  $i$  if for all  $m$  with  $1 \leq m \leq n$ ,  $P$  (sub-)m-requires  $i$ .*

For example,

`#3; b2.set:false; !; !` (sub)-1-requires  $\delta$ , but (sub)-2-requires `b2.set:false`.

Let  $c = (i, P)$  be an instruction sequence component. By default we assume that  $P$  subrequires  $i$  unless explicitly stated otherwise. We further call  $i$  the interface of  $c$  and  $P$  the body of  $c$ . So for an instruction sequence component  $c$  it may for instance be the case that its body (sub-)(1, 7)-requires its interface. Here are some elementary connections:

- if  $P$  subrequires  $i$  then for all  $n$ ,  $P$  sub- $n$ -requires  $i$ ,
- conversely, if for all  $n$ ,  $P$  sub- $n$ -requires  $i$  then  $P$  subrequires  $i$ ,
- if for  $m = 1, \dots, \ell(P)$ , where  $\ell(P)$  is the length of  $P$  (its number of instructions),  $P$   $m$ -requires  $i_m$  then  $P$  requires  $+_{m=1}^{\ell(P)} i_m$ .

Further decoration of an inseq component with for instance information about its inseq notation is possible but will not be considered here.

## 4 Service Components

For a service  $H$  we assume that it offers processing for a number of methods collected in a method interface. Thread-service composition is briefly explained in [PZ06]; inseq-service composition was introduced in [BP02] (where services were called ‘state machines’) and defined as the composition obtained after thread extraction of the program term involved. The definition of a service  $H$  includes that of its *reply function*, the function that determines the reply to a call of any of its methods (which in general is based on initialization assumptions and on its history, i.e., the sequence of earlier calls). Furthermore, there is a so-called *empty* service  $\emptyset$  that has the empty method interface  $\delta$ .

A method interface (MI) is either  $\delta$  or is created by  $+$  and  $\cdot$  from constants in  $V_{LDC}$ .

**Example 2.** A stack over values 0, 1 and 2 can be defined as a service with methods `push:i`, `topeq:i`, and `pop`, for  $i = 0, \dots, 2$ , where `push:i` pushes  $i$  onto the stack and yields `true`, the action `topeq:i` tests whether  $i$  is on top of the stack, and `pop` pops the stack with reply `true` if it is non-empty, and otherwise does nothing and yields `false`.

With  $\beta$  ranging over  $(V_{LDC})^*$ , the reply function  $F$  of the stack is informally defined by

$$\begin{aligned}
F(\beta \text{ push}:i) &= \text{true} \quad \text{and generates a stack with } i \text{ on top} \\
F(\beta \text{ pop}) &= \begin{cases} \text{true} & \text{if the stack generated by } \beta \text{ is non-empty} \\ & \text{and then pops this stack,} \\ \text{false} & \text{otherwise,} \end{cases} \\
F(\beta \text{ topeq}:i) &= \begin{cases} \text{true} & \text{if the stack generated by } \beta \text{ has } i \text{ on top} \\ \text{false} & \text{otherwise.} \end{cases}
\end{aligned}$$

For a formal definition of a stack as a service, see e.g. [PZ06].

**Definition 6.** A service  $H$  *provides* interface  $i$  if  $H$  offers a reply function for all elements of  $i$ , and  $H$  *superprovides*  $i$  if for some  $j \sqsupseteq i$ ,  $H$  provides  $j$ .

For a *service component*  $(j, H)$  we assume that  $H$  superprovides the method interface  $j$  unless explicitly stated otherwise.

Below we consider two forms of inseq-service composition: the *use* and the *apply* composition. We first discuss the use operator. For an inseq  $P$  a use application is written

$$P /_{\beta} H$$

where  $\beta$  is a focus. A use application always yields a thread, which is reasonable because it is upon the execution of an inseq that its generated actions may call for some service and use its reply for subsequent execution. The use operator simply drops the service upon termination or deadlock. Use compositions are defined in [BP02] and take a thread as their left argument, but inseqs can be used instead by setting  $P /_{\beta} H = |P| /_{\beta} H$ . Below we lift use compositions to the level of components.

**Example 3.** The stack described in Example 2 provides the method interface  $j$  defined by

$$j = (\text{push} : + \text{topeq} : )(0 + 1 + 2) + \text{pop}. \quad (3)$$

Let  $P$  be a program defined in  $\text{PGLB}_{i+c.j}$  for the FMI  $i$  defined in Example 1. Then  $P$  can use the stack defined in Example 2 via focus  $c$ . If we only push the value 0 (so the stack behaves as a counter), we can write  $C(n)$  for a stack holding  $n$  times the value 0 (so  $C(0)$  represents the empty stack). With the defining equations from [BP02] it follows that

$$\begin{aligned}
(c.\text{push}:0; P) /_c C(n) &= P /_c C(n+1), \\
(-c.\text{pop}; !; P) /_c C(0) &= S, \\
(-c.\text{pop}; !; P) /_c C(n+1) &= P /_c c C(n).
\end{aligned}$$

Instructions in  $\text{PGLB}_{i+c.j}$  with foci different from  $c$  are distributed over use applications, e.g.,

$$(+g.\text{set}:true; f.\text{get}; P) /_c C(n) = ((f.\text{get}; P) /_c C(n)) \trianglelefteq g.\text{set}:true \trianglerighteq (P /_c C(n)).$$

**Definition 7.** For an instruction sequence component  $(i, P)$ , a service component  $(j, H)$ , and some  $\beta \in V_{LDC}^+$ , the **use composition**

$$(i, P)/\beta (j, H)$$

is **matching** if  $\frac{\partial}{\partial\beta}(i) \sqsubseteq j$  (note the period that occurs in  $\frac{\partial}{\partial\beta}$ ), in which case

$$(i, P)/\beta (j, H) = \left(\frac{\bar{\partial}}{\partial\beta}(i), P/\beta H\right).$$

The use composition  $(i, P)/\beta (j, H)$  is **non-matching** if  $\frac{\partial}{\partial\beta}(i) \not\sqsubseteq j$  and then

$$(i, P)/\beta (j, H) = (\delta, D).$$

**Example 4** (Example 3 continued). We find for inseq component  $(i + c.j, c.push:0; P)$  that

$$(i + c.j, c.push:0; P)/c (j, C(n)) = (i, P/c C(n+1)),$$

while  $(i, c.push:0; P)/c (j, C(n)) = (\delta, D)$ .

The *apply composition* of a thread  $T$  and a service  $H$  is written

$$T \bullet_{\beta} H,$$

and always yields a service. Thus threads are used to alter the state of a particular service, and only finite threads without deadlock (D) do this in a meaningful way. Apply compositions are defined in [BP02] and take a thread as left-argument, and a typical defining equation is

$$S \bullet_{\beta} H = H.$$

However, we can use inseqs as left-arguments by defining  $P \bullet_{\beta} H = |P| \bullet_{\beta} H$ . Below we lift apply compositions to the level of components.

**Definition 8.** For an instruction sequence component  $(i, P)$ , a service component  $(j, H)$ , and some  $\beta \in V_{LDC}^+$ , the **apply composition**

$$(i, P) \bullet_{\beta} (j, H)$$

is **matching** if  $i \sqsubseteq \beta.j$  and then

$$(i, P) \bullet_{\beta} (j, H) = (j, P \bullet_{\beta} H).$$

The apply composition  $(i, P) \bullet_{\beta} (j, H)$  is **non-matching** if  $i \not\sqsubseteq \beta.j$  and then

$$(i, P) \bullet_{\beta} (j, H) = (\delta, \emptyset)$$

where  $\emptyset$  is the empty service.

**Example 5.** For the stack and the interfaces  $i$  and  $j$  defined in Examples 2–4, we find

$$(c.j, c.push:0; c.push:0; !) \bullet_c (j, C(n)) = (j, C(n+2)),$$

while  $(i, c.push:0; c.push:0; !) \bullet_c (j, C(n)) = (\delta, \emptyset)$ .

## 5 Plots and Related Work

By allowing interfaces which are not minimal (subrequired) more options arise for finding concise interface notations. One might question the plausibility of components with interfaces which are larger than strictly necessary. Here are some plots that might lead to that situation.

1.  $(i, P)$  with  $P$  the result of complex projections that are not yet computed.
2.  $(i, P)$  with  $P$  projected to PGLB in fragments (JIT projection). At no point in time the complete interface of  $P$  is known.
3.  $(i, P)$  claims name space while preventing redesign of relevant methods of the services used. This creates degrees of freedom for redesign of  $P$ . For example, if  $P$  uses booleans  $\mathbf{b1}, \dots, \mathbf{b50}$ , one can reserve  $\mathbf{b1}, \dots, \mathbf{b100}$  to facilitate future ideas to optimize  $P$ .
4.  $(i, P)$  is one of a series inseq components that one may want to plug in in an execution architecture [BP07]. The interface  $i$  is kept simple to facilitate a quick check.

Without any doubt, more plots that justify the existence of inseq components can be thought of. However, as stated earlier, the main argument for the introduction of (right) progression rings introduced in this paper is to provide a flexible and practical notation for interface specification by means of an appropriate abstract datatype specification.

In [BM07], a process component is taken as a pair of an interface and a process specifiable in the process algebra ACP [BW90, Fok00]. In that paper interfaces are formalized by means of an *interface group*, which allows for the distinction between expectations and promises in interfaces of process components, a distinction that comes into play in case components with both client and server behaviour are involved. However, in our approach the interaction between instruction sequences (or threads) and services is not sufficiently symmetric to use the group structure.

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