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The Forward and Reverse Shock Dynamics of Cassiopeia A

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Abstract

We report on proper motion measurements of the forward- and reverse shock regions of the supernova remnant Cassiopeia A (Cas A), including deceleration/acceleration measurements of the forward shock. The measurements combine 19 yr of observations with the Chandra X-ray Observatory, using the 4.2–6 keV continuum band, preferentially targeting X-ray synchrotron radiation. The average expansion rate is $0.218 \pm 0.029\% \text{ yr}^{-1}$ for the forward shock, corresponding to a velocity of $\approx 5800 \text{ km s}^{-1}$. The time derivative of the proper motions indicates deceleration in the east, and an acceleration up to $1.1 \times 10^{-4} \text{ yr}^{-2}$ in the western part. The reverse shock moves outward in the east, but in the west it moves toward the center with an expansion rate of $-0.0225 \pm 0.0007 \% \text{ yr}^{-1}$, corresponding to $-1884 \pm 17 \text{ km s}^{-1}$. In the west, the reverse shock velocity in the ejecta frame is $\gtrsim 3000 \text{ km s}^{-1}$, peaking at $\approx 8000 \text{ km s}^{-1}$, explaining the presence of X-ray synchrotron emitting filaments there. The backward motion of the reverse shock can be explained by either a scenario in which the forward shock encountered a partial, dense, wind shell, or one in which the shock transgressed initially through a lopsided cavity, created during a brief Wolf–Rayet star phase. Both scenarios are consistent with the local acceleration of the forward shock. Finally we report on the proper motion of the northeastern jet, using both the X-ray continuum band, and the Si XIII K-line emission band. We find expansion rates of, respectively, $0.21\%$ and $0.24\% \text{ yr}^{-1}$, corresponding to velocities at the tip of the X-ray jet of $7830–9200 \text{ km s}^{-1}$.

Unified Astronomy Thesaurus concepts: Supernova remnants (1667); Shocks (2086); Galactic cosmic rays (567); Stellar mass loss (1613); X-ray astronomy (1810)

1. Introduction

The early dynamical evolution of supernova remnants (SNRs) is as much determined by the density structure of the progenitor’s circumstellar medium (CSM) as by the velocity distribution of the supernova (SN) ejecta. The CSM properties and SN explosion energy determine the expansion velocity of the blast wave, whereas the structure of the SN ejecta determine at what rate energy from the freely expanding, cold ejecta, is added to the hot shell after the ejecta are shocked by the reverse shock. The dynamical evolution of young SNRs is, therefore, of interest as it reveals information about both the structure of the CSM, determined by the SN progenitor mass-loss history, as well as the structure of the ejecta, which provides insights into the SN explosion properties.

The presence of a reverse shock in young SNRs was first pointed out by McKee (1974). The hydrodynamics of young SNRs was further explored by Chevalier (1982), who showed that self-similar solutions for the expansion and relative locations of the forward and reverse shocks with respect to the contact continuity exist if one assumes that the freely expanding SN ejecta density has a velocity profile of the form $\rho_c \propto v^{-n}$, with $5 < n \lesssim 12$. Under these conditions the forward and reverse shock radii evolve as $R_{fs} \propto R_{ns} \propto t^{m}$, with $m = (n - 3)/(n - s)$ the expansion parameter. The parameter $s$ here indicates the density structure of the CSM, $\rho_{cs} \propto r^{-s}$, with $s = 0$ a uniform density profile, and $s = 2$ corresponding to the density profile of a steady SN progenitor wind. Of course, the CSM structure may be more complicated than described by an $s = 0$ or $s = 2$ model.

According to Chevalier’s model, valid for the earliest SNR phase, the reverse shock expands outward in the observer frame. This analytical model is no longer valid once the reverse shock has reached the core of the ejecta density distribution, where the density is assumed to be more or less uniform—but decreasing with time as $r^{-3}$. Once most of energy of the SN ejecta has been processed by the reverse shock, and is contained in the hot SNR shell, the SNR has entered the Sedov–Taylor phase, for which the forward shock is expected to expand as $R_{fs} \propto t^{2/(5-s)}$.

The Truelove & McKee (1999) model describes the transition in the expansion from the early phase to the Sedov–Taylor phase. For the $s = 0$ models the reverse shock eventually reverses its course and moves backward also in the frame of the observer. For SNRs expanding in a steady wind, i.e., $s = 2$, semi-analytical models (Hwang & Laming 2012; Micelotta et al. 2016; Tang & Chevalier 2017), but also hydrodynamical simulations (Orlando et al. 2016, 2021), show that the reverse shock keeps expanding outward for 2000–3000 yr, with the reverse shock velocity in the frame of the ejecta being more or less constant.

The most important example of a young core-collapse SNR, and one that is most likely expanding in a dense steady wind, is Cassiopeia A (Cas A). Its forward shock has reach a radius of $\approx 2.8$, conventionally corresponding to 2.8 pc at the distance of 3.4 kpc (Reed et al. 1995). Cas A is the result of a Type Ib SN (Krause et al. 2008; Rest et al. 2011) that occurred around the year 1672 (Thorstensen et al. 2001), but went unnoticed at the time. The SN ejecta contains very little hydrogen (Fesen & Becker 1991), and the total ejecta mass is estimated to be around $2–4 M_{\odot}$ (Vink et al. 1996; Willingale et al. 2003; Laming & Temim 2020). The thermal X-ray emission from Cas A is dominated by the shocked ejecta, with prominent emission lines from intermediate mass elements (IMEs; Si, S, Ar) and iron. The IMEs and Fe have distinct spatial distributions and
Doppler shift profiles (Willingale et al. 2001). In the optical, ejecta knots can be separated based on spectra rich in [S II] or in [O III] line emission, with optical knots dominated by [S II] lines being preferentially found in the northeast to southwest direction.

One important feature of the ejecta distribution is the presence of two oppositely placed jet-like structures, which are visible in the optical (van den Bergh & Dodd 1970; Fesen & Milisavljevic 2016), X-rays (Hwang et al. 2004; Vink 2004), and infrared (Hines et al. 2004). The jets stand out clearly because they happen to move almost perpendicular to our line of sight (Fesen et al. 2006). X-ray and optical spectroscopy reveal that they are rich in IMEs, particularly in silicon and sulfur. Another peculiar morphological feature is the presence of ring-like features in three-dimensional space (sky coordinates + Doppler velocity), as revealed by optical Doppler velocity mapping (Milisavljevic & Fesen 2013). These rings are likely the result of the inflation of radioactive $^{56}$Ni/$^{56}$Co bubbles, pushing aside the nonradioactive IME ejecta (Blondin et al. 2001).

The observed deviations from spherical symmetry mostly relate to asymmetries in the explosion itself (e.g., Rest et al. 2011). The outer shock wave, as marked by the thin rims of X-ray synchrotron radiation, appears nearly circular, and suggests a relatively spherically symmetric CSM density distribution. However, in the optical the CSM is traced by some high density knots, the so-called quasi-stationary flocculi (QSFs; Minkowski 1957). These knots have typical velocities of $\sim$150 km s$^{-1}$ (Kamper & van den Bergh 1976), and are nitrogen-rich, while still containing hydrogen. These QSFs appear to be the products of a pre-SN mass-loss phase, and their distribution is not spherically symmetric (e.g., Lawrence et al. 1995; Fesen et al. 2001; Alarie et al. 2014; Koo et al. 2018), with a large concentration in the northern part, and a protruding arc of QSFs in the southwest part. Weil et al. (2020) recently reported the detection of faint optical emission from clumpy CSM outside the current shock radius.

Here we report on an expansion study of Cas A with archival Chandra X-ray Observatory (Chandra; Weisskopf et al. 2003) data, spanning 19 yr of observations. The goal is to both understand the dynamics of the forward shock and the reverse shock, and the northeastern jet. We limit this study to the X-ray continuum band of 4.2–6 keV, which for the forward shock, but likely also for part of the reverse shock (Hughes 1999; Heldner & Vink 2008; Uchiyama & Aharonian 2008), is dominated by X-ray synchrotron emission from plasma immediately downstream of the shock. An alternative hypothesis for the hard X-ray emission from (part of) the reverse shock region is nonthermal bremsstrahlung caused by suprathermal electrons; see, for example, Asvarov et al. (1990), Laming (2001), Vink & Laming (2003), and the discussions in Vink (2008a) and Grefenstette et al. (2015).

This study is a follow-up of many previous X-ray expansion measurements focusing on different regions and using different measurements, starting with the Einstein High Resolution Imager (HRI) and ROSAT HRI observations (Koralesky et al. 1998; Vink et al. 1998), and various measurements with Chandra (Delaney & Rudnick 2003; DeLaney et al. 2004; Patnaude & Fesen 2009; Sato et al. 2018). For the forward shock these studies indicate an average expansion rate of typically 0.2 % yr$^{-1}$, faster than the radio expansion rates based on proper motion of bright knots, $\sim$0.1 % yr$^{-1}$, but with a large intrinsic scatter (Tuffs 1986; Anderson & Rudnick 1995). A radio expansion measurement based on the overall fitting of the expansion in the UV plane at 151 MHz listed a much faster expansion rate of 0.19%–0.24% yr$^{-1}$ (Agüeros & Green 1999).

The main results reported here consist of direct proper motion measurements in azimuthal sectors, with annuli encompassing the forward and reverse shock regions, and a separate measurement of the northeastern jet. The results for the reverse shock show a wide variety in expansion properties, which does not agree with the theoretical expectations.

We also report on a measurement of the acceleration/deceleration of the expansion, albeit with some caution given the sensitivity of these measurements to systematic errors.

### 2. Method and Data

For certain periods of time the expansion of an SNR can be described by a self-similar solution model of the form

$$ R = R_0 \left( \frac{t}{t_0} \right)^m, $$

with $R$ a certain radius—e.g., the forward shock radius, $R_f$—$R_0$ the reference radius, corresponding to age $t_0$, and with $m$ the expansion parameter. For example, for a point explosion into a uniform density medium one expects the evolution to be described by $m = 2/5$, corresponding to the Sedov–Taylor model. Previous X-ray expansion measurements of Cas A indicate that $m \approx 0.7$ (Koralesky et al. 1998; Vink et al. 1998; Delaney & Rudnick 2003; Patnaude & Fesen 2009), close to $m = 2/3$, expected for an SNR evolving inside a steady progenitor wind with $\rho(r) \propto r^{-2}$, i.e., $s = 2$ case.

The method used here measures the expansion projected onto the plane of the sky by comparing X-ray images based on observations in different years. One reference image is stretched or shrunk by the expansion factor $f$ until it provides a good fit to an image based on a later, respectively an earlier observation. We can approximate the development of the expansion factor by a Taylor series around a reference time $t_0$,

$$ f(t) = \frac{R(t)}{R_0} = 1 + a(t - t_0) + \frac{1}{2} b(t - t_0)^2 + \ldots, $$

with $t_0$ being the age of the SNR at time the reference image (the model image) was taken. For a self-similar evolution we expect

$$ a = \left( \frac{1}{R} \frac{dR}{dt} \right)_{t=t_0} = \frac{m}{t_0}, $$

$$ b = \left( \frac{1}{R} \frac{d^2R}{dt^2} \right)_{t=t_0} = \frac{m(m - 1)}{t_0^2}. $$

We will refer to $a$ as the expansion rate, and will use the units % yr$^{-1}$. Its inverse is the expansion timescale $\tau_{\exp} \equiv a^{-1}$,

$$ \tau_{\exp} \equiv a^{-1}, $$

which gives the age of the SNR if no deceleration has taken place over its lifetime, i.e., $m = 1$. This shows that the expansion parameter can be estimated as $m = t_0/\tau_{\exp}$. But strictly speaking this is only valid if the SNR shock radius evolved as Equation (1) for the entire life of the SNR. In other words, a mismatch between $m$ as derived from $a$ or $\tau_{\exp}$, and $b$ is an indication that the the self-similar model may not be a valid approximation.
Table 1
Summary of the Observations Used

<table>
<thead>
<tr>
<th>Epoch</th>
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<th>Δt</th>
<th>Exposure</th>
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<tr>
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<td>50.1</td>
</tr>
</tbody>
</table>

The parameter $b$ is the second derivative and can be labeled the acceleration parameter. However, as indicated in Equation (3) for $m < 1$ one expects the value of $b$ to be negative. So we define the deceleration rate for Cas A, at an age of $t_0 = 331$ yr (in 2004) and $m \approx 0.7$ is expected be around $-b \approx 1.9 \times 10^{-6}$ yr$^{-2}$.

2.2. Expansion Measurement Method

There are several ways to measure expansions from images; for example, extracting radial profiles and comparing them (Delaney & Rudnick 2003; Patnaude & Fesen 2009)—fitting in one dimension—or by stretching/shrinking one image (the model image) with respect to an image of another epoch, and measuring what stretching factor provides a best fit (Koralewsky et al. 1998; Vink et al. 1998). This means that the fitting is done in two dimensions. We apply the latter method, using our self-developed C++ code, which we named multisectionexpansion. The input for the code consists of a text file containing a list of all necessary input files (images and mask files, see below) and optional input parameters. After the fitting procedure, a new input file is automatically generated for iterative purposes, which also includes best-fit parameters. Other output files are postscript files with expansion plots (generated using the pgplot library, Pearson 2011), and text and LATEX-formatted files with the fitting results. The core of the C++ code goes back to one of the earliest X-ray expansion measurements (Vink et al. 1998), but has been substantially updated, first for an expansion measurements of Kepler’s SNR (Vink 2008b), and even more substantially for the current measurements. The most significant change is that instead of comparing one observation set with a model for a given epoch, it makes a joint fit to observations from different epochs simultaneously, greatly increasing the sensitivity.

Like in previously published proper motion measurements, using earlier versions of the code, the fitting is done by stretching (regridding) the model image (or part of it) by a factor $f$ (Equation (2)) and comparing them to another image using the Cash statistic (Cash 1979), which is the log-likelihood function for Poisson distributions:

$$L_k = -2 \ln \mathcal{L} = -2 \ln \Pi_i \Pi_j P_{i,j,k}$$
$$= -\sum_{i,j} \left[ n_{ijk} \ln \bar{n}_{ijk} - \bar{n}_{ijk} - \ln(n_{ijk}!) \right].$$

Here $k$ refers to the epoch number, $i, j$ refers to the pixel coordinates, $n_{ijk}$ is the count number in pixel $ij$ of epoch $k$, and $\bar{n}_{ijk}$ is the predicted pixel photon count for a given epoch, as derived from the model image. The best-fit model is the one that minimizes $L_k$. As the term $\ln(n_{ijk}!)$ is based on the count image alone, which does not depend on model parameters, the term is omitted when comparing different models.

Apart from the model and reference images a set of mask images are provide consisting of pixels with values 0 or 1, which is used to select only those parts of the images for which the model is evaluated. This was already part of the earlier version of the code.

The novelty of our approach is that instead of comparing the model image with one other image, the new code compares all epochs simultaneously, by optimizing the sum of the log-likelihoods for all model image comparisons. That is, we obtain oversampling of factor 4, with a four times larger image size, for the reasons explained below.

The parameter $b$ is the second derivative and can be labeled the acceleration parameter. However, as indicated in Equation (3) for $m < 1$ one expects the value of $b$ to be negative. So we define the deceleration rate for Cas A, at an age of $t_0 = 331$ yr (in 2004) and $m \approx 0.7$ is expected be around $-b \approx 1.9 \times 10^{-6}$ yr$^{-2}$. The novelty of our approach is that instead of comparing the model image with one other image, the new code compares all epochs simultaneously, by optimizing the sum of the log-likelihoods for all model image comparisons. That is, we obtain oversampling of factor 4, with a four times larger image size, for the reasons explained below.
the best-fit model by minimizing

\[ L_{\text{tot}} = \sum_k L_k, \]

with \( k \) the epoch number, and with for each \( k \) using a model image \( k \) characterized by the expansion rate parameter, \( a \) —and for some regions \( b \)—and time difference \( \Delta t \), according to Equation (2). This greatly decreases the error of the measurement, as now the photon counts in 17 images are combined to give a single log-likelihood value. The model images are based on the VLP observations. For the center of expansion we use the expansion center measured by Thorstensen et al. (2001), based on optical proper motion studies of the fastest moving knots, which are probably least decelerated. The input model images are four times oversampled. So after the applying the stretching factor the model images are rebinned by a factor 4, to bring the pixel size to that of the comparison images. This reduces interpolation artefacts caused by shifting pixels within a subpixel scale. After this procedure the normalization has to be adapted to match that of the comparison images. This is done by normalizing the sum of the pixel values for the model images to that of the comparison images within each region of interest, using the mask images.

The Cash statistic cannot be used for model input values of zero —due to the \( \tilde{n}_{ijk} \) term in the statistic. To avoid zero value pixels, we first produced a master mask, which was closely cropped around Cas A, as not to compare regions that lie far outside Cas A and for which the count statistic is low. Moreover, we smoothed the image slightly with a Gaussian of \( \sigma = 0.5^\prime \)08, and we added a small number to each pixel of 0.01 to avoid pixels with zero values (see also Vink 2008b).

### 2.3. Correcting for Pointing Errors

Apart from stretching the model image, the code also applies a displacement correction to the model image for each epoch \( (\Delta x_k, \Delta y_k) \) in order to correct for pointing errors. These can be of the order of 0.6 for Chandra (90% containment radius Weisskopf et al. 2003). In fact, blinking images with the ds9 viewer (Joye & Mandel 2003) reveals pointing errors by eye. The field of Cas A does not contain stable point sources that can be reliably used to realign images, with the exception of the central point source in Cas A, the putative neutron star left behind by the SN explosion. This point source is off center, and likely has its own proper motion of a few hundred kilometers per second (Mayer & Becker 2021).

We experimented with several solutions for co-aligning the images, including using centroiding of the central point source. In the end we settled on an iterative procedure: we fitted the expansion parameters \((a \text{ and } b)\) for all sectors (masks) associated with the forward shock, without using a pointing correction. We used the forward shock masks, as the errors in the pointing are small compared to the displacement of the shock. Then, in a separate procedure, we fitted all sectors combined to find the optimal values for \( \Delta x_k, \Delta y_k \) for each epoch, using the best-fit expansion parameters. These best fit \( \Delta x_k, \Delta y_k \) were then used for finding a new optimal expansion solution for individual sectors. Three iterations provided a stable fit for all \( \Delta x_k, \Delta y_k \), which are listed in Table 2. The table shows that typically the corrections are less than 0.55 pixels \((0.027)\), which is better than the expected Chandra performance of 0.6. Note that this is a correction with respect to the reference image, whereas the 0.6 Chandra error refers to the absolute celestial coordinate reconstruction.

### 2.4. Selected Regions

The main aim of this study is the dynamics of both the forward and reverse shocks. The Chandra VLP X-ray image in the continuum band shows that the nonthermal filaments associated with the forward shock roughly lie on a circle with radius of 2/8 (2.8 pc); Figure 1. For an explosion date around 1672 this amounts to an average velocity of \( \approx 8000 \text{ km s}^{-1} \). For the expansion study of the forward shock we created the master mask by smoothing the Chandra VLP continuum image \((4.2--6 \text{ keV})\) with a Gaussian of 2 pixels, and selecting all pixels with a value of more than 1.0. Then a dilation algorithm was applied to add 4 pixels around the periphery of the SNR, plugging some isolated holes in the mask, and removing pixel clusters not connected to the SNR. Having created the master mask, the forward shock region was selected from it by overlaying an annulus with an inner radius of 2/4 and an outer radius of 3'. Inspecting the continuum image, some small circular regions were removed, as they appeared to contain structure associated with ejecta knots, or the reverse shock region. The selected annuli can be seen in Figure 1 (left) and the final mask in Figure 1 (right). The forward shock mask was then further divided in segments with opening angles of 20°, as indicated by the color coding in the map on the right.

The reverse shock location is less easily identified. Helduer & Vink (2008) used the location of nonthermal filaments and a deprojection method to infer that the reverse shock has a radius of 1/9, but shifted to the west with respect to the explosion center of Cas A; see Gotthelf et al. (2001) for an earlier estimate of 1/6. The disadvantage of this X-ray determination is that the 4--6 keV continuum band is a mix of synchrotron radiation and thermal bremsstrahlung. More recently, Arias et al. (2018) used free-free absorption in the radio below 100 MHz to map the unshocked ejecta. This confirmed that the reverse shock has a radius of 1/9, but with a center shifted 14° to the west of the explosion center as estimated by Thorstensen et al. (2001).

### Table 2

<table>
<thead>
<tr>
<th>Epoch</th>
<th>( \Delta x )</th>
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<td>17</td>
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</table>

**Note.** \( \Delta x, \Delta y \) are in Chandra ACIS-S pixel units \((0.0^\prime.492)\) with respect to the reference VLP image.
et al. (2001). The free-free absorption map shows a more complete map of the outline of the reverse shock than the results of Helder & Vink (2008) in that it includes the eastern part. The corresponding free expansion velocities for the ejecta reaching the reverse shock correspond to \( \approx 4400 \text{ km s}^{-1} \) in the east and \( \approx 6600 \text{ km s}^{-1} \) in the west. Interestingly, a similar discrepancy in velocity is seen for the optical knots in Doppler velocity, with the backside velocity being best fit with velocities \( \sim 6000 \text{ km s}^{-1} \), and the front side with velocities of \( \sim 4000 \text{ km s}^{-1} \) (e.g., Lawrence et al. 1995; Reed et al. 1995; Milisavljevic & Fesen 2013).

Note that most of the optical fast moving knots are located within this inferred reverse shock radius. The reason is that these optical knots flare up for tens of years after having encountered the reverse shock—so they mark the location of the reverse shock in three-dimensions—but many are seen in projection inward of the projected reverse shock radius. Only the outer most ejecta knots should be close to the projected radius of the reverse shock. This can be verified by applying the conversion of projected radius to motion in the plane of the sky as inferred by Milisavljevic & Fesen (2013), \( S = 0^\circ022 \text{ km s}^{-1} \), to their best-fit outer most velocities of \( \approx 4000-6000 \text{ km s}^{-1} \), giving a radius ranging from \( \approx 1^\circ5-2^\circ2 \).

To select the region of the reverse shock we use an annulus of \( 1^\circ67-2^\circ18 \), encompassing the reverse shock location obtained by Arias et al. (2018), as indicated in Figure 1 in red. The red/white colored sectors in Figure 1 (right) show the 18 segments for the reverse shock regions. Like for the forward shock each segment has an opening angle of \( 20^\circ \), centered on the explosion center, with the central position angle (PA) starting at PA = \( 10^\circ \), i.e., the first segment starts due north and ends at PA = \( 20^\circ \).

2.5. Optimization Methods

For finding the optimal solution for expansion measurements per region, the code searches for the lowest value of \( L_{\text{tot}} \) (Equation (6)) in each region (i.e., for each mask). For the reverse shock region and the jet region we only optimized for the best value of the expansion rate, \( a \). For the forward shock region we optimized jointly for \( a \) and the second derivative, the acceleration parameter \( b \).

The code has the options to simply scan in fixed steps the parameter space from \( a_{\text{min}} \), \( a_{\text{max}} \), or by a more efficient way, but with the drawback that a local rather than a global minimum is found. The latter method is the default option after having tested both options. The efficient way consists of first determining \( L_{\text{tot}} \) for six values of \( a \), and selecting the values of \( a \) corresponding to the lowest values of \( L_{\text{tot}} \). These are then used to determine a parabola through these three points \((a, L)\), which is then used to guess the minimum of the parabola. The actual value of \( L_{\text{tot}} \) for this inferred minimum is then determined, and compared to the other three values of \((a, L)\), and the worst one is discarded. The remaining three values are then used for a next iteration, until the improvement in \( L_{\text{tot}} \) is \( \Delta L < 4 \). It can be that the parabola is inverted or no valid solutions are found, in which case a random point is chosen and the procedure is repeated. If this happens repeatedly, a scan of the parameter space is used.

For finding the parameter \( b \) for the forward shock sectors, we used initially a similar efficient procedure as for determining \( a \), in an iterative procedure: determine \( a \), and fix it to the best-fit value, and then determine \( b \), and then repeat this procedure. But here we did find that sometimes the absolute minimum is not found, due to a strong statistical correlation between parameters \( a \) and \( b \). So instead we scanned the parameter space in \( a \) and \( b \), but optimized computing time by using a mesh-refinement method: we first scan \( L \) on a very coarse grid of \( 5 \times 5 \) values in \( a \) and \( b \). We then refine the grid by a factor of \( 3 \times 3 \) and continue the scan of the parameter space for those grid values for which \( \Delta L = L - L_{\text{min}} \) is above a given threshold. A refinement to a parameter grid of \( 135 \times 135 \) values was found to be sufficient, but in a few cases (see Appendix A) we also
performed an additional refinement to 405 × 405 grid points. The starting values of \(a\) was found using the efficient method described above. The initial threshold for a refinement of the likelihood search was \(\Delta L = 2048\), the final threshold was set to \(\Delta L = 16\). The result of the mesh-refinement procedure, as well as the statistical correlation between \(a\) and \(b\) can be judged from the examples in Figure 10. The resulting grid values of \(\Delta L\) are also used to determine the measurement error, with \(\Delta L = 1, 4, 9\) corresponding to, respectively, 1\(\sigma\), 2\(\sigma\), and 3\(\sigma\) confidence intervals. See also the log-likelihood curves in Figure 8.

For the separate optimization of the misalignment correction of the images (Section 2.3) a simple scan of \((\Delta x, \Delta y)\) was used, iteratively using first a coarse grid and then zooming in on the best value with a smaller grid size.

### 3. Results

We discuss the results of our X-ray expansion study in the 4.2–6 keV band for the forward shock and reverse shock regions separately. The results for both shock regions are combined in Figure 2 as a function of PA. In Figure 3 we illustrate in a graphical way the expansion of both shocks on top of the X-ray image of Cas A. As an illustration for how well the best-fit expansion models fit the data we show in Figure 3 two residual maps, of the model with the 2019 observation. On the left, the model is simply the unaltered 2004 VLP observation, whereas in the right-hand image the model consists of the sum of the best-fit models for all 37 sectors.

By comparing the models with individual observations one can indeed see brightness changes over time (see also Patnaude & Fesen 2007; Uchiyama & Aharonian 2008; Patnaude et al. 2011; Sato et al. 2017), and occasionally also filaments that change in shape, perhaps due to variable expansion rates along the filaments. These type of changes are source systematic errors, and cause most of the residuals seen in Figure 3 (right).

In Appendix A the profiles of \(\Delta L\) are shown as a function of \(a\)—and for the forward shock \(b\)—which shows that the best-fit values are well defined; although some local minima indicate that there are some ambiguities, possibly due to substructure in some sectors.

![Figure 2](image.png)

**Figure 2.** Expansion rates for the forward and reverse shock regions. The grayscale shows the corresponding expansion parameter, defined here as \(m = t_0/\tau_{\text{exp}}\), determined using \(t_0 = 332\) yr. North corresponds to PA = 0°, and PA is defined counterclockwise.

![Figure 3](image.png)

**Figure 3.** Same image as Figure 1 but now with a spider diagram overlaid that illustrates the expansion rate as a function of PA. For the forward shock (green) the radial extent of the spider diagram is linearly proportional to the expansion rate. For the reverse shock (red) the radial coordinate provides the expansion relative to the dashed circle—inside the circle indicates a motion toward the interior.

### 3.1. Forward Shock Region

For determining the proper motion of the forward shock expansion we used two methods, one with solving also for the deceleration of the shock—\(b\) in Equation (3)—and one with assuming \(b = 0\). The best-fit parameters including the measurement of the deceleration rate are summarized in Table 3.

For the best-fit values of \(a\), fitting also for \(b\) does not make much of a difference for the expansion rates. For example, solving for \(b\) gives an average expansion rate of \(0.218\% \pm 0.029\%\) yr\(^{-1}\), compared to \(0.219 \pm 0.030\%\) yr\(^{-1}\) when keeping \(b = 0\)—the errors indicate the rms of the variations. These values correspond to expansion timescales of \(\tau_{\text{exp}} = 457 \pm 60\) yr, or \(m = t_0/\tau_{\text{exp}} = 0.73 \pm 0.10\).

There is quite some variation in the expansion parameter as a function of PA, with for a PA of 30° an expansion parameter of \(m = 0.919 \pm 0.003\), which approaches free expansion (\(m = 1\)). The slowest expansion is measured for a PA of 190° (south) with \(m = 0.506 \pm 0.05\). As illustrated in Appendix B, the expansion rates are relatively robust to the choice of pointing corrections.

Figure 5 shows the measured deceleration rates (= \(– b\)). Fitting for \(b\) does significantly improve the fitting results for each PA, with the exception of PA = 270° for which the best-fit value is anyway \(b \approx 0\). For the other PAs the improvement ranges from \(\Delta L = –4.7\) for PA = 50° to \(\Delta L = –400\) for PA = 250°. For the sum of \(L\) over all 18 sectors of the forward shock region, we have \(\Delta L_\text{PA}L = –1155\).

The average of the best-fit values for the expansion rate derivative is \(b = (– 0.21 \pm 4.94) \times 10^{-3}\) yr\(^{-2}\), which shows there is a larger spread in \(b\) than the average value. Contrary to expectations, there appear to be portions of the forward shock that are accelerating—in particular around PA = 190° and 250°. Moreover, the spread is about an order of magnitude larger than...
the expected value of $|b| \approx 2 \times 10^{-6} \text{ yr}^{-2}$ (see Section 2). For each individual PA, however, the detection is determined with an error of the order of $(1-2) \times 10^{-6} \text{ yr}^{-2}$. So for several individual PAs we can report a statistically significant measurement of a deceleration or an acceleration.

There are some reason to be cautious about the values of the deceleration rates. First, because the measured values are larger than the theoretical expectations, and therefore require more scrutiny. More importantly, the measured deceleration rates are more sensitive to systematic errors than the expansion rates. Potential systematic errors are (1) within each shock region multiple filaments exists with potentially different dynamics; (2) filaments brighten and dim, and new filaments may appear over the time span of interest (2000–2019), so one should be aware that the shock front is not uniquely defined during the whole observation period; and, last but not least, (3) unlike the expansion rate $a$ the deceleration rate is more sensitive to misalignments of images of different epochs. We illustrate this by showing in Appendix B also the measured deceleration rates without solving for $(\Delta x, \Delta y)$ for each individual epoch (see Section 2.3).

Note. For $v_{\text{obs}}$ and $m$ a radius of 2.18 and an SNR age of 333 yr are assumed.

### 3.3. Northeastern Jet

The measurement of the northeastern jet stands apart from the other expansion measurements reported here, in that one cannot claim to measure the expansion of a shock front. Instead the measured proper motions concern the movement of plasma in the jet, mostly caused by the movements of bright features within the overall jet envelope. In this sense the rational for using the 4.2–6 keV energy band is not valid, also because, as far as we know, the continuum is thermal rather than nonthermal radiation. In the continuum band the northeastern jet is relatively faint, but the jet is rich in IMEs. For that reason we report here both an expansion in the continuum band, to be consistent with the measurements of the forward and reverse shock regions, and a measurement in the 1.75–1.97 keV band, which is dominated by K-shell emission from Si XIII. The results are summarized in Table 5. The measurements in the two bands are not consistent with each other, with $a = 0.2041 \text{ yr}^{-1}$ for the continuum band, and $a = 0.2401 \text{ yr}^{-1}$ for the Si XIII band. But the expansion rates are within a range similar to that of the forward shock.

### 4. Discussion

We reported here X-ray proper motion measurements for the forward and reverse shock regions, as well as for the northeastern jet. The two shock regions are each divided in 18 annular segments with opening angles of 20°. This is different from various previous studies (Delaney & Rudnick 2003; Patnaude & Fesen 2009; Sato et al. 2018), most of which focus on individual filaments. We
reported in Vink et al. that a reverse shock velocity directed toward the center is reported.

For a (reported here the proper motions in terms of the expansion rate vs $t$), and they are somewhat larger than those reported by Patnaude and 300°.

The forward shock proper motions reported here generally agree with previous measurements. They are comparable with those measured by Delaney & Rudnick (2003), who reported an average expansion rate of $0.20 \pm 0.07$ yr$^{-1}$ ($m = 0.66$), and they are somewhat larger than those reported by Patnaude & Fesen (2009), but for fewer filaments. The forward shock expansion is also consistent with the 151 MHz radio expansion measurement by Agüeros & Green (1999), reporting a dynamical timescale of 400–500 yr, which corresponds to 0.25%–0.20% yr$^{-1}$.

The proper motions near the reverse shock reported here show that the reverse shock is moving outward in most regions, but is negative for PAs between 260° and 300°. This is not the first time that a reverse shock velocity directed toward the center is reported. In X-rays the first hints for much lower velocities in the west were reported in Vink et al. (1998), but negative velocities of filaments associated with the reverse shock were reported by Delaney et al. (2004) and more recently by Sato et al. (2018). Sato et al. 2018 reported projected velocities of 2100–3800 km s$^{-1}$, toward the center. These X-ray measured proper motions find their radio-band counterpart in the proper motion measurements of individual knots (Bell 1977; Tuffs 1986; Anderson & Rudnick 1995), which provide a much slower expansion rate than the overall expansion of the SNR in the radio as reported by Agüeros & Green (1999).

In fact, Anderson & Rudnick (1995) even reported proper motions toward the center in the west. With this in mind, it is clear that these radio knots, and the overall bright radio ring they are embedded in, contain relativistic electrons accelerated by the reverse shock, a conclusion already obtained by Helder & Vink (2008) and Uchiyama & Aharonian (2008).

For the discussion on the implications of our reverse shock measurements, we assume that in the western part the X-ray continuum from the reverse shock region is dominated by synchrotron radiation, rather than nonthermal bremsstrahlung (Laming 2001; Vink & Laming 2003). X-ray synchrotron radiation from the western reverse shock region is currently the more prevalent hypothesis (e.g., Helder & Vink 2008; Uchiyama & Aharonian 2008; Vink 2008a; Grefenstette et al. 2015; Sato et al. 2018), but definitive proof is lacking. In the near future X-ray polarization measurements with the Imaging X-ray Polarimeter Explorer (Weisskopf et al. 2021) may be able to provide a definitive proof for a synchrotron radiation nature by detecting polarized X-ray radiation from the western region, provided the local magnetic-field turbulence is not too high (Bykov et al. 2020).

In the case of X-ray synchrotron radiation the measured proper motions most likely reflect the proper motion of the reverse shock itself, as X-ray synchrotron radiation requires a strong shock, and the highly relativistic electrons need to be close to the shock on account of the rapid synchrotron cooling rate. For nonthermal bremsstrahlung it is plausible that they may be powered by internal, secondary shocks as well, allowing for the possibility that the measured proper motions are not directly linked to the reverse shock proper motion. However, as we will discuss below, there are also hints from

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**Table 4**

Expansion Measurements of the Reverse Shock Region

<table>
<thead>
<tr>
<th>PA (°)</th>
<th>$f$ (% yr$^{-1}$)</th>
<th>$\tau_{\text{exp}}$ (yr)</th>
<th>$m$</th>
<th>$v_{\text{obs}}$ (km s$^{-1}$)</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.2275 ± 0.0005</td>
<td>439.8$^{+1.0}_{-0.9}$</td>
<td>0.7574 ± 0.0018</td>
<td>4179 ± 10</td>
<td>132,400.8</td>
</tr>
<tr>
<td>30</td>
<td>0.1364 ± 0.0011</td>
<td>733.3$^{+2.3}_{-3.3}$</td>
<td>0.5451 ± 0.0036</td>
<td>2506 ± 20</td>
<td>72,990.0</td>
</tr>
<tr>
<td>50</td>
<td>0.2140 ± 0.0008</td>
<td>476.2$^{+3.3}_{-1.7}$</td>
<td>0.7127 ± 0.0026</td>
<td>3932 ± 14</td>
<td>125,390.4</td>
</tr>
<tr>
<td>70</td>
<td>0.1814 ± 0.0023</td>
<td>551.2$^{+2.0}_{-0.9}$</td>
<td>0.6041 ± 0.0075</td>
<td>3333 ± 42</td>
<td>119,088.9</td>
</tr>
<tr>
<td>90</td>
<td>0.1663 ± 0.0018</td>
<td>601.2$^{+2.4}_{-3.4}$</td>
<td>0.5539 ± 0.0058</td>
<td>3056 ± 32</td>
<td>122,083.8</td>
</tr>
<tr>
<td>110</td>
<td>0.1748 ± 0.0013</td>
<td>572.2$^{+3.2}_{-3.2}$</td>
<td>0.5819 ± 0.0043</td>
<td>3211 ± 23</td>
<td>105,591.0</td>
</tr>
<tr>
<td>130</td>
<td>0.2053 ± 0.0016</td>
<td>487.1$^{+3.9}_{-3.9}$</td>
<td>0.6837 ± 0.0054</td>
<td>3772 ± 30</td>
<td>127,765.6</td>
</tr>
<tr>
<td>150</td>
<td>0.1704 ± 0.0012</td>
<td>586.7$^{+2.7}_{-2.7}$</td>
<td>0.5675 ± 0.0040</td>
<td>3131 ± 22</td>
<td>110,526.2</td>
</tr>
<tr>
<td>170</td>
<td>0.1479 ± 0.0009</td>
<td>675.9$^{+2.2}_{-1.8}$</td>
<td>0.4926 ± 0.0030</td>
<td>2718 ± 17</td>
<td>101,548.7</td>
</tr>
<tr>
<td>190</td>
<td>0.1188 ± 0.0012</td>
<td>841.8$^{+3.3}_{-3.3}$</td>
<td>0.3956 ± 0.0039</td>
<td>2183 ± 22</td>
<td>110,024.0</td>
</tr>
<tr>
<td>210</td>
<td>0.1090 ± 0.0008</td>
<td>917.7$^{+4.6}_{-4.6}$</td>
<td>0.3628 ± 0.0026</td>
<td>2002 ± 14</td>
<td>111,771.7</td>
</tr>
<tr>
<td>230</td>
<td>0.0929 ± 0.0013</td>
<td>1076.4$^{+5.3}_{-5.3}$</td>
<td>0.3094 ± 0.0044</td>
<td>1707 ± 25</td>
<td>127,131.4</td>
</tr>
<tr>
<td>250</td>
<td>0.0679 ± 0.0014</td>
<td>1471.8$^{+11.8}_{-11.8}$</td>
<td>0.2262 ± 0.0048</td>
<td>1248 ± 26</td>
<td>88,390.1</td>
</tr>
<tr>
<td>270</td>
<td>−0.1023 ± 0.0012</td>
<td>−977.5$^{+11.3}_{-11.3}$</td>
<td>−0.3407 ± 0.0039</td>
<td>−1880 ± 22</td>
<td>55,628.8</td>
</tr>
<tr>
<td>290</td>
<td>−0.0226 ± 0.0007</td>
<td>−4425.0$^{+24.9}_{-15.3}$</td>
<td>−0.0753 ± 0.0022</td>
<td>−415 ± 12</td>
<td>109,937.4</td>
</tr>
<tr>
<td>310</td>
<td>0.1062 ± 0.0008</td>
<td>941.4$^{+7.2}_{-7.2}$</td>
<td>0.5357 ± 0.0027</td>
<td>1952 ± 15</td>
<td>149,720.9</td>
</tr>
<tr>
<td>330</td>
<td>0.1865 ± 0.0012</td>
<td>536.1$^{+3.5}_{-3.5}$</td>
<td>0.6211 ± 0.0040</td>
<td>3427 ± 22</td>
<td>130,213.0</td>
</tr>
<tr>
<td>350</td>
<td>0.2285 ± 0.0004</td>
<td>437.7$^{+0.7}_{-0.7}$</td>
<td>0.7608 ± 0.0012</td>
<td>4198 ± 7</td>
<td>142,704.4</td>
</tr>
</tbody>
</table>

Mean | 0.134 ± 0.084 | 0.446 ± 0.279 | 2459 ± 1540 |

**Note.** For $v_{\text{obs}}$ and $m$ a radius of 1° is used—but not centered on the explosion center—and an SNR age of 333 yr are assumed.

---

**Table 5**

Expansion Measurements of the Northeastern Jet

<table>
<thead>
<tr>
<th>Energies (keV)</th>
<th>$a$ (% yr$^{-1}$)</th>
<th>$\tau_{\text{exp}}$ (yr)</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cont.</td>
<td>4.2–6.0</td>
<td>0.2041 ± 0.0004</td>
<td>489.9 ± 0.88</td>
</tr>
<tr>
<td>Si-K</td>
<td>1.75–1.97</td>
<td>0.2401 ± 0.0003</td>
<td>416.5 ± 0.45</td>
</tr>
</tbody>
</table>

---
optical observations of a reverse shock that moves toward the center—but puzzlingly more extensively so than found in our study. Finally, the nonthermal X-ray filaments in the western part coincide with the expected location of the reverse shock as determined by the radio-absorption measurements (Arias et al. 2018).

The expansion rates can be converted into a (projected) velocity using the distance to Cas A of 3.4 kpc (Reed et al. 1995), and using the shock radii. For the forward shock we use a radius of 2.8 pc, which conveniently corresponds to 2.8 pc. Note that often 2.5 pc is used, but 2.8 pc is closer to the location of the filament in the Chandra observations.

For the reverse shock we used the estimate of its location obtained from radio-absorption measurements of unshocked ejecta gas by Arias et al. (2018) for the conversion of expansion rate to velocity. The location of the reverse shock is approximated by a 1.9 pc radius, but centered on R.A. = 23°23′26″, decl. = 58°48′54″, which is ~14° toward the (north)west of the explosion center derived by Thorstensen et al. (2001). One assumption is that the velocities measured by us are largely confined to the plane of the sky. Although this will not be the case everywhere, by measuring in annuli targeting the outer radius, and the inferred projected reverse shock position, we hope to have minimized the chance for large velocity components along the line of sight.

The inferred velocities are listed in Tables 3 and 4, and plotted in Figure 6. For the forward shock the velocities range from 4040–7340 km s$^{-1}$, the latter at a PA of 30°, with an expansion parameter of $m = 0.91$, which is close to free expansion ($m = 1$). The maximum observed proper motion for the reverse shock region corresponds to 4200 km s$^{-1}$ in the north, but at a PA of 270° the projected reverse shock velocity in the observer frame is ~1880 km s$^{-1}$, which is somewhat smaller than the values reported in this region by Sato et al. 2018. However, Sato et al. (2018) handpicked the filament, and the value reported by us corresponds to a larger region. Indeed if we focus on this region, as done in Figure 7 we see that the residuals are still large, even after expansion correction, around the regions picked by Sato et al. (2018); their W1 and W2. We note that the residuals around W1 are also reflecting brightness changes and changes in the shape of the filaments.

For the forward shock the inferred velocities of the X-ray synchrotron filaments correspond to the shock velocity $V_{rs}$, but for the reverse shock velocity we have to make a distinction between the velocity in the frame of the observer (red solid line in Figure 6) and the velocity difference between the shock front and the unshocked ejecta. The latter the defines the thermodynamical and particle acceleration properties of the shock.

For the reverse shock the unshocked gas close to the shock front moves with the free expansion velocity $V_{ej} = R_{ej}/t$, whereas the reverse shock in the frame of the observer can be expressed as $V_{rs,obs} = dR_{rs}/dt$. This gives for the reverse shock velocity in the frame of the ejecta,

$$|V_{rs}| = \left| \frac{R_{rs}}{t} - \frac{dR_{rs}}{dt} \right|,$$

with $t$ the age of the SNR. In Figure 6 $V_{rs}$ is shown as a red dashed line. For the western part the free expansion velocity of the ejecta close to the reverse shock is $R_{ej}/t_0 \approx 6300$ km s$^{-1}$, whereas for the eastern part it is $R_{ej}/t_0 \approx 4900$ km s$^{-1}$. The difference is due to the different projected distances of the reverse shock location with respect to the explosion center (Arias et al. 2018).

4.1. Reverse Shock Velocity and X-ray Synchrotron Radiation

X-ray synchrotron radiation from the vicinity of SNR shocks requires fast acceleration, and hence fast shocks, as the electrons responsible for the radiation—typically with energies $\gtrsim 10$ TeV—have a relatively short loss timescale (e.g., Ginzburg & Syrovatskii 1965; Vink 2020):

$$\tau_{\text{syn,loss}} \approx 20 \left( \frac{E}{10 \text{ TeV}} \right)^{-1} \left( \frac{B}{250 \mu G} \right)^{-2} \text{yr},$$

with $B \approx 250 \mu G$ the typical post-shock magnetic field for Cas A and $E$ the electron energy. This sets the typical timescale for brightness variations in X-ray synchrotron, but it also means that the acceleration rate needs to be faster than the synchrotron loss rate. Using the theory of diffusive shock acceleration, while incorporating radiative energy losses, one can show that the typical photon energy, $h\nu$, relates to the shock velocity as (e.g., Aharonian & Atoyan 1999; Zirakashvili & Aharonian 2007; Vink 2020)

$$h\nu \lesssim 3\eta^{-1} \left( \frac{V_{rs}}{3000 \text{ km s}^{-1}} \right)^2 \text{keV},$$

with $\eta \gtrsim 1$ a parameter that is inversely proportional to the magnetic-field turbulence on length scales of the electron gyroradius. So X-ray synchrotron radiation in the 4.2–6 keV band requires shock velocities that are above 3000 km s$^{-1}$, in conjunction with a highly turbulent magnetic field.

The forward shock velocity reported here, and in previous studies, is everywhere well above this limit. However, for the reverse shock region this is not the case. Note that $V_{rs}$ for the reverse shock refers to $V_{rs}$ in Equation (7), and is depicted as a dashed line in Figure 6. Apart from the difference in reverse shock motion, $dR_{rs}/dt$, also the difference in location with respect to the explosion center adds to the difference in shock velocity of about 1400 km s$^{-1}$ between the eastern and western parts.

It is clear that $V_{rs}$ is only sufficiently high for X-ray synchrotron emission for PA $\sim 180^\circ$–310°, which explains why indeed only in the western part of Cas A the reverse shock X-ray continuum emission is dominated by synchrotron radiation (see Helder & Vink 2008, Figure 6), and even why both X-ray and radio synchrotron radiation is very bright around a PA of 270°: the shock velocity there is $V_{rs} \approx 8000$ km s$^{-1}$, exceeding even the forward shock velocity. However, there are two caveats, which are related to each other. First of all, if X-ray synchrotron emission is only originating from the reverse shock regions between PA $\sim 180^\circ$–310°, then one rationale for selecting the 4.2–6 keV band—namely, that we wanted to focus on X-ray synchrotron radiation as it traces the shock front—is not entirely valid. The other caveat is that studies of the optical knots near the reverse shock (Fesen et al. in preparation, see also Fesen et al. 2019), shows evidence that all around the SNR the reverse shock is either at a standoff or returning toward the center, clearly at odds with our study where the reverse shock in the observer frame has a proper motion corresponding to 2000–4000 km s$^{-1}$ outside PAs of 180°–310°. Note that the optical emission from the reverse shock is caused by ejecta clumps, entering the reverse shock and lighting up for a few
years. So the motion of the reverse shock in the optical refers to the pattern of the locations where the knots start are appearing, rather than relying on direct proper motion measurements. So what is the potential effect of having a region dominated by thermal bremsstrahlung rather than synchrotron radiation in the continuum band? Due to the short radiative loss timescale, synchrotron radiation is confined to a region close to the shock front, and moves along with the shock front. For thermal bremsstrahlung the shock front itself will be the onset of the emission, but the shocked plasma itself remains visible for a long time. So likely the proper motion is a combination of the proper motion of the shock front, and of the plasma motion behind it. The plasma motion with respect to the shock front is given by $\Delta v = V_{rs}/\chi$, with $\chi = 4$ the shock compression ratio—assuming that the cosmic ray acceleration is not very efficient, in which case $\chi > 4$ (e.g., Berezhko & Ellison 1999; Vink 2020). This means that the shock plasma in the frame of the observer will move with

$$v_{\text{plasma,obs}} = \frac{dR_{\text{rs}}}{dt} + \frac{V_{rs}}{\chi}. \quad (10)$$

We can conservatively estimate the magnitude of the plasma velocities on the proper motions of the eastern part by assuming that the reverse shock is stationary, i.e., $dR_{\text{rs}}/dt = 0$ and $V_{rs} = |R_{\text{rs}}|/\chi$, and that the proper motion reflects solely the plasma velocity. In that case $v_{\text{plasma,obs}} = R_{\text{rs}}/(\chi t) \approx 1400$ km s$^{-1}$, for $R_{\text{rs}} = 1.9$ pc and $\chi = 4$. Clearly, the measured velocities in the eastern part are larger than 1400 km s$^{-1}$, with velocities as high as 4200 km s$^{-1}$, and an average shock velocity of 2460 km s$^{-1}$. So, although we may have overestimated the reverse shock velocity outside the western part in our frame, the reverse shock cannot be stationary or returning backward for PAs $0^\circ–220^\circ$, or from $310^\circ–360^\circ$. An alternative explanation for the difference between the optically determined reverse shock proper motion, and the proper motions reported here, may be that using the optical ejecta knots for determining the reverse shock velocity biases locally toward stationary or returning reverse shocks. Another observational bias may be that most of the optical knots are projected toward the inside of the projected reverse shock outer radius. For the X-ray proper motions we selected the movements near the edge of the reverse shock, whereas for optical measurements there may be a bias to select knots lying well inside the inner radius of the reverse shock region used by us.

There may well be more regions of the reverse shock going backward missed in our study, as we concentrated on the projected reverse shock region. For example, optical studies show that the backside of the optical shell seems to move faster than the front side (Reed et al. 1995; Milisavljevic & Fesen 2013), which likely has a similar origin as the displacement of the reverse shock location in the plane of the sky. It could well be that the backside indeed also has a reverse shock moving inward. This may also cause X-ray synchrotron emission from the backside of the shell. But since we observe the narrow filaments on the backside face-on, rather than edge-on, the X-ray synchrotron may be less easily identified. Figure 6 in Helder & Vink (2008), which is based on purely spectral analysis does give a hint of more extended X-ray synchrotron emission projected to the inside. But their figure still indicates a lack of diffuse X-ray synchrotron emission from the eastern part.

### 4.2. Why Did the Reverse Shock Reverse Its Direction?

The standard scenario for the evolution of Cas A is that of a remnant of a core-collapse SN of Type IIb (Krause et al. 2008; Rest et al. 2011), with a low ejecta mass of $2–4 M_\odot$ evolving in the clumpy, dense wind of its progenitor (Vink et al. 1996; Chevalier & Oishi 2003; Willingale et al. 2003; Vink 2004; Hwang & Laming 2012; Weil et al. 2020). The observed deviations from spherical symmetry, like the jet and ejecta rings, are usually attributed to asymmetries in the explosion itself, for which light echoes from different directions indeed provide evidence (Rest et al. 2011). The pre-shock density at the forward shock is $n \approx 1$ cm$^{-3}$ (Lee et al. 2014), corresponding to $\rho_{\text{csm}} \approx 2 \times 10^{-24}$ g cm$^{-3}$. The density is expected to fall off as $\rho(r) = M/4\pi r^2 v_w$, implying a mass-loss rate of $M \approx 2.5 \times 10^{-5} M_\odot$ yr$^{-1}$ for an assumed wind velocity of $v_w = 10$ km s$^{-1}$. For such a $1/r^2$ CSM density profile, the reverse shock is not expected to move back to the center at the current age of Cas A, but rather when its age is 2000–3000 yr; see Figure 3 in Micelotta et al. (2016) for a semi-analytical model, and Figure 4 in Orlando et al. (2021) for the hydrodynamical simulation of Cas A. So the question arises why the reverse shock in the western part of Cas A does not agree with well-understood evolutionary expansion models.

One can obtain some intuitive understanding of the reverse shock dynamics by starting with the well-known Rankine–Hugoniot relation $P_1 + \rho_1 v_1^2 = P_0 + \rho_0 v_0^2$, expressing momentum flux conservation, with subscript 0 referring to the unshocked gas, and subscript 1 to the post-shock gas. For a strong shock $P_0$ can be neglected, and $v_0 = V_s$. Using the shock compression ratio $\chi = \rho_1/\rho_0 = v_0/v_1$, and using for the density the ejecta density at the reverse shock ($\rho_0 = \rho_{ej,0}$) we can transform this into (see Vink 2020):

$$P_{rs,1} = \left(1 - \frac{1}{\chi}\right) \rho_{ej,0} V_{rs}^2. \quad (11)$$

$P_{rs,1}$ refers to the pressure in the shocked gas, which is of the same order, but somewhat lower (Chevalier 1982) than the pressure in the shell just behind the forward shock $P_{bs,1} = (1 - 1/\chi) \rho_{\text{csm,0}} V_{bs}^2$. Writing $P_{rs,1} = \xi P_{bs,1}$, with $\xi \approx 0.5$, and assuming the compression ratios are similar, we see that

$$\rho_{ej,0} V_{rs}^2 = \xi \rho_{\text{csm,0}} V_{bs}^2. \quad (12)$$

For $V_{rs}$ we can insert Equation (7) and $V_{bs} = m R_{bs}/t$. The turning around of the reverse shock happens when $dR_{rs}/dt = 0$. We conclude, therefore, that the reverse shock is moving inward when (Vink 2020)

$$\rho_{ej,0} < \xi \rho_{\text{csm,0}} m^2 \left(\frac{R_{bs}}{R_{rs}}\right)^2 \approx 1.1 \xi \rho_{\text{csm,0}}.$$  

(13)

for which we used $m = 0.7$, $R_{ns} = 1.9$ pc, and $R_{bs} = 2.8$ pc. For the average ejecta density we can write

$$\rho_{ej,0} \approx 9.5 \times 10^{-24} \left(\frac{M_{ej}}{4 M_\odot}\right) \left(\frac{R_{rs}}{1.9 \text{ pc}}\right)^{-3} \text{g cm}^{-3}. \quad (14)$$
Combining Equations (13) and (14), we see that the ejecta density needs to be a factor of 2–5 lower than the average density for a CSM density of $2 \times 10^{-24}$ g cm$^{-3}$, in order for the reverse shock to move toward the center.

So under the hypothesis that the reverse shock moves inward due to relatively low ejecta density in the western part, the implication is that the ejecta density is much lower than $2 \times 10^{-24}$ g cm$^{-3}$, or, alternatively, the forward shock is moving through dense material, or has done so recently.

The ejecta density in the western part is already somewhat lower, as here the reverse shock is furthest from the explosion center. The low density could be more enhanced, if locally there is a large bubble, inflated by radioactive $^{56}$Ni/$^{56}$Co during the SN phase. Such bubbles give rise to ring-like structures surrounding the inflated bubbles, which have been identified in the three-dimensional layout of the optical knots (Milisavljevic & Fesen 2013). Indeed, there is a high concentration of Fe-rich ejecta close the western part of the reverse shock (e.g., last panel of Figure 1 in Hwang & Laming 2012).

The hypothesis that the reverse shock in the western part moves toward the center due to a large ejecta bubble seems, therefore, plausible. A problem is, however, that there are similar large concentrations of Fe-rich ejecta, and also ejecta rings, in the northern part and in the southeastern part, in regions for which we do not measure an inward movement of the reverse shock. Moreover, the hydrodynamical simulations of Cas A’s evolution by Orlando et al. (2021)—based on a realistic ejecta distributions that include underdense regions caused by $^{56}$Ni/$^{56}$Co decay— show that the reverse shock is expected to move outward with 2000–4000 km s$^{-1}$ everywhere. These velocities are consistent with the measurements reported here outside PAs in the range of $\sim 220^\circ$–$320^\circ$.

Alternatively, the reverse shock in the western part may have been affected by the local CSM structure. There is a two decade old discussion on the possibility of an interaction of the western part of the SNR with a molecular cloud at the western edge of Cas A (e.g., Keohane et al. 1996; Kilpatrick et al. 2014). There is indeed a molecular cloud projected toward the western part of Cas A, and the southwestern (counter)jet is much more irregularly shaped than the northeastern jet, which could be the result of some form of interaction of the southwestern jet with a density enhancement (Schure et al. 2008). However, there is no direct evidence that the Cas A SNR shock has a direct interaction with the molecular cloud seen in absorption. Such an interaction should lead to bright thermal X-ray emission in the western part due to the high local density, which is not observed. Nor is there evidence for a direct interaction between the forward shock and the molecular cloud as probed by CO molecular line studies (Zhou et al. 2018).

In a follow-up study Orlando et al. (2022) studied the possibility of the interaction of the forward shock with an asymmetric mass-loss shell with which the forward shock interacted in the recent past, based on a preliminary presentation of the work reported here. Their simulations can indeed reproduce the reversal of the reverse shock motion in the current epoch of the SNR evolution. The interaction of the forward shock with the shell creates an increase in pressure in the shell, and induces reflected shocks, which enhance the shell pressure at the reverse shock.

A constraint on when the encounter with the putative shell could have taken place comes from early radio maps of Cas A: the radio bright region in the western part, likely associated with the strong reverse shock encountered there in the frame of the ejecta, was already present in the first radio synthesis map of Cas A produced in the early 1960s by Ryle et al. (1965). So likely the reverse shock has been going back in that region prior to 1960.

On the other hand, the optical morphology underwent quite some changes over the last 50 yr (Patnaude & Fesen 2014), with the northern shell already being prominent in the 1950s, and the southeastern part only appearing in the 1970s. This may provide a window on the history of the reverse shock development and its possible connection to the the CSM structure. A complication is that the connection between the evolution of the optical structures and the structures seen in X-ray or radio is not fully understood. For example, the emergence of optical knots in certain regions seems to precede the local X-ray brightening of more diffuse emission by several years (Patnaude & Fesen 2014). Also note that the southeastern shell, emerging in the 1970s, already has a radio counterpart in the earliest radio map (Ryle et al. 1965).

The debris of this overrun shell may be associated with the quasi-stationary flocculi in the western part of Cas A (Koo et al. 2018). If true, the current reverse shock dynamics may reveal some information about the mass-loss history of the Cas A progenitor, which could be combined with the optical information about unshocked CSM as reported by Weil et al. (2020) to shed light on the nature of the progenitor star.

Another type of CSM structure that could have affected the evolution of the reverse shock is if the progenitor went through a short Wolf–Rayet (W-R) star phase. The progenitor was a stripped star—the SN being a SN IIb— and it is possible that it exploded as a W-R star. In the case of a late W-R phase of the progenitor, the fast ($1000–2000$ km s$^{-1}$) W-R wind creates a low density cavity. After the explosion the forward shock initially moves fast through the cavity until it reaches the W-R/ red-supergiant wind boundary, and then rapidly decelerates. Once more energy is transferred to the shocked red-supergiant wind, the shock reaccelerates.

The possible CSM configurations, and the subsequent SNR evolution of Cas A inside the wind structure, was modeled by van Veelen et al. (2009), who concluded that the W-R phase must have been absent, or relatively short-lived ($\leq 15,000$ yr). The main argument was that for a longer W-R phase the reverse shock would be moving backward, which was thought not to be the case in 2009.

Now that we know that the reverse shock moves back, at least in the western part, one should reconsider this scenario. Indeed, also in this scenario the forward shock could be still accelerating, in agreement with our findings (see Figure 8 in van Veelen et al. 2009). The difficulty for this scenario is how to explain that the W-R wind cavity is aspherical, given the fast wind speed. One possibility is that the progenitor star was moving through the interstellar medium, creating a lopsided wind region, which also affected the shape of the inner cavity (see Weaver et al. 1977; Meyer et al. 2021).

The effect of a W-R phase on the hydrodynamics of the jets in Cas A was investigated by Schure et al. (2008), who concluded that the jets would not survive the encounter was the edge of the cavity for a W-R phase that lasted longer than $\approx 2500$ yr. Interestingly, the northeastern jet in Cas A is well defined in X-rays, whereas the southwestern jet looks broken up. This is also in agreement with the idea that there was a W-R wind cavity, elongated in the southwestern direction. The late encounter of the southwestern jet partially broke and then went
up the southwestern jet, whereas the northeastern jet encountered the boundary layer early enough and pierced through it unhindered.

The nature of the progenitor of Cas A is still a mystery. In particular, the cause of its large mass-loss rate is often attributed to a closely interacting binary system. But there is no evidence for a surviving companion star (Kerzendorf et al. 2019). Nevertheless, the shock dynamics reported here provide important hints on the late mass-loss history of the progenitor, be it in the form of a partial, asymmetric shell from episodic mass loss, an aspherical cavity created by a brief W-R phase wind, or perhaps even a combination of both.

4.3. Deceleration versus Acceleration of the Forward Shock

Interestingly, the interaction of the forward shock with a dense mass-loss shell (Orlando et al. 2022) or the edge of a W-R wind cavity (van Veelen et al. 2009) initially leads to a strong deceleration of the forward shock, followed by an acceleration once the shock has penetrated through the shell. Indeed, one of the surprising results of the deceleration/acceleration measurements reported here is that the forward shock appears to be accelerating around PAs of 180° (south) and 250° (west), see Figure 4. The latter agrees with the location of a returning reverse shock, and it also the location where several tests of the deceleration/acceleration measurements provide robust results (Figure 11).

On the other hand, we do find a rather strong deceleration toward the northern part of the SNR, an order of magnitude stronger than expected based on Equation (3). In the simulations of Orlando et al. (2022) it is assumed that the mass-loss shell is denser in the west. But the strong deceleration in the north may potentially indicate that the shell is not partial, but rather aspherical, and that in the north the shock is just encountering the shell and decelerating.

Clearly following up the deceleration/acceleration measurements with further monitoring of Cas A with Chandra is important, to make the measurements more precise and confirm these results, with even longer expansion base lines.

The reverse shock motion itself could be potentially explained by the asymmetries in the ejecta distribution, but with some difficulties. However, the combination of an inward moving reverse shock coinciding with an acceleration of the forward shock in the western region is more naturally explained by a more complex CSM structure.

4.4. Proper Motion of the Northeastern Jet

The measurements of northeastern jet regions reported is the first measurement of the motion of this jet measured in X-rays. The X-ray measurement motions reveal more about the dynamics of the shock heated gas, which is a combination of shocked ejecta and shocked CSM. Previous measurements of the jet velocities are based on optical measurements, which rely on dense optical knots, which are expected to have velocities close to the free expansion velocities $R/t$. Indeed, Fesen et al. (2006) and Fesen & Milisavljevic (2016) report proper motions up to $\sim 15,000 \text{ km s}^{-1}$. Note that the optical knots traces the northeastern jet out to 5'/3—further out than the X-ray counterpart, which goes out to 4'. For our measurement we used measurements out to a radius of 3'/9.

The inclusion of a measurement of the northeast jet region stands apart from the rest of the measurements here, which focus on the forward and reverse shock, whereas the X-ray structure of the jet shows a number of bright streaks, which are likely associated with ejecta material. For that reason we deviated from the overall setup for the measurements, and included X-ray images from 1.75–1.94 keV, dominated by Si XII line emission. The measurements in the continuum band and Si-K line emission band differ substantially, with expansion rates of 0.2% yr$^{-1}$ for the continuum band, and 0.24% yr$^{-1}$ for the Si-K band. This difference may be real and related to the fact that Si-K is almost exclusively associated with shocked ejecta, whereas the continuum emission is likely a mix of thermal bremsstrahlung from shocked ejecta, plus more diffuse thermal bremsstrahlung from shocked CSM. Either way, both expansion rates are consistent with an expansion parameter of $m = 0.68–0.8$ similar to, or even larger than, the average expansion parameter of the forward shock. The similarity of the expansion parameters suggest that the northeast jet is still moving through the red-supergiant wind of the progenitor (see Schure et al. 2008).

The tip of the jet in X-rays is around 3'/9, which, combined with the expansion parameter and age of Cas A of 332 yr (in 2004), suggests an expansion velocity of $\approx 7830–9200 \text{ km s}^{-1}$.

5. Conclusions

We reported here on the proper motion of the forward and reverse shock regions of Cas A along the entire projected edges of both shocks using Chandra observations in the 4.2–6 keV.
passages, with proper motion measurements by Sato et al.


equation derivative of the proper motion of the forward shock, i.e., the sensitivity, which allowed us to measure the second-order images, spanning 19 yr in observations, greatly increased the observations obtained in 2004. The combination of many and comparing them to a model based on the Chandra VLP continuum band. We used a method in which the proper motions are measured by directly combining 17 observations and comparing them to a model based on the Chandra VLP observations obtained in 2004. The combination of many images, spanning 19 yr in observations, greatly increased the sensitivity, which allowed us to measure the second-order derivative of the proper motion of the forward shock, i.e., the acceleration/deceleration rate.

For the forward shock we report an average expansion rate of 0.218% yr\(^{-1}\), varying from 0.152%–0.276% yr\(^{-1}\), corresponding to a mean expansion parameter of \(m = 0.73\) with variations from \(m = 0.51–0.91\). The mean is somewhat higher than reported by Patnaude & Fesen (2009) and close to what was reported by DeLaney et al. (2004), but both did not use the entire edge of the SNR and cannot be compared one to one. The second-order derivatives show large variations, and values higher than predicted by a model in which the shock moves through a smooth CSM with a \(1/r^2\) density profile. The average deceleration rate we found is \((0.2 \pm 4.9) \times 10^{-5}\) yr\(^{-2}\). The eastern part of the forward shock appears to be decelerating, whereas in the western part, around PAs of 190\(^{\circ}\) and 250\(^{\circ}\), the shock appears to be accelerating.

The reverse shock is moving outward in the frame of the observer in the eastern part of the SNR, with typical expansion rates of 0.1%–0.2% yr\(^{-1}\), but is lower between PAs of 190\(^{\circ}\) and 310\(^{\circ}\), with negative expansion rates for the range of 260\(^{\circ}\)–300\(^{\circ}\):

\[ V_r < 3000 \text{ km s}^{-1} \]

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\[ V_r < 3000 \text{ km s}^{-1} \]

We thank several colleagues for helpful discussions on Cas A: with Martin Laming on nonthermal bremsstrahlung and the possible implications of multiple reflected/secondary shocks, with Salvatore Orlando on the effects of mass-loss shells on the dynamics of the forward and reverse shocks, and with Rob Fesen and Danny Milisavljevic on the optical emission from Cas A. J.V. is partially supported by funding from the European Union’s Horizon 2020 research and innovation program under grant agreement No. 101004131 (SHARP). D.J. P and D.C. acknowledge support from NASA contract NAS8-03060.

Facility: Chandra X-ray Observatory (CXO).

Software: The software for measuring the expansion can be downloaded at https://zenodo.org/record/5593154. We have made used of the FITS-image viewer ds9 (Joye & Mandel 2003), and the subroutine libraries ppplot (Pearson 2011) and cftisio https://heasarc.gsfc.nasa.gov/docs/software/fitsio.

Appendix A

Log-likelihood Profiles

Figures 8 and 9 show the log-likelihood profiles, \(\Delta L\), for the forward and reverse shock regions, respectively. For the reverse shock \(L\) only depends on the expansion rate \(a\). For the forward shock a two-dimensional fit was employed, fitting both the expansion and acceleration rates \(a\) and \(b\), and both the two-dimensional likelihood contours and the likelihood curves as a function of \(a\) and \(b\) are shown, marginalized over \(b\) and \(a\), respectively. The curves for \(b\) appear rather ragged, which is caused by the finite stepsize in \(b\).
Figure 8. Log-likelihood ($\Delta L = -2\Delta \ln L$) distributions relative to the minimum values given as a contour plot with levels $\Delta L = 1, 4, 9$ (corresponding to $1\sigma, 2\sigma, 3\sigma$ confidence regions) and the distribution of $\Delta L$ as a function of $a$ and the deceleration parameter $-b$, marginalized over the other parameter.
In Figure 10 color-coded log-likelihood maps—\(\Delta L\) as a function of \(a\) and \(b\)—are shown, which were calculated at a smaller stepsize in \(a\) and \(b\). Log-likelihood maps of those forward shock regions are shown that appear to have substructure in the maps, possibly caused by filaments with different \(a\) and \(b\) colocated within the same region. The map with \(PA = 250^\circ\) is shown as a counter example, displaying a rather smooth likelihood map.

**Appendix B**

**Robustness of the Best-fit Expansion and Deceleration**

For our best-fit solutions, presented in the main text, we made a correction for image misalignments (pointing errors) as explained in Section 2.3, with the values for \(\Delta x\), \(\Delta y\) listed in Table 2. Moreover, we fitted for the acceleration/deceleration (parameter \(b\)) for the forward shock. In Figure 11 we show the effects of assuming no pointing misalignments (for all epoch \(\Delta x = 0\), \(\Delta y = 0\)) and for not fitting for \(b\), but setting it to \(b = 0\). For the expansion parameters there are clear differences in measured expansion parameters, but the general trends are not affected, indicating that the results are robust. For the forward shock differences in measured \(a\) are below 0.04\% yr\(^{-1}\), and for the reverse shock region the deviation on \(a\) are even below 0.02\% yr\(^{-1}\).

The difference in total \(L\) for all 18 forward shock regions between the best-fit value and one for which \(\Delta x = 0\), \(\Delta y = 0\) is \(\Delta L = 3803\), for 34 extra degrees of freedom (corresponding to \(\Delta x, \Delta y\) for 17 epochs). The difference in log-likelihood if one does not fit for \(b\), but set the values for all sectors to \(b = 0\), is \(\Delta L = 1156\), for 18 degrees of freedom.

Clearly, solving for both \(\Delta x\), \(\Delta y\), and \(b\) leads to a much better fit, and the preferred values for the expansion are those listed in Tables 3 and 4. For comparison we do list the forward shock expansion rates for \(b = 0\) in Table 6.
Figure 11. Left panel: the expansion rate measurements of the forward and reverse shock regions employing different settings. The (black forward shock) and red (reverse shock) lines correspond to the measurements discussed in the main text, and are shown in Figure 2. The cyan line shows the best-fit expansion rates if no correction to the individual pointings is applied. The blue line shows the solution if no there is no deceleration measurement performed, i.e., setting \( b = 0 \). Right panel: the measured deceleration parameter for the forward shock region shown for the optimal fit (black line), a solution with \( \Delta x = 0 \), \( \Delta y = 0 \) for all epochs, and for two subsets of the observations, namely, the odd and even numbered observations.

### Table 6

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<th>PA (°)</th>
<th>( f ) (% yr(^{-1}))</th>
<th>( \tau_{\text{exp}} ) (yr)</th>
<th>( m )</th>
<th>( v_{\text{obs}} ) (km s(^{-1}))</th>
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Mean: 0.219 ± 0.030

0.728 ± 0.099

5817 ± 787

### ORCID iDs

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### References
