On the functioning of markets with frictions

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This thesis proposes strategic complementarities that arise in markets with search frictions as explanation for market volatility. In markets with search frictions, externalities are the norm and not the exception. Taking any resulting positive feedback loops into account is crucial for understanding recent developments in labor, goods and housing markets. Search frictions can also explain the co-existence of different organizational forms in markets. This thesis shows that there is a role for institutions to improve the functioning of such markets, and that these can simultaneously increase equity and efficiency.

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On the Functioning of Markets with Frictions   Florian Jos Ton Sniekers

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On the Functioning of Markets with Frictions

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University of Amsterdam and Vrije Universiteit Amsterdam

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ON THE FUNCTIONING OF MARKETS WITH FRICTIONS

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Contents

CHAPTER 1

Introduction

Economic activity in labor, goods and housing markets varies substantially over time. The number of unemployed workers, unfilled vacancies, unsold inventories, or houses for sale can easily double in the course of a few years. Similarly, at one moment finding a job or selling a house can take a couple of days, while at another moment either can take more than two years. Search frictions provide an explanation for the coexistence of unemployed workers and unfilled vacancies. Because search frictions result in trading delays, they can also help us understand time-on-market and how this can vary over time. Finally, search frictions make the occurrence of trade risky and dependent on the actions of others. The functioning of markets therefore relies on the strategies of all market participants, and the institutions that govern their interactions.

Indeed, markets with search frictions are generally characterized by externalities. When the optimal strategy of one agent increases in the strategies of others, strategic complementarities arise (Cooper and John, 1988). In such situations, agents have incentives to coordinate their actions and to behave in similar ways. In markets with search frictions, shifts in the coordination of the actions of economic agents can thus provide an explanation for the fluctuations in economic activity. This thesis studies strategic complementarities in labor, goods, and housing markets.

The seminal paper of Diamond (1982) is the first to show that search frictions can result in a coordination problem. In a model of a goods market, he shows that there exist different levels of economic activity that agents can coordinate on, if the probability that agents meet a trading partner is increasing in the number of potential trading partners. Intuitively, when agents expect few potential trading partners to be around, they only pursue the few most profitable projects, and consequently only few trading partners will actually be around. Conversely, if many trading partners are expected to be available, many projects are worth to be carried out, and as a result there will actually be many trading partners. Studying the dynamics of this model, Diamond and Fudenberg (1989) show that it can generate self-fulfilling fluctuations in economic activity. If there are multiple markets with search frictions, the expected benefits of trade in one market can depend on the presence and behavior of trading partners in another market. Howitt and McAfee (1992) and Kaplan and Menzio (2016) show in related models with interactions between frictional labor and goods markets how exogenous shifts in expectations about the level of economic activity can be self-fulfilling and result in more irregular fluctuations.[1]

In Chapter 2 of this thesis, I apply a similar demand externality to explain the cyclical behavior of unemployment and vacancies. As a result of this externality, revenue per worker-firm match is increasing in aggregate employment.[2] The benefits of match formation in the labor market therefore depend on the actions of others in the labor market, because changes in aggregate employment affect the revenues that can be generated in the goods market. Expectations of higher future employment and thus higher revenue per match motivate current investments in matching, and are therefore self-fulfilling. For that reason, multiple perfect foresight equilibrium paths exist. Some of these paths never converge to a steady state, but to a stable limit cycle. Cycles exist because there is not only positive feedback, but also congestion. When employment is high and firms open many vacancies, it takes more time and resources to fill a single vacancy. These costs are no longer justified when firms foresee an end to the boom, and they cut back on recruiting. Employment starts to fall, together with revenue per match, until the labor market becomes so slack that hiring picks up again. The resulting limit cycle resembles the empirically observed counterclockwise

[1] Howitt and McAfee (1987) identify another reason why search frictions may result in multiple levels of economic activity. They point out that search frictions give rise to a bilateral monopoly across matched trading partners. As a result, any division of the surplus of a match is possible, and every division results in a different level of economic activity. Building on this insight, Farmer (2012) assigns a role to self-fulfilling expectations in selecting the level of economic activity. Kashiwagi (2014) applies this idea to the U.S. housing market. In this thesis, the division of the surplus is either determined by Nash bargaining, by a fixed price, or by competitive search. The results therefore do not rely on indeterminacy resulting from a bilateral monopoly.

[2] Alternatively, this relationship can result from increasing returns in production as in Mortensen (1999), or increasing returns in matching as in Diamond (1982).

cycles around the Beveridge curve - the negative relationship between unemployment and vacancies. I calibrate these 'Beveridge cycles' and show that both unemployment and vacancies are as persistent as the data, without losing any of the amplification of the standard search and matching model of Pissarides (1985). Persistence is the result of calibrating the cycle to the average duration of the business cycle, and does not rely on a (persistent) stochastic process. The fluctuations are generated by the interplay of the positive demand externality and the negative congestion externality, and this endogenous mechanism reduces the need for exogenous shocks in explaining fluctuations in unemployment and vacancies.

As is clear from the literature cited above, demand externalities or increasing returns to scale in matching are well-known sources of strategic complementarities. Chapter 3 of this thesis, which is joint work with Espen Moen and Plamen Nenov, shows that moving owner-occupiers in the housing market are impelled to coordinate their search behavior for another reason, not previously identified. Moving owner-occupiers must buy a new house and sell their current one. They can choose to buy first or to sell first, but search frictions result in costly delays for both buyers and sellers. Time-on-market depends on the ratio of buyers to sellers. When moving owner-occupiers would like to make the period in between the two transactions as short as possible, they want to conclude their second transaction as fast as possible after the first one, and therefore accept a longer time-on-market at their first transaction. If owners buy first, there will be more buyers than sellers in the market. Consequently, on average it will take long to find a suitable house, and buying first is optimal. Conversely, if owners sell first, the buyer-seller ratio is low. Houses are thus expected to be for sale for a long time, and selling first is optimal. As a result, there are two steady state equilibria: one in which everybody buys first, and another one in which everybody sells first.

The strategic complementarity above exists without an explicit role for prices. When prices are endogenous, resulting from Nash bargaining or competitive search, an additional strategic complementarity arises. Moving owner-occupiers who already bought first are relatively desperate to sell their old house, because of the costs of holding two houses. For that reason, they are willing to sell at a relatively low price. The possibility to exploit the impatience of such an owner of two houses increases other moving owner-occupiers' incentives to enter the market as a buyer. However, doing so implies that they buy first, also ending up with two houses. Similarly, any presence of impatient buyers motivates other moving owner-occupiers to sell first, and to become desperate buyers themselves. Indeed, when trading partners come in different kinds, the composition of the pool of potential trading partners matters for the expected benefits of searching for a partner.[3] We also show that if moving owner-occupiers expect housing prices to rise, their incentives to buy first are stronger. Buying first implies that a household has a long position on housing in between the two transactions. Any anticipated rise in house prices thus makes it less costly to buy first irrespective of the buyer-seller ratio in the market. Conversely, a moving owner-occupier that sells first has a short position and benefits when prices fall. When prices increase in market tightness, expectations can therefore destabilize the housing market and lead to sudden self-fulfilling fluctuations in prices and market tightness. Such fluctuations are quantitatively relevant, as they could explain almost a doubling of the time-to-sell and price decreases up to 30 percent as the result of a switch from a 'buy first' to a 'sell first' equilibrium. The co-movements of these variables are in line with data for Copenhagen, a metropolitan area for which we can reconstruct the fraction of households that buy first.

In the data for Copenhagen, however, the fraction of households that buy first never reaches zero or one hundred percent, and the transition from this fraction's minimum to its maximum takes many years. For that reason, in Chapter 4 of this thesis, I build on the analysis in the previous chapter but relax the perfect coordination of moving owner-occupiers by assuming they neither know market tightness nor the fraction of households that buy first. Although the number of sellers can simply be observed from the stock of houses for sale, the number of buyers does not unambiguously show itself in the market. As a result, the crucial variable that determines optimal behavior - the ratio of buyers to sellers - is imperfectly observed. Instead, owners learn the behavior of others in a process of random contagion *à la* Lux (1995). If contagion is strong enough, multiple steady state equilibria exist with fractions of moving owner-occupiers that buy first unequal to zero or one hundred percent. However, housing prices are low when moving owner-occupiers in majority sell first. Such episodes offer profit opportunities to speculators that can buy houses cheap, rent them out, and bring them on the market later. Therefore I extend the model to allow for boundedly rational speculators that

[3]A similar strategic complementarity was first identified by Burdett and Coles (1998). They show that the expectation of a favorable composition can make matched agents separate from their current partner, to search for a better match. When such separation decisions actually improve the composition of the pool of potential trading partners, separation is actually beneficial to matched agents. In this thesis, separation is exogenous.

buy houses when they are cheap. When moving owner-occupiers take the presence of such speculators into account, the fraction of households that buy first can fluctuate perpetually in a way that closely matches the empirically observed housing cycle of Copenhagen, and presumably other local housing markets. The fraction of owners that buy first moves slowly, and in tandem with the number of transactions and housing prices, and in the opposite direction of the time-on-market and the stock of houses for sale.

The interaction between frictional goods and labor markets in Chapter 2 is of reduced form. Chapter 5, which is joint work with Piotr Denderski, features these markets explicitly in order to highlight unemployment risk in the labor market and the risk of not selling in the goods market. We propose a theory of the self-employment rate and the role of firms in the functioning of markets, which is based on the trade-off between these risks and the opportunities for insurance against them. Because both risks increase in the number of individuals facing them, their choices for self- or payroll employment are strategic substitutes. Equilibrium is therefore unique, and can feature the co-existence of self- and payroll employment. However, equilibrium is inefficient under risk-aversion. One reason for this, identified by Acemoglu and Shimer (1999), is that firms offer market insurance by paying lower wages, increasing the job finding probability. In our model, such market insurance additionally distorts the career choice in favor of payroll employment. We present an additional source of inefficiency, thus far not identified in the literature, which is driven by the ability of the self-employed to self-insure by setting lower prices. Because search is directed, lower prices increase the probability of selling. Despite a partial pricing response by firms, self-insurance steals business away from firms, distorting the allocation of buyers in the goods market. Consequently, firms' probabilities and benefits of trade in the goods market are smaller, and they decrease their investments in match formation in the labor market. As a result, unemployment risk increases and the career choice is distorted in favor of self-employment. Combining self- and market insurance results in a self-employment rate that is generally either too high or too low. We show that unemployment benefits and benefits for the self-employed that fail to sell, paid for by taxes that differ between self- and payroll employed, can target each of the margins of inefficiency and balance the budget. In the presence of search frictions, equity and efficiency may therefore move in the same direction, and there is a role for institutions to improve the functioning of markets.

This thesis shows once more that externalities are the norm, and not the exception, in markets with search frictions. Taking any resulting inefficiencies and positive feedback loops into account is crucial for a deeper understanding of recent developments in labor, goods and housing markets.

CHAPTER 2

Persistence and volatility of Beveridge cycles

2.1 Introduction

The dynamic relation between vacancies and unemployment is characterized by counterclockwise cycles in the *unemployment, vacancy rate*-plane. After removing any long-term trends with an HP-filter, the cycles for the United States are presented in Panel 1a of Figure 1. It shows that the cycles mostly consist of movements parallel to an almost perfectly inverse relationship between vacancies and unemployment - the Beveridge curve - where downturns trace out a lower path than recoveries. The figure also shows that vacancies are about as volatile and persistent as unemployment.

These observations are confirmed in the summary statistics of the unemployment and vacancy rate. Table 1 reports standard deviations, autocorrelations, and cross-correlations of these and other variables: market tightness v/u, the job finding rate f, the destruction rate δ, and labor productivity y. The standard deviations of the unemployment and vacancy rate v are similar, their autocorrelations are about 0.95, and their correlation is almost -0.9. Note also that the standard deviation of productivity is almost ten times smaller than that of the unemployment and vacancy rate. Finally, the average (unfiltered) unemployment and quarterly job finding and destruction rates are 0.0587, 1.738, and 0.100 respectively.

In the canonical search and matching model of Pissarides (1985), unemployment is a state variable while vacancies respond to shocks immediately. As a result, the

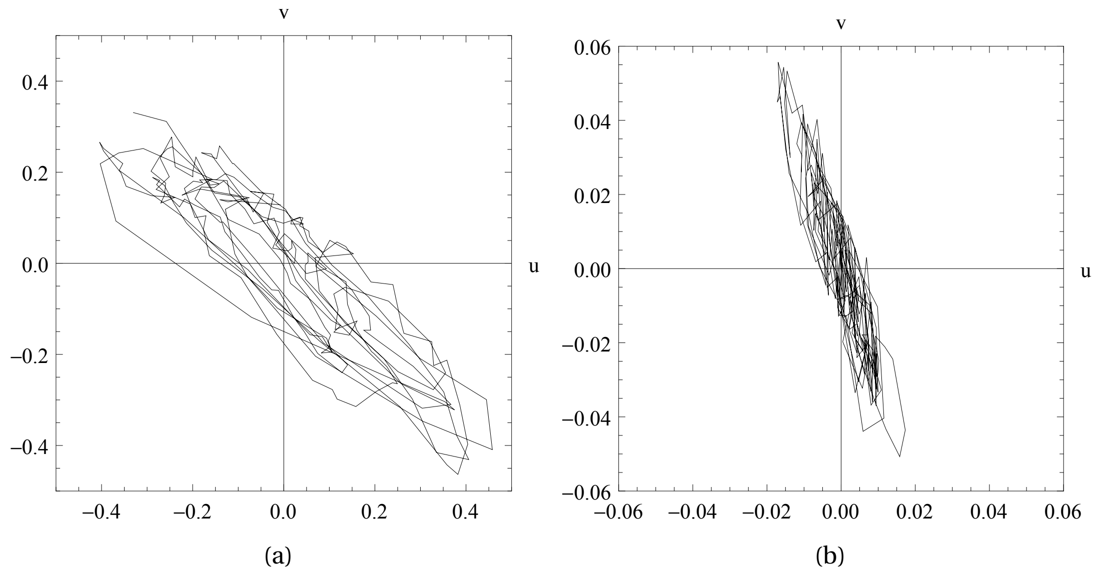

Figure 1: Cyclical component of the Beveridge curve for (a) the United States, 1951-2014, and (b) a simulation of Shimer (2005).

Notes: The seasonally adjusted unemployment rate u is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The seasonally adjusted vacancy rate v is obtained by dividing a measure of vacancies by the labor force from the CPS. Following Daly et al. (2012), the former is the Composite Help-Wanted Index constructed by Barnichon (2010) for the 1951-2000 period, rescaled to equal the JOLTS in December 2000, which is used from that month onwards. I thank Bart Hobijn for providing me with these data. The data are quarterly averages of a monthly series. The simulation is a representative realization with productivity shocks only. Both data and simulation are expressed in logs as deviations from an HP trend with smoothing parameter 10^5.

model captures the counterclockwise direction of the cycles along the Beveridge curve. However, the model fails to describe these cycles in at least two dimensions. First, as is well-known, the model lacks sufficient amplification to generate the observed volatility in unemployment and vacancies in response to realistic exogenous shocks in productivity and/or the destruction rate (Shimer, 2005). Second, the model lacks cyclical responses, as can be seen comparing the panels of Figure 1. Panel 1b plots a realization of a detrended Beveridge curve as simulated by Shimer (2005), in his calibration with productivity shocks only. The simulation not only lacks unemployment volatility (note the scaling differences across the panels), but it also mainly features near

		u	v	v/u	f	δ	y
Standard deviation		0.195	0.178	0.362	0.175	0.073	0.02
Quarterly autocorr.		0.946	0.946	0.948	0.926	0.735	0.894
	u	1	−0.888	−0.974	−0.962	0.64	−0.35
	v		1	0.969	0.887	−0.647	0.327
Correlation matrix	v/u			1	0.953	−0.662	0.349
	h				1	−0.522	0.333
	s					1	−0.504
	y						1

Table 1: Summary statistics, quarterly US data, 1951-2014.

Notes: Labor market tightness v/u is the ratio of the seasonally adjusted quarterly unemployment rate u and vacancy rate v, both described below Figure 1. Appendix 2.7.A describes the construction of the job finding rate f and destruction rate δ from the monthly seasonally adjusted employment, unemployment, and short-term unemployment rate as provided by the BLS from the CPS. Average labor productivity y is seasonally adjusted real average output per person in the non-farm business sector constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

vertical dynamics, in contrast to the data in Panel 1a. As a result, vacancies in the model are neither as persistent as in the data, nor as persistent as unemployment.[4]

The current paper explains the cyclical behavior of unemployment and vacancies with an endogenous cycle driven by demand externalities. Demand externalities result from spillovers across a monopolistic goods market and a labor market with search frictions, in which high aggregate unemployment feeds back to low demand for output and thus a low revenue per worker-firm match. Combined with the delay in matching and the congestion externality that are standard in search models of the labor market, such feedback can give rise to equilibrium paths in vacancies and unemployment that never converge to a steady state, but converge to a deterministic periodic closed path. I refer to these rational expectations limit cycles in vacancies and unemployment as Beveridge cycles. Expectations of high revenue per match are self-fulfilling because

[4]The lack of propagation in Shimer (2005) can also be understood from a univariate regression of labor market tightness (or the job finding rate) on productivity, which results in an R^2 of 1.00.

such expectations make firms open vacancies, resulting in higher employment and - via demand externalities - higher revenue per match in the future. However, congestion in a tight labor market may make hiring so costly that firms may reduce vacancies before reaching a steady state. In that case, the equilibrium path turns and job destruction takes over from job creation, unemployment increases and revenue per match falls. Hiring picks up again when the labor market becomes sufficiently slack.

Rather than modeling the goods market explicitly, I assume that revenue per match is a function of aggregate employment. I add this reduced-form relationship to a Pissarides (2000) search and matching model with variable search intensity and analyze the global dynamics of the model. Theoretically, the occurrence of a Bogdanov-Takens bifurcation shows that Beveridge cycles exist and are stable for a range of parameter values. This implies that a small perturbation to a parameter, or introducing a small amount of risk aversion, would not eliminate cycles. Quantitatively, I investigate whether these cycles can match the empirically observed standard deviation and autocorrelation of unemployment and vacancies. Calibrating the cycle to the average duration of the business cycle, the simulated autocorrelations of both unemployment and vacancies closely match their observed autocorrelations. Persistence is an endogenous feature of the Beveridge cycle, and results from the neighborhood of a steady state with saddle-path stability. In particular, it is not a result of a persistent stochastic process for productivity, and it does not compromise volatility. Indeed, the model is subject to the same (lack of) amplification mechanisms as the standard model, but generates its volatility in revenue per match endogenously, reducing the need for exogenous shocks. Variable search intensity and a high positive value of leisure are important for the empirical performance of the model. The flow value of unemployment does not only contribute to amplification, but also brings the size of the required demand externalities in line with the literature. The existence of limit cycles does not rely on variable search intensity or a positive value of leisure.

My model is formally equivalent to the model of Mortensen (1999), who shows the existence of limit cycles in employment and the surplus of a match. He gives the intuition of the counterclockwise cycles along the Beveridge curve, but does not perform a calibration. I present the model in the standard search and matching variables - unemployment and labor market tightness - and add a positive value of leisure, which has a non-trivial impact on the existence and characteristics of equilibria in this nonlinear model. In Mortensen (1999), following Mortensen (1989), the feedback from aggregate employment on revenue per match results from increasing returns in production. Alternatively, this feedback can result from increasing returns in the matching function for the goods market as in Diamond (1982) and Howitt and McAfee (1987). Diamond and Fudenberg (1989) and Boldrin et al. (1993) show that such increasing returns can result in endogenous cycles, but their models do not contain vacancies. In a reduced form, however, the three interpretations of the relationship between aggregate employment and revenue per match are equivalent, and capture the three examples of Cooper and John (1988) that can result in strategic complementarities: production technology, matching technology, and agents' demands.

With respect to the latter, Heller (1986) and Roberts (1987) show that demand externalities can generate multiple equilibria. Exploiting equilibrium multiplicity, Howitt and McAfee (1992) propose belief shocks to switch from one equilibrium path to another. Drazen (1988) shows that demand externalities can generate endogenous cycles in firm match value and unemployment. However, he assumes an equal number of vacancies and unemployed and thus rules out cycles in the two. Recently, novel interactions between frictional labor and product markets have been proposed to generate endogenous cycles or multiplicity. Beaudry et al. (2015) obtain demand externalities from unemployment risk and precautionary savings, and embed the resulting limit cycle in a model with exogenous disturbances. They estimate this model and show that the endogenous cycle can explain U.S. business cycle fluctuations in output and employment well, provided that the productivity shocks make the cycle sufficiently irregular. They do not investigate the dynamics of vacancies. Kaplan and Menzio (2016) argue that the unemployed do not only spend less, but are also more likely to pay low prices for the same goods. They use the combination of these effects - referred to as shopping externalities - to explain the outward shift of the Beveridge curve since the Great Recession. In particular, this shift results from the transitional dynamics after a belief shock switches coordination to an equilibrium with higher unemployment and smaller markups.

The literature on the outward shift of the Beveridge curve is growing. In Ravn and Sterk (2012), lower aggregate demand results from precautionary savings in the wake of higher unemployment, and further reduces job finding prospects. Heterogeneity in search efficiency introduces negative duration dependence and an outward shift in the Beveridge curve. In Eeckhout and Lindenlaub (2015), sorting makes on-the-job search improve the composition of job searchers, boosting labor demand and justifying on-the-job search. The shift in the Beveridge curve results from a switch from an equilibrium without to an equilibrium with on-the-job search. Consistent with my model and suggested by Panel 1a of Figure 1, Diamond and Sahin (2015) show that outward shifts in the Beveridge curve after a trough are common in U.S. historical data, and not likely to be persistent.

My paper is also related to the large literature that proposes mechanisms that increase the amplification of the standard search and matching model. Mortensen and Nagypal (2007) provide an overview. Recently, Gomme and Lkhagvasuren (2015) have shown that procyclical search intensity increases amplification by making the net flow value of leisure countercyclical, and as the result of a strategic complementarity in search and recruiting activity. My model features procyclical search intensity, but, given the elasticity of the matching function, constrains its effect to the estimated elasticity of the job finding rate with respect to labor market tightness. A few other papers address the persistence of the standard model. Fujita and Ramey (2007) show that cyclical responses can be generated by the introduction of sunk costs of vacancy creation. They show, however, that spreading out the impact of a shock in such a way results in a counterfactually high cross correlation between labor market variables and productivity across time. Coles and Kelishomi (2011) achieve persistence of vacancies by replacing the free entry condition by search for business opportunities and time to build an identified opportunity into a vacancy. Dromel et al. (2010) and Petrosky-Nadeau (2014) address propagation by credit frictions, and therefore follow Fujita and Ramey in focusing on the costs of vacancy creation. In contrast, persistence in my model results from a self-reinforcing effect on the benefits of vacancy creation, while maintaining free entry of vacancies.

The calibration of Chéron and Decreuse (2016) is most closely related to mine. They show that phantoms - traders that are still present in the market but that are no longer available for trade - can result in a labor market matching function with increasing returns to scale in the short run and constant returns in the long run, which results in excess volatility and self-fulfilling fluctuations. They also calibrate a deterministic limit cycle, and can explain the persistence of unemployment and labor market tightness if wages are rigid, if phantoms mostly consist of vacancies, and if they 'haunt' the market for a long time. Note that the demand externalities in my model are similar to decreasing, not increasing, returns to scale in the labor market. Ellison et al. (2014) show that such locally decreasing returns to scale in labor market matching can increase amplification and persistence. However, with respect to their quantitative results, they focus on saddle-path dynamics. Sterk (2016) shows that skill losses upon unemployment can result in multiple steady states, and slower dynamics in their neighborhoods result in more persistence. Technically, the same mechanism delivers persistence for the Beveridge cycle, but the model of Sterk (2016) features a unique equilibrium. Only Chéron and Decreuse (2016) calibrate a deterministic model to quantify its performance in explaining labor market dynamics, while Beaudry et al. (2015) show that fundamental shocks on top of a deterministic cycle can reproduce the spectrum of the data.

Finally, my paper is related to applications of the Bogdanov-Takens bifurcation in economics. Using this bifurcation, Benhabib et al. (2001) show that an active monetary policy rule in the presence of a zero lower bound results in indeterminacy and can direct an economy into a liquidity trap. In models with search frictions, phase diagrams in Howitt and McAfee (1988), Coles and Wright (1998), and Kaplan and Menzio (2016) also suggest the occurrence of a Bogdanov-Takens bifurcation, although these authors do not explore this possibility.

2.2 Model

This section presents a Pissarides (2000) equilibrium search and matching model with variable search intensity and feedback from employment to revenue per match. Time is continuous and lasts forever, and there is no aggregate uncertainty.

2.2.1 Preferences, markets, and choices

The economy consists of a measure one of infinitely lived workers and an endogenous measure of firms owned by workers. All firms have access to the same technology. They maximize expected profits and discount future profits at rate r. Workers are endowed with an indivisible unit of homogeneous labor every period. They are risk-neutral too and maximize lifetime utility, discounted at the same rate r. At time t, an endogenous measure n_t of workers is employed, and the remainder $u_t = 1 - n_t$ is unemployed. Unemployed workers receive a flow utility $z > 0$ that is independent of labor market conditions. It captures the combination of the unemployment benefit, the stigma of unemployment, the value of home production and the pure value of leisure that come with unemployment.

Matches of a single worker and firm produce consumption goods that are sold for a one-period IOU in a goods market. A firm's receipts are split in a wage w_t and profits, are immediately transferred to its employee and owners respectively, and must be spent in the same period on consumption goods produced by other worker-firm matches. Rather than modeling the goods market explicitly, I propose a reduced-form relationship between the flow revenue y_t of a worker-firm match and *aggregate* employment. Assuming a constant elasticity, $y_t = \phi(n_t) = \phi(1-u_t) = \phi_0(1-u_t)^{\alpha}$. I normalize ϕ_0 to one and assume that $\alpha > 0$, so that revenue is increasing in aggregate employment. As explained in the introduction, several interpretations can be given to the effect of aggregate employment on revenue per match, but throughout this paper I refer to this effect as a demand externality. In this interpretation the goods market is characterized by imperfect competition. Revenue per match increases in aggregate employment because when employment is high, customers spend more, possibly also in any given trade.

The labor market is characterized by search frictions, so that the formation of worker-firm matches takes both time and resources. The total measure of matches m_t formed in a certain period is given by the common Cobb-Douglas matching function $m(v_t, s_t u_t) = m_0 v_t^{\eta}(s_t u_t)^{1-\eta}$, with $0 < \eta < 1$. Inputs of this function are aggregate recruiting activity represented by the vacancy rate v_t (scaled by the labor force), and aggregate search effort given by the unemployment rate u_t, times the search intensity s_t of the unemployed. Normalize m_0 to one and define labor market tightness θ_t as the ratio of the inputs of the matching function: $\theta_t \equiv v_t/(s_t u_t)$.[5] An individual unemployed worker finds a job at the Poisson rate $v_t^{\eta}(s_t u_t)^{1-\eta}/u_t = s_t\theta_t^{\eta}$. Similarly, individual vacancies are filled at a rate $v_t^{\eta}(s_t u_t)^{1-\eta}/v_t = \theta_t^{\eta}/\theta_t$. Matches are destroyed at an exogenous rate $\delta > 0$. The surplus of a match is divided by Nash bargaining.

There are two choices to be made in this economy. First, at any moment in time t, unemployed workers choose the intensity s_t at which they search for jobs. Search intensity comes at increasing and strictly convex costs $c(s_t) = c_0 s_t^{\gamma}$, with $\gamma > 1$. Simply scaling search intensity, I normalize c_0 to one.[6] The discouraged worker effect - unemployed workers stop looking for a job if the prospect of finding one is very bad - is modeled by s_t, because the labor force is fixed while search intensity will be seen to

[5]In this paper I only study the cases where $s_t, u_t > 0$, so that θ is always defined.

[6]The model allows for normalization of m_0 and c_0, because the level of the ratio of vacancies to unemployment v_t/u_t and the value of search intensity s_t are intrinsically meaningless.

increase with labor market tightness. Combined with the gross value of leisure z, the net flow utility of the unemployed is $z - s_t^\gamma$.[7] Second, at any moment in time, potential firms freely decide whether to enter the labor market by opening a single vacancy at a flow cost $k > 0$. Simultaneously any existing firm decides freely whether to withdraw its (unfilled) vacancy from the market.

2.2.2 Asset values of workers and firms

Free entry of vacancies implies that in equilibrium the value of opening an additional vacancy cannot be positive. As described above, an individual vacancy costs k per period and is filled at a rate θ_t^η / θ_t. Defining J_t as the value of a worker to a firm, it must hold that

$$k \geq \frac{\theta_t^\eta}{\theta_t} J_t, \tag{2.1}$$

and $\theta_t \geq 0$ with complementary slackness. The value of opening an additional vacancy is therefore only negative if the stock of vacancies is zero. In the remainder of this paper I focus on equilibria with economic activity, i.e. with a positive level of labor market tightness.

An unemployed worker enjoys a flow utility $z - s_t^\gamma$ and finds a job at the Poisson rate $s_t \theta_t^\eta$. The value U_t of unemployment to an individual worker is therefore described by

$$rU_t = z - s_t^\gamma + s_t \theta_t^\eta (W_t - U_t) + \dot{U}_t, \tag{2.2}$$

where W_t is the value of a job to a worker.

The flows to a firm are revenues $(1 - u_t)^\alpha$ minus wage w_t, which is the periodical income to the worker. The asset price equations of a job J_t and W_t, to a firm and a worker respectively, are then

$$rJ_t = (1 - u_t)^\alpha - w_t - \delta J_t + \dot{J}_t, \tag{2.3}$$

$$rW_t = w_t - \delta (W_t - U_t) + \dot{W}_t. \tag{2.4}$$

The wage is determined by Nash bargaining over the surplus of a match $p_t \equiv J_t + W_t - U_t$, with worker's bargaining power equal to $\beta \in (0,1)$ and separation $(U_t, 0)$ as threat point. The firm's rent is therefore equal to its share of the surplus:

$$J_t = (1 - \beta)(J_t + W_t - U_t), \quad \text{and} \quad \dot{J}_t = (1 - \beta)\left(\dot{J}_t + \dot{W}_t - \dot{V}_t\right), \tag{2.5}$$

[7] Note that z does not affect $c(s_t)$, which seems a reasonable assumption for risk-neutral agents. This assumption highlights the effect of the value of leisure on vacancy creation rather than labor supply.

where the latter follows because wages are continuously renegotiated.[8]

As in (2.2), an unemployed worker's net expected income from search activity g_t is $s_t\theta_t^{\eta}(W_t - U_t) - s_t^{\gamma}$. He optimally chooses his search intensity s_t, taking into account that the surplus of a match will be divided by Nash bargaining. Using (2.1) with equality, net expected income from search activity can be expressed in terms of labor market tightness:

$$g(\theta_t) = \max_{s_t}\left[\frac{\beta}{1-\beta}s_t k\theta_t - s_t^{\gamma}\right]. \tag{2.6}$$

Finally, add (2.3) to (2.4), subtract (2.2), and substitute $g(\theta_t)$, to obtain the law of motion for match surplus

$$\dot{p}_t = (r+\delta)p_t - (1-u_t)^{\alpha} + z - g(\theta_t). \tag{2.7}$$

2.2.3 Equilibrium behavior and dynamics

Unemployed workers choose the intensity s_t at which they search for jobs. Balancing the benefits of search with the costs, from (2.6) optimal search intensity $s^*(\theta_t)$ is given by

$$s^*(\theta_t) = \left(\frac{\beta}{1-\beta}\frac{k}{\gamma}\theta_t\right)^{\frac{1}{\gamma-1}}. \tag{2.8}$$

As referred to above, optimal intensity increases in tightness, and is positive for $\theta > 0$. Together with the stocks of vacancies and unemployed workers, it determines the measure of matches formed at any instant. Combined with the destruction of existing jobs, unemployment evolves according to a differential equation in unemployment and tightness:

$$\dot{u}_t = \delta(1-u_t) - s^*(\theta_t)\theta_t^{\eta}u_t. \tag{2.9}$$

By opening or closing vacancies, firms translate changes in expectations about the value of a worker to the firm in changes in labor market tightness with perfect foresight. Indeed, for any positive level of tightness, (2.1) implies

$$J_t = \frac{k\theta_t}{\theta_t^{\eta}}. \tag{2.10}$$

[8] Pissarides (2009) shows that the crucial assumption for job creation is that wages of *new* matches are given by this rule. How rents in ongoing jobs are split is inconsequential for job creation, and thus for the dynamics in this model. Coles and Wright (1998) have shown that outside a steady state, Nash bargaining no longer necessarily corresponds to the outcome of strategic bargaining with appropriately defined threat points as the time between offers goes to zero, as it would in a stationary environment (Binmore et al., 1986). Consequently, the division rule in this paper should not be interpreted as the outcome of strategic bargaining, but as an axiomatic solution.

Differentiating with respect to time, the value of a worker to a firm and labor market tightness move in tandem:

$$\dot{J}_t = \frac{k\dot{\theta}_t(1-\eta)}{\theta_t^{\eta}}. \tag{2.11}$$

Substituting (2.5) and (2.10) into (2.7), the value of a worker to a firm evolves according to

$$\dot{J}_t = (r+\delta)\frac{k\theta_t}{\theta_t^{\eta}} - (1-\beta)\left[(1-u_t)^{\alpha} - z + g(\theta_t)\right]. \tag{2.12}$$

Finally, combine (2.11) and (2.12) into a second differential equation in unemployment and tightness:

$$\dot{\theta}_t = (r+\delta)\frac{\theta_t}{1-\eta} + (1-\beta)\frac{\theta_t^{\eta}}{k(1-\eta)}\left[g(\theta_t) + z - (1-u_t)^{\alpha}\right]. \tag{2.13}$$

Equilibrium can now be defined as in:

Definition 2.1. A perfect foresight equilibrium with economic activity is a pair of functions $\{u_t, \theta_t\}$ such that:

1. For all $t \geq 0$, unemployment $u_t \in [0,1]$ evolves according to (2.9);
2. For all $t \geq 0$, labor market tightness $\theta_t > 0$ evolves according to (2.13);
3. $\lim_{t\to\infty}\theta_t$ is finite and u_0 is given.

In the presence of search frictions, an equilibrium is not necessarily efficient. Positive externalities of search and recruiting activity occur for trading partners for whom matching is *more* likely because of the availability of more (effective) trading partners. Negative externalities of search and recruiting activity occur for searchers of the same type, for whom matching is *less* likely because of increased congestion for trading partners. These externalities only cancel once the net private returns from search and recruiting activity equal the net social returns. Proposition 2.2 states this happens if the familiar Hosios (1990) condition is satisfied. Note that the Hosios condition only concerns the search externalities, not the demand externalities. The proof in Appendix 2.7.B extends the efficiency results of the Pissarides (2000) model with variable search intensity to out-of-steady state dynamics.

Proposition 2.2. *Suppose revenue per match is independent of unemployment and constant, i.e. $y_t = y$. Then search intensity s_t and labor market tightness θ_t are efficient if and only if the bargaining power of firms $1-\beta$ is equal to the elasticity of the matching function η.*

Thus, $1-\beta=\eta$ is the efficient sharing rule for a social planner that takes the demand externalities as given, just as firms and workers do, but does internalize search externalities. In fact, the demand externalities result in multiple equilibria for the same fundamentals, and Mortensen (1999) shows that these equilibria can be Pareto-ranked. The next section presents steady-state equilibria, whereas the section after that presents the non-stationary equilibrium of the Beveridge cycle that encloses one of these steady states.

2.3 Steady state equilibria

In this section, I present the steady-state equilibria of the model economy, and study their stability. I show that if there exists a steady state with economic activity for $z>0$, then there are generically multiple of them. Knowledge of the stability of these steady states helps to understand the Beveridge cycle that ultimately explains the data.

2.3.1 Nullclines

A steady state is a pair of functions (u_t, θ_t) in which both unemployment and labor market tightness are constant. Unemployment is constant on the $\dot{u}_t = 0$-locus or unemployment nullcline, where (2.9) is equal to zero:

$$u_t = \frac{\delta}{\delta + s^*(\theta_t)\,\theta_t^{\eta}}, \tag{2.14}$$

with $s^*(\theta_t)$ as given in (2.8). One can see that in steady state all workers are unemployed if tightness is zero, and that steady-state unemployment decreases in tightness via the job finding rate.

Tightness is constant on the $\dot{\theta}_t = 0$-locus or tightness nullcline. Equalizing (2.13) to zero, the tightness nullcline for equilibria with economic activity can also be expressed as unemployment in terms of tightness:

$$u_t = 1 - \left[(r+\delta)\frac{k\theta_t}{(1-\beta)\theta_t^{\eta}} + g(\theta_t) + z\right]^{\frac{1}{\alpha}}. \tag{2.15}$$

Again unemployment decreases in tightness: at a lower level of unemployment, revenue will be higher, and therefore in equilibrium firms open more vacancies. From (2.15) with $\theta_t = 0$, we can see that the nullcline crosses the $\theta = 0$-axis at

$$u_{\theta=0} = 1 - z^{1/\alpha}. \tag{2.16}$$

At this level of unemployment, revenue per worker-firm match equals the gross value of leisure, and firms do not want to open any vacancies. For smaller values of leisure, wages are lower. As a result, there is more surplus of a match, and firms open more vacancies. Figure 2 shows representative unemployment and tightness nullclines, and indicates the location of $u_{\theta=0}$ for some $z > 0$.[9]

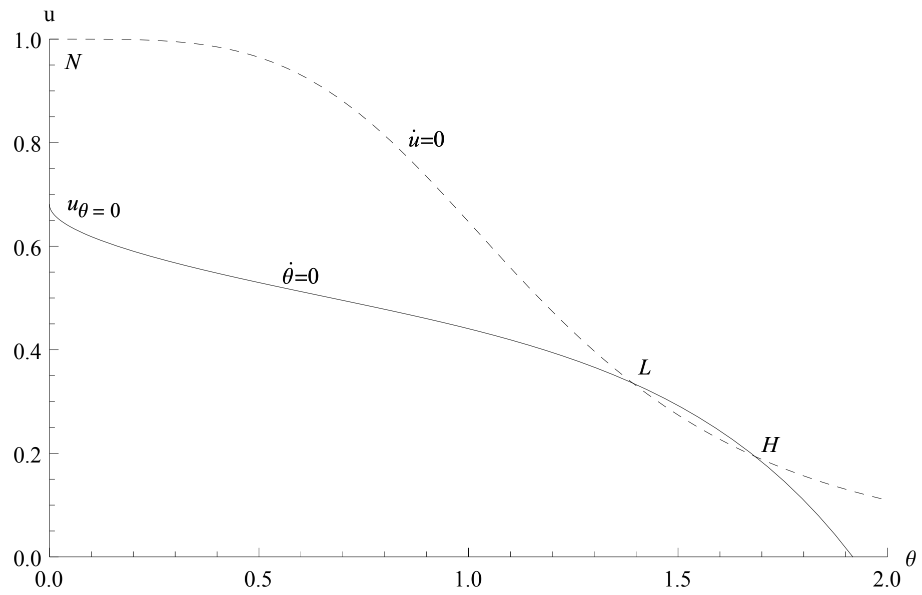

Figure 2: Nullclines of unemployment (dashed) and labor market tightness, resulting in the three steady states N, L, and H.

Notes: Parameters are $k = 0.55$, $\delta = 0.1$, $r = 0.012$, $\eta = 0.5$, $\gamma = 1.29$, $\alpha = 0.3$, $\beta = 0.5$, and $z = 0.71$.

2.3.2 Existence of steady states

A steady state with economic activity exists where the two downward-sloping nullclines intersect. Figure 2 shows two steady states with activity: a steady state L with a relatively low but positive employment and labor market tightness, and a steady state H with high employment and tightness. Due to the complementary slackness condition, there is also always a steady state N without economic activity at $\theta = 0$ and $u = 1$. However, it is

[9] Since unemployment can be expressed more easily as a function of tightness than the reverse, I plot unemployment on the vertical axis and tightness on the horizontal. Once I later plot vacancies to unemployment, unemployment will be on the horizontal axis as is common in the literature. With these conventions, counterclockwise cycles in unemployment and vacancies thus correspond to clockwise cycles in unemployment and tightness.

not generally possible to give explicit solutions for steady states with economic activity, and they may not always exist. In particular, for $z > 0$ there are only steady states with economic activity if recruiting costs k are small enough. If not, the unemployment nullcline will lie entirely above the tightness nullcline, and the steady state without activity is the only steady state.[10]

As soon as recruiting costs become small enough, the two nullclines touch and a *saddle-node bifurcation* occurs. In a bifurcation the qualitative properties of a dynamical system change as the result of a change in one or more parameters, k in this case.[11] Bifurcations are of interest because regions in the parameter space delimited by bifurcations are therefore structurally stable, i.e. the qualitative dynamics are invariant to small perturbations of the parameters. The qualitative change as the result of a saddle-node bifurcation is the emergence of two additional steady states: a saddlepoint and an *antisaddle* (node or focus).[12] Depending on the shape of the nullclines, saddle-node bifurcations can happen multiple times. As a result, if any steady state with economic activity exists for $z > 0$, there is generically an even number of them. Focusing on this empirically relevant case, Proposition 2.3 states a sufficient condition for the existence of exactly two steady states with economic activity. The proof is in Appendix 2.7.B.

Proposition 2.3. *Suppose k is small enough to guarantee the existence of a steady state with economic activity. Then if $\alpha \leq 1$ and $z > \left(1 - \frac{\delta}{\delta+(r+\delta)(1-\eta)\eta(\gamma-1)}\right)^{\alpha}$, exactly two of them exist.*

The sufficient condition in Proposition 2.3 ensures that the unemployment nullcline is convex and the tightness nullcline is concave on the relevant sections, but my calibrations as presented in Subsection 2.5.1 shows that this condition is not necessary. In the next subsection I characterize the stability of the set of steady-state equilibria with activity.

[10]If $z = 0$, at least one steady state with activity exists if $\eta + (\gamma - 1)^{-1} < 1$, independent of other parameters.

[11]Alternatively, for a given k and α, the gross value of leisure z must be small enough. See e.g. Kuznetsov (2004) for more on bifurcation theory.

[12]An antisaddle is a focus if its eigenvalues are complex, thus if $\mathrm{tr}^2 < 4\det$, and a node otherwise.

2.3.3 Stability of steady states

The local stability of the steady states with economic activity can be studied by linearizing the dynamical system given by (2.9) and (2.13). In a steady state with economic activity, the Jacobian matrix is

$$\left.\begin{pmatrix} \frac{\partial\dot{\theta}_t}{\partial\theta_t} & \frac{\partial\dot{\theta}_t}{\partial u_t} \\ \frac{\partial\dot{u}_t}{\partial\theta_t} & \frac{\partial\dot{u}_t}{\partial u_t} \end{pmatrix}\right|_{\dot{\theta},\dot{u}=0} = \begin{pmatrix} r+\delta+s^*(\theta_t)\,\theta_t^{\eta}\frac{\beta}{(1-\eta)} & (1-\beta)\frac{\theta_t^{\eta}}{k(1-\eta)}\alpha\,(1-u_t)^{\alpha-1} \\ -s'(\theta_t)\,\theta_t^{\eta}u_t - s^*(\theta_t)\,\eta\theta_t^{\eta-1}u_t & -\delta-s^*(\theta_t)\,\theta_t^{\eta} \end{pmatrix}, \tag{2.17}$$

with

$$s'(\theta_t) = \frac{1}{\theta_t(\gamma-1)}\left(\frac{\beta}{1-\beta}\frac{k}{\gamma}\theta_t\right)^{\frac{1}{\gamma-1}}.$$

We see that $\partial\dot{\theta}_t/\partial\theta_t > 0$, $\partial\dot{\theta}_t/\partial u_t > 0$, $\partial\dot{u}_t/\partial\theta_t < 0$, and $\partial\dot{u}_t/\partial u_t < 0$ in steady state. Depending on whether the product of the diagonal elements $(\partial\dot{\theta}_t/\partial\theta_t)(\partial\dot{u}_t/\partial u_t)$ or the product of the cross-diagonal elements $(\partial\dot{\theta}_t/\partial u_t)(\partial\dot{u}_t/\partial\theta_t)$ is more negative, the determinant is negative or positive respectively. If and only if the determinant of the Jacobian matrix at a steady state is negative, it has eigenvalues of different signs and thus saddle path dynamics (see e.g. Kuznetsov (2004, p. 49)). If and only if the determinant is positive, the eigenvalues are of equal sign and the steady state is an antisaddle. Since only an antisaddle can feature surrounding oscillatory dynamics, Proposition 2.4 states a necessary condition for endogenous cycles.

Proposition 2.4. *A steady state with economic activity is an antisaddle if and only if the unemployment nullcline crosses the tightness nullcline from above.*

Proof. The slopes of the nullclines in any of the steady states with economic activity are given by

$$\frac{du_t}{d\theta_t}|_{\dot{u}_t=0} = -\frac{\partial\dot{u}_t}{\partial\theta_t}/\frac{\partial\dot{u}_t}{\partial u_t} \quad\text{and}\quad \frac{du_t}{d\theta_t}|_{\dot{\theta}_t=0} = -\frac{\partial\dot{\theta}_t}{\partial\theta_t}/\frac{\partial\dot{\theta}_t}{\partial u_t}.$$

Now we see that

$$\frac{\partial\dot{\theta}_t}{\partial\theta_t}\frac{\partial\dot{u}_t}{\partial u_t} > \frac{\partial\dot{\theta}_t}{\partial u_t}\frac{\partial\dot{u}_t}{\partial\theta_t}$$

if and only if the former is steeper than the latter. Since both terms are negative by the sign restrictions, this corresponds to the unemployment nullcline crossing the tightness nullcline from above. □

As can be seen in Figure 2, steady state L is the intersection of the unemployment nullcline crossing the tightness nullcline from above. As a result, only this steady state can feature endogenous oscillatory dynamics enclosing the steady state. Steady state H is a saddlepoint, so that there is an equilibrium saddlepath leading to it.

The trace of the Jacobian matrix in the antisaddle indicates whether the dynamics locally converge to it (the trace is negative and the antisaddle is a sink), diverge from it (positive trace; source), or neither (zero trace; center). The trace is given by the sum of the diagonal elements of the Jacobian matrix (2.17), and is thus

$$\frac{\partial \dot{\theta}_t}{\partial \theta_t} + \frac{\partial \dot{u}_t}{\partial u_t} = r + s^* (\theta_t) \theta_t^\eta \left(\frac{\beta}{1-\eta} - 1 \right). \tag{2.18}$$

We see that the trace is exactly $r > 0$ if $\beta = 1 - \eta$, thus if the Hosios condition is satisfied. The trace is also always positive for $\beta > 1 - \eta$. As a result, the antisaddle is unstable in these cases. However, as we will again see in the next section, the trace can take either sign for $\beta < 1 - \eta$.

2.4 Beveridge cycle equilibria

In this section I show that the steady states and equilibrium paths leading to them are not the only equilibria. In particular, I show that there exists a stable *limit cycle* for a range of values for workers' bargaining power. A limit cycle is a periodic orbit enclosing an antisaddle such that at least one other path converges to it as time approaches positive infinity (the cycle is stable) or negative infinity (the cycle is unstable). Since the limit cycle in this paper results in enduring endogenous fluctuations in vacancies and unemployment, I refer to it as a Beveridge cycle.

The existence of a stable Beveridge cycle follows from the occurrence of a *Bogdanov-Takens bifurcation*. This bifurcation generically occurs in a system of two or more parameters, in which a Hopf bifurcation, a saddle-loop bifurcation, and a saddle-node bifurcation occur in a single point in the parameter space. This Bogdanov-Takens point is important because it is an organizing center for the dynamics: it characterizes the qualitative dynamics of the system in the neighborhood of this point. The next subsection presents examples of the relevant kinds of qualitative dynamics, as demarcated by a Hopf and a saddle-loop bifurcation respectively. These examples also show the existence of multiple equilibria for the same initial unemployment rate. Next, I show the occurrence of the bifurcations more formally. Finally, I describe the economics of the Beveridge cycle.

2.4.1 Examples of oscillatory dynamics

Figure 3 plots phase diagrams for four different values of the workers' bargaining power β. These phase diagrams zoom in on the steady states with economic activity, and the nullclines are dashed. Starting off from a small β in Panel 3a, the antisaddle is a stable focus, so that it attracts oscillating paths from initial conditions for unemployment outside itself. Notice that the equilibrium path towards the antisaddle is locally indeterminate, and that, in between the two outer paths of the panel there must also be an equilibrium saddlepath leading to the saddlepoint.

Increasing β, at some critical value the eigenvalues of the Jacobian matrix at the antisaddle become purely imaginary, and a Hopf bifurcation occurs. In a Hopf bifurcation, a periodic orbit emerges out of an antisaddle, and inherits its stability. Because the antisaddle of Panel 3a is stable, in this Hopf bifurcation it becomes unstable and gives rise to a stable limit cycle. Panel 3b presents this limit cycle. Equilibrium is still locally indeterminate and in between the two depicted paths there is again a saddlepath.

The limit cycle grows for a larger and larger workers' bargaining power until it coincides with the stable and unstable manifolds of the saddlepoint. When this happens, a saddle-loop bifurcation occurs, which is depicted in Panel 3c. At this bifurcation the periodic orbit connects the saddlepoint with itself, and is therefore called a *homoclinic orbit*. At the saddle-loop bifurcation the basin of attraction outside the periodic orbit has disappeared, except for the remaining saddlepath below the saddlepoint.

For larger values of β as in Panel 3d, periodic orbits do not exist any longer. The antisaddle remains unstable, but now the paths originating from its neighborhood will no longer be bounded, except for the saddlepath. Equilibrium is, however, still locally indeterminate.

2.4.2 Hopf, saddle-loop and Bogdanov-Takens bifurcations

The phase diagrams suggest the occurrence of a Hopf and a saddle-loop bifurcation. In this subsection I confirm the occurrence of these bifurcations. Moreover, I show that parameter combinations for which these bifurcations occur come together in a Bogdanov-Takens point. As a result, these bifurcations fully describe the behavior of the dynamical system in the neighborhood of this point in the parameter space.

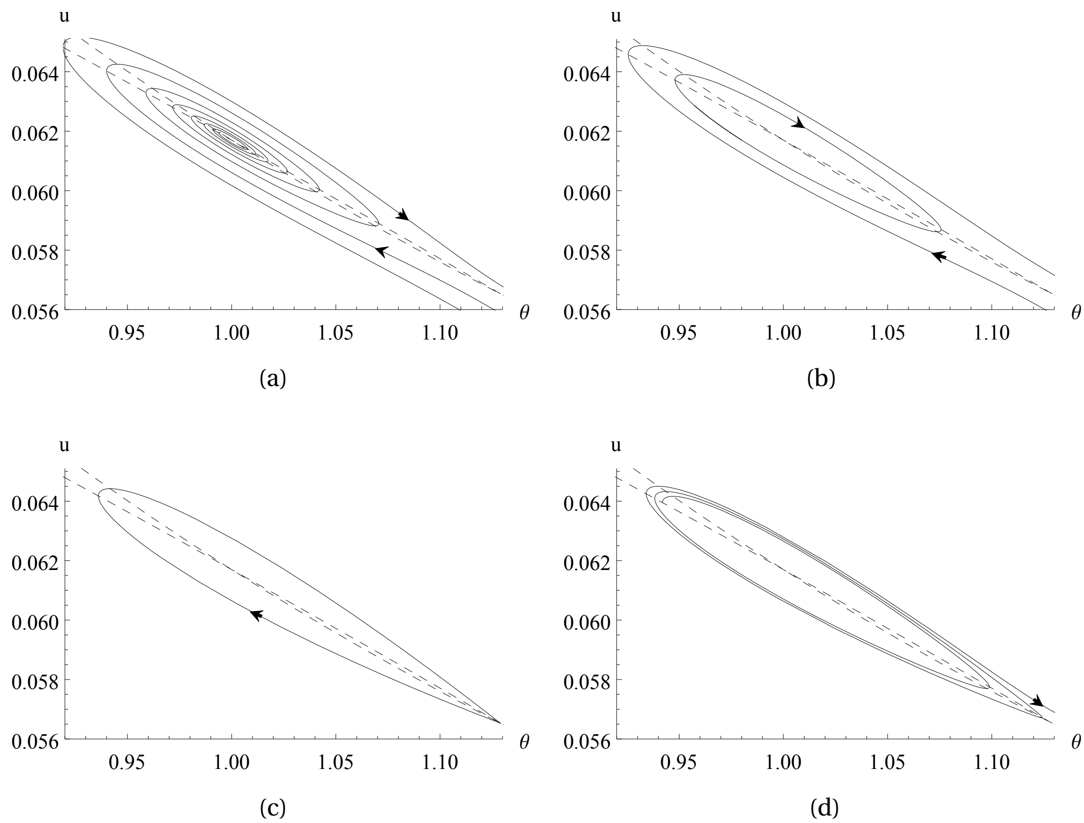

Figure 3: Representative phase diagrams for different values for β.

Notes: (a) Stable steady state for $\beta = 0.85$; (b) Stable limit cycle for $\beta = 0.89294$; (c) Homoclinic orbit of the saddle-loop bifurcation for $\beta \approx \beta_{SL} \approx 0.8930$; (d) No closed orbits at efficient bargaining for $\beta = 0.9$. Nullclines are dashed, intersecting twice. k and c_0 used to normalize steady state L tightness to 1. Other parameter values from Table 2, first column. $\beta_{Hopf} \approx 0.89290$.

A *Hopf bifurcation* occurs when the eigenvalues of the Jacobian matrix at the antisaddle cross the imaginary axis, so that the trace becomes zero. Remember from discussion of Equation 2.18 that the trace is always positive for $\beta \geq 1 - \eta$. Proposition 2.5 shows that it can become zero for a sufficiently small workers' bargaining power.

Proposition 2.5. *Under the regularity condition that the job finding rate in the antisaddle exceeds the discount rate $r > 0$, there exists a $\beta_{Hopf} \in (0, 1 - \eta)$ for which the antisaddle undergoes a Hopf bifurcation. As a result, there is a limit cycle in a one-sided neighborhood of β_{Hopf}.*

Proof. In any antisaddle the real parts of the eigenvalues are of the same sign, so that if the trace has a simple zero in the antisaddle, the eigenvalues cross the imaginary axis. From (2.18) we see that the trace has a simple zero at $\beta_{Hopf} = (1-\eta)\left(1 - r / \left(s^*(\theta_t)\,\theta_t^{\eta}\right)\right) \in (0, 1-\eta)$ if $s^*(\theta_t)\,\theta_t^{\eta} > r > 0$. Therefore, for $s^*(\theta_t)\,\theta_t^{\eta} > r$ in the antisaddle there exists a $0 < \beta_{Hopf} < 1-\eta$ for which a Hopf bifurcation occurs. As a result, a limit cycle exists in a one-sided neighborhood of β_{Hopf}. □

Proposition 2.5 implies that the antisaddle is a sink for $\beta < \beta_{Hopf}$ and a source for $\beta > \beta_{Hopf}$. For only one of these inequalities there exists a limit cycle enclosing the antisaddle for β sufficiently close to β_{Hopf}, but the proposition does not tell in which case. If the limit cycle exists for the left-sided neighborhood of β_{Hopf} it must be unstable, because the antisaddle is stable, and vice versa for the right-sided neighborhood.

While the limit cycle coincides with the antisaddle and the period of the cycle approaches zero as $\beta \to \beta_{Hopf}$, in a so-called *saddle-loop bifurcation* the cycle assumes its maximal size. A saddle-loop bifurcation occurs when the stable and the unstable manifolds of a saddlepoint connect to form a homoclinic orbit, a path that connects a steady state with itself. Because of the neighborhood of the saddlepoint, the period of the cycle approaches infinity as the cycle approaches the homoclinic orbit. In Hamiltonian systems (where all orbits are level curves) homoclinic orbits are a generic phenomenon, but in systems where the trace is generically nonzero the existence of a homoclinic orbit is not robust to small perturbations of a single parameter. In such systems the existence of a homoclinic orbit can be proven by perturbing a Hamiltonian system, and then the Andronov-Leontovich theorem (see e.g. Kuznetsov (2004, p. 200)) states that a limit cycle bifurcates on one side of the homoclinic orbit. Proposition 2.6 extends the result of Mortensen (1999) on the existence of a homoclinic orbit to the case of a positive value of leisure.[13] The proof is in Appendix 2.7.B.

Proposition 2.6. *Suppose that parameters are such that two steady states with economic activity exist for $z > 0$, $r = 0$, and $\beta = 1-\eta$, and define θ_H and u_H as market tightness and unemployment in the saddlepoint H respectively. Suppose also that $(1 - u_H + \alpha z^{1/\alpha})^{\alpha+1} < (1-\alpha)\left[(1-u_H)\left(z + k\theta_t^{1-\eta}/(1-\beta)\right) - u_H g(\theta_H)\right]$. Then there exist a $\beta_{SL} < 1-\eta$ such that*

[13] Mortensen (1999) shows that (2.18) holds globally, not only in steady states, and as a result his system can be described by a Hamiltonian function when the Hosios condition holds and $r = 0$. Moreover, Bendixson's criterion (Guckenheimer and Holmes, 1983, p. 44) then rules out limit cycles whenever both $r > 0$ and $\beta \geq 1-\eta$.

for a sufficiently small $r > 0$ a homoclinic orbit exists. Moreover, there exists a family of stable limit cycles for a one-sided neighborhood of β_{SL}.

Proposition 2.6 states that there exists an orbit that connects the saddlepoint with itself for $\beta = \beta_{SL}$, and $(1-u_H+\alpha z^{1/\alpha})^{\alpha+1} < (1-\alpha)\left[(1-u_H)\left(z+k\theta_t^{1-\eta}/(1-\beta)\right)-u_H g(\theta_H)\right]$ is necessary and sufficient to guarantee that $\theta_t > 0$ on this homoclinic orbit. While the homoclinic orbit itself is not a robust phenomenon, it is of interest because it implies that a family of limit cycles can be found in the neighborhood of β_{SL}. However, although Proposition 2.6 tells us that these limit cycles are stable, it cannot tell us whether these cycles occur for $\beta > \beta_{SL}$ or $\beta < \beta_{SL}$. Moreover, we do not know yet whether $\beta_{Hopf} < \beta_{SL}$, or the other way around.

Given the limited number of ways that limit cycles can appear or disappear, if $\beta_{Hopf} < \beta_{SL}$ stable limit cycles would exist for the right-sided neighborhood of β_{Hopf}, delimited by β_{SL}. If $\beta_{Hopf} > \beta_{SL}$, stable limit cycles would exist for the right-sided neighborhood of β_{SL}, upon collision (in a saddle-node bifurcation of cycles) annihilating an unstable limit cycle that would exist for the left-sided neighborhood of β_{Hopf}. To rule out this latter case, and to show that a stable limit cycle exists for a range of values for the workers' bargaining power $\beta \in (\beta_{Hopf}, \beta_{SL})$, I show the occurrence of a Bogdanov-Takens bifurcation. A necessary condition for this bifurcation is Bogdanov-Takens singularity, which requires that both eigenvalues are zero in a steady state. Proposition 2.7 states that such a point in parameter space exists. The proof is in Appendix 2.7.B.

Proposition 2.7. *Define Bogdanov-Takens singularity as a steady state with two zero eigenvalues but a nonzero Jacobian matrix. There exists a point in parameter space that satisfies this singularity, and it is unique for $\alpha < 1$.*

Under certain genericity conditions this singularity is sufficient for the occurrence of a Bogdanov-Takens bifurcation. These conditions require that one can 'travel' through the Bogdanov-Takens point by varying the parameters. To suggest that this is the case, I present the bifurcation diagram of Figure 4.[14] The figure shows combinations of parameters α and β for which bifurcations occur. The regions bounded by these bifurcations can be represented by qualitatively similar phase diagrams, where (a), (b),

[14] A formal proof requires the construction of a normal form and is too involved to present here, see e.g. Kuznetsov (2004, p. 322).

and (d) correspond to the respective panels of Figure 3, reprinted for convenience. The gray solid curve represents combinations of α and β for which a saddle-node bifurcation occurs, so that above the curve (in region (0)) no steady states with economic activity exist while below it there are two of them. The bold curve depicts the occurrence of a saddle-loop bifurcation as in Panel 3c. Right of this curve is the familiar region (d) in which all equilibria are either steady states or saddlepaths, and in which the dots indicate efficient bargaining. The dashed curve represents combinations of α and β for which a Hopf bifurcation occurs, so that left of this curve (in region (a)) there is a continuum of equilibria spiraling towards antisaddle L. Region (b), bounded by the Hopf and the saddle-loop bifurcations, features the Beveridge cycle. This region increases in r. Finally, there is a Bogdanov-Takens point BT where the Hopf bifurcation curve and the saddle-loop bifurcation curve connect tangentially to the saddle-node bifurcation curve, as they should for a Bogdanov-Takens bifurcation to occur.

Consequently, for an elasticity of the demand externalities α in the left-sided neighborhood of the Bogdanov-Takens point, there exists a family of limit cycles enclosed by a Hopf and saddle-loop bifurcation. The Bogdanov-Takens bifurcation rules out another bifurcation that could change the stability of these cycles. Because Proposition 2.6 shows that the limit cycles born at the saddle-loop bifurcation are stable, the entire family is stable. This implies that $\beta_{Hopf} < \beta_{SL}$, because a stable limit cycle requires a positive trace in antisaddle L, which only occurs for $\beta > \beta_{Hopf}$. As a result, a stable Beveridge cycle exists for $\beta \in (\beta_{Hopf}, \beta_{SL})$. Because the Beveridge cycle exists for a range of values for the parameters, it is structurally stable. The cycle will thus continue to exist under small perturbations of the parameters, or upon introduction of some risk aversion.

2.4.3 The economics of the Beveridge cycle

The analysis above implies that for $\beta < \beta_{SL}$, starting from a sufficiently low initial condition for unemployment, expectations select which steady state will be reached in the long run. Moreover, also for $\beta > \beta_{SL}$ multiple equilibria exist in the neighborhood of the antisaddle due to local indeterminacy. Obviously, multiple self-fulfilling expectations are the result of the feedback from aggregate employment to the revenue per match generated by the demand externality, but they also require delays in matching. An expectation that unemployment will be low makes firms open vacancies, because low

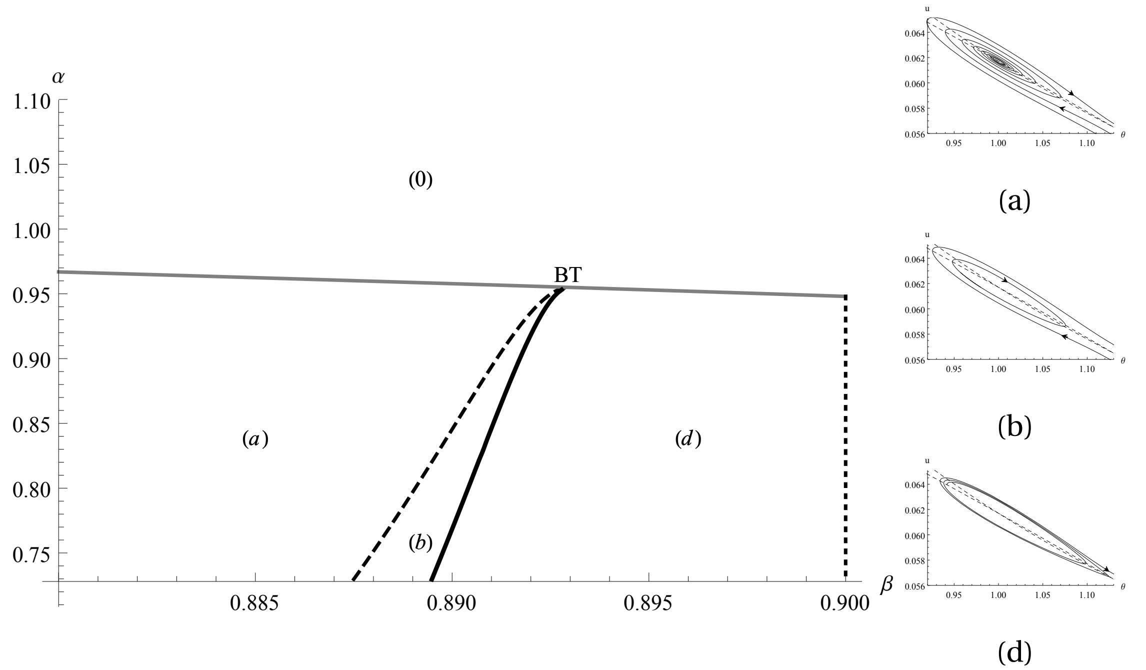

Figure 4: Bifurcation diagram of the Bogdanov-Takens bifurcation, with α and β as bifurcation parameters.

Notes: Other parameters from Table 3, first column. The gray curve corresponds to the saddle-node bifurcation, the bold curve to the saddle-loop bifurcation, and the dashed curve to the Hopf bifurcation. Regions (a), (b), and (d) refer to the panels of Figure 3, reprinted for convenience, and (d) extends beyond efficient bargaining at $\beta = 0.9$ (dotted curve). BT refers to the Bogdanov-Takens-point, and there are no steady states with economic activity in region (0). The homoclinic orbit hits the u-axis for $\alpha \approx 0.73$.

unemployment implies a high revenue per match. This expectation is self-fulfilling because more vacancies now result in lower future unemployment.

However, rational expectations Beveridge cycles do not only require positive feedback and a propagation mechanism, but also a congestion externality. Howitt and McAfee (1988) argue that in models with only a positive externality, because of positive discounting antisaddles are unstable and no cycles exist. However, they show that a negative externality can overturn this effect. This negative externality is a standard ingredient in search models of the labor market, where matching is less likely for a potential trader when many other potential traders search on the same side of the market. The congestion externality makes firms with expectations of low unemployment open vacancies immediately, not only because revenue per match will be higher in the future and matching takes time, but also because hiring will be more costly in the future. Proposition 2.2 shows that when $\beta = 1-\eta$, thick-market externalities on one side of the market exactly cancel congestion externalities on the other side of the market. It follows from Propositions 2.5 and 2.6 that Beveridge cycles exist for a range of value for workers' bargaining power that satisfy $\beta < 1-\eta$. This condition is required because free entry of firms drives the evolution of market tightness, and if $\beta < 1-\eta$, congestion occurs primarily on the firms' side of the market. Besides, variable search intensity is not essential for the existence of the cycle, but is only helpful for the calibration.

It follows from Proposition 2.4 that oscillations enclose a steady state where the unemployment nullcline crosses the tightness nullcline from above. At this antisaddle, the strategic complementarity from the demand externality dominates the strategic substitute from the congestion externality: when firms increase their vacancies, this results in such a large decrease in unemployment in the future, that revenue per match will increase so much that firms would want to open even more vacancies. As a result, firms overshoot the steady state level of tightness of the antisaddle for the alluringly high revenue per match at high aggregate employment levels. However, with so many vacancies, unemployment decreases fast and it starts to take a long time to fill an individual vacancy. Consequently, while unemployment still decreases but firms foresee an end to the boom, they do not want to spend valuable resources on vacancies that are hard to fill. Expecting higher unemployment in the future and thus smaller benefits of a filled vacancy, firms reduce labor market tightness. As a result, at some point fewer matches are made than jobs are destroyed, and unemployment increases. Higher unemployment feeds back to lower revenue per match, so that firms also overshoot the steady state level of tightness in the trough. With so few vacancies, however, a single vacancy is filled very fast. So while unemployment still increases but firms foresee an end to the trough, they are willing to spend some resources on vacancies that will be filled very soon and pay off at the higher employment levels of the future. Job matching takes over from job destruction again and unemployment decreases, completing the cycle.

The relative size of the discount rate r and the distortion of the Hosios condition, $\beta < 1-\eta$, determines whether oscillations diverge, converge, or form a closed orbit. The asset pricing equation of the system, (2.13), shows that for a high r and given flow values, equilibrium requires large capital gains or losses, i.e. fast movements in market tightness. For that reason, for a higher r and a given η, stability of the antisaddle requires a smaller β, or more congestion. Alternatively, for a given r the oscillations converge for a small β, form closed orbits for an intermediate β, and diverge for a large β, as in Figure 3. Remember that for a given r, the cycle is largest for β_{SL}. Otherwise, fix $\beta \in (\beta_{Hopf}, \beta_{SL})$ and compare Beveridge cycles for different discount rates. The cycle is small for a small discount rate because when firms are patient, they heavily respond to expected future changes in the revenue per match. A limit cycle is then only consistent with equilibrium if revenue per match does not vary too much over the cycle, thus if the cycle is small.

As the result of self-fulfilling expectations, the model features multiple equilibria, which results in the problem of equilibrium selection. Because of the evidence on the cyclical dynamics as presented in Panel 1a of Figure 1, I take the dynamics of the Beveridge cycle to be the relevant dynamics to explain the actual data. The stability of the limit cycle further supports its plausibility as a data-generating process, although its basin of attraction may be small. Figure 4 shows that the set of β's that give rise to a Beveridge cycle has a positive but small measure, so that the proposed data-generating process is not very robust to changes in β. On the other hand, for virtually all $\beta < \beta_{Hopf}$ the dynamics oscillate but eventually settle down in the antisaddle, as in Panel 3a of Figure 3. Especially if β is smaller than but close to β_{Hopf}, it may take a very long time to reach the steady state, so that many business cycles could be explained with one exogenous shock in fundamentals or beliefs. Kaplan and Menzio (2016) provide an example of the latter. I focus on the limit cycle and therefore I do not exploit the additional degrees of freedom that exogenous shocks provide. Instead, I use the fixed period of the limit cycle to calibrate the model. The next section presents the quantitative contribution of this paper.

2.5 Quantitative results

In this section I calibrate the model to the average duration of the business cycle and assess its quantitative performance in describing unemployment and vacancies over the business cycle. I compare the model-generated data to the actual data and to data generated by the canonical search and matching model of Pissarides (1985) with productivity shocks. Because the latter features constant search intensity, I also calibrate a model of the Beveridge cycle without variable search intensity. Next, I discuss robustness to alternative calibrations, including alternative targets for the duration of the business cycle. Finally, I show that the model-generated data move in the expected direction upon changes in unemployment benefits.

2.5.1 Calibration

I calibrate the parameters of the model using data on the duration of the business cycle. Depending on its measurement, the typical cycle in Panel 1a of Figure 1 lasts between 18 and 28 quarters. Because productivity in the model moves with employment, the duration of the NBER business cycle provides an alternative calibration target that avoids some arbitrary choices. Figure 5 shows the duration of the NBER business cycles falling entirely in the sample period from 1951 to 2014. Irrespective of whether the cycle is measured from peak to peak or from trough to trough, the average cycle lasts roughly 24 quarters.

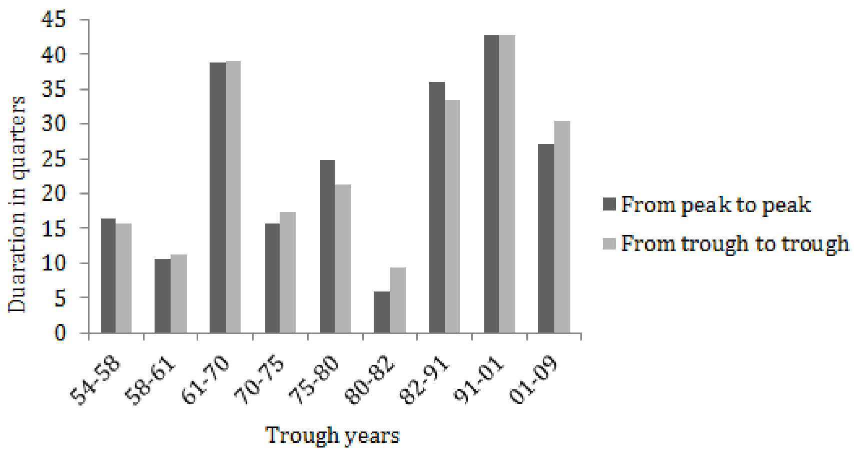

Figure 5: Duration in quarters of all completed NBER business cycles between 1951 and 2014.

Source: http://www.nber.org/cycles.html

The parameters that describe workers' preferences are discount rate r, gross value of leisure z, and the elasticity of the search cost function γ. Firms' technology is described by the elasticity of revenue per match with respect to aggregate employment α, the vacancy creation cost k, and the job destruction rate δ. Matching and bargaining are described by the elasticity of the matching function with respect to vacancies η, and workers' bargaining power β.

The bifurcation analysis of the preceding section makes clear that β is important for the existence of the Beveridge cycle. I choose the β that is closest to efficient bargaining but still gives rise to a limit cycle with a computationally significant basin of attraction outside its path. This limit cycle is close to the homoclinic orbit, the largest closed orbit possible taking the other parameters as given.[15] By moving β further away from efficient bargaining in the direction of the Hopf bifurcation, the limit cycle can be made arbitrarily small, but then the Beveridge cycle can explain little volatility. My calibration strategy therefore approaches an upper bound to deterministic volatility.

I choose a period in the model to be one quarter. Given β, I set α such that the duration of the Beveridge cycle corresponds to the average duration of the NBER business cycle over the sample period. The value of r corresponds to an annual interest rate of 4 percent. I choose the value for δ to equal the observed average quarterly job destruction rate, and the value for k so that the average unemployment rate is the same in the model and in the data.

In the model, the elasticity ε of the job finding rate with respect to the vacancy-to-unemployment ratio is $\varepsilon = \eta + (1-\eta)/\gamma$. Therefore I choose η and γ such that their combination results in an estimated elasticity of 0.46. To distinguish the role of variable search intensity from the role of the matching function, I exploit that (by definition) non-participants do not search for jobs but still find jobs at cyclical rates. Assuming that non-participants and unemployed workers find jobs via the same matching function, and that the higher job finding rate of the unemployed is the result of their search effort, allows me to disentangle η and γ. I use a matching function with ranking (similar to Blanchard and Diamond (1994)) to ensure that non-participants do not create congestion for unemployed workers, as in the model. Appendix 2.7.A describes this procedure and the data used in more detail.

I choose z such that I match an average flow value of leisure of 0.71 as in Hall and Milgrom (2008). In my model, however, the flow benefit of leisure consists of a gross value of leisure z and variable search costs s_t^γ. On top of that, while average productivity is commonly normalized to 1, productivity in my model never reaches 1. For that reason, the relevant calibration target of my model is not z, but the net value of leisure relative to output:

$$\zeta = \frac{z - s^* (\theta_t)^\gamma}{(1-u_t)^\alpha}.$$

[15]As pointed out in section 2.4, the period of the homoclinic orbit approaches infinity, but because it has no basin of attraction outside itself, sampling from this cycle is computationally unstable.

I calibrate the parameters of the Pissarides (1985) model using the same targets. I use the same β as for the Beveridge cycle, because in the standard model it is unrelated to the size of the cycle. However, I choose the parameters of the stochastic process for productivity such that the standard deviation and autocorrelation of revenue per match are the same for the Beveridge cycle and the Pissarides (1985) model. Following the RBC literature's convention for stochastic processes, I match the statistics in logs as deviations from a linear trend.[16] I repeat this procedure to match the targets of the Beveridge cycle without variable search intensity.

The calibration strategy described above results in the parameters of Table 2. All calibrations of the Beveridge cycle result in two steady states with economic activity. My calibration procedure to disentangle γ and η results in an estimated γ of 2.5, which is a bit higher than the quadratic cost function that the literature (e.g. Gomme and Lkhagvasuren (2015)) generally assumes.[17] The target for ε then implies $\eta = 0.1$, so that the Hosios condition corresponds to $\beta = 0.9$. In both the Pissarides (1985) model and the model of the Beveridge cycle without variable search intensity, $\eta = \varepsilon$. When on top of that average revenue per match is equal to one, as in the Pissarides (1985) model, $z = \zeta$.

To match the average unemployment rate over the cycle, my benchmark calibration results in a demand externality of 6.9. This value is considerably higher than Kaplan and Menzio (2016)'s estimate of shopping and demand externalities that implies $\alpha = 1.15$, and lies completely out of the range of estimates of increasing returns to scale, which provide an upper bound of $\alpha = 0.17$ (Harrison, 2001). As can be seen in the last column of Table 2, this problem only deteriorates for a fixed search intensity. The following two alternative calibration strategies show that either a high value of leisure, or a high elasticity of the job finding rate with respect to tightness, result in demand externalities that are of the same order of magnitude as Kaplan and Menzio (2016).

The estimation of the elasticity of the job finding rate ε is very sensitive to the choices made and the data selected. As a result, in the literature it takes almost any value between zero and one, see Petrongolo and Pissarides (2001). Because the quantitative results are also sensitive to ε, I consider an alternative calibration, exploiting that from December 2000 onwards the JOLTS provides an alternative source of the job finding

[16] An HP-filter with a lower smoothing parameter biases autocorrelation coefficients to such an extent that an Ornstein-Uhlenbeck process cannot match the filtered autocorrelation of the Beveridge cycle.

[17] Note, however, that $\gamma = 2$ is not compatible with a positive elasticity of the matching function for an estimated elasticity $\varepsilon < 0.5$.

	Beveridge cycle	Pissarides I	$\gamma \to \infty$	Pissarides II
r	0.012	0.012	0.012	0.012
z	0.527	0.71	0.345	0.71
γ	2.5	-	-	-
α	6.887	-	11.05	-
k	$2.079 * 10^{-8}$	$1.155 * 10^{-2}$	$4.561 * 10^{-2}$	$7.974 * 10^{-2}$
δ	0.1	0.1	0.1	0.1
η	0.1	0.46	0.46	0.46
β	0.893	0.893	0.536	0.536

Table 2: Calibrated parameters for the benchmark calibration, the Pissarides (1985) model (Pissarides I), the Beveridge cycle without variable search intensity ($\gamma \to \infty$), and the Pissarides (1985) model that uses the same targets as the the Beveridge cycle without variable search intensity (Pissarides II)

Notes: For Pissarides I, the Ornstein-Uhlenbeck volatility parameter is $\sigma = 0.02139$, and its persistence parameter is $\gamma = 0.032$. For Pissarides II, $\sigma = 0.0546$ and $\gamma = 0.0459$.

probability. Regressing the log of this job finding probability on the log of labor market tightness for the relevant sample period results in $\varepsilon = 0.84$. To disentangle γ and η, I set $\gamma = 1.29$ as estimated by Burdett et al. (1984).

Similarly, the literature disagrees on the flow value of leisure. For that reason, I also consider Hagedorn and Manovskii (2008)'s target of 0.955 for the average ζ over the cycle. Finally, for the benchmark calibration targets of $\varepsilon = 0.46$ and $\zeta = 0.71$, I also calibrate the duration of the Beveridge cycle to a lower and upper bound of 18 and 28 quarters, respectively, as suggested by the evidence in Panel 1a of Figure 1.

The parameters resulting from these alternative calibrations are presented in Table 3. Remember that the estimate of Kaplan and Menzio (2016) corresponds to $\alpha = 1.15$, so that for $\zeta = 0.955$ the calibrated demand externalities are smaller than theirs. Of course, in this case the calibrated value for z is relatively high. For $\varepsilon = 0.84$, α is somewhat above the estimate of Kaplan and Menzio (2016), but in the same order of magnitude. Besides, the calibration strategy with $\gamma = 1.29$ results in $\eta = 0.3$, which is more common in the literature without variable search intensity. Finally, a Beveridge cycle of shorter duration requires demand externalities that are slightly higher than the benchmark parameter value, while a longer duration is obtained for a smaller α.

	$\zeta = 0.955$	$\varepsilon = 0.84$	18 quarters	28 quarters
r	0.012	0.012	0.012	0.012
z	0.909	0.839	0.490	0.538
γ	2.5	1.29	2.5	2.5
α	1.064	1.342	7.500	6.706
k	$1.936 * 10^{-11}$	$1.226 * 10^{-3}$	$2.943 * 10^{-8}$	$1.862 * 10^{-8}$
δ	0.1	0.1	0.1	0.1
η	0.1	0.3	0.1	0.1
β	0.893	0.695	0.893	0.893

Table 3: Calibrated parameters for four alternative targets.

Notes: $\varepsilon = 0.45$ and $\zeta = 0.71$ unless specified otherwise.

A high value of leisure and a high elasticity of the job finding rate reduce the calibrated value for α, because both decrease the unemployment rate, or increase the job finding rate, in the antisaddle. Consequently, smaller demand externalities are required to target an average unemployment rate of 0.587. A higher elasticity and more variable search intensity change the unemployment nullcline so that the same market tightness results in lower unemployment. A higher value of leisure shifts the tightness nullcline to the left, because for a high ζ, the surplus of a match is lower and firms open fewer vacancies. The next subsection presents the model-generated data.

2.5.2 Performance

In this subsection I present the data generated by the Beveridge cycle, and compare it to the actual data, the data generated by the calibrated Pissarides (1985) model, and the data that result from a Beveridge cycle without variable search intensity. To assess the quantitative performance of the Beveridge cycle, I sample 256 (quarterly) observations from the calibrated Beveridge cycle, and compute time series for unemployment, vacancies, the ratio of vacancies to unemployment, the job finding rate $f_t = s^*(\theta_t)\theta_t^{\eta}$, and revenue per match $y_t = (1-u_t)^{\alpha}$. More specifically, for each variable I draw time series for each of the 24 different starting points around the cycle, and report the average statistics. As for the original data, I take logs and deviations from an HP trend with smoothing parameter 10^5. Table 4 presents the standard deviation, autocorrelation and cross-correlations of the variables of the model.

		u	v	v/u	f	y
Standard deviation		0.043	0.060	0.101	0.047	0.019
Quarterly autocorr.		0.942	0.934	0.939	0.939	0.940
Correlation matrix	u	1	−0.938	−0.979	−0.979	−1.000
	v		1	0.989	0.989	0.938
	v/u			1	1.000	0.979
	h				1	0.979
	y					1

Table 4: Summary statistics of the Beveridge cycle calibrated to $\zeta = 0.71$ and $\varepsilon = 0.45$.

Notes: f stands for the job finding rate, y for revenue per match. Statistics are averages from 24 samples of 256 quarters across the cycle. All variables are in logs as deviations from an HP trend with smoothing parameter 10^5.

Comparing the model-generated data to the actual data in Table 1, some features stand out. First, the endogenous fluctuations in revenue per match almost account for the full observed volatility of productivity. However, in logs as deviations from a linear trend the standard deviation of revenue per match is also only 0.019, compared to 0.041 in the data, so that the demand externality amounts to less than half of the fluctuations in productivity. The HP-filtered time series for revenue per match is somewhat more persistent than in the data, but in logs as deviations from a linear trend the autocorrelation coefficient is also only 0.940, compared to 0.976 in the data. These differences can be understood from the fact that the Beveridge cycle features only fluctuations in the business cycle frequency domain. Besides, by construction, revenue per match is much more negatively correlated with unemployment than observed productivity.

Second, the Beveridge cycle explains about 22 per cent of the observed volatility in unemployment, and more than one-third of the observed volatility in vacancies. Due to the strong negative correlation between vacancies and unemployment, both in the model and in the data, the standard deviation of the ratio of vacancies to unemployment is about the same as the sum of the standard deviations of vacancies and unemployment, both in the model and in the data. As a result, the model-generated standard deviation of the ratio of vacancies to unemployment and the job finding rate also fall short compared to the data.

Third, the autocorrelation of unemployment, vacancies, the ratio of vacancies to unemployment, and the job finding rate are about the same in the model and in the data. It is important to note that this persistence is an endogenous result of calibrating the Beveridge cycle to the duration of the average business cycle, and is not generated by feeding in a persistent stochastic process. The existence of a saddlepoint in the neighborhood of the Beveridge cycle slows down the dynamics and therefore allows the cycle to be calibrated to the duration of the business cycle. To assess to what extent the persistence of the labor market variables is driven by the Beveridge cycle rather than simply the persistence of the fluctuations in revenue per match, it is useful to compare the model-generated data of the Beveridge cycle and the Pissarides (1985) model.

Table 5 reports the summary statistics of the Pissarides (1985) model, using the code from Shimer (2005) but the parameters from Table 2, second column. In particular, the standard deviation and autocorrelation of revenue per match in logs as deviations from a linear trend are the same as for the Beveridge cycle. Note that the autocorrelation of unemployment approaches that of the Beveridge cycle and the observed data, but that vacancies, and to a smaller extent the ratio of vacancies to unemployment and the job finding rate, are not as persistent. Similarly, the standard deviation of unemployment lags behind that of the Beveridge cycle, in absolute terms, but also relative to filtered productivity. The ratios of the standard deviations across the labor market variables are the same for the Beveridge cycle and the Pissarides (1985) model, resulting in a similar slope of the Beveridge curve. Finally, in both models the cross-correlations are generally higher than in the data.

The differences between the Beveridge cycle and the Pissarides (1985) model can also be seen graphically in Figure 6. Panel 6a contains 25 simulated quarterly observations that complete a Beveridge cycle, HP-filtered and connected by straight lines. The shape of the Beveridge cycle is similar to the observed cycles in Panel 1a of Figure 1, and it rotates counterclockwise. Panel 6b of Figure 6 plots a representative realization of 256 quarters of the Pissarides (1985) model. The model-generated Beveridge curve is somewhat less volatile than the Beveridge cycle, but, most importantly, lacks cyclical dynamics. Although there are some swings parallel to the inverse relationship between vacancies and unemployment, most of the dynamics is almost vertical.

Figure 6 also shows that the Beveridge curve generated by both the Beveridge cycle and the Pissarides (1985) model is too steep compared to the data. It is important to note, however, that both the Beveridge cycle and the Pissarides (1985) model feature a

		u	v	v/u	f	y
Standard deviation		0.018	0.027	0.044	0.02	0.013
Quarterly autocorr.		0.931	0.781	0.873	0.873	0.873
Correlation matrix	u	1	−0.900	−0.963	−0.963	−0.962
	v		1	0.984	0.984	0.983
	v/u			1	1.000	0.999
	f				1	0.999
	y					1

Table 5: Summary statistics of the Pissarides (1985) model calibrated to the same targets as the Beveridge cycle.

Notes: Code from Shimer (2005), where the Ornstein-Uhlenbeck volatility parameter is $\sigma = 0.02139$ and its persistence parameter is $\gamma = 0.032$, to target the standard deviation and autocorrelation of productivity in logs as deviations from a linear trend of the 24 quarter Beveridge cycle. All variables are in logs as deviations from an HP trend with smoothing parameter 10^5. The table reports averages across 10000 simulations.

constant job destruction rate and no transitions into employment from employment or non-participation. It is well known that job destruction shocks can flatten out the Beveridge curve, but only at the cost of a counterfactually low correlation between vacancies and unemployment. Other mechanisms can also increase the volatility of unemployment relative to the volatility of vacancies. It follows from Elsby et al. (2015) that the participation margin contributes to unemployment volatility and thus reduces the slope of the Beveridge curve. Menzio and Shi (2011) show that on-the-job search affects firms' incentives to post vacancies and can sharply reduce the volatility of vacancies over the cycle. However, Eeckhout and Lindenlaub (2015) show that in the presence of sorting, on-the-job search can also contribute to the volatility of vacancies and even result in self-fulfilling fluctuations.

Both the absence of fluctuations in job destruction, and the absence of transitions into employment from employment or non-participation in the model of the Beveridge cycle imply that the model-generated data should not exactly match the observed volatility of both unemployment and vacancies. For instance, Pissarides (2009) argues that in reality one-third to one-half of the volatility in unemployment is driven by fluctuations in the inflow into unemployment rather than the outflow. As a result, a

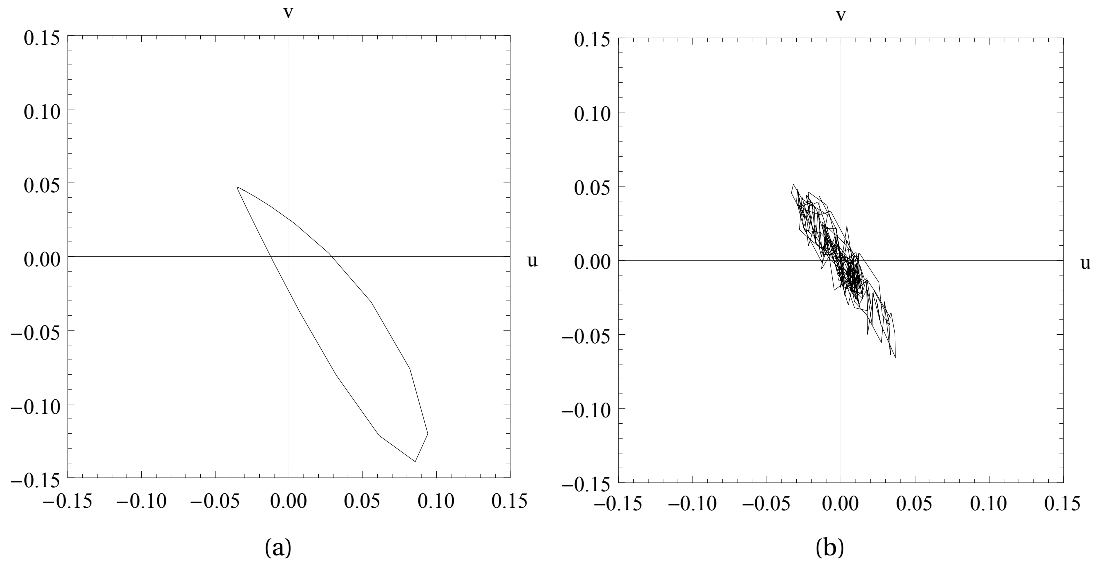

Figure 6: Simulated dynamics of unemployment and vacancies for (a) the Beveridge cycle, and (b) the Pissarides model.

Notes: Panel a plots 25 quarterly datapoints from the calibrated Beveridge cycle. Panel b plots one simulation of 256 quarters based on code from Shimer (2005), calibrated to the same targets as the Beveridge cycle. Simulated data are in logs as deviations from an HP trend with smoothing parameter 10^5, connected by straight lines.

model with constant job destruction should explain at most two-thirds of the volatility in unemployment.

Summing up, vacancies are more persistent over the Beveridge cycle than in the Pissarides (1985) model, without being less volatile. In fact, both unemployment and vacancies are somewhat more volatile over the Beveridge cycle than in the Pissarides (1985) model, also relative to HP-filtered productivity. To investigate the source of additional volatility, I also compare the data generated by the Pissarides (1985) model with data generated by the Beveridge cycle without variable search intensity. Table 6 presents the standard deviations and autocorrelations from the Beveridge cycle without variable search intensity, and the Pissarides (1985) model calibrated to the same targets. The former features counterfactually large fluctuations in revenue per match as the result of the large demand externalities. However, the table shows that *relative* to the HP-filtered volatility of revenue per match, the volatility of the labor market variables is about the same for the Pissarides (1985) model and the Beveridge cycle without variable search intensity. As a result, the Beveridge cycle does not feature any new amplification mechanisms. Variable search intensity contributes to amplification because it makes the net flow value of leisure countercyclical and because search and recruiting activity are strategic complements.

		u	v	v/u	f	y
Bev. cycle, $\gamma \to \infty$	Standard deviation	0.061	0.087	0.145	0.067	0.044
	Quarterly autocorr.	0.931	0.918	0.926	0.926	0.927
Pissarides II	Standard deviation	0.045	0.066	0.109	0.05	0.031
	Quarterly autocorr.	0.926	0.763	0.864	0.864	0.863

Table 6: Summary statistics of the Beveridge cycle without variable search intensity ($\gamma \to \infty$), and the Pissarides (1985) model recalibrated to feature the same β and the same standard deviation and autocorrelation of revenue per worker in logs as deviations from a linear trend (Pissarides II).

Regarding persistence, in both the Pissarides (1985) model and the Beveridge cycle (with or without variable search intensity), the autocorrelation of labor market tightness and the job finding rate is about the same as the autocorrelation of the (HP-filtered) revenue per match. As a result, the higher persistence of these variables may be attributed to the higher persistence of revenue per match after filtration. However, in the Pissarides (1985) model the autocorrelation of vacancies is substantially smaller than that of labor market tightness and the job finding rate, unlike the Beveridge cycle. The feature of the Beveridge cycle that vacancies are almost as persistent as unemployment (and the other variables) is exactly what results in the counterclockwise cycles in unemployment and vacancies. The next subsection shows that this feature is robust to alternative targets for the duration of the cycle.

2.5.3 Robustness

This subsection presents the model-generated data resulting from the four alternative calibration strategies. Table 7 presents the standard deviation and autocorrelations of the data generated by the Beveridge cycle calibrated to 18 and 28 quarters. Not surprisingly, the 18-quarter Beveridge cycle results in a smaller autocorrelation coefficient than the 24-quarter cycle, and larger demand elasticities result in a smaller standard deviation. Equivalently, the 28-quarter Beveridge cycle features smaller volatility but more persistence. Note, however, that the relative persistence of vacancies compared to the other variables survives under alternative targets for the duration of the cycle, although it is higher for longer cycles.

		u	v	v/u	f	y
18 quarters	Standard deviation	0.095	0.143	0.231	0.106	0.047
	Quarterly autocorr.	0.888	0.855	0.875	0.875	0.879
28 quarters	Standard deviation	0.027	0.037	0.063	0.029	0.011
	Quarterly autocorr.	0.960	0.957	0.959	0.959	0.960

Table 7: Summary statistics of the Beveridge cycle calibrated to 18 and 28 quarters, respectively.

The calibration results in Table 3 have shown that either a high value of leisure, or a high elasticity of the job finding rate with respect to tightness, result in demand externalities that are of the same order of magnitude as Kaplan and Menzio (2016). The summary statistics of Table 8 show the familiar result that both also contribute to amplification. Although smaller demand externalities result in only a fraction of the volatility in revenue per match, the volatility of unemployment falls to a much smaller extent. The standard deviation of unemployment is about the same in the benchmark calibration and the calibration with $\varepsilon = 0.84$, and still more than three-quarters of the benchmark result in the calibration with $\zeta = 0.955$. Moreover, the persistence of vacancies does not suffer from additional amplification. Note, however, that a high elasticity of the job finding rate with respect to tightness results in a low volatility of vacancies compared to unemployment.

As discussed above, a high value of leisure reduces the required externalities because it lowers the steady state L unemployment rate. Although such comparative statics might seem counterintuitive, I show that the dynamics of the Beveridge cycle move in the opposite 'intuitive' direction. Increasing the value of leisure to $z = 0.5277$ while keeping all other parameters fixed, steady state unemployment decreases from 0.062 to 0.060. This is the comparative statics effect described above. However, I claim that the dynamical system of the Beveridge cycle is the data-generating process. Sampling from

		u	v	v/u	f	y
$\zeta = 0.955$; $\varepsilon = 0.45$	Standard deviation	0.033	0.046	0.079	0.036	0.002
	Quarterly autocorr.	0.946	0.940	0.944	0.944	0.945
$\zeta = 0.71$; $\varepsilon = 0.84$	Standard deviation	0.042	0.015	0.054	0.045	0.004
	Quarterly autocorr.	0.952	0.934	0.951	0.951	0.951

Table 8: Summary statistics of the Beveridge cycle calibrated to $\zeta = 0.955$ and $\varepsilon = 0.45$, and $\zeta = 0.71$ and $\varepsilon = 0.84$, respectively.

the slightly displaced Beveridge cycle, the average unemployment rate over the cycle increases from 0.059 to 0.060. Consequently, my model predicts a positive effect of the unemployment benefit on observed unemployment, as most economists would expect.

This argument does not rely on adjustment dynamics, but compares datapoints on two different Beveridge cycles. Figure 7 shows a time series of unemployment over 256 quarters, connected by straight lines, resulting from the Beveridge cycle with the calibrated value of leisure. As can be seen in this figure, the calibrated cycle spends most of its time on segments of the cycle with low unemployment rates. A Beveridge cycle for a higher value of leisure spends its time more evenly over the cycle. This nonlinear effect dominates the displacement of steady state L and its enclosing Beveridge cycle. As a result, a Beveridge cycle with a high value of leisure produces a lower average unemployment rate than a cycle with a high value of leisure, even though steady state L moves in the opposite direction.

Finally, the time series in Figure 7 is very regular, much more so than actual data. However, exogenous shocks in fundamentals or beliefs can cause variations in amplitude and period of the cycle, without altering its driving mechanism. Beaudry et al. (2015) show that adding exogenous shocks on top of a deterministic cycle can reproduce the spectrum of business cycle fluctuations in output and employment. Whether the persistence of vacancies survives such additional shocks is a question for future research.

2.6 Conclusion

Mortensen (1999) presents a parsimonious model to show that multiple Pareto-ranked cycles and steady states can coexist, and that different expectations can be self-fulfilling

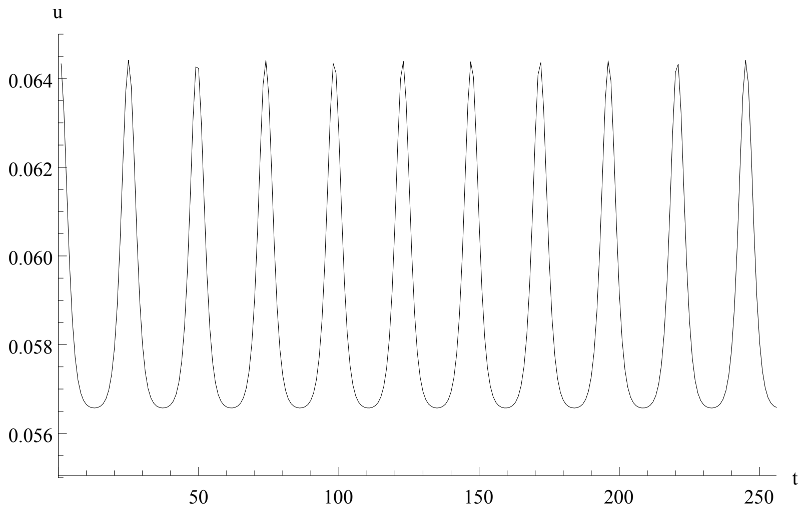

Figure 7: Simulated time series of unemployment.

Notes: 256 quarterly datapoints from the calibrated Beveridge cycle, connected by straight lines.

and result in each of these equilibria. By presenting a Bogdanov-Takens bifurcation, I show that a stable limit cycle - the Beveridge cycle - exists for a range of values for the workers' bargaining power enclosed by a Hopf and a saddle-loop bifurcation. I calibrate this Beveridge cycle to the average duration of the business cycle. The calibrated cycle looks qualitatively similar to the observed counterclockwise cycles in the *unemployment, vacancy rate*-plane. In addition, it can account for the persistence of unemployment and vacancies for plausible parameter values, but suffers from the same lack of amplification as the Pissarides (1985) model. However, volatility can be generated endogenously by the interplay of demand and congestion externalities.

A limitation of this study is that the range of parameter values that results in a limit cycle is small. Although my calibration focuses on this purely deterministic Beveridge cycle, for a much bigger set of parameter values a single shock can result in counterclockwise fluctuations that are able to explain many business cycles but eventually settle down into a steady state. Separation rate and productivity shocks provide a natural complement to the endogenous mechanism of this paper. On top of that, the indeterminacy of equilibrium allows for belief shocks, which directly result in another level of labor market tightness by the opening or closing of vacancies by firms. However, while additional exogenous shocks can result in more irregular time series than those generated by my calibration, they also provide additional degrees of freedom.

2.7 Appendix

2.7.A Data and calibration

I follow Shimer (2005) in the construction of the monthly job finding probability F_t and job destruction probability Δ_t. In particular,

$$F_t = 1 - \frac{u_{t+1} - u^s_{t+1}}{u_t},$$
$$\Delta_t = \frac{u^s_{t+1}}{e_t\left(1 - \frac{1}{2}F_t\right)},$$

where u^s denotes the short-term unemployment rate and e denotes the employment rate. Following Elsby et al. (2009), I inflate the short-term unemployment rate by 1.16 from January 1994 onwards to correct for changes in the way the CPS measures unemployment duration. These probabilities are subsequently transformed in job finding and job destruction rates according to

$$f_t = -\log(1 - F_t),$$
$$\delta_t = -\log(1 - \Delta_t),$$

respectively. I add three monthly rates to obtain the quarterly rate. Regressing the HP-filtered job finding rate on the HP-filtered vacancy-to-unemployment ratio results in an estimate of 0.46.

In my model, the elasticity of the job finding rate with respect to the vacancy-to-unemployment ratio is $\eta + (1-\eta)/\gamma$. I exploit the cyclicality in the job finding rate of non-participants (that by definition do not search) to isolate the elasticity of the matching function. I assume that non-participants and unemployed workers find jobs according to a matching function with the same parameters, but that the unemployed have a superior ranking. In particular, denoting the number of non-participants, unemployed workers and vacancies by N, U and V, respectively, the total number of matches in any period is given by $\mu_0 V^\eta (N + sU)^{1-\eta}$. Consistent with the model, the number of unemployed workers finding a job is given by $\mu_0 V^\eta (sU)^{1-\eta}$, which is not affected by the number of non-participants. Consequently, the number of non-participants finding a job is given by $\mu_0 V^\eta (N + sU)^{1-\eta} - \mu_0 V^\eta (sU)^{1-\eta} = \mu_0 V^\eta \left((N+sU)^{1-\eta} - (sU)^{1-\eta}\right)$. Note that unless $\eta = 0$, unemployed workers do create congestion for non-participants. This variant of Blanchard and Diamond (1994), similar in spirit to Blanchard and Diamond

(1989, p. 32), can be justified by the search effort of the unemployed that allows them to form all potential matches before non-participants arrive.

I take data on the job finding rate of non-participants from Elsby et al. (2015), using their classification error adjusted ("deNUNified") and time-aggregation adjusted hazard rates. These hazard rates are based on monthly gross worker flows, which the BLS provides from February 1990 onwards. The data from June 1967 and December 1975 were tabulated by Joe Ritter and made available by Hoyt Bleakley. This leaves a gap of fifteen years. This data was constructed by Robert Shimer. For additional details, please see Shimer (2012). Extending the series of Elsby et al. (2015) with recent data from the BLS, I have monthly job finding rates from 1967 to 2014.

A first-stage regression of the job finding rate of the *unemployed* from these same sources on the vacancy-to-unemployment ratio results in an elasticity of 0.45, virtually the same elasticity as for job-finding rate based on short-term unemployment available for 1951-2014. In my nonlinear second-stage regression of the job finding rate of non-participants I therefore impose that $\eta + (1-\eta)/\gamma = 0.45$. Moreover, I impose that the scale parameter of the matching function is the same, so that any differences in the *level* of the job finding rate of the unemployed from that of non-participants results from the search intensity and the superior ranking of the unemployed. Using the matching function with ranking above and the expression for optimal search intensity in (2.8), the logarithm of the job finding rate of non-participants is given by

$$\mu - \frac{1-\eta}{\gamma}c + \eta\log(V) - \log(N) + \log\left(\left(U\left(\frac{\mathrm{e}^c V}{U}\right)^{1/\gamma} + N\right)^{1-\eta} - \left(U\left(\frac{\mathrm{e}^c V}{U}\right)^{1/\gamma}\right)^{1-\eta}\right),$$

where μ is the estimated constant in the first-stage regression of the job finding rate of the *unemployed* on the vacancy-to-unemployment ratio, and where c is a constant of the second-stage regression that takes out the difference in the level of the job finding rates of the unemployed and non-participants that results from the search intensity of the former. Minimizing the sum of squared residuals under the restriction that $\eta + (1-\eta)/\gamma = 0.45$, results in $\eta = 0.0975$ and $\gamma = 2.537$. Rounding off at one decimal results in a combination of η and γ that is consistent with the estimated elasticity for the job-finding rate based on short-term unemployment available for 1951-2014: $0.1 + (1-0.1)/2.5 = 0.46$.

2.7.B Proofs

Proof of Proposition 2.2

Social welfare is given by

$$\int_0^\infty \mathrm{e}^{-rt}\left[(1-u_t)\,y + u_t(z-s_t^\gamma) - k\theta_t s_t u_t\right]dt. \tag{2.19}$$

The social planner maximizes this function by choosing both the socially efficient level of labor market tightness and search intensity, subject to the law of motion of unemployment given in (2.9). First-order conditions for the optimal θ_t and s_t, where μ_t denotes the co-state variable for the constraint on the dynamics of u_t, are

$$-\mathrm{e}^{-rt}\left[y-z+s_t^\gamma+k\theta_t s_t\right]+\mu_t\left[\delta+s_t\theta_t^\eta\right]-\dot{\mu}_t=0, \tag{2.20}$$

$$-\mathrm{e}^{-rt}ks_t u_t+\mu_t s_t u_t \eta\theta_t^{\eta-1}=0, \tag{2.21}$$

$$-\mathrm{e}^{-rt}\left[\gamma s_t^{\gamma-1}u_t+k\theta_t u_t\right]+\mu_t u_t\theta_t^\eta=0. \tag{2.22}$$

Both (2.21) and (2.22) can be rewritten to yield μ_t, so that

$$\mu_t=\frac{\mathrm{e}^{-rt}\theta_t k}{\eta\theta_t^\eta}=\frac{\mathrm{e}^{-rt}k\theta_t}{\eta\theta_t^\eta} \tag{2.23}$$

$$=\frac{\mathrm{e}^{-rt}\left[\gamma s_t^{\gamma-1}+k\theta_t\right]}{\theta_t^\eta}. \tag{2.24}$$

The efficient search intensity s_t is therefore given by

$$\frac{1-\eta}{\eta}k\theta_t=\gamma s_t^{\gamma-1}.$$

Comparing this expression with the privately chosen intensity in (2.8), search intensity is efficient if and only if $\beta=1-\eta$, the Hosios condition.

The expression for μ_t in (2.23) can be used to derive

$$\dot{\mu}_t=\frac{\mathrm{e}^{-rt}k\dot{\theta}_t-r\mathrm{e}^{-rt}k\theta_t}{\eta\theta_t^\eta}-\frac{\mathrm{e}^{-rt}k\theta_t\eta^2\frac{\theta_t^\eta}{\theta_t}\dot{\theta}_t}{\eta^2(\theta_t^\eta)^2}=\frac{\mathrm{e}^{-rt}k[(1-\eta)\dot{\theta}_t-r\theta_t]}{\eta\theta_t^\eta}. \tag{2.25}$$

Substituting (2.23) and (2.25) into (2.20) and rearranging, yields

$$\frac{(1-\eta)k\dot{\theta}_t-rk\theta_t}{\eta\theta_t^\eta}=\frac{k\theta_t\left[\delta+s_t\theta_t^\eta\right]}{\eta\theta_t^\eta}-\left[y-z+s_t^\gamma+k\theta_t s_t\right],$$

$$\Leftrightarrow\frac{(1-\eta)k\dot{\theta}_t}{\theta_t^\eta}=\frac{k\theta_t\left[\delta+r\right]}{\theta_t^\eta}-\eta\left[y-z+s_t^\gamma-\frac{1-\eta}{\eta}k\theta_t s_t\right],$$

$$\Leftrightarrow\dot{\theta}_t=\frac{\theta_t}{1-\eta}\left[\delta+r\right]-\frac{\eta\theta_t^\eta}{(1-\eta)k}\left[y-z+s_t^\gamma-\frac{1-\eta}{\eta}k\theta_t s_t\right].$$

Comparing this expression with the privately chosen tightness in (2.13), taking into account the definition of $g(\theta_t)$ in (2.6), labor market tightness is efficient if and only if $\beta = 1-\eta$.

Proof of Proposition 2.3

Proof. The second derivative with respect to θ_t of the tightness nullcline in (2.15) is

$$\begin{aligned}\frac{d^2 u_t}{d\theta_t^2} = &-\frac{1}{\alpha}\left[\frac{(r+\delta)k\theta_t}{(1-\beta)\theta_t^{\eta}} + g(\theta_t) + z\right]^{\frac{1-\alpha}{\alpha}}\left[\frac{k\beta s^*(\theta_t)}{(\gamma-1)(1-\beta)\theta_t} - \frac{(r+\delta)k(1-\eta)\eta}{(1-\beta)\theta_t^{\eta}\theta_t}\right] \\ &-\frac{1-\alpha}{\alpha^2}\left[\frac{(r+\delta)k\theta_t}{(1-\beta)\theta_t^{\eta}} + g(\theta_t) + z\right]^{\frac{1}{\alpha}-2}\left[\frac{(r+\delta)k(1-\eta)}{(1-\beta)\theta_t^{\eta}} + \frac{k\beta s^*(\theta_t)}{(1-\beta)}\right]^2.\end{aligned}$$

One can see that for $\alpha \leq 1$, the *tightness* nullcline is concave at least on the segment of the nullcline for which

$$s^*(\theta_t)\theta_t^{\eta} > (r+\delta)(1-\eta)\eta(\gamma-1).$$

Define ξ as the job finding rate equal to $(r+\delta)(1-\eta)\eta(\gamma-1)$, and $\chi \equiv \eta + (\gamma-1)^{-1}$. Now note that the *unemployment* nullcline is convex or has the shape of a negative logistic function. In particular, differentiate (2.14) twice with respect to θ, to obtain

$$\frac{d^2 u_t}{d\theta_t^2} = \frac{\delta\chi\left(\frac{\beta}{1-\beta}\frac{k}{\gamma}\right)^{\frac{1}{\gamma-1}}\theta_t^{\chi-2}\left[\delta(1-\chi) + (1+\chi)\left(\frac{\beta}{1-\beta}\frac{k}{\gamma}\right)^{\frac{1}{\gamma-1}}\theta_t^{\chi}\right]}{\left[\delta + \left(\frac{\beta}{1-\beta}\frac{k}{\gamma}\right)^{\frac{1}{\gamma-1}}\theta_t^{\chi}\right]^3}.$$

For $\chi \leq 1$ the second derivative is positive for all $\theta_t > 0$, so that the unemployment nullcline is convex. For $\chi > 1$, the second derivative can be positive or negative, depending on θ_t. More specifically, for $\chi > 1$ there exists a unique inflection point at the positive labor market tightness given by

$$\theta^* = \left[\frac{\delta(\chi-1)}{(1+\chi)\left(\frac{\beta}{1-\beta}\frac{k}{\gamma}\right)^{\frac{1}{\gamma-1}}}\right]^{\frac{1}{\chi}}.$$

Consequently, for $\chi > 1$ the unemployment nullcline is concave for $0 < \theta_t < \theta^*$, and convex for all $\theta_t > \theta^*$, so that as a whole it has the shape of a negative logistic function.

Given that the unemployment nullcline is convex or negative logistic, if any steady state with economic activity exists, generically exactly two steady states with economic activity exist if the tightness nullcline lies below the unemployment nullcline for any potential non-concave segment of the former. In that case, the concave segment of the tightness nullcline intersects at most twice with the unemployment nullcline. A sufficient condition for any non-concave segment of the tightness nullcline to lie below the unemployment nullcline is the maximum unemployment rate giving rise to any vacancy creation ($u_{\theta=0}$ as given by (2.16)) to be lower than the unemployment rate consistent with the job finding rate ξ. Consequently, assuming the existence of a steady state in the positive quadrant, for $\alpha \leq 1$ generically exactly two steady states exist if

$$z > \left(1 - \frac{\delta}{\delta + \xi}\right)^{\alpha}.$$

□

Proof of Proposition 2.6

This proof and the next can be more concisely written after a change in coordinates from labor market tightness θ_t to match surplus p_t. To point out the similarities with Mortensen (1999), I also change u_t to n_t. Lemma 2.8 then first shows equivalence between Mortensen's system in p_t and n_t and the one presented here. It is proven by the recognition that there is a smooth one-to-one correspondence between employment and unemployment, and surplus and tightness respectively. Following the definition of Kuznetsov (2004, p. 42), two smooth systems $\dot{x} = \mu(x)$, $x \in \mathbb{R}^n$ and $\dot{y} = \nu(y)$, $y \in \mathbb{R}^n$ are not only topologically equivalent, but also smoothly equivalent if (1) an invertible map $f : \mathbb{R}^n \to \mathbb{R}^n$ exists such that $y = f(x)$, if (2) this map is smooth together with its inverse, and if (3) f can be used to change coordinates such that holds that $\mu(x) = M^{-1}(x)\nu(f(x))$, where $M(x) = df(x)/dx$ is the Jacobian matrix of $f(x)$ at x. As a result, f is not only a homeomorphism, but also a diffeomorphism.

Lemma 2.8. *The dynamical system in unemployment u_t and labor market tightness θ_t and Mortensen (1999)'s dynamical system in employment n_t and surplus p_t for $\beta(\theta_t) = \beta$ and a positive value of leisure z are smoothly equivalent for all equilibria with economic activity.*

Proof. My dynamical system in u_t and θ_t is for all equilibria with economic activity given by the two smooth differential equations in (2.13) and (2.9), for convenience reprinted below

$$\begin{aligned} \dot{\theta}_t &= (r+\delta)\frac{\theta_t}{1-\eta} + (1-\beta)\frac{\theta_t^\eta}{k(1-\eta)}\left[g(\theta_t) + z - (1-u_t)^\alpha\right], \\ \dot{u}_t &= \delta(1-u_t) - s^*(\theta_t)\,u_t\theta_t^\eta, \end{aligned}$$

with $u_t \in [0,1]$ and $\theta_t > 0$. The dynamical system of Mortensen (1999) extended with z follows from (2.7) and the definition of the labor force. For all interior equilibria, it is given by the following two smooth differential equations

$$\dot{p}_t = (r+\delta)p_t + g(p_t) + z - n_t^\alpha, \tag{2.26}$$

$$\dot{n}_t = h(p_t)(1-n_t) - \delta n_t, \tag{2.27}$$

with $p_t > 0$ and $n_t \in [0,1]$, and where $h(p_t) = s^*(\theta_t)\theta_t^\eta$ and $g(p_t) = g(\theta_t)$, for the invertible map defined by

$$p_t = \frac{k\theta_t}{(1-\beta)\theta_t^\eta}, \tag{2.28}$$

$$n_t = 1 - u_t. \tag{2.29}$$

Nash bargaining implies $J_t = (1-\beta)p_t$, so that the first equation follows from the free-entry condition in (2.1), while the second is true by definition. Both equations are smooth together with their inverses, so that they satisfy the second requirement as well. The Jacobian matrix of this diffeomorphism is given by

$$M(x) = \begin{pmatrix} \frac{k(1-\eta)}{(1-\beta)\theta_t^\eta} & 0 \\ 0 & -1 \end{pmatrix}.$$

If we apply the map in (2.28) and (2.29), then indeed

$$\begin{pmatrix} (r+\delta)\frac{\theta_t}{1-\eta} + (1-\beta)\frac{\theta_t^\eta}{k(1-\eta)}\left[g(\theta_t) + z - (1-u_t)^\alpha\right] \\ \delta(1-u_t) - s^*(\theta_t)\,u_t\theta_t^\eta \end{pmatrix} = \begin{pmatrix} \frac{(1-\beta)\theta_t^\eta}{k(1-\eta)} & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} (r+\delta)\frac{k\theta_t}{(1-\beta)\theta_t^\eta} + g(\theta_t) + z - (1-u_t)^\alpha, \\ s^*(\theta_t)\,u_t\theta_t^\eta - \delta(1-u_t) \end{pmatrix},$$

so that the two systems also satisfy the last of the three requirements. As a result, they are smoothly equivalent as long as $\theta > 0$. □

Lemma 2.8 shows that the two systems is the same system written in different coordinates, retaining the same eigenvalues of the corresponding equilibria and the same periods of the corresponding limit cycles (Kuznetsov, 2004, p. 42). I can thus prove Proposition 2.6 in n_t and p_t.

Proof. The system in (2.26) and (2.27) is characterized by Hamiltonian dynamics if the discount rate r is zero and the sharing rule is efficient. The Hamiltonian function is

$$H\left(p_t, n_t\right) = \int_0^{n_t} \phi(x)dx + (1-n_t)[g\left(p_t\right) + z] - \delta p_t n_t, \tag{2.30}$$

as can be checked by noting that $\partial H/\partial p_t = \dot{n}_t$ and $\partial H/\partial n_t = -\dot{p}_t$ for $\beta = 1-\eta$ and $r = 0$. Indeed, remember that $h\left(p_t\right) = s^*\left(\theta_t\right)\theta_t^{\eta}$ and $g\left(p_t\right) = g\left(\theta_t\right)$ for the map in (2.28), so that $g\left(p_t\right) = \int_0^p h(q)dq$.

Although a homoclinic orbit generically exists in this Hamiltonian system, for $z > 0$ part of this homoclinic orbit may fall outside the positive quadrant. Define $n_{\theta=0} \equiv 1 - u_{\theta=0}$ as the employment level at the intersection of the tightness nullcline with the unemployment axis (thus with $u_{\theta=0}$ as defined in (2.16)). Note that $(1-u_H+\alpha z^{1/\alpha})^{\alpha+1} < (1-\alpha)\left[(1-u_H)\left(z + k\theta_t^{1-\eta}/(1-\beta)\right) - u_H g\left(\theta_H\right)\right]$ is equivalent to $H\left(p_H, n_H\right) < H\left(0, n_{\theta=0}\right)$. If and only if the latter holds, the homoclinic orbit is entirely situated in the positive quadrant. Because in a Hamiltonian system all equilibrium paths are level curves, combinations of n and p on the homoclinic orbit have the same value of the Hamiltonian as the saddlepoint on it. The laws of motion in (2.26) and (2.27) show that the antisaddle L is a local minimum in the system. Moving along the continuum of surrounding closed orbits, the largest possible closed orbit in the positive quadrant lies on $n_{\theta=0}$. Consequently, with $H\left(p_H, n_H\right) < H\left(0, n_{\theta=0}\right)$, a homoclinic orbit connecting H to itself lies entirely in the positive quadrant.[18]

For a small perturbation towards positive discounting and a smaller than efficient β two steady states with economic activity continue to exist by continuity. Melnikov perturbation (see e.g. Guckenheimer and Holmes (1983, p. 184)) shows that the same

[18] Substituting (2.16) into (2.7) it can be checked for $z, \alpha > 0$ that $H\left(0, n_{\theta=0}\right) < H(0,0) = z$, so that $H\left(p_H, n_H\right) < H\left(0, n_{\theta=0}\right)$ implies $H\left(p_H, n_H\right) < H(0,0)$. The latter ensures that the saddlepoint on the homoclinic orbit is steady state H rather than the no-trade steady state.

holds for the homoclinic orbit. The differential vector system allowing for a small distortion such that $r > 0$ and $\beta < 1-\eta$ is defined by

$$\dot{x}_t = F(x_t) + \varepsilon G(x_t) \text{ with } x_t = \begin{pmatrix} p_t \\ n_t \end{pmatrix},$$

$$F(x_t) = \begin{bmatrix} F_1(x_t) \\ F_2(x_t) \end{bmatrix} = \begin{bmatrix} -\frac{\partial H(p_t,n_t)}{\partial n_t} \\ \frac{\partial H(p_t,n_t)}{\partial p_t} \end{bmatrix} = \begin{bmatrix} \delta p_t + \int_0^{p_t} h(q)dq - n_t^\alpha + z \\ h(p_t)[1-n_t] - \delta n_t \end{bmatrix},$$

$$G(x) = \begin{bmatrix} G_1(x) \\ G_2(x) \end{bmatrix} = \begin{bmatrix} rp_t + g(p_t) - \int_0^{p_t} h(q)dq \\ 0 \end{bmatrix},$$

where ε is a small positive number. $F(x_t)$ is the Hamiltonian vector field, and $\varepsilon G(x_t)$ is a perturbation attributable to positive discounting and a smaller than efficient bargaining power.

Because the perturbation is time independent, the Melnikov function $M(p_t, n_t)$ is simply

$$M(p_t, n_t) = \int_\Gamma \left[r + h(p_t)(\frac{\beta}{1-\eta} - 1) \right] dp_t dn_t,$$

where $\Gamma = \{x \in \mathbb{R}^2 | H(x) \leq H(p_H, n_H)\}$ is the area enclosed by the homoclinic orbit in the Hamiltonian system. Note that the Melnikov function is independent of ε. Now $\beta_{SL} < 1-\eta$ can be chosen to target any sufficiently small $r = \hat{r} > 0$ with

$$\hat{r} = \frac{\int_\Gamma \left[h(p_t)\left(1 - \frac{\beta_{SL}}{1-\eta}\right)\right] dp_t dn_t}{\int_\Gamma dp_t dn_t}.$$

For $r = \hat{r}$ the Melnikov function has a simple zero at β_{SL}, so that for a sufficiently small distortion a homoclinic orbit in p_t and n_t continues to exist and remains in the positive quadrant. By Lemma 2.8 the same must hold for the system in θ_t and u_t.

According to the Andronov-Leontovich theorem, a family of limit cycles bifurcates on one side of this homoclinic orbit, and these are stable if the trace of the Jacobian matrix at saddlepoint H is negative. Because the homoclinic orbit is proven for a perturbed Hamiltonian system, the trace is only based on the distortion, and is simply equal to ε times the integrand of the Melnikov function at H:

$$\text{tr}(H) = \varepsilon \left[r + h(p_H)\left(\frac{\beta}{1-\eta} - 1\right)\right].$$

Given that $\beta < 1-\eta$, the integrand of the Melnikov function is monotonically decreasing in p_t. Consequently, when the Melnikov function is zero, the tr(H) is negative for values of β in the neighborhood of β_{SL}, so that the limit cycles are stable. □

Proof of Proposition 2.7

Proof. Because the proof is more concisely written in surplus than in tightness, I present it for Mortensen (1999)'s system extended with $z > 0$. Remember that by Lemma 2.8 the two systems are smoothly equivalent so that the eigenvalues are the same. The nullclines of the dynamical system in (2.26) and (2.27) are

$$(r+\delta)p_t + g\left(p_t\right) + z = (n_t)^\alpha \tag{2.31}$$

$$n_t = \frac{h\left(p_t\right)}{h\left(p_t\right)+\delta}, \tag{2.32}$$

and its nonzero Jacobian matrix is

$$J = \begin{pmatrix} r+\delta+g'\left(p_t\right) & -\alpha\frac{(n_t)^\alpha}{n_t} \\ h'\left(p_t\right)(1-n_t) & -h\left(p_t\right)-\delta \end{pmatrix}.$$

Both eigenvalues are zero if and only if both the determinant and the trace are zero, so that

$$\mathrm{tr} = r + g'\left(p_t\right) - h\left(p_t\right) = 0, \tag{2.33}$$

$$\det = \alpha\frac{(n_t)^\alpha}{n_t}h'\left(p_t\right)(1-n_t) - (r+\delta+g'\left(p_t\right))(h\left(p_t\right)+\delta) = 0. \tag{2.34}$$

Remember that $g\left(p_t\right) = \beta/(1-\eta)\int_0^{p_t} h(q)dq$, and moreover that $h\left(p_t\right) = s^*\left(\theta_t\right)\theta_t^\eta$ so that using the map in (2.28) $h'\left(p_t\right)p_t/h\left(p_t\right) = (1-\eta+\eta\gamma)/((1-\eta)(\gamma-1)) \equiv \kappa$. Substituting (2.33) and the elasticities into (2.34) yields

$$\alpha\frac{(n_t)^\alpha}{n_t}\kappa\frac{h\left(p_t\right)}{p}(1-n) = (h\left(p_t\right)+\delta)^2.$$

Substituting the nullclines of (2.31) and (2.32),

$$\alpha\kappa\delta\frac{(r+\delta)p_t + g\left(p_t\right)+z}{p_t} = (h\left(p_t\right)+\delta)^2.$$

Consequently, both eigenvalues are non-degenerately zero in steady state if the function

$$B(p_t) = (h\left(p_t\right)+\delta)^2 - \alpha\kappa\delta\left[r+\delta+\frac{h\left(p_t\right)-r}{1+\kappa}+\frac{z}{p_t}\right], \tag{2.35}$$

has a simple zero. Because $\lim_{p_t\to\infty}B(p_t) = \infty$, $\lim_{p_t\to 0}B(p_t) = -\infty$, and $B(p_t)$ is continuous, this condition is satisfied by the Intermediate Value Theorem. Moreover, for $\alpha < 1$ the condition is satisfied only once, because $B'(p_t) > 0$ if $2 > \alpha\kappa/(1+\kappa)$.[19] □

[19]The same holds if $z = 0$, but then $\lim_{p_t\to 0}B(p_t) = \delta^2 - \alpha\delta\kappa\left(\delta+\kappa r/(\kappa+1)\right)$, so that (2.35) can only be zero for a sufficiently large α and κ. A sufficient condition met by Mortensen's numerical example and my calibration is $\alpha\kappa > 1$.

CHAPTER 3

Buying first or selling first in housing markets[20]

3.1 Introduction

A large number of households move within the same local housing market every year. Many of these moves are by owner-occupiers who buy a new property and sell their old housing unit. However, it takes time to transact in the housing market, so a homeowner that moves may end up either owning two units or being forced to rent for some period, depending on the sequence of transactions. Either of these two alternatives may be costly.[21] There is anecdotal evidence that the incentives to "buy first" (buy the new property before selling the old property) or "sell first" (sell the old property before buying the new one) may depend on the state of the housing market.[22] If the transaction sequence decisions of moving owner-occupiers in turn affect housing

[20]This chapter is based on Moen et al. (2015)

[21]The following quote from Realtor.com, an online real estate broker, highlights this issue: "If you sell first, you may find yourself under a tight deadline to find another house, or be forced in temporary quarters. If you buy first, you may be saddled with two mortgage payments for at least a couple months." (Dawson (2013))

[22]A common realtor advice to homeowners that have to move is to "buy first" in a "hot" market, when there are more buyers than sellers and prices are high or expected to increase, and "sell first" in a "cold" market, when there are more sellers than buyers and house prices are depressed or expected to fall. Anundsen and Larsen (2014) provides evidence on the response of the intentions of owner-occupiers to buy first or sell first to the state of the housing market using survey data for Norway. In Section 3.2 we provide direct evidence for this link using data for the Copenhagen housing market.

market conditions, there could be powerful equilibrium feedbacks with important consequences for housing market dynamics.

In this chapter we study a tractable equilibrium model of the housing market, which explicitly features trading delays and a transaction sequence decision for moving owner-occupiers. We show that the transaction sequence choices of moving homeowners can have powerful effects on the housing market and can lead to large fluctuations in the stock of houses for sale, time-on-market, transaction volume, and prices.

In the model, agents continuously enter and exit a local housing market. They have a preference for owning housing over renting, and consequently, search for a housing unit to buy. The market is characterized by a frictional trading process in the form of search-and-matching frictions. This leads to a positive expected time-on-market for buyers and sellers, which is affected by the tightness in the market – the ratio of buyers to sellers. Once an agent becomes an owner-occupier, he may be hit by an idiosyncratic preference shock over his life cycle. In that case he becomes "mismatched" with his current house and wants to move internally in the same housing market. To do that, the mismatched owner has to choose optimally the order of transactions – whether to buy a new housing unit first and then sell his old unit (buy first), or sell his old unit first and then buy (sell first). Given trading delays, this may lead to the agent becoming a double owner (owning two housing units) or a forced renter (owning no housing unit) for some time, which is costly. The expected time of remaining in such a state depends on the time-on-market for sellers and buyers, respectively.

If the costs incurred by a double owner or a forced renter are high relative to the costs of a mismatch, the mismatched owner prefers buying first over selling first whenever there are *more* buyers than sellers in the market, or more generally, when the buyer-seller ratio is high. The intuition for this behavior is the following. First, whenever it is more costly to be a double owner or a forced renter compared to being mismatched, a moving owner wants to minimize the delay between the two transactions. Second, when there are more buyers than sellers, the expected time-on-market is low for a seller and high for a buyer. Consequently, if a mismatched owner buys first he expects to spend a longer time as a buyer, and hence to remain mismatched longer. However, once he buys, he expects to stay with two houses for a short time while searching for a buyer for his old property. Conversely, choosing to sell first in that case implies a short time-to-sell and a short time of remaining mismatched but a longer time of searching to buy a new unit afterwards. Since the flow costs in between the two transactions are higher than during mismatch, buying first clearly dominates selling first in that case.

However, the order of transactions by moving owner-occupiers affects the buyer-seller ratio. Specifically, when mismatched owner-occupiers buy first, they crowd the buyer side of the market, and so, the market ends with more buyers than sellers in steady state. Nevertheless, this high buyer-seller ratio is consistent with the incentives of mismatched owners to buy first. Conversely, when all mismatched owners sell first, there are more sellers than buyers in steady state. However, a low buyer-seller ratio is consistent with the incentives of mismatched owners to sell first. Therefore, the interaction between the behavior of mismatched owners and the buyer-seller composition of the housing market creates a *strategic complementarity* in their transaction sequence decisions, which in turn may lead to multiple steady state equilibria. In one steady state equilibrium (a “Sell first” equilibrium), mismatched owners prefer to sell first, the market tightness is low and the expected time-on-market for sellers is high. In the other steady state equilibrium (a “Buy first” equilibrium), mismatched owners prefer to buy first, the market tightness is high and the expected time-on-market for sellers is low.[23]

To illustrate this strategic complementarity more clearly, we first assume that prices are fixed across steady states, or alternatively, that the rental price is equal to the house price times the discount rate, in which case prices do not influence the transaction sequence decision. After explaining this channel and its implications, in a separate section, we endogenize house prices. First, we assume that the steady state price level is an increasing function of the buyer-seller ratio. We show that even with a rental price that is constant, multiplicity exists as long as the responsiveness of the house price to the buyer-seller ratio is not too high. After clarifying this countervailing effect, we show that there can exist multiple equilibria when house prices are (endogenously) determined by Nash bargaining, and hence, both differ across trading pairs and respond to changes in the buyer-seller ratio. Specifically, the main channel that drives equilibrium multiplicity in our benchmark model – the strategic complementarity resulting from the interplay between mismatched owners' transaction sequence decisions and the stock-flow conditions that determine the equilibrium market tightness – is still present in that environment. We also show that there can be equilibrium multiplicity even in an environment with competitive search where agents can trade off prices and time-on-market (queue length).

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[23]Note that we derive this multiplicity under the assumption of a constant returns to scale matching function. Therefore, the strategic complementarity does not arise from increasing returns to scale in matching as in Diamond (1982).

We also analyze switches between the “Buy first” and “Sell first” equilibria. Such switches lead to fluctuations in the housing market. Specifically, moving from the “Buy first” to the “Sell first” equilibrium is associated with an increase in the for-sale stock and time-on-market for sellers, and a drop in transactions. This behavior is broadly consistent with evidence on the housing cycle. In particular, explaining the negative comovement between the for-sale stock and transactions has been a challenge for search-based models of the housing market (Diaz and Jerez, 2013). As we show in a simple numerical example, the fluctuations generated by these switches can be substantial. Also, when prices are determined by Nash bargaining, we show that there can be substantial house price fluctuations arising from these equilibrium switches.

Finally, we show that when house prices respond to changes in the buyer-seller ratio, there can exist equilibria with self-fulfilling fluctuations in prices and market tightness. Since mismatched owners are more likely to buy first (sell first) when they expect price appreciation (depreciation), they end up exerting a destabilizing force on the housing market. For example, if agents expect prices to depreciate, they are more likely to sell first. However, this decreases the buyer-seller ratio, which in turn drags down house prices, and thus, confirms the agents’ expectations.

Related literature. The chapter is related to the growing literature on search-based models of the housing market and fluctuations in housing market liquidity, initiated by the seminal work of Wheaton (1990). This foundational paper is the first to consider a frictional model of the housing market to explain the existence of a “natural” vacancy rate in housing markets and the negative comovement between deviations from this natural rate and house prices. In that model, mismatched homeowners must also both buy and sell a housing unit. However, the model implicitly assumes that the cost of becoming a forced renter with no housing is prohibitively large, so that mismatched owners always buy first. As we show in our chapter, allowing mismatched owners to endogenously choose whether to buy first or sell first has important consequences for the housing market.

The chapter is particularly related to the literature on search frictions and propagation and amplification of shocks in the housing market (Krainer (2001), Novy-Marx (2009), Caplin and Leahy (2011), Diaz and Jerez (2013), Head et al. (2014), Ngai and Tenreyro (2014), Guren and McQuade (2013), Anenberg and Bayer (2015), Ngai and Sheedy (2015)). This literature shows how search frictions naturally propagate aggregate shocks due to the slow adjustment in the stock of buyers and sellers. Additionally, they can amplify price responses to aggregate shocks, which in Walrasian models would be fully absorbed by quantity responses.[24]

Diaz and Jerez (2013) calibrate a model of the housing market in the spirit of Wheaton (1990) where mismatched owners must buy first, as well as a model where they must sell first. They show that each model explains some aspects of the data on housing market dynamics, pointing to the importance of a model that contains both choices. Other models of the housing market assume that the sequence of transactions is irrelevant, which implicitly assumes that the intermediate step of a transaction sequence for a moving owner is costless (Caplin and Leahy (2011), Ngai and Tenreyro (2014), Head et al. (2014), Guren and McQuade (2013), Ngai and Sheedy (2015)).

Ngai and Sheedy (2015) model an endogenous moving decision based on idiosyncratic match quality as an amplification mechanism of sales volume. The paper shows how the participation decisions of mismatched owners in the housing market can explain why time-on-market for sellers can decrease while the stock of houses for sale increases at the same time, as was the case during the housing boom of the late 90s and early 2000s. In our model we assume that mismatched owners always participate and instead focus on their transaction sequence decisions. The implications we draw from our analysis are therefore complementary to the insights in their paper.

Anenberg and Bayer (2015) is a recent contribution that is close to our chapter, particularly in terms of motivation. The chapter studies a rich quantitative model of the housing market to explain the large volatility of internal movements by existing owners, which they argue is the main driver of fluctuations in housing transactions. The authors calibrate their model to data for Los Angeles and show how shocks to the flow of new buyers can be amplified through the decisions of existing owners and can lead to substantial fluctuations in prices and volume. Similar to our model, existing owners in their model also have to both buy and sell in the same market. Unlike our model, however, existing owners in their model simultaneously search both on the buyer and seller side, so they do not choose to bias their search decisions. With some probability, both a buy and a sell offer may arrive in the same period, in which case an owner has to choose to conduct one of the two transactions and forfeit the possibility of conducting the other. Otherwise, whether a buy offer arrives before a sell offer or

[24] The chapter is also broadly related to the Walrasian literature on house price dynamics and volatility (Stein (1995), Ortalo-Magne and Rady (2006), Glaeser et al. (2014).

vice versa is exogenous from the point of view of individual owners. In contrast, in our model, mismatched owners choose which side of the market to enter and whether to receive only offers from buyers or from sellers. This endogenous choice by existing owners to bias their search decisions influences market tightness and buyer and seller times-on-market, which in turn influence the choices of other existing owners. The strategic complementarity resulting from this feedback between market tightness and the decisions of mismatched owners leads to multiplicity, self-fulfilling fluctuations, and housing market volatility in our model. Therefore, the theoretical focus of our chapter and the different mechanism that we explore make our chapter complementary to this study.

The paper is also related to the literature on multiple equilibria and self-fulfilling fluctuations as the result of search frictions. Multiple equilibria in that literature arise mainly from increasing returns to scale in matching (Diamond (1982)) or from the interactions between several frictional markets (Howitt and McAfee (1988)).[25] In contrast, multiplicity in our model arises in a single market with constant returns to scale in matching. Other sources of multiplicity in models with search frictions include an indeterminacy in the division of the match surplus (Howitt and McAfee (1987), Farmer (2012), Kashiwagi (2014)) or the interaction between the outside option of matched market participants and their endogenous separation decisions (Burdett and Coles (1998), Coles and Wright (1998), Burdett et al. (2004), Moen and Rosén (2013), Eeckhout and Lindenlaub (2015)). In our paper, the division of the match surplus is determined by a fixed price, by Nash bargaining, or by competitive search, so that the indeterminacy of a bilateral monopoly is not exploited. Also, separation is exogenous in our framework.

Section 3.5 that extends our results to environments where prices are endogenously determined, relates the chapter to the literature on Nash bargaining and competitive search in the housing market. In particular, Albrecht et al. (2007) study a search model of the housing market where buyers and sellers may become "desperate" if they search unsuccessful for too long. Because prices in their model are determined by Nash bargaining, the presence of agents with heterogeneous flow values while searching results in compositional effects that also arise in the extension of our model with Nash

[25] Similar papers include Drazen (1988), Diamond and Fudenberg (1989), Mortensen (1989), Howitt and McAfee (1992), Boldrin et al. (1993), Mortensen (1999), Kaplan and Menzio (2016), Chéron and Decreuse (2016), and the previous chapter, among others.

bargaining. However, in their model equal numbers of buyers and sellers enter the market at an exogenous rate, so that there is no transaction sequence decision.[26] Finally, our extension in Section 3.5.3 to a model of the housing market with competitive search relates the chapter to recent models of competitive search in housing and asset markets (Diaz and Jerez (2013), Lester et al. (2015), Albrecht et al. (2016), and Lester et al. (2013)).

The rest of the chapter is organized as follows. In the next section we present some motivating facts using individual level data from Denmark. We also explain why modeling the transaction sequence of moving owners as an explicit choice rather than a passive outcome of simultaneous search on both sides of the market is important for consistency with the data. Section 3.3 presents the basic model of the housing market, and Section 3.4 characterizes the decisions of mismatched owners and discusses the equilibrium multiplicity and the implications of equilibrium switches. Section 3.5 shows that there can be equilibrium multiplicity in an environment where prices are determined by Nash bargaining and also in an environment with competitive search, and discusses some additional extensions. Section 3.6 shows how the incentives of mismatched owners to buy first or sell first depend on price expectations and shows that there can exist equilibria with self-fulfilling fluctuations in house prices and tightness, while section 3.7 concludes.

3.2 Motivating facts

We start by providing some motivating facts about the transaction sequence decisions of owner-occupiers for Copenhagen, Denmark. We focus on the Copenhagen urban area for the period 1992-2010. We use the Danish ownership register, which records the property ownership of individuals and legal entities as of January 1st of a given year. We combine that with a record of property sales for each year. The unique owner and property identifiers give us a matched property-owner data set, which we use to keep track of the transactions of individuals over time. We focus on individual owners who are recorded as the primary owner of a property.

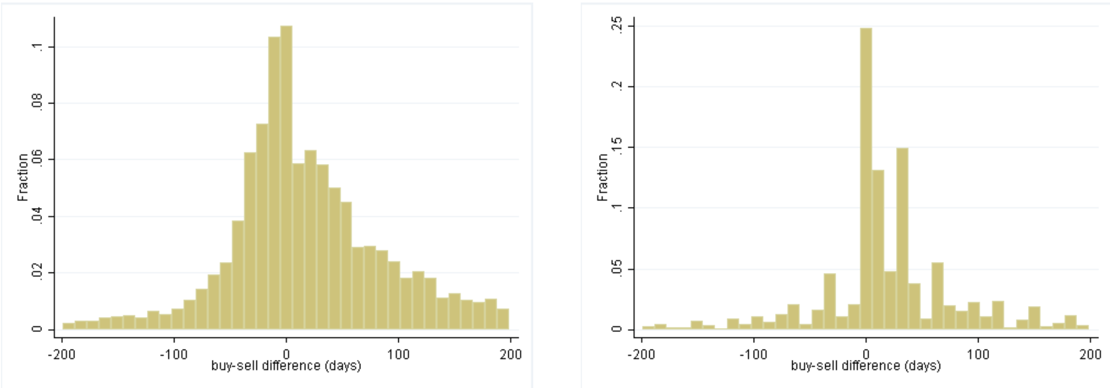
[26] Maury and Tripier (2014) study a modification of the Wheaton (1990) model, in which mismatched owners can buy and sell simultaneously, which they use to study price dispersion in the housing market. However, they do not consider the feedback from buying and selling decisions on the stock-flow process and on market tightness. This feedback is key for the mechanisms we explore in our chapter.

We use the ownership records of individual owners over time to identify owner-occupiers who buy and sell in the Copenhagen housing market.[27] We then use the property sales record to determine the agreement dates (the dates the sale agreement is signed) and closing dates (the dates the property formally changes ownership) for the two transactions. Based on those, we construct a variable that measures the time difference between the sale of the old property and the purchase of the new property. If the difference is positive, the owner-occupier buys first, if it is negative he sells first.

Figure 8 shows the distribution of the time difference between the agreement dates (Panel 8a) and closing dates (Panel 8b) for owner-occupiers who both buy and sell in Copenhagen during our sample period. There is substantial dispersion in the time difference between agreement dates, which suggests that a large fraction of these homeowners cannot synchronize the two transactions on the same date. Specifically, there is substantial mass even in the tails of the distribution. Examining the difference in closing dates shows a similar picture. Even though the distribution is more compressed in that case, since homeowners try to a greater extent to synchronize the closing dates, so that they occur on the same day or in a close interval, a large fraction of homeowners face a time difference of a month or more in between closing the two transactions. Overall, these distributions suggest that for homeowners that buy and sell in the same housing market the time difference between transactions can be substantial, confirming the anecdotal evidence cited in the introduction.

Another important observation is that the two distributions are right skewed, so moving homeowners tend to buy first during our sample period. This is confirmed when we examine the time series behavior of the fraction of homeowners that are identified as buying first in a given year from 1993-2008, as Figure 9 shows. Similar to Figure 8, the left-hand panel (Panel 9a) is based on agreement dates, while the right-hand panel (Panel 9b) is based on closing dates. Both panels also contain a price index for single family homes for the Copenhagen housing market. As the figure shows, the fraction of owners that buy first is not constant over time but exhibits wide variations going from a low of around 0.3 in 1994 to a high of 0.8 in 2006 and then back to a low of around 0.4 in 2008. This fraction tracks closely the house price index increasing over most of the sample period and peaking in the same year. It is then followed by a substantial drop as

[27] Appendix 3.8.A contains detailed information on the data used and on the procedure for identifying owner-occupiers that buy and sell. Given the way we identify these owner-occupiers, we have a consistent count for the number of owners who buy first or sell first in a given year for the years 1993 to 2008.



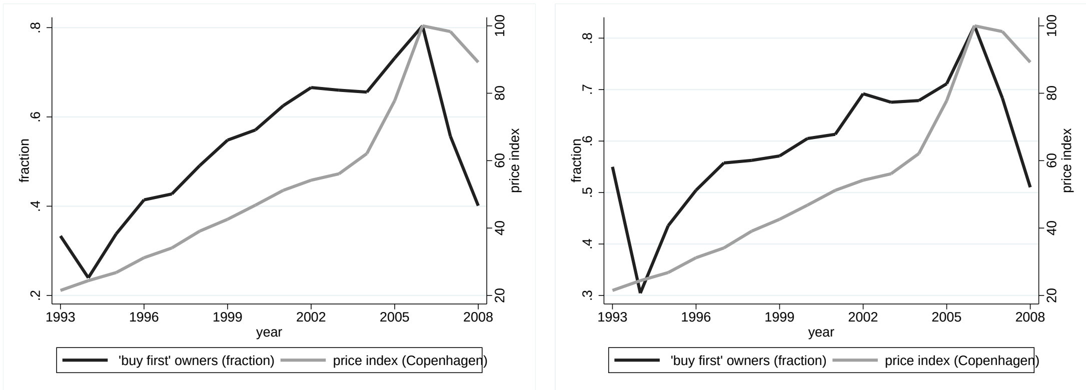
(a) (b)

Figure 8: Distribution of the time difference between “sell” and “buy” agreement dates (a) and closing dates (b) for homeowners who both buy and sell in Copenhagen (1993-2008).

house prices start to decline after 2006. Therefore, Figure 9 suggests that the decisions to buy first may be related to the state of the housing market.

A closer examination of the period 2004-2008 strengthens this conjecture. Specifically, Figure 10 illustrates the fluctuations in key housing market variables like the for-sale stock, seller time-on-market, transaction volume and prices for Copenhagen in the period 2004-2008. It also includes our constructed fraction of buy first owners for Copenhagen in the period 2004-2008. During the first half of this period seller time-on-market (TOM), and the for-sale stock are low, while transaction volume and the fraction of buy first owners are high. There is a switch in all of these series around the 3rd quarter of 2006 and a quick reversal during which seller time-on-market and the for-sale stock increase rapidly, while the fraction of moving owners that buy first drops. Transaction volume is also lower during the second half of this period. Prices increase during the first half of the period and then decline.

We take these three exhibits as indication that there is a non-trivial transaction sequence choice for owner-occupiers that move in the same housing market, that the time difference between the two transactions can be substantial, and that the decision to buy first or sell first is related to the state of housing markets. These facts motivate our theoretical study below.



(a) (b)

Figure 9: Fraction of owners who "buy first" and housing market conditions in Copenhagen (1993-2008). Panel a is based on agreement dates, and Panel b is based on closing dates.

Notes: The series on the fraction of "buy first" owners is from own calculations based on registry data from Statistics Denmark. See Appendix 3.8.A for a description on how we identify an owner that buys and sells in Copenhagen (a buyer-and seller) as a "buy first" ("sell first") owner. We compute annual counts of the number of "buy first" and "sell first" owners by looking at the year of the first transaction for each of these owners. The fraction of "buy first" owners is then the proportion of buyer-and-seller owners that buy first. For Panel a the identification of an owner as buy first/sell first is based on the difference in the two agreement dates. For Panel b, it is based on the difference in the two closing dates. The price index is a repeat sales price index for single family houses for Copenhagen (Region Hovedstaden) constructed by Statistics Denmark.

3.2.1 Why an explicit transaction sequence choice?

Before proceeding with our model, we make the following important conceptual point. Suppose that rather than explicitly choosing how to conduct the sequence of transactions, moving owners always enter both sides of the market simultaneously. Then they simply take whichever trading opportunity comes first. Thus, they are *observed* to "buy first" whenever they happen to meet a seller before a buyer and vice versa. We refer to this as a simultaneous search strategy.

Suppose that the market is frictional and there are trading delays. With standard assumptions on the matching technology, the rate at which a buyer meets a seller is decreasing in the buyer-seller ratio in the market, while the rate at which a seller meets a buyer is increasing in the buyer-seller ratio. Hence, a simultaneous search strategy

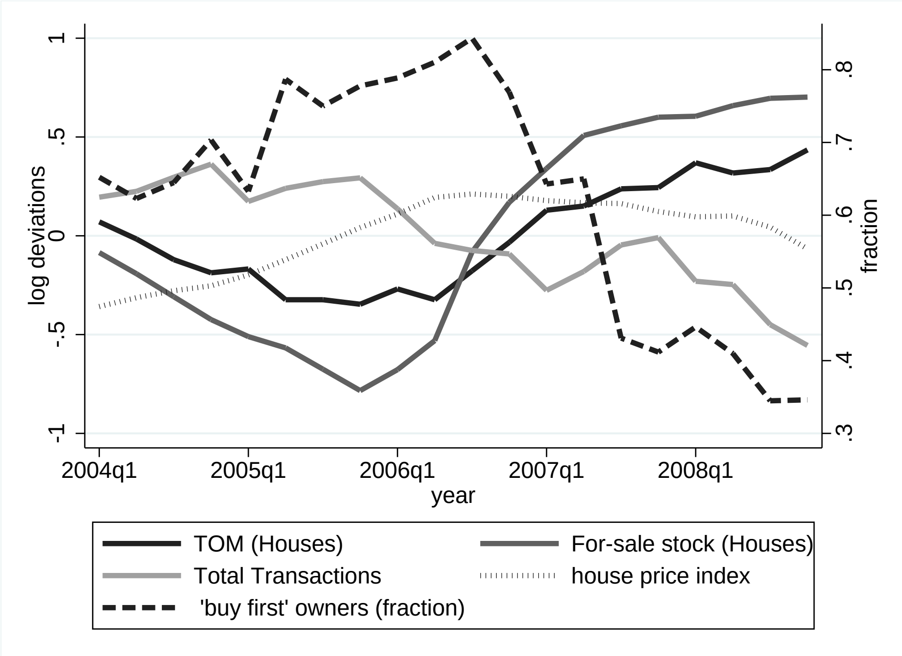


Figure 10: Housing market dynamics, Copenhagen 2004-2008

Notes: Data on seller time-on-market (TOM) and the for-sale stock is from the Danish Mortgage Banks' Federation (available at http://statistik.realkreditforeningen.dk/BMSDefault.aspx). These series are shown in log deviations from their sample mean. The total transaction volume is from Statistics Denmark. It is in log deviations from the sample mean after controlling for seasonal effects by quarter-in-year dummies. The fraction of buy first owners is from own calculations based on registry data from Statistics Denmark. See Appendix 3.8.A for a description on how we identify an owner that buys and sells in Copenhagen (a buyer-and-seller) as a "buy first" ("sell first") owner. We compute quarterly counts of the number of "buy first" and "sell first" owners by looking at the quarter of the first transaction for each of these owners. The fraction of "buy first" owners is then the proportion of buyer-and-seller owners that buy first. The identification of an owner as buy first/sell first is based on the difference in the two agreement dates.

implies that the observed fraction of agents that buy first is decreasing in the buyer-seller ratio. The (steady state) average time-on-market is the inverse of the meeting rate of sellers, and hence is also decreasing in the buyer-seller ratio. It follows that if the agents use a simultaneous search strategy, the fraction of owners that are observed to buy first and the seller time-on-market should move in tandem – one should observe fewer owners "buying first" whenever seller time-on-market is low and vice versa. However, this is counterfactual, in view of Figure 10.[28]

This simple example shows (without reference to the optimization decision of agents) that to be consistent with the data, mismatched owners must explicitly choose to (predominantly) search only on one side of the market, thus steering the sequence of their transactions, rather than to search simultaneously on both sides and having the sequence of their transactions be determined by the exogenous arrival of trading counterparties. Moreover, as we show in our model, when agents' optimizing decisions are taken into account, under a naturally satisfied parametric assumption on preferences, mismatched owners rationally choose to bias their search towards one side of the market in a way that leads to aggregate behavior that is consistent with the data.

3.3 Model

In this section, we set up the basic model of a housing market characterized by trading frictions and re-trading shocks that will provide the main insights of our analysis.

3.3.1 Preferences

Time is continuous. The housing market consists of a unit measure of durable housing units that do not depreciate, and a unit measure of households, which we refer to as agents. The agents are risk neutral and can borrow and lend freely at interest rate $r > 0$. When an agent buys a house and becomes a homeowner, he receives a flow utility of $u > 0$. We say that the homeowner is *matched*. With a Poisson rate γ the matched homeowner is hit by a taste shock, and becomes *mismatched* with his current housing unit. In that case the homeowner obtains a flow utility of $u - \chi$, for $0 < \chi < u$. A mismatched owner has to move to another housing unit in order to become matched

[28] To show this formally, below we denote the buyer-seller ratio by θ, the rate at which sellers meet buyers by $\mu(\theta)$, where $\mu(\theta)$ is increasing in θ, and the rate at which a buyer meets a seller by $q(\theta) = \frac{\mu(\theta)}{\theta}$, where $q(\theta)$ is decreasing in θ. If owners follow a simultaneous search strategy, in steady state, the ratio of owners observed to buy first relative to those observed to sell first is

$$\frac{q(\theta)}{\mu(\theta)} = \frac{1}{\theta}. \tag{3.1}$$

Therefore, this ratio is decreasing in θ. Since θ and sellers' (average) time-on-market, $\frac{1}{\mu(\theta)}$, are negatively related, the fraction of owners observed to buy first and seller time-on-market should move in tandem as θ changes.

again. Taste shocks of this form are standard in search models of the housing market (Wheaton, 1990), and create potential gains from trading.[29]

A mismatched owner can choose to *sell first* (and become a *mismatched seller*) – selling the housing unit he owns first and then buying a new one. Alternatively, he can choose to *buy first* (becoming a *mismatched buyer*) – buying a new housing unit first and then selling his old one. Finally, the mismatched owner may choose to not enter the housing market and stay mismatched.[30]

We also assume that a mismatched owner cannot synchronize the two transactions (the selling and buying). For example, he cannot exchange houses with another mismatched owner, as there is no double coincidence of housing wants among owners.[31] Instead, a mismatched owner has to conduct the two transactions in a sequence. A mismatched buyer ends up holding two housing units simultaneously for some period. In this case we say that he becomes a *double owner*. Similarly, a mismatched seller ends up owning no housing. In that case the agent becomes a *forced renter*.

The utility flows during the transaction period (when the agent is a double owner or a forced renter) are key for our results. We assume that a double owner receives a flow utility of $u_2 < u$, while a forced renter receives a flow utility of $u_0 < u$. These flows do not include the cost of renting a house for a forced renter, or rental income from renting out the second house for the double owner (as will be clear below, we assume that the double owner rents out the second housing unit). For the double owner, u_2 includes

[29]Rather than introducing segmentation in the housing stock, we treat all housing units as homogeneous, so that mismatched owners participate in one integrated market with other agents. Although in reality agents move across housing market segments (whether spatial or size-based) in response to a taste shock of the type we have in mind, modeling explicitly several types of housing would substantially reduce the tractability of the model. Furthermore, defining empirically distinct market segments is not straightforward as in reality households often search in several segments simultaneously (Piazzesi et al., 2015).

[30]See our discussion in Section 3.2.1 for why an explicit choice of buying first versus selling first is necessary for explaining the patterns we observe in the data. Also, in Section 3.5.4, we explicitly allow a mismatched owner to search as a buyer and seller simultaneously, subject to a fixed time endowment. We show that, given the preference assumptions we make, mismatched owners rationally choose to bias their search towards only one side of the market depending on the market tightness, so the restriction to either only buying first or selling first is without loss of generality in this case. Finally, notice that searching simultaneously as both a buyer and seller does not mean that the agent can synchronize the two transactions, only that he chooses to receive offers from both potential buyers and sellers.

[31]This is similar to the lack of double coincidence of wants used in money-search models (Kiyotaki and Wright (1993)). In reality, some moving homeowners may be able to synchronize the two transactions as is also evident from Figure 8. Allowing some mismatched owners to synchronize their buy and sell transactions would reduce the tractability of the model without changing its qualitative predictions.

maintenance costs, costs (pecuniary and non-pecuniary) associated with renting out the second house – reflecting unmodelled frictions in the rental market, costs associated with bridge loans (over and above the interest rate), reflecting unmodelled frictions in the financial market, and so on. For the forced renter, u_0 includes relocation costs to a new temporary quarter, inconveniences and monetary costs associated with short-term renting, such as, for example, rent-back agreements, where a former owner rents his old house at a premium from the new owner, and other unmodelled frictions in the rental market, costs of storage of furniture, etc. As will be clear below, a driving assumption in our analysis is that u_2 and u_0 are less than $u - \chi$, i.e., the flow utilities to an agent during the transaction period are lower than when living as mismatched in his own house. For tractability, we also assume that a double owner does not experience mismatching shocks. This ensures that the maximum holdings of housing by an agent will not exceed two units in equilibrium.

Agents are born and die at the same rate g. New entrants start out their life without owning housing, and receive a flow utility $u_n < u$. Also, we assume that $u_n \geq u_0$, so that forced renters do not obtain a higher utility flow than new entrants. After a death/exit shock, an agent exits the economy immediately and obtains a reservation utility normalized to 0. If he owns housing, his housing units are taken over by a real-estate firm, which immediately places them for sale on the market.[32] Real-estate firms are owned by all the agents in the economy with new entrants receiving the ownership shares of exiting agents. Given the exit shock, agents effectively discount future flow payoffs at a rate $\rho \equiv r + g$. For notational convenience, we will directly use ρ later on.

Finally, agents without a house rent a dwelling. A landlord can simultaneously rent out a unit and have it up for sale. Hence double owners rent out one of their units, as do real-estate firms. The rental price is denoted by R.

3.3.2 Trading frictions and aggregate variables

The housing market is subject to trading frictions. These frictions are captured by a standard constant returns to scale matching function $m(B(t), S(t))$, mapping a stock $B(t)$ of searching buyers and a stock $S(t)$ of searching sellers to a flow m of new matches. We assume that there is random matching, so different types of agents meet with

[32] For simplicity, we assume that agents are not compensated for their housing upon exiting the economy. In Section 3.5.4, we discuss an extension to the a case where exiting agents are compensated for their housing by the real-estate firms.

probabilities that are proportional to their mass in the population of sellers or buyers. Directed search is discussed in Section 3.5.3. We define the market tightness in the housing market as the buyer-seller ratio, $\theta(t) \equiv B(t)/S(t)$. Additionally, $\mu(\theta(t)) \equiv m(B(t)/S(t),1) = m(B(t),S(t))/S(t)$ is defined as the Poisson rate with which a seller meets a buyer. Similarly, $q(\theta(t)) \equiv m(B(t),S(t))/B(t) = \mu(\theta(t))/\theta(t)$ is the rate with which a buyer meets a seller.

Beside the market tightness, $\theta(t)$, which will be relevant for agents' equilibrium payoffs, we keep track of a number of stock variables. Specifically, those include the new entrants (denoted by $B_n(t)$), matched owners ($O(t)$), mismatched buyers ($B_1(t)$), mismatched sellers ($S_1(t)$), double owners ($S_2(t)$), forced renters ($B_0(t)$) and the housing units sold by real-estate firms ($A(t)$). Therefore, the total measure of buyers is $B(t) = B_n(t)+B_0(t)+B_1(t)$ and the total measure of sellers is $S(t) = S_1(t)+S_2(t)+A(t)$. Since the total population is constant and equal to 1 in every instant, it follows that

$$B_n + B_0 + B_1 + S_1 + S_2 + O = 1. \tag{3.2}$$

Also, since the housing stock does not shrink or expand over time, the following housing ownership condition holds in every instant,

$$O + B_1 + S_1 + A + 2S_2 = 1. \tag{3.3}$$

Summing up, the life-cycle of an agent in the model proceeds as follows. An agent begins his life as a new entrant. With rate $q(\theta)$, he becomes a matched owner. Once matched, he becomes mismatched with rate γ. A mismatched owner chooses to either buy first (mismatched buyer) or sell first (mismatched seller). A mismatched buyer becomes a double owner with rate $q(\theta)$, who in turn sells and reverts to being a matched owner with rate $\mu(\theta)$. A mismatched seller becomes a forced renter with rate $\mu(\theta)$ and after that moves to being a matched owner with rate $q(\theta)$. In every stage of life an agent may exit the economy with rate g.

3.3.3 House price and rental price determination

We begin our analysis by assuming that the house price p is fixed and does not vary with the market tightness θ (or, as a special case, that the rental price is equal to the house price times the discount rate ρ, see the end of this subsection). However, in the equilibria we consider, the price p lies in the bargaining set of all actively trading pairs.

We progressively relax this assumption by assuming that p varies with θ in a reduced form-way in Section 3.5.1 and by assuming that prices are determined by symmetric Nash bargaining in Section 3.5.2 or in a competitive search equilibrium in Section 3.5.3. The main insights of our analysis hold in those environments as well, although at a significant reduction in tractability.[33]

In this paper, we do not explicitly model the rental market. Since there are equally many houses as there are agents in the economy, and all houses are either occupied by the owner or rented out, the supply of houses for rent is equal to the demand for houses for rent independently of the price (as long as all agents prefer to own rather than to rent) and independently of the transaction sequence of the agents. It follows that if the rental market is competitive, the rental price is indeterminate. In what follows, we assume that the rental price is constant, independently of θ. Furthermore, given the assumption that a real-estate firm can rent out a housing unit without costs, we require that

$$\rho p \geq R, \tag{3.4}$$

as otherwise the real estate firms would want hold on to their houses forever.

As alluded to in the beginning of this subsection, an interesting special case, which is our benchmark case below, arises when $R = p\rho$. As we show below, in this case, the (steady state) house price does not influence the sequence of transactions. A higher price makes it more attractive to sell first due to discounting, while higher rental price makes it more attractive to buy first. If $R = \rho p$, the two effects cancel out.

3.4 Steady state equilibria

We start by characterizing steady state equilibria of this economy. We first discuss the value functions of different types of agents in a candidate steady state equilibrium.

[33] Although the assumption that house prices are independent of θ is made for convenience, it may also be an equilibrium outcome in some environments. Given that the price is assumed to lie in the bargaining sets of all trading pairs, it can be derived as the market clearing price in a competitive market with frictional entry of traders. In particular, as in Duffie et al. (2005) or Rocheteau and Wright (2005), the total measure of participants in that competitive market is determined by the matching function $M(B,S)$. The transaction price in our case will be indeterminate, and this opens up for a price that is independent of θ. Also, under certain conditions, a unique fixed price that does not vary with tightness or across trading pairs can be microfounded as resulting from bargaining between heterogeneous buyers and sellers, in which the buyer has full bargaining power but does not know the type of the seller. As shown in Appendix 3.8.F, take-it-or-leave-it offers from buyers under private information about the seller's type can generate a fixed price that is equal to the present discounted value of rental income.

3.4.1 Value functions

We have a number of value functions for different agents in this economy. Specifically, we use the notation V^x, for the value function of a new entrant ($x = Bn$), a forced renter ($x = B0$), a mismatched buyer or seller ($x = B1$ or $x = S1$), a double owner ($x = S2$) and real-estate firm holding one housing unit ($x = A$). Finally, we denote the value function of a matched owner by V. Given these notations, we have a standard set of Bellman equations for the agents' value functions in a steady state equilibrium.

First of all, for a mismatched buyer we have

$$\rho V^{B1} = u - \chi + q(\theta) \max\left\{-p + V^{S2} - V^{B1}, 0\right\}, \tag{3.5}$$

where $u - \chi$ is the flow utility from being mismatched. Upon matching with a seller, a mismatched buyer purchases a housing unit at price p, in which case he becomes a double owner, incurring a utility change of $V^{S2} - V^{B1}$.

A double owner has a flow utility of $u_2 + R$ while searching for a counterparty. Upon finding a buyer, he sells his second unit and becomes a matched owner. Therefore, his value function satisfies the equation[34]

$$\rho V^{S2} = u_2 + R + \mu(\theta)\left(p + V - V^{S2}\right). \tag{3.6}$$

The value function of a mismatched seller is analogous to that of a mismatched buyer apart from the fact that a mismatched seller enters on the seller side of the market first and upon transacting becomes a forced renter. Therefore,

$$\rho V^{S1} = u - \chi + \mu(\theta) \max\left\{p + V^{B0} - V^{S1}, 0\right\}. \tag{3.7}$$

Finally, for a forced renter we have

$$\rho V^{B0} = u_0 - R + q(\theta)\left(-p + V - V^{B0}\right). \tag{3.8}$$

The remaining value functions are straightforward and are given in Appendix 3.8.B.

[34]We present the value functions of double owners and forced renters assuming that they always trade at the price p, since that will always be the case in the steady state equilibria we consider. For example, for the case of a double owner we have $V + p \geq (u_2 + R)/\rho$. Appendix 3.8.B provides a set of sufficient conditions for this to hold.

3.4.2 Optimal choice of mismatched owners

In a steady state equilibrium, the optimal decision of mismatched owners depends on the simple comparison

$$V^{B1} \gtreqless V^{S1}. \tag{3.9}$$

We can substitute for V^{B0} and V^{S2} from equations (3.8) and (3.6) into the value functions for a mismatched buyer and seller to obtain

$$V^{B1} = \max\left\{\frac{u-\chi}{\rho}, \frac{u-\chi}{\rho+q(\theta)} + \frac{q(\theta)\left(u_2-(\rho p-R)\right)}{\left(\rho+\mu(\theta)\right)\left(\rho+q(\theta)\right)} + \frac{q(\theta)\mu(\theta)}{\left(\rho+\mu(\theta)\right)\left(\rho+q(\theta)\right)}V\right\}, \tag{3.10}$$

and

$$V^{S1} = \max\left\{\frac{u-\chi}{\rho}, \frac{u-\chi}{\rho+\mu(\theta)} + \frac{\mu(\theta)\left(u_0+(\rho p-R)\right)}{\left(\rho+\mu(\theta)\right)\left(\rho+q(\theta)\right)} + \frac{q(\theta)\mu(\theta)}{\left(\rho+\mu(\theta)\right)\left(\rho+q(\theta)\right)}V\right\}. \tag{3.11}$$

We define the effective utility flow for a forced renter as $\tilde{u}_0 \equiv u_0 + \Delta$, and for a double owner as $\tilde{u}_2 \equiv u_2 - \Delta$, where

$$\Delta \equiv \rho p - R. \tag{3.12}$$

In the special case in which $R = \rho p$, $\tilde{u}_0 = u_0$ and $\tilde{u}_2 = u_2$. Hence in this case, the housing price does not influence the flow value (including incomes/expenses from renting) of double owners or forced renters.

Our analysis focuses on the empirically relevant and realistic case, in which being mismatch gives a higher flow value than being a double owner or a forced renter:

Assumption 3.A1: $u - \chi \geq \max\{\tilde{u}_0, \tilde{u}_2\}$**.**

Anecdotal evidence points to the mismatch state as not particularly costly for the majority of homeowners. As Ngai and Sheedy (2015) argue, mismatch may be so small for many homeowners that they prefer not to move and save on the transaction costs. On the other hand, a comparison between the difference in agreement and closing dates from Figure 8 shows that moving homeowners tend to minimize the delay between the closing of the two transactions with many transactions either occurring simultaneously or within a short period. This suggests that delays between transactions are particularly costly for moving homeowners.

In the special case with $R = \rho p$, Assumption 3.A1 can be written as $u - \chi > \max\{u_0, u_2\}$. If, in addition, $u_0 = u_2 = c$, the assumption boils down to $u - \chi \geq c$.

Define $D(\theta) \equiv V^{B1} - V^{S1}$ as the difference in value between buying first and selling first. Assuming that it is optimal for both a mismatched buyer and mismatched seller to transact, we have

$$D(\theta) = \frac{\mu(\theta)}{\left(\rho + q(\theta)\right)\left(\rho + \mu(\theta)\right)}\left[\left(1 - \frac{1}{\theta}\right)\left(u - \chi - \tilde{u}_2\right) - \tilde{u}_0 + \tilde{u}_2\right]. \tag{3.13}$$

In the case where $\tilde{u}_0 = \tilde{u}_2 = c$, equation (3.13) simplifies to

$$D(\theta) = \frac{\left(\mu(\theta) - q(\theta)\right)\left(u - \chi - c\right)}{\left(\rho + q(\theta)\right)\left(\rho + \mu(\theta)\right)}. \tag{3.14}$$

Hence, in this simple case, buying first is preferred whenever $\mu(\theta) > q(\theta)$. The expected time on the market for a buyer and a seller are $1/q(\theta)$ and $1/\mu(\theta)$, respectively. Hence buying first is preferred, if and only if, time-on-market is higher for a buyer than for a seller. The reason is that a mismatched owner has to undergo two transactions on both sides of the market before he becomes a matched owner. Given this, a mismatched owner wants to minimize the expected time in the situation that is relatively more costly. Since it is more costly to be a double owner or a forced renter than to be mismatched, a mismatched owner would care more about the expected time-on-market for the second transaction and would want to minimize the delay between the two transactions.

As an example, suppose that $\theta < 1$ and consider the schematic representation of a mismatched owner's expected payoffs in Figure 11. If the agent buys first (top part of Figure 11), he has a short expected time-on-market as a buyer. However, he anticipates a long expected time-on-market in the next stage when he is a double owner and has to dispose of his old housing unit. In contrast, selling first (bottom part of Figure 11) implies a long expected time-on-market until the agent sells his property but a short time-on-market when the agent is a forced renter and has to buy a new property. Since $u - \chi > c$, it is more costly to remain in the second stage for a long time (as a double owner or forced renter) rather than to be mismatched and searching. Therefore, selling first is strictly preferred to buying first in that case.

We now formally characterize the optimal action of a mismatched owner given a steady state market tightness θ. We adopt the notation $\theta = \infty$ for the case where the buyer-seller ratio is unbounded. We define

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2}{u - \chi - \tilde{u}_0}. \tag{3.15}$$

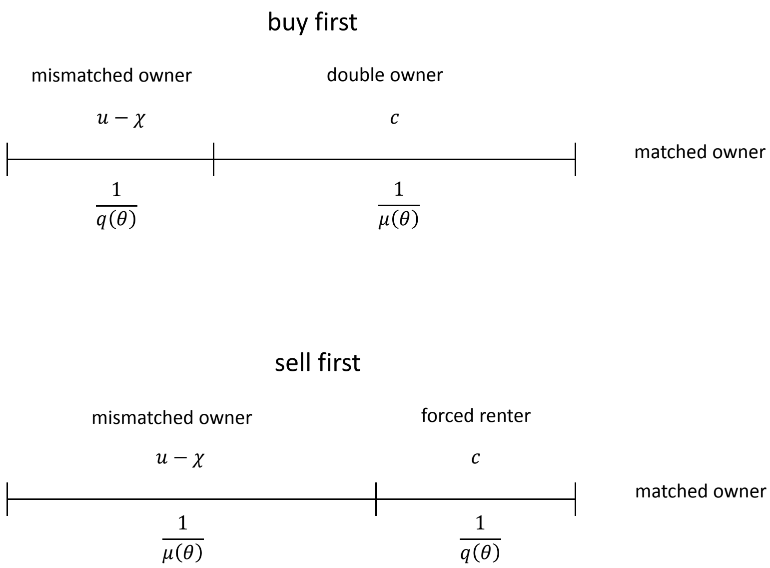


Figure 11: Buying first versus selling first when $\theta < 1$.

Note that if $\tilde{u}_2 = \tilde{u}_0$, then $\tilde{\theta} = 1$, while if $\tilde{u}_2 > \tilde{u}_0$, then $\tilde{\theta} < 1$, and vice versa if $\tilde{u}_2 < \tilde{u}_0$.[35] The following lemma fully characterizes the incentives of mismatched owners to buy first or sell first given a steady state market tightness θ.

Lemma 3.1. *Let $\tilde{\theta}$ be as defined in (3.15). Then for $\theta \in (0,\infty)$, $V^{B1} > V^{S1} \iff \theta > \tilde{\theta}$ and $V^{B1} = V^{S1} \iff \theta = \tilde{\theta}$. For $\theta = 0$ and $\theta = \infty$, $V^{B1} = V^{S1} = (u-\chi)/\rho$.*

Proof. See Appendix 3.8.C. □

Lemma 3.1 shows that, in general, as θ increases, the incentives to buy first are strengthened. For high values of θ, buying first dominates selling first. For low values of θ, selling first dominates buying first.

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[35] In what follows we will additionally assume that at $\theta = \tilde{\theta}$, both $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$, so that a mismatched owner is strictly better off from transacting at $\theta = \tilde{\theta}$. This removes uninteresting steady state equilibria in which mismatched owners never transact. Assumption 3.A2 in Appendix 3.8.B gives a sufficient condition for this.

3.4.3 Steady state flows and stocks

We turn next to a description of the steady state equilibrium stocks and flows of this model. The full set of equations for these flows are included in Appendix 3.8.B. Here we just make some important observations on the stock-flow process in the model. First, combining the population and housing ownership conditions (3.2) and (3.3) we get that

$$B_n(t) + B_0(t) = A(t) + S_2(t). \tag{3.16}$$

Since there are equally many agents and houses, the stocks of agents without a house (forced renters and new entrants) must be equal to the stock of double owners and real-estate firms. This identity implies that in a candidate steady state equilibrium where all mismatched owners buy first (so that there are no forced renters), the market tightness, denoted by $\overline{\theta}$ satisfies

$$\overline{\theta} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n} > 1.$$

Similarly, if $\underline{\theta}$ denotes the market tightness in a candidate steady state where all mismatched owners sell first (so that there are no double owners), we have that

$$\underline{\theta} = \frac{B_n + B_0}{A + S_1} = \frac{A}{A + S_1} < 1.$$

Therefore, $\underline{\theta} < 1 < \overline{\theta}$. This points to possibly wide variations in market tightness arising from changes in the behavior of mismatched owners. In Lemma 3.2 we show that $\overline{\theta}$ solves

$$\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma}\right)\theta + \left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g}\right) = \frac{1}{g} + \frac{1}{\gamma}, \tag{3.17}$$

and $\underline{\theta}$ solves

$$\left(\frac{1}{\mu(\theta) + g} + \frac{1}{\gamma}\right)\frac{1}{\theta} = \frac{1}{g} + \frac{1}{\gamma}. \tag{3.18}$$

These two equations arise from the flow conditions and population and housing conditions if all mismatched owners buy first and sell first, respectively.

Lemma 3.2. *Let $\overline{\theta}$ and $\underline{\theta}$ denote the steady-state market tightness when all mismatched owners buy first and sell first, respectively. Then $\overline{\theta}$ and $\underline{\theta}$ are unique. Furthermore, $\overline{\theta} > 1$, $\underline{\theta} < 1$, and $\overline{\theta}$ is increasing in γ and $\underline{\theta}$ is decreasing in γ.*

Proof. See Appendix 3.8.C. □

It is illustrative to consider a limit economy with small flows, where $g \to 0$ and $\gamma \to 0$ but the ratio $\gamma/g = \kappa$ is kept constant in the limit. From equations (3.17) and (3.18), we have that

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \overline{\theta} = 1 + \kappa, \tag{3.19}$$

and

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \underline{\theta} = \frac{1}{1 + \kappa}. \tag{3.20}$$

Thus, the more important mismatched owners are in housing transactions (the higher is $\kappa = \gamma/g$), the larger the variation in market tightnesses from changes in mismatched owners' actions.

3.4.4 Equilibrium characterization

We now combine the observations on the optimal choice of mismatched owners and the steady state stocks from the previous two sections to characterize equilibria of our model.

Proposition 3.3. *Consider the above economy. Let $\tilde{\theta}$ be defined by condition (3.15), and $\overline{\theta}$ and $\underline{\theta}$ be defined by (3.17) and (3.18) with $\overline{\theta}, \underline{\theta} \in (0, \infty)$.*

1. *If $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, the model exhibits multiple steady state equilibria: an equilibrium with $\theta = \overline{\theta}$, in which mismatched owners buy first; an equilibrium with $\theta = \underline{\theta}$, in which mismatched owners sell first; and an equilibrium with $\theta = \tilde{\theta}$, in which the mismatched owners randomize between buying and selling.*
2. *If $\tilde{\theta} < \underline{\theta}$, there exists a unique steady state equilibrium in which all mismatched owners buy first.*
3. *If $\tilde{\theta} > \overline{\theta}$, there exists a unique steady state equilibrium in which all mismatched owners sell first.*

Proof. See Appendix 3.8.C. □

Therefore, depending on the flow payoffs $\tilde{u}_0$ and $\tilde{u}_2$, there can exist multiple equilibria or a unique equilibrium. Note that the mixing equilibrium is unstable, in the sense that a small deviation from $\tilde{\theta}$ implies that all mismatched owners want to buy first if the deviation is upwards, and to sell first if the deviation is downwards.

In the special case, with $\tilde{u}_0 = \tilde{u}_2 = c$, multiple equilibria always exists:

Corollary 3.4. *Consider the above economy and suppose that $\tilde{u}_0 = \tilde{u}_2 = c$. Then there exist three steady state equilibria: a sell first equilibrium with $\theta = \underline{\theta}$, in which mismatched owners sell first; a buy first equilibrium with $\theta = \overline{\theta}$, in which mismatched owners buy first; and an equilibrium with $\theta = 1$, in which the mismatched owners are indifferent between buying first and selling first, and half of them buy first.*

Proof. See Appendix 3.8.C. □

Intuitively, the equilibrium multiplicity arises because the feedback from the transaction sequence decisions of mismatched owners to the steady state equilibrium market tightness creates a form of *strategic complementarity* in their actions. When mismatched owners are buying first, the steady state buyer-seller ratio is high, so that it is individually rational for any mismatched owner to buy first. Conversely, when mismatched owners are selling first, the steady state buyer-seller ratio is low, and it is individually rational to sell first. Figure 12 illustrates this equilibrium multiplicity and the equilibrium value functions of mismatched owners.[36]

Figure 13 shows examples in which only one equilibrium may exists. In Panel 13a, $\tilde{\theta} > \overline{\theta}$, so that only a "Sell first" equilibrium exists with $\theta = \underline{\theta}$. Panel 13b shows the opposite case when $\tilde{\theta} < \underline{\theta}$, so that only a "Buy first" equilibrium exists with $\theta = \overline{\theta}$.

Since $\tilde{\theta}$ depends on flow payoffs of mismatched owners, shocks to these payoffs can lead to equilibrium switches.[37] Apart from payoff shocks, equilibrium switches may also occur because of changes in agents' believes. Next, we discuss the implications of such equilibrium switches for transaction volume, time-on-market, and the stock of houses for sale.

[36] For illustrative purposes, in the figures below we assume that V^{B1} and V^{S1} as defined in (3.10) and (3.11) are single peaked, i.e. there is a $\hat{\theta}^{B1}$, s.t. V^{B1} is increasing for $\theta < \hat{\theta}^{B1}$ and decreasing for $\theta > \hat{\theta}^{B1}$ and similarly for V^{S1}.

[37] As an example of such a payoff shock, suppose that the payoff of a double owner, u_2, includes costs associated with obtaining a mortgage that allows him to finance the downpayment on his new property prior to the sale of his old property. When financial markets function normally, these costs are relatively low. Suppose that in that case $\tilde{u}_2 > \tilde{u}_0$ and $\tilde{\theta} < \underline{\theta}$. Therefore, the "Sell first" equilibrium does not exist. Conversely, suppose that there is a shock to financial markets, so that obtaining a bridging mortgage becomes very costly, and thus $\tilde{u}_2 < \tilde{u}_0$ and $\tilde{\theta} > \overline{\theta}$. As a result, after the shock, buying first is no longer optimal and the "Buy first" equilibrium no longer exists.

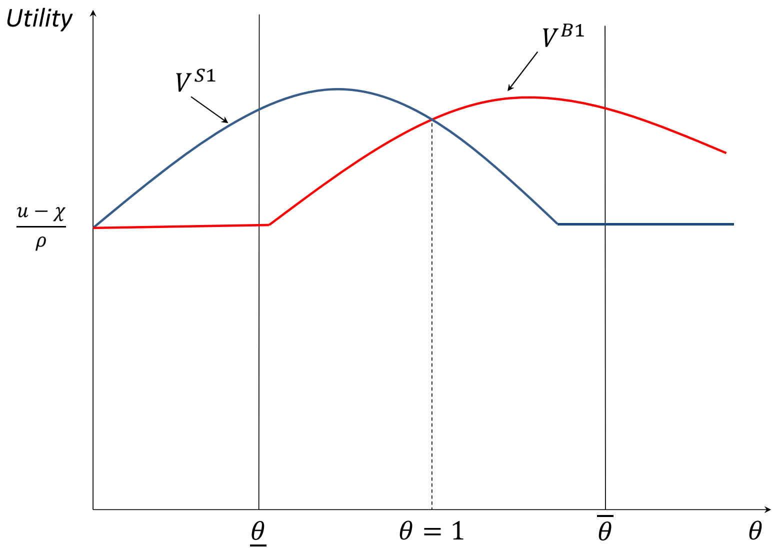


Figure 12: Equilibrium multiplicity with $\tilde{u}_0 = \tilde{u}_2 = c$.

3.4.5 Equilibrium switches

To simplify the analysis we consider the limit economy introduced in Section 3.4.3, where $g \to 0$ and $\gamma \to 0$ and $\gamma/g = \kappa$, $\overline{\theta} = 1+\kappa$, and $\underline{\theta} = 1/(1+\kappa) = 1/\overline{\theta}$. Suppose that the economy starts in a "Buy first" equilibrium with market tightness $\theta = \overline{\theta}$. In that case

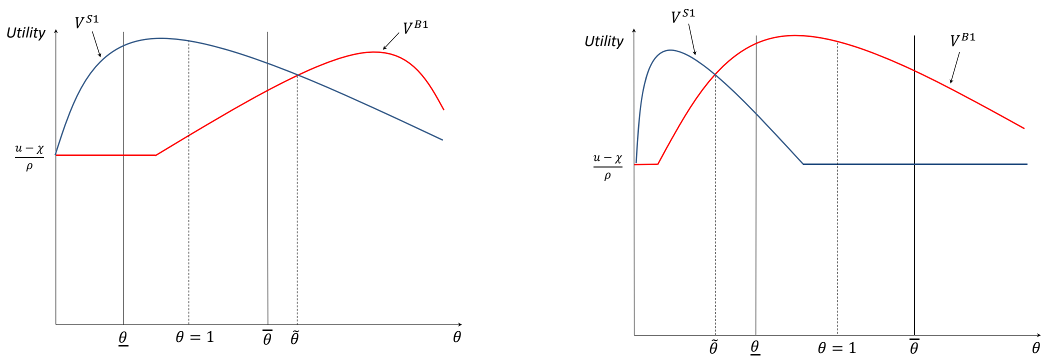
$$\overline{\theta} = \frac{\overline{B}}{\overline{S}} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n}, \tag{3.21}$$

where $\overline{B}$ and $\overline{S}$ denote the stocks of buyers and sellers in the "Buy first" equilibrium. Suppose that the whole stock of mismatched owners, B_1, decide to sell first rather than buy first. In that case, the new market tightness becomes

$$\theta' = \frac{B'}{S'} = \frac{B_n}{B_n + B_1} = \underline{\theta},$$

where B' and S' denote the stocks of buyers and sellers immediately after the switch. Hence, the tightness jumps directly to its new steady state equilibrium value with no dynamic adjustment in θ.

What are the implications of this switch? First of all, clearly average time-on-market for sellers, $1/\mu(\theta)$, increases. Second, consider the ratio of the stock of sellers before



(a) (b)

Figure 13: Examples with unique equilibria in the case of $\tilde{\theta} > \overline{\theta}$ (a) and $\tilde{\theta} < \underline{\theta}$ (b).

and after the switch. That ratio is exactly $\underline{\theta}$, which is less than 1. Therefore, there is an increase in the stock of houses for sale since some of the previous buyers become sellers. Finally, transaction volume may also fall depending on the shape of the matching function. Specifically, suppose that we have a Cobb-Douglas matching function, so $m(B,S) = \mu_0 B^\alpha S^{1-\alpha}$, for $0 < \alpha < 1$, and consider the ratio of transaction volumes before and after the switch

$$\frac{\mu\left(\overline{\theta}\right)}{q\left(\underline{\theta}\right)} = \frac{\mu_0 \overline{\theta}^\alpha}{\mu_0 \underline{\theta}^{\alpha-1}} = (1+\kappa)^{2\alpha-1}.$$

Hence, transaction volume falls after the equilibrium switch if $\alpha > 1/2$ and increases if $\alpha < 1/2$. The reason is that for $\alpha > 1/2$ buyers are more important than sellers in generating transactions. When mismatched owners switch from buying first to selling first this leads to a reduction in the number of buyers and an increase in the number of sellers, and hence, to a fall in the transaction rate. Genesove and Han (2012) estimate a value of $\alpha = 0.84$. At that value, transaction volume would drop after the switch from a "Buy first" equilibrium to a "Sell first" equilibrium.

Consequently, a switch from a "Buy first" equilibrium to a "Sell first" equilibrium implies behavior for key housing market variables – the for-sale stock, average time-to-sell, and transaction volume – that is broadly consistent with evidence on housing cycles (Diaz and Jerez (2013), Guren (2014)). This behavior is also consistent with the evidence on the housing cycle in Copenhagen as shown in Figure 10. The negative comovement between the for-sale stock and transactions is of particular interest since search-based models of the housing market usually have trouble generating this comovement (Diaz and Jerez, 2013).

3.4.6 Quantitative relevance

In this section we provide a simple numerical example to assess the quantitative relevance of our mechanism. Specifically, we examine the switch from the "Buy first" to the "Sell first" equilibrium in the limit economy.

We use the calibration of Head et al. (2014) to determine plausible values for the rate of mismatch, γ, and the entry and exit rate, g. We use the annual fraction of owners that move across U.S. counties, which according to the Census Bureau is around 3.2 percent, to determine g. The fraction of owners that stay in the same county conditional on moving is 60 percent, or around 1.5 times more owners move within the same county than across counties. Therefore, we set $g = 0.035$ and $\gamma = 0.0525$. This implies an average duration for homeowners of around 11.4 years and an annual turnover rate of 8%. These flows give us values of the market tightness in the limit economy of $\lim \overline{\theta} = 2.5$ and $\lim \underline{\theta} = 0.4$, respectively. Finally, we assume that the matching function is Cobb-Douglas so that $\mu(\theta) = \mu_0 \theta^{\alpha}$. We choose $\alpha = 0.84$, following Genesove and Han (2012).[38]

With these numbers, switching from the "Buy first" to the "Sell first" equilibrium in the limit economy leads to a decrease in the market tightness by around 85%. This is associated with a 150% increase in the for-sale stock, and a 350% increase in average time-to-sell. In addition, the transaction rate falls by around 50% immediately after the switch.

3.5 Endogenous prices

In this section we relax the assumption that prices are fixed, and analyze the existence of multiple equilibria with endogenous prices. First we impose an exogenous relationship between the steady state market tightness and the housing and rental prices in that

[38] The small values of γ and g suggest that the focus on the limit economy in Section 3.4.5 is reasonable. See the working paper version of this chapter (Moen et al. (2015)) for a comparison of tightnesses in the limit economy and away from the limit. Also note that the value of μ_0 is irrelevant for the numerical results in this section. In Section 3.5.2, where we discuss the quantitative implications for house prices, we use a value of $\mu_0 = 4$, which gives a seller time-on-market of 6 weeks in a "Buy first" equilibrium.

market, and derive conditions for equilibrium multiplicity. Then we go one step further, and show that there can be multiple equilibria in an environment where prices are determined by Nash bargaining, and even in an environment with competitive search where agents can trade-off prices and time-on-market. With endogenous price formation, the model becomes algebra-intensive, as we have to keep track of which agent types trade with one another. For that reason, the details of the analysis are deferred to Appendix 3.8.D.

3.5.1 Prices depend on market tightness

When deriving conditions for multiplicity, we assumed that prices were exogenous, and hence unaltered by the transaction sequence decision of the agents. Now, we allow housing prices in steady state to depend on θ. We thus write $p = p(\theta)$, with $p'(\theta) > 0$. We may also write the rent R as a function of θ, $R = R(\theta)$, with $R'(\theta) \geq 0$. Recall that the flow values of double owners and forced renters depend on $\Delta = \rho p - R$. Thus, we can write $\Delta = \Delta(\theta)$. It seems reasonable to assume that $\Delta'(\theta) \geq 0$ (otherwise, there is no countervailing effect from endogenous prices).

As we already pointed out in Section 3.4.1, in the case with $R = \rho p$, $\Delta'(\theta) = 0$, and so, assuming that house prices are exogenous is without loss of generality, since different (steady state) prices do not influence the flow value of double owners or forced renters. However, a countervailing effect arises if $\Delta'(\theta) > 0$. Specifically, from Equation (3.13), the decision whether to buy or to sell first depends on the sign of the following expression:

$$\tilde{D}(\theta) = \frac{\theta - 1}{\theta}\left(u - \chi - u_2 + \Delta(\theta)\right) + u_2 - u_0 - 2\Delta(\theta).$$

We normalize $u_2 - u_0$ so that $\tilde{D}(1) = 0$, which requires that $u_2 - u_0 = 2\Delta(1)$. In order for the "Buy first" and "Sell first" equilibria to exist, we must have that (recall that $\overline{\theta}$ and $\underline{\theta}$ denote the steady state tightness in the "Buy first" and "Sell first" equilibria, respectively):

$$\frac{\overline{\theta} - 1}{\overline{\theta}}[u - \chi - u_2 + \Delta(\overline{\theta})] \geq 2(\Delta(\overline{\theta}) - \Delta(1)),$$

and

$$\frac{1 - \underline{\theta}}{\underline{\theta}}[u - \chi - u_2 + \Delta(\underline{\theta})] \geq 2(\Delta(1) - \Delta(\underline{\theta})).$$

In the two conditions above, the left-hand side broadly reflects the strategic complementarity effect identified in Section 3.4. A higher θ makes buying more time-consuming and selling less time-consuming, and since the agent's utility flow is higher whenever mismatched compared to being a double owner or a forced renter, this favors buying first.

The right-hand side reflects how a higher value of θ changes the difference in the flow values of being a forced renter compared with a double owner. Note that if $R \leq \rho p$, it is beneficial for the agent, everything else equal, to buy late and sell early. We refer to this as a discounting effect. A higher θ increases p, and unless R increases at the same rate, this strengthens the discounting effect and makes it more attractive to sell first. Due to our normalization, at $\theta = 1$ the discounting effect is exactly balanced by a difference in the income flows u_2 and u_0. However, for $\theta > 1$, the discounting effect becomes stronger, which favors selling first to buying first. Similarly, for $\theta < 1$, the discounting effect becomes weaker, which favors buying first to selling first.

Therefore, our multiple equilibrium result requires that the house price (or more generally, Δ) should not be too sensitive to changes in θ, so that the discounting effect is weaker than the strategic complementarity effect. Notice that the two conditions always hold if u_2 (and also u_0 given the normalization) is sufficiently low. In that case the strategic complementarity effect always dominates for values of θ that are consistent with a steady state equilibrium.

In this subsection the relationship between price and tightness is assumed to be exogenous. Next, we demonstrate that our results still hold in a model with Nash bargaining.

3.5.2 Prices determined by Nash bargaining

In this section we assume that prices are determined by symmetric Nash bargaining, so that buyers and sellers split the surplus of a match equally between them, as in Pissarides (2000). Therefore, there is no longer a single transaction price p, but prices depend on the types of the trading counterparties. To simplify the stock-flow conditions, we again consider a limit economy with small flows, where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is constant, and where $\overline{\theta} = 1 + \kappa$ and $\underline{\theta} = 1/(1+\kappa)$. Additionally, and for analytical tractability, we consider parameter restrictions which ensure that all trading surpluses are positive in both a "Buy first" and "Sell first" candidate equilibrium, except for the surplus between a mismatched buyer and a mismatched seller. We give the relevant parameter restrictions in Appendix 3.8.B.

Note that the real-estate firms are different from the other agents in the economy, as they receive no utility from owning a house, and their gain from transacting is the price, which is a transfer and hence does not influence the match surplus. As a result, the equilibrium allocation becomes asymmetric, and tilts towards the "Buy first" equilibrium even with $u_2 = u_0$. It turns out that symmetry, in the sense that mismatched owners are indifferent between buying first and selling first if $\theta = 1$, is re-established if $u_n - u_0 = u - u_2$. In what follows we, therefore, assume that this is the case.

"Buy first" equilibrium

Consider a "Buy first" steady state equilibrium candidate with a market tightness of $\theta = \overline{\theta} > 1$. In that candidate equilibrium, the sellers with positive measure are the double owners and real-estate firms, while the buyers with positive measure are the mismatched owners and new entrants. In the limit economy, the outflow rate of mismatched agents is equal to the outflow rate of new entrants, so $B_1/B_n = \gamma/g = \kappa$. Hence, the shares of new entrants and mismatched buyers in the pool of buyers are $1/\overline{\theta}$ and $1 - 1/\overline{\theta}$, respectively. Furthermore, in the limit, as there is no death, the shares of real-estate firms and double owners in the pool of sellers are also $1/\overline{\theta}$ and $1 - 1/\overline{\theta}$, respectively.

Given these shares and since buyers and sellers split the match surplus evenly, it follows that the value function of a mismatch buyer is (given $\rho \to r$ in the limit)

$$rV^{B1} = u - \chi + \frac{1}{2}q(\overline{\theta})\left[\frac{1}{\overline{\theta}}\Sigma_{AB1} + (1 - \frac{1}{\overline{\theta}})\Sigma_{S2B1}\right],$$

where $\Sigma_{AB1} = V^{S2} - V^{B1} - V^A$ is the match surplus when a mismatched buyer meets a real-estate firm, and $\Sigma_{S2B1} = V - V^{B1}$ is the match surplus when a mismatched buyer meets a double owner.

Consider a mismatched owners who deviates (permanently) and sells first.[39] Since a meeting between a mismatched buyer and a mismatched seller is assumed to lead to negative surplus, it follows that the value function of a deviator is simply

$$rV^{S1} = u - \chi + \frac{1}{2}\mu\left(\overline{\theta}\right)\frac{1}{\overline{\theta}}\Sigma_{S1Bn}.$$

[39] Examining permanent deviations here is without loss of generality, since a temporary deviation can only dominate a permanent deviation if and only if no deviation dominates the permanent deviation.

It follows that the difference between the value of buying first and selling first, $D(\overline{\theta}) = V^{B1} - V^{S1}$, can be written as

$$D(\overline{\theta}) = \frac{\frac{1}{2}q\left(\overline{\theta}\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)} \left(\frac{u_n - u_0}{r + \frac{1}{2}q\left(\overline{\theta}\right)} - \frac{1}{\overline{\theta}} \frac{u - u_2}{r + \frac{1}{2}\mu\left(\overline{\theta}\right)} \right). \tag{3.22}$$

Given our assumptions on utility flows, $D\left(\overline{\theta} = 1\right) = 0$ for $\kappa = 0$. An increase in $\overline{\theta}$ (equivalently, an increase in κ) leads to a increase in $D\left(\overline{\theta}\right)$ since the expression in parenthesis, increases. This increase comes from two effects. First, $\mu\left(\overline{\theta}\right)$ increases and $q\left(\overline{\theta}\right)$ decreases, so the second term in the parenthesis decreases (given that $u_2 < u - \chi < u$) and the first term increases (since then $u_n > u_0$). This effect is tightly linked to the strategic complementarity effect in our benchmark model in Section 3.4. Specifically, as before, an increase in $\overline{\theta}$ increases the value of buying first given a lower expected time-on-market for double owners, while it decreases the value of selling first, given a higher expected time-on-market for forced renters.

Second, the fraction of new entrants and real-estate firms, $1/\overline{\theta}$, decreases. Therefore, buyers are more likely to meet double owners and sellers are more likely to meet mismatched buyers. However, the trading surplus for a buyer is higher when matched with a double owner compared to a match with a real-estate firm. Similarly, the trading surplus is lower for a seller when matched with a mismatched buyer compared to a new entrant. This compositional effect on both the buyer and seller sides of the market additionally strengthens the incentives to buy first.

"Sell first" equilibrium

Consider a "Sell first" steady state equilibrium candidate with a market tightness of $\theta = \underline{\theta} < 1$. In that candidate equilibrium, the sellers with positive measure are the mismatched owners and real-estate firms, while the buyers are the forced renters and new entrants. In the limit economy, we have that the shares of forced renters and new entrants in the pool of buyers are $\underline{\theta}$ and $1 - \underline{\theta}$, respectively. These are also the respective shares of real-estate firms and mismatched owners.

In this equilibrium candidate, the gain from deviating to (permanently) buying first for a mismatched owner is $D(\underline{\theta}) = V^{B1} - V^{S1}$, which is given by

$$D\left(\underline{\theta}\right) = \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(\underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)} - \frac{u - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\right). \tag{3.23}$$

Given our assumptions on the utility flows, $D(\underline{\theta} = 1) = 0$ for $\kappa = 0$. A decrease in $\underline{\theta}$ (increasing κ) decreases D, and hence, makes it more attractive to sell first.

We conclude that the model with Nash bargaining exhibits multiple equilibria, as stated in the following proposition.

Proposition 3.5. *Consider the limit economy with prices determined by symmetric Nash bargaining. Suppose that conditions 3.B1-3.B4 given in Appendix 3.8.D hold. Then there exists a steady state equilibrium, in which all mismatched owners buy first and the equilibrium market tightness converges to* $\overline{\theta} = 1 + \kappa$. *Also, there exists a steady state equilibrium, in which all mismatched owners sell first and the equilibrium market tightness converges to* $\underline{\theta} = 1/(1+\kappa)$.

Proof. See Appendix 3.8.D. □

As already mentioned, we have to make restrictions on the parameter set (conditions 3.B2-3.B4 in Appendix 3.8.D), to ensure that our assumptions regarding the matching sets are satisfied in equilibrium. These restrictions are only sufficient and the same matching sets may emerge for more general parameter values. Also, we conjecture that if these restrictions are not satisfied, and other matching sets emerge, these may also have multiple steady state equilibria with the structure described in Proposition 3.5.[40]

Numerical example

To gain some additional insight into when multiple equilibria are possible with endogenously determined prices, and to show that conditions 3.B2-3.B4 are only sufficient for multiple equilibria, we consider a simple numerical example, in which conditions 3.B2-3.B4 need not be satisfied.[41] Figure 14 plots the difference $D(\theta) = V^{B1} - V^{S1}$ in a

[40]Simulations confirm that multiple equilibria exist also for parameter values that do not satisfy conditions 3.B1-3.B4.

[41]One can also relax the symmetry condition 3.B1. Relaxing 3.B1 changes the value of θ for which a mismatched owner is indifferent between buying first and selling first. Thus, it essentially shifts the curve in Figure 14 to the left or to the right.

candidate “Sell first” equilibrium for $\theta \leq 1$ and a candidate “Buy first” equilibrium for $\theta > 1$. The figure illustrates several important points.

First, multiple steady state equilibria can be sustained even if the steady state market tightness fluctuates substantially between the candidate “Buy first” and “Sell first” equilibria. Using the stock-flow calibration from Section 3.4.6 produces substantial variation in market tightness across the two steady state equilibria as illustrated by the red and blue vertical lines on the Figure. This in turn means that there can be substantial price effects from switches across the two steady state equilibria. Specifically, for the numerical example that produces the figure, average house prices decline by around 32% between the “Buy first” and the “Sell first” equilibrium. Therefore, our mechanism can also lead to quantitatively significant fluctuations in house prices when those are endogenously determined.

Second, unlike the benchmark case with exogenously fixed prices, endogenously determined prices imply that for sufficiently low (sufficiently high) market tightness, buying first starts to dominate selling first (and vice versa). This reflects the discounting effect discussed in section 3.5.1, which can outweigh the strategic complementarity effect, so that multiple equilibria cease to exist. Nevertheless, as the numerical example illustrates, with Nash bargaining, the price response must be very large for the discounting effect to start dominating. Specifically, in the example in Figure 14, average house prices must decline by (approximately) more than 50% between a candidate “Buy first” and “Sell first” equilibria for the discounting effect to dominate.

3.5.3 Competitive search

In competitive search equilibrium, the sellers post prices, and buyers direct their search towards the sellers they find most attractive, taking into account that better terms of trade mean a longer expected waiting time before trade occurs. The market splits up in submarkets, and the different agents choose which submarket to enter. As shown in Garibaldi et al. (2016), the most patient buyers (who are most willing to trade off a short waiting time for a low price) will search for the most impatient sellers (who are most willing to trade off a low price for a short waiting time). Analogously, the least patient buyers search for the most patient sellers.

In Appendix 3.8.E we derive sufficient conditions on parameter values for multiple competitive search equilibria to exist. The economic mechanisms play out in a slightly

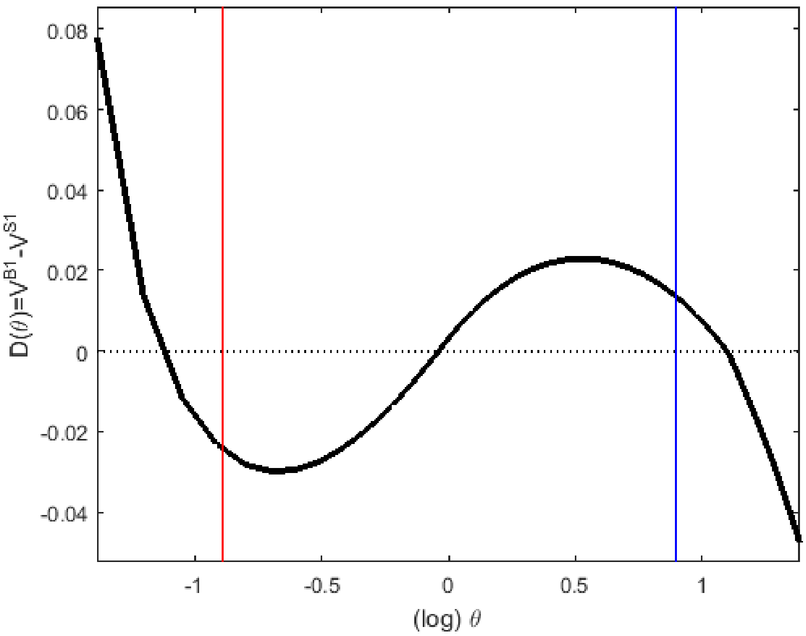


Figure 14: A plot of $D(\theta)$ (solid black) against (log) θ.

Notes: Preference parameters used for the example: $u = 2$, $u_0 = 1.55$, $u_2 = 1.85$, $u_n = 1.7$, $\chi = 0.14$. Rental price $R = 0.1$. Vertical red and blue lines denote steady state market tightnesses in "Buy first" and "Sell first" equilibria using the stock-flow calibration from Section 3.4.6.

different way with competitive search than with wage bargaining. With wage bargaining, the strategic complementarity works through the market tightness: if more agents buy first, this increases the buyer-to seller ratio in the market, and makes it more attractive to buy first. In competitive search equilibrium, the strategic complementarities are more involved. Single agents trade off waiting time and terms of trade. At market level, more buy-first agents will influence the buyer-to-seller ratio in the submarket(s) for mismatched buyers. This will not directly affect the sell-first agents, as they search in different submarkets. However, the asset values of all agents will change, thereby influencing the value of selling first and the speed at which sell-first agents transact.

In Appendix 3.8.E we derive the competitive search equilibrium allocation when the cost of being mismatched, χ, is low, the flow utility of being a double owner, u_2, and as in 3.5.2 we assume that new entrants enjoy a strictly higher flow utility than forced renters: $u_n > u_0$. In the market constellations we study, a submarket with new entrants and real estate firms always operates.

In the "Buy first" equilibrium, the buyers are mismatched owners and new entrants, while the sellers are real estate firms and double owners as in Panel 15a of Figure 15, where blue indicates sellers and red buyers. The most patient buyer is the mismatched owner, while the most impatient sellers are the double owners. Hence these agents always transact. The least patient buyers are the new entrants, while the most patient sellers are the real estate firms. Hence submarkets for real estate firms and new entrants will always exist. In addition, a market for new entrants and double owners will also open.

Note that the match surplus Σ_{B1S2} between a mismatched buyer and a double owner is given by

$$\Sigma_{B1S2} = V + V^{S2} - V^{B1} - V^{S2} = V - V^{B1} > 0,$$

so that there is trade in this market.

Now consider a mismatched owner that deviates and sells first. For a small χ, this seller will be more patient than both the real estate firms and the double owners, and will, therefore, transact with the most impatient buyers among the non-deviating buyers, the new entrants. It will then become a new buyer type – a forced renter – that is even more impatient than the new entrant because $u_n > u_0$. It will, therefore, transact with the most patient sellers, which are the real estate firms. Given that a forced renter is more impatient than a new entrant, real estate firms are willing to open a new submarket for the deviating agent. In particular, the value from being a forced renter, V^{B0}, maximizes his gain from search given the value of the real estate firm.

The match surplus between the deviating mismatched owner and the new entrant, Σ_{BnS1}, can be written as

$$\Sigma_{BnS1} = V + V^{B0} - V^{Bn} - V^{S1}.$$

Note that also $\lim_{\chi\to 0} V^{S1} = \lim_{\chi\to 0} V$. Moreover, for $u_0 < u_n$, V^{B0} is strictly lower than V^{Bn}, also in the limit as $\chi \to 0$. As a result, the match surplus Σ_{BnS1} is negative for small values of χ, so that the mismatched owner cannot gain by deviating and the "Buy first" equilibrium exists.

In the "Sell first" equilibrium, the sellers are mismatched homeowners and real estate firms, while the buyers are new entrants and forced renters. The most patient buyers are the new entrants, and the most patient sellers - the mismatched owners. The active submarkets will be between new entrants and real estate firms, mismatched buyers and forced renters, and forced renters and real estate firms. The markets (together with a deviating agent) are illustrated in Panel 15b of Figure 15.

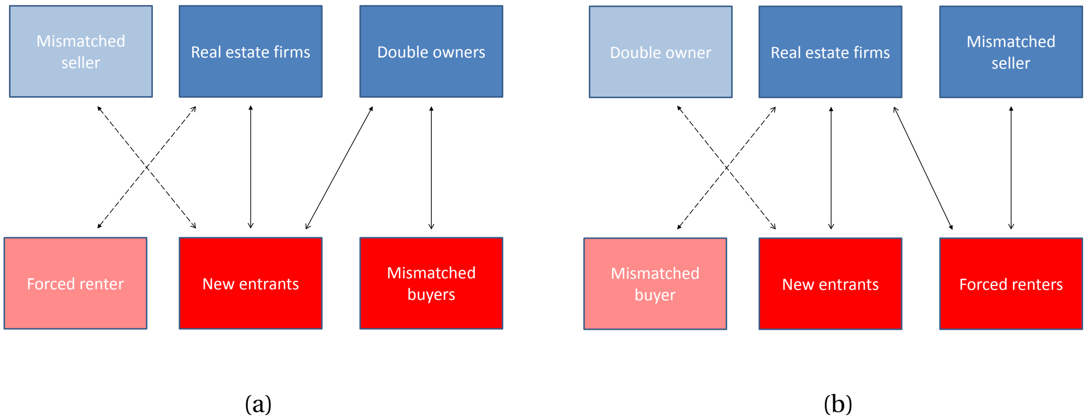


Figure 15: Equilibrium market segments (solid colors) and deviators (weaker colors) for “Buy first” (a) and “Sell first” (b) competitive search equilibria.

The asset values in the real-estate-new agents are as in the “Buy first” equilibrium, so that this market is active. For a real estate agent, the surplus of a transaction with a (more impatient) forced renter is even larger than with a new entrant, so that there are benefits of trade in this market as well. Hence, a forced renter obtains the same value V^{B0} as the (deviating) forced renter in the “Buy first” equilibrium. The match surplus between a forced renter and a mismatched seller, Σ_{B0S1}, is given by

$$\Sigma_{B0S1} = V + V^{B0} - V^{S1} - V^{B0} = V - V^{S1} > 0.$$

Now, consider a mismatched agent that deviates and buys first. This buyer will be more patient than both new entrants and forced renters, and will thus transact with the real estate firm, the most impatient seller. He will then become a double owner, thus, becoming more impatient than the real estate firm, and will therefore transact with the new entrants. A new submarket will open up, and the expression for the asset value V^{S2} is the same as for the double owner in the “Buy first” equilibrium. However, for the same reasons as in the “Buy first” equilibrium, the match surplus between the mismatched buyer and the real estate firm, Σ_{B1A}, is negative for low values of u_2. It follows that the deviation is unprofitable. We conclude that the model still exhibits multiple equilibria, as stated in the following proposition.

Proposition 3.6. *Consider the economy with competitive search, and suppose that χ and u_2 are small and that $u_0 < u_n$. Then the economy exhibits multiple equilibria. In one* *equilibrium, all mismatched owners buy first. In another equilibrium, all mismatched owners sell first.*

Proof. See Appendix 3.8.E. □

We have no reason to believe that the condition on χ is necessary to obtain multiple equilibria. However, without this assumption the model becomes less tractable, as it is not clear from the outset what market constellations will then be realized.

3.5.4 Additional extensions

Our benchmark model assumes that a mismatched owner has to choose to enter the housing market either as a buyer or as a seller. In Appendix F, we examine the optimal behavior of a mismatched owner that can choose to be both a buyer and a seller at the same time. We show that a mismatched owner strictly prefers to either only enter as a buyer or as a seller for any $\theta \neq \tilde{\theta}$, where $\tilde{\theta}$ is defined as in Equation (3.15). Intuitively, since the decision to enter as both a buyer and seller depends ultimately on the value from entering as a buyer only and the value from entering as a seller only, whenever entering as a buyer only is dominated by entering as a seller only, then entering as both a buyer and seller is also dominated by entering as a seller only, and vice versa.

Additionally, we show that our main results continue to hold even assuming that homeowners are compensated for the value of their housing units when they exit the economy when g is sufficiently small. As can be seen in Appendix 3.8.F, in that case a modified version of Equation (3.15) determines the critical value of $\tilde{\theta}$ below which a mismatched owner is better off selling first. In the limit, as $g \to 0$, that modified version converges to the value of $\tilde{\theta}$ from Equation (3.15).

3.6 House price fluctuations

In this section, we first examine the implications of expected changes in the house price for the behavior of mismatched owners. We then construct dynamic equilibria with self-fulfilling fluctuations in prices and tightness. In the entire section we study our benchmark case, in which $R = \rho p$.

3.6.1 Exogenous house price movements

We first show that expected future changes in the house price affect the incentives of mismatched owners to buy first or sell first. Consider a simple, exogenous process for the price p. With rate λ the house price p changes to a permanent new level p_N.[42] We compare the utility from buying first relative to selling first for a mismatched owner before the price change. If the price change occurs between the two transactions, the mismatched owner will make a capital gain of $p^N - p$ if he buys first and a capital loss of the same amount if he sells first. If the shock happens before the first transaction or after the second transaction, it will not influence the decision to buy first or sell first. To simplify the exposition we assume that $u_0 = u_2 = c$.[43]

The price risk associated with the transaction sequence decision creates asymmetry in the payoff from buying first or selling first. Specifically, at $\theta = 1$, the difference between the two value functions $D(\theta) = V^{B1} - V^{S1}$ takes the form

$$D(1) = \frac{\mu(1)}{\left(\rho + q(1) + \lambda\right)\left(\rho + \mu(1) + \lambda\right)} 2\lambda \left(p_N - p\right). \tag{3.24}$$

An expected price decrease, leads to a higher value of V^{S1} relative to V^{B1}, even if matching rates for a buyer and a seller are the same. Consequently, $V^{S1} > V^{B1}$ even for some values of $\theta > 1$. If the expected price decrease is sufficiently large, so that even at $\theta = \overline{\theta}$, $D\left(\overline{\theta}\right) < 0$, then selling first will dominate buying first for any value of θ that is consistent with equilibrium. Similarly, a sufficiently large expected price increase, will imply that $D\left(\underline{\theta}\right) > 0$, so buying first will dominate selling first for any value of θ that is consistent with equilibrium. We summarize these observations in the following

Proposition 3.7. *Consider the modified economy with an exogenous house price change. Then for every $\lambda > 0$ and $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, a mismatched owner prefers to sell first for sufficiently low values of p_N. Analogously, a mismatched owner prefers to buy first for sufficiently high values of p^N.*

Proof. See Appendix 3.8.C. □

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[42] Since we assume that $p = \frac{R}{\rho}$, one can think of a permanent change in the equilibrium rental rate to R_N, which leads to a house price change to $p_N = R_N / \rho$.

[43] We assume that θ remains constant over time, so the only change occurs in the house price p. Also, for this exercise, we implicitly assume that $\gamma \to 0$, so that V is independent of the house price p.

3.6.2 Self-fulfilling house price fluctuations

In the steady-state analysis, we showed that the model may exhibit multiple equilibria, with different market tightnesses. If a high market tightness is associated with a high price, this may lead to a destabilizing effect on housing prices. In this subsection we assume that the housing price depends on the buyer-seller ratio as in section 3.5.1 and on expected future prices, and show that this may lead to self-fulfilling house price fluctuations.

Suppose that $X(t) \in \{0,1\}$ follows a two-state Markov chain. $X(t)$ starts in $X(t)=0$ and with Poisson rate λ transitions permanently to $X(t)=1$. The realization of $X(t)$ plays the role of a sunspot variable. The price in state 1 is assumed to be given by a smooth function $p_1 = f(\theta_1)$. The price in state 0 is assumed to be implicitly given by a smooth function $p_0 = f(\theta_0, \lambda(p_1 - p_0))$, increasing in both arguments, and with $f(\theta, 0) \equiv f(\theta)$. We take these relationships as exogenous and reduced form to illustrate the equilibrium consequences of the interaction of housing prices and market liquidity conditions with the transaction decisions of mismatched owners. Again we look at a limit economy as γ and g go to zero, keeping $\kappa = \gamma/g$ fixed.

We consider an equilibrium in which the economy starts out in a "Buy first" regime ($X(t)=0$) in which 1) mismatched owners prefer to buy first and the market tightness is $\theta_0 = \overline{\theta}$, and 2) agents expect that with rate λ, the economy permanently switches to a "Sell first" regime with market tightness $\theta_1 = \underline{\theta}$. In that second regime, 1) mismatched owners strictly prefer to sell first, and 2) agents expect that the economy will remain in the "Sell first" regime forever. Since $\bar{\theta} > \underline{\theta}$, it follows that $p_0 > p_1$.[44] It is straightforward to show that as $\lambda \to 0$, the pay-offs when buying first and selling first converge to the pay-offs without regime switching. Hence, in the limit, buying first in state 0 is an equilibrium strategy if $\bar{\theta} > \tilde{\theta}$, while selling first is an equilibrium strategy in state 1 if $\underline{\theta} < \tilde{\theta}$, where $\tilde{\theta}$ is defined by proposition 3.3. Hence, multiple equilibria exists whenever

$$1+\kappa > \frac{u-\chi-u_2}{u-\chi-u_0} > \frac{1}{1+\kappa}. \tag{3.25}$$

This equation is clearly satisfied when $u_2 = u_0$.

Proposition 3.8. *Consider the limit economy with $g \to 0$, $\gamma \to 0$ and $\gamma/g = \kappa$, and with the sunspot process described above. Suppose further that equation (3.25) is satisfied.*

[44]Suppose $p_0 \le p_1$. Then $p_0 = f(\bar{\theta}, \lambda(p_1 - p_0)) \ge f(\bar{\theta})$. But then $p_0 \ge f(\bar{\theta}) > f(\underline{\theta}) = p_1$, a contradiction.

Then there is a $\overline{\lambda}$, such that for $\lambda < \overline{\lambda}$, there exists a dynamic equilibrium characterized by two regimes $x \in \{0, 1\}$. In the first regime, $\theta_0 = \overline{\theta}$ and mismatched owners buy first. In the second regime, $\theta_1 = \underline{\theta}$, $p_1 < p_0$, and mismatched owners sell first. The economy starts in regime 0 and transitions to regime 1 with rate λ.

Proof. Follows from the discussion above. □

Intuitively, if agents expect the change in regimes to occur sufficiently soon (λ is high), then it can be optimal for mismatched owners to sell first in the first regime despite the high market tightness, speculating on regimes changing in between their two transactions. This, however, is inconsistent with equilibrium. Therefore, an equilibrium with a transition between the two regimes exists only for a sufficiently low regime switching rate λ.

3.7 Institutional details and conclusion

In this section we compare the process of housing sales in several countries, and then provide brief concluding comments.

3.7.1 Institutional details

Actual housing markets in different countries differ in their institutional characteristics. Naturally, our model of the housing market abstracts from many of these peculiarities. As a result, the fit between the model and the way that houses are bought and sold may vary across countries. Nonetheless, we think our model captures essential elements of housing transactions for many countries, including Denmark, Norway, the Netherlands, and the United States. In these countries, the institutional set-up for the process of housing transactions is such that homeowners are concerned about the order of buying and selling, at least to some extent. In principle, the same issue occurs in the United Kingdom, but there the phenomenon of housing chains (see Rosenthal (1997)) may provide a way to accommodate the risks associated with moving in the owner-occupied housing market. Because of the widespread usage of housing chains in the UK, our model may be less suited for capturing the way houses are bought and sold in that country. Instead it describes more closely the housing markets of countries where housing chains are rare or non-existent.[45]

Additionally, in England and Wales buyers and sellers are not legally bound to an agreed transaction until late in the process, so that both sides easily renege on offers (Rosenthal, 1997). As a result, if a household is not able to complete a second transaction as fast as desired, it may just withdraw from a first transaction in order to avoid the costly period in between. As shown in Appendix G, in Table 9 for buyers and Table 10 for sellers, commitment to an agreed transaction is significantly larger outside the UK. The tables show whether the law requires a grace period, what the penalty is for reneging during and after this grace period, which conditions that allow to dissolve a contract are usually included in the contract, and for what period parties can still refer to these conditions. In Denmark (where our transactions data are from) for instance, only buyers enjoy a grace period of 6 days, in which they can cancel the transaction at a cost of one percent of the transaction price. Afterwards, buyers are liable for the full amount, while sellers can be taken to court if they do not transfer the house. Sometimes purchase offers allow for contingencies such as the ability to secure financing or the approval of one's own lawyer, but referral to these conditions requires proof and is restricted in time. The picture that emerges from the tables is that in Denmark, Norway, the Netherlands, and the United States it may be very costly to renege on a transaction once a purchase offer has been made or a conditional contract has been signed.

3.7.2 Conclusion

The transaction sequence decision of moving owner-occupiers depends on housing market conditions, such as the expected time-on-market for buyers and sellers and expectations about future house price appreciation. However, these decisions in turn exert important effects on the buyer-seller ratio of the housing market. This creates a coordination problem for moving owner occupiers, resulting in multiple equilibria. Equilibrium switches are associated with large fluctuations in the stock of units for sale, average time-on-market, transactions, and prices.

The tractable equilibrium model that we study in this paper to show these effects is deliberately simplified, and so lacks heterogeneity in many important dimensions.

[45]There is anecdotal evidence that innovations in mortgage financing in recent years may have decreased the importance of housing chains in the UK market as well.

In particular, there is no heterogeneity in the costs of being a double owner versus a forced renter, which are likely to vary substantially across households and also to vary over time in response to aggregate shocks. In addition, we assumed constancy of the rate of mismatch and entry into and exit from the market. Nevertheless, endogenous fluctuations in γ and g are likely to additionally amplify and propagate aggregate shocks. Enriching the model along these dimensions will allow for a detailed quantitative model of the housing market, which can be taken to the data. We view this as an important step for future research.

3.8 Appendix

3.8.A Data Description

We use two data sets. The first (EJER) is an ownership register which contains the owners (private individuals and legal entities) of properties in Denmark as of the end of a given calendar year. The data set contains unique identifiers for owners (which, unfortunately, cannot be matched with other datasets beyond EJER for different years). It also contains unique identifiers for each individual property. The second data set (EJSA) contains a record of all property sales in a given calendar year. The majority of transactions include information on the sale price, sale (agreement), and takeover (closing) dates. Furthermore, they contain the property identifiers used in the EJER data-set, which allows for linking of the two datasets. The first data set is available from 1986 (recording ownership in 1985) until 2010 (recording ownership at the end of 2009), while the second is available from 1992 to 2010. Therefore, we effectively use data from 1991 (for ownership as of January 1, 1992) to 2009 (for ownership as of January 1, 2010).

We focus on the Copenhagen urban area (Hovedstadsområdet). We take the definition of the Copenhagen urban area as containing the following municipalities (by number): 101, 147, 151, 153, 157, 159, 161, 163, 165, 167, 173, 175, 183, 185, 187, 253, 269.[46]

We restrict attention to private owners and also to the primary owner of a property in a given year (whenever a property has more than one owners). Furthermore, we examine transactions where the new owner is a private individual and which have a

[46]Due to a reform in 2007, which merged some municipalities and created a new one, we omit municipality 190 for consistency.

non-missing agreement date. We drop properties that are recorded to transact more than once in a given year. We also remove property-year observations for which no owner is recorded. This leaves us with a total of 3312520 property-year observations. These comprise 199812 unique properties and 345943 unique individual owners over our sample period.

To identify an individual owner as a buyer-and-seller we rely on the information from the ownership register across consecutive years. First of all, we use the information on ownership over consecutive years to determine the counterparties for each recorded transaction in our sample. We then identify an individual owner as a buyer-and-seller if he is recorded to buy a new property and sell an old property within the same year or over two consecutive years. An old property is defined as a property which an individual is registered as owning over at least 2 consecutive years.[47] Also, we do not count individuals that are recorded as holding two properties for two or more consecutive years, which we treat as purchases for investment purposes.

We conduct this for individuals that are recorded as owning at most 2 properties at the end of any calendar year in our sample. This comprises the large majority of individual owners in our sample. In particular, in a given year in our sample from 1991-2009 there are on average only around 0.4% of individual owners who own more than two properties in the Copenhagen. Therefore, the majority of individuals hold at most 1 or 2 properties over that period. In particular, on average, around 1.6% of individual owners hold two properties at the end of a calendar year in our sample. Interestingly, around 5% of the recorded owners of two properties at the end of a calendar year are also identified as a buyer-and-seller according to our identification procedure described above with that number going up to almost 14% at the peak of the housing boom in 2006.

For each individual owner that has been identified as buyer-and-seller, we compute the time period (in days) between the agreement data for sale of the old property and the agreement date for the purchase of the new property. Similarly, we compute the time period (in days) between the closing date that of the buyer-and-seller's old property by the new owner and the closing date for his new property. We then denote a buyer-and-seller for which the time period between agreement dates is negative (sale date

[47] We make this restriction in order not to misclassify as a buyer-and-seller an individual who acquires a house, for example as a bequest (which is not recorded as a transaction), which he ends up selling quickly and then buys a new house with the proceeds from the sale. Adding back those agents has a very small effect on the pattern we uncover.

is before purchase date) as "selling first" and a buyer-and-seller for which the time period is positive (sale date is after purchase date) as "buying first". We also do the same classification but based on closing dates rather than agreement dates. Given the way we identify a buyer-and-seller, we have a consistent count for the number of owners who "buy first" vs. "sell first" in a given year for the years 1993 to 2008.

In principle, and as Figures 8 and 9 show, working with either of the two identifications produces similar results. This is not surprising given that the time difference between the agreement dates and closing dates are highly correlated with a correlation coefficient of 0.9313. Figure 16 visualizes this strong correlation by plotting a scatter plot of the two time differences.

3.8.B Equilibrium concept and parameter restrictions for the basic model

First of all, the steady state value functions for a new entrant, a matched owner, and a real estate firm satisfy the following equations:

$$\rho V^{Bn} = u_n - R + q(\theta)\left(-p + V - V^{Bn}\right), \tag{3.26}$$

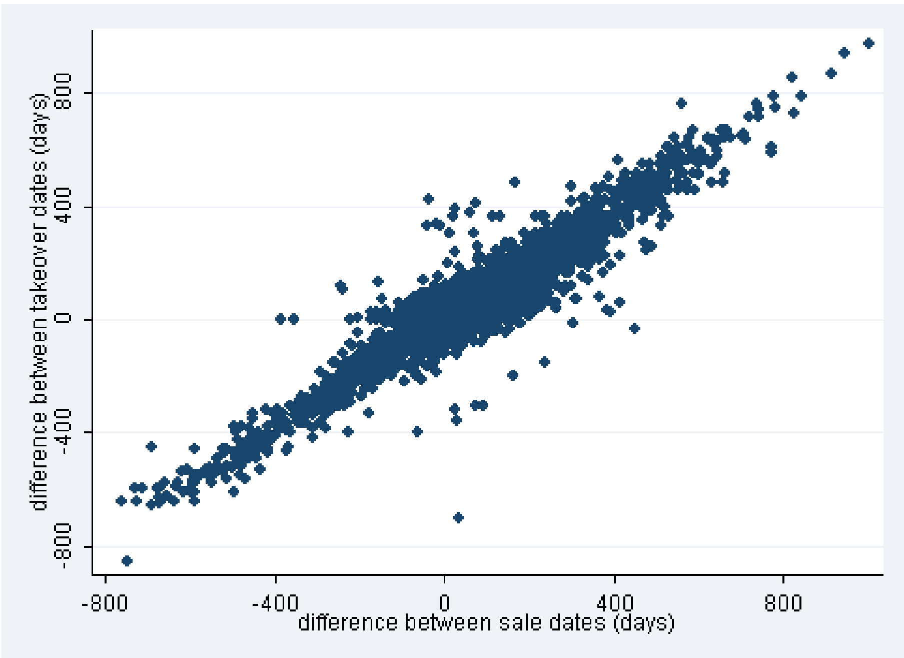

Figure 16: Difference in agreement dates vs. difference in closing dates, Copenhagen (1993-2008)

$$\rho V = u + \gamma\left(\max\left\{V^{B1}, V^{S1}\right\} - V\right), \tag{3.27}$$

and

$$\rho V^{A} = R + \mu(\theta)\left(p - V^{A}\right). \tag{3.28}$$

Importantly, in every steady state equilibrium, V satisfies $V \geq \tilde{V}$, where $\tilde{V} = \frac{u}{\rho+\gamma} + \frac{\gamma}{\rho+\gamma} V^{m}$, with $V^{m} = \frac{u-\chi}{\rho}$. Hence, $\tilde{V}$ is the value of a matched owner who never transacts. Therefore, $V \geq \tilde{V} = \frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}$ in any steady state equilibrium.

Parameter restrictions

Sufficient conditions for new entrants, forced renters and double owners to prefer transacting and becoming matched owners are given by

$$\frac{u_n - R}{\rho} \leq \tilde{V} - p, \tag{3.29}$$

$$\frac{u_0 - R}{\rho} \leq \tilde{V} - p, \tag{3.30}$$

and

$$\frac{u_2 + R}{\rho} \leq \tilde{V} + p. \tag{3.31}$$

Since $u_n \geq u_0$, we can disregard (3.30), as it is implied by (3.29). Conditions (3.29) and (3.31) imply restrictions for the values of the house price, p, that are sufficient for these agents to be willing to transact at p, namely $p \in \left[\frac{u_2}{\rho} - \tilde{V} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]$.

From (3.28) a real estate firm is willing to transact if and only if $p \geq \frac{R}{\rho}$. Therefore, equilibrium is defined for a house price p, that satisfies

$$p \in \left[\max\left\{\frac{u_2}{\rho} - \tilde{V}, 0\right\} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]. \tag{3.32}$$

For $u - \chi \geq \max\{u_0, u_2\}$, which is the condition we will use to characterize equilibria, it follows that $\frac{u_2}{\rho} - \tilde{V} < 0$ and so the set for prices is given by

$$p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho}\right]. \tag{3.33}$$

Finally, a sufficient condition for $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$ at $\theta = \tilde{\theta}$, with $\tilde{\theta}$ as defined in (3.15) is

Assumption 3.A2: $\frac{u-\chi}{\rho} < \frac{u-\chi}{\rho+\mu(\tilde{\theta})} + \frac{\mu(\tilde{\theta})}{(\rho+\mu(\tilde{\theta}))(\rho+q(\tilde{\theta}))}\tilde{u}_0 + \frac{\mu(\tilde{\theta})q(\tilde{\theta})}{(\rho+\mu(\tilde{\theta}))(\rho+q(\tilde{\theta}))}\left(\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}\right)$.

Note that $\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho} \leq V$, $\forall\theta$, so the right hand side of this expression is lower than the value of V^{S1} at $\theta = \tilde{\theta}$.

Steady state flow conditions

Before moving to our formal definition, it is necessary to describe the flow conditions that the aggregate stock variables defined in Section 3.3.2 must satisfy. We have that in a steady state equilibrium, given a market tightness θ, the steady state values of B_n, B_0, B_1, S_1, S_2, O, and A must satisfy the following system of flow conditions:

$$g = \left(q(\theta) + g\right) B_n, \tag{3.34}$$

$$\mu(\theta) S_1 = \left(q(\theta) + g\right) B_0, \tag{3.35}$$

$$\mu(\theta) S_2 + q(\theta)(B_n + B_0) = \left(\gamma + g\right) O, \tag{3.36}$$

$$\gamma x_b O = \left(q(\theta) + g\right) B_1, \tag{3.37}$$

$$\gamma x_s O = \left(\mu(\theta) + g\right) S_1, \tag{3.38}$$

$$q(\theta) B_1 = \left(\mu(\theta) + g\right) S_2, \tag{3.39}$$

$$g(O + B_1 + S_1 + 2S_2) = \mu(\theta) A, \tag{3.40}$$

$$x_b + x_s = 1, \tag{3.41}$$

where x_b, and x_s are the equilibrium fractions of mismatched buyers and sellers, respectively. Apart from these conditions, the aggregate variables must satisfy the population constancy and housing ownership conditions (3.2) and (3.3). Finally, the equilibrium market tightness θ, satisfies

$$\theta = \frac{B}{S} = \frac{B_n + B_0 + B_1}{S_1 + S_2 + A}. \tag{3.42}$$

Equilibrium definition

We define a steady state equilibrium for this economy in the following way:

Definition 3.9. A steady state equilibrium consists of a house price p, equilibrium rental rate R, value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^{A}, market tightness θ, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, and aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A such that:

1. The house price $p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho}\right]$;
2. The equilibrium rental rate $R \in [0, u_0]$;
3. The value functions satisfy equations (3.5)-(3.8) and (3.26)-(3.28) given θ, p, and R;
4. Mismatched owners choose $x \in \{b, s\}$, to maximize $\overline{V} = \max\left\{V^{B1}, V^{S1}\right\}$ and the fractions x_b, and x_s reflect that choice, i.e.

$$x_b = \int_i I\{x_i = b\}\, di,$$

   where $i \in [0,1]$ indexes the i-th mismatched owner, and similarly for x_s;
5. The market tightness θ solves (3.42) given B_n, B_0, B_1, S_1, S_2, O, and A;
6. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (3.34)-(3.40) given θ and mismatched owners' optimal decisions reflected in x_b and x_s.

3.8.C Proofs

Proof of Lemma 3.1

First of all, note that the function $D(\theta)$, defined in (3.13) crosses zero only at $\theta = \tilde{\theta}$. To see this, notice that

$$\lim_{\theta \to 0} D(\theta) = \frac{\tilde{u}_2 - (u - \chi)}{\rho} < 0,$$

and

$$\lim_{\theta \to \infty} D(\theta) = \frac{u - \chi - \tilde{u}_0}{\rho} > 0.$$

Away from these two limiting values, $D(\theta) > 0$, whenever

$$\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 > 0,$$

which is equivalent to $\tilde{\theta} < \theta$. Therefore, $D(\theta) > 0$ if and only if $\theta \in (\tilde{\theta}, \infty)$ and $D(\theta) < 0$ if and only if $\theta \in (0, \tilde{\theta})$. Therefore, $D(\theta) = 0$, if and only if

$$\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2 = 0,$$

or $\theta = \tilde{\theta}$. Note that $D(\theta)$ fully summarizes the incentives of a mismatched owner to buy first/sell first apart from at $\theta = 0$ and $\theta = \infty$. To see this, let

$$\begin{aligned}
\tilde{V}^{B1} &= \frac{u-\chi}{\rho + q(\theta)} + \frac{q(\theta)\,\tilde{u}_2}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\,\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))} V - \frac{u-\chi}{\rho} \\
&= \frac{q(\theta)}{\rho + q(\theta)} \left(\frac{\tilde{u}_2}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)} V - \frac{u-\chi}{\rho} \right),
\end{aligned}$$

and

$$\begin{aligned}
\tilde{V}^{S1} &= \frac{u-\chi}{\rho + \mu(\theta)} + \frac{\mu(\theta)\,\tilde{u}_0}{(\rho + \mu(\theta))(\rho + q(\theta))} + \frac{q(\theta)\,\mu(\theta)}{(\rho + \mu(\theta))(\rho + q(\theta))} V - \frac{u-\chi}{\rho} \\
&= \frac{\mu(\theta)}{\rho + \mu(\theta)} \left(\frac{\tilde{u}_0}{\rho + q(\theta)} + \frac{q(\theta)}{\rho + q(\theta)} V - \frac{u-\chi}{\rho} \right).
\end{aligned}$$

The functions $\tilde{V}^{B1}$ and $\tilde{V}^{S1}$ give the difference between the value of transacting and never transacting for a buyer first and seller first, respectively.

By Assumption 3.A2, at $\tilde{\theta}$, $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$, so at $\theta = \tilde{\theta}$, $\tilde{V}^{B1} > 0$, and $\frac{\tilde{u}_2}{\rho+\mu(\theta)} + \frac{\mu(\theta)}{\rho+\mu(\theta)} V - \frac{u-\chi}{\rho} > 0$. Furthermore, this latter inequality holds for any $\theta > \tilde{\theta}$, and so $\tilde{V}^{B1} > 0$ for any $\theta > \tilde{\theta}$. Therefore, for any $\theta > \tilde{\theta}$, a mismatched buyer is better off transacting than not transacting. Similarly, for $\theta < \tilde{\theta}$ the mismatched seller is better off transacting than not transacting.

Therefore, for $\theta \in (0, \infty)$, if $D(\theta) > 0$, a mismatched owners is better off buying first (and transacting) compared to selling first (and transacting or not transacting) and similarly, if $D(\theta) < 0$, a mismatched owner is better off selling first (and transacting) compared to buying first (and transacting or not transacting). At $D(\theta) = 0$, he is indifferent between buying first (and transacting) and selling first (and transacting).

Finally, clearly if $\theta \to \infty$, $\tilde{V}^{B1} \to 0$, so $V^{B1} \to \frac{u-\chi}{\rho} = V^{S1}$. Similarly, if $\theta \to 0$, $\tilde{V}^{S1} \to 0$, and so $V^{S1} \to \frac{u-\chi}{\rho} = V^{B1}$. □

Proof of Lemma 3.2

For the case where mismatched owners buy first ($x_s = 0$), the stock-flow conditions are

$$g = \left(q(\theta) + q\right) B_n,$$

$$\gamma O = \left(q(\theta) + g\right) B_1,$$

$$q(\theta) B_1 = \left(\mu(\theta) + g\right) S_2,$$

$$g = \left(\mu(\theta) + g\right) A,$$

$$B_n + B_1 + S_2 + O = 1,$$

and

$$B_n = A + S_2.$$

It follows that $B_n = \frac{g}{q(\theta)+g}$ and $A = \frac{g}{\mu(\theta)+g}$, or $A = \frac{q(\theta)+g}{\mu(\theta)+g} B_n$, so $S_2 = \frac{g}{q(\theta)+g} - \frac{g}{\mu(\theta)+g}$. Therefore, from the equation for θ, we have that $B_1 = (\theta - 1) B_n$ and $O = \frac{1}{\gamma}\left(q(\theta) + g\right)(\theta - 1) B_n$. Substituting into the population constancy condition, we have that

$$\theta B_n + B_n - \frac{q(\theta) + g}{\mu(\theta) + g} B_n + \frac{1}{\gamma}\left(q(\theta) + g\right)(\theta - 1) B_n = 1,$$

which, after substituting for B_n and re-arranging we can write as

$$\left(\frac{1}{q(\theta) + g} + \frac{1}{\gamma}\right)\theta + \left(\frac{1}{q(\theta) + g} - \frac{1}{\mu(\theta) + g}\right) = \frac{1}{g} + \frac{1}{\gamma}.$$

This is exactly equation (3.17). At $\theta = 1$, the left-hand side equals

$$\frac{1}{q(1) + g} + \frac{1}{\gamma} < \frac{1}{g} + \frac{1}{\gamma}.$$

Furthermore, note that $\left(\frac{1}{q(\theta)+g} + \frac{1}{\gamma}\right)\theta$ is strictly increasing in θ and also unbounded. Similarly, $\left(\frac{1}{q(\theta)+g} - \frac{1}{\mu(\theta)+g}\right)$ is strictly increasing in θ as well. Therefore, the left-hand side of (3.17) is strictly increasing in θ, unbounded, and lower than the right-hand side for $\theta = 1$. Therefore, it has a unique solution for $\theta > 1$. We call this solution $\overline{\theta}$. Furthermore, by the Implicit Function Theorem, it immediately follows that $\overline{\theta}$ is increasing in γ.

For the case where mismatched owners sell first ($x_s = 1$) the stock-flow conditions become

$$g = \left(q(\theta) + q\right) B_n,$$

$$\mu(\theta) S_1 = \left(q(\theta) + g\right) B_0,$$

$$\gamma O = \left(\mu(\theta) + g\right) S_1,$$

$$g = \left(\mu(\theta) + g\right) A,$$

$$B_n + B_0 + S_1 + O = 1,$$

and

$$B_n + B_0 = A.$$

It follows that $A = \frac{g}{\mu(\theta)+g} = B_0 + B_n$, $S_1 = \frac{1-\theta}{\theta} A$ and $O = \frac{1}{\gamma}\left(\mu(\theta) + g\right)\frac{1-\theta}{\theta} A$. Therefore, substituting for these in the population constancy condition, we have that

$$\frac{1}{\theta} A + \frac{1}{\gamma}\left(\mu(\theta) + g\right)\frac{1-\theta}{\theta} A = 1.$$

Substituting for A, we obtain an equation for θ of the form

$$\left(\frac{1}{\mu(\theta) + g} + \frac{1}{\gamma}\right)\frac{1}{\theta} = \frac{1}{g} + \frac{1}{\gamma},$$

which is equation (3.18). At $\theta = 1$, the left-hand side equals

$$\frac{1}{\mu(1) + g} + \frac{1}{\gamma} < \frac{1}{g} + \frac{1}{\gamma}.$$

Note also that $\left(\frac{1}{\mu(\theta)+g} + \frac{1}{\gamma}\right)\frac{1}{\theta}$ is strictly decreasing in θ and goes to 0 as $\theta \to \infty$. Also it asymptotes to ∞ as $\theta \to 0$. Therefore, the equation has a unique solution for $\theta < 1$. We call this solution $\underline{\theta}$. By the Implicit Function Theorem, it immediately follows that $\underline{\theta}$ is decreasing in γ. □

Proof of Proposition 3.3

Clearly, Lemma 3.2 that determines the values of $\overline{\theta}$ and $\underline{\theta}$ is independent of the agents' payoffs. With regard to Item 1, a direct application of Lemma 3.1 shows that if $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, then at $\theta = \underline{\theta}$ a mismatched owner is (weakly) better off selling first, at $\theta = \overline{\theta}$ he is (weakly) better off buying first, and at $\theta = \tilde{\theta}$ he is indifferent. If mismatched owners are indifferent, they can randomize, such that $\theta = \tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$. This follows since the stock-flow conditions ensure a continuous relation between the fraction of agents buying first, x_b, and θ, and observing that $\theta = \underline{\theta}$, if and only if $x_b = 0$ and, $\theta = \overline{\theta}$, if and only if $x_b = 1$, so that for $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, $x_b \in [0, 1]$. Also, by Assumption 3.A2, they are strictly better off from transacting than not transacting. Consequently, agents' actions are optimal given θ and the steady state value of θ is consistent with agents' actions. Considering Item 2, by the same logic a steady state equilibrium in which mismatched owners buy first and $\theta = \overline{\theta}$ exists. To see that it is the only symmetric steady state equilibrium, remember from Lemma 3.1 that mismatched owners only sell first for $\theta < \tilde{\theta}$, which contradicts $\tilde{\theta} < \underline{\theta}$. The same logic applies to Item 3. □

Proof of Corollary 3.4

For $\tilde{u}_0 = \tilde{u}_2 = c$, $\tilde{\theta} = 1$. Because Lemma 3.2 shows that $\overline{\theta} > 1$ when all mismatched owners buy first, and $\underline{\theta} < 1$ if they sell first, $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$. Then an application of Proposition 3.3 gives the buy first equilibrium and the sell first equilibrium. To see that $\theta = 1$ if half of the mismatched owners buy first, take $x_s = x_b = \frac{1}{2}$. We have

$$\gamma \frac{1}{2} O = \left(q(\theta) + g\right) B_1, \tag{3.43}$$

and

$$\gamma \frac{1}{2} O = \left(\mu(\theta) + g\right) S_1. \tag{3.44}$$

At $\theta = 1$, $\mu(\theta) = q(\theta) = \mu(1)$, so $B_1 = S_1$. Also, $B_n = A = \frac{g}{\mu(1)+g}$ and $B_0 = S_2 = S_1 \frac{\mu(1)}{\mu(1)+g}$. Finally, population constancy implies that

$$2S_1 \frac{\mu(1)}{\mu(1)+g} + 2S_1 + 2S_1 \frac{\mu(1)+g}{\gamma} = \frac{\mu(1)}{\mu(1)+g},$$

which is satisfied for some $S_1 \in \left(0, \frac{1}{2}\right)$. Because mismatched owners are indifferent at $\theta = 1$ for $\tilde{u}_0 = \tilde{u}_2 = c$ by Lemma 3.1, an equilibrium in which half of the mismatched owners buy first exists at $\theta = 1$. □

Proof of Proposition 3.7

Consider the difference between the two value functions, $D(\theta) = V^{B1} - V^{S1}$ assuming that the mismatched owner transacts in both cases.

$$D(\theta) = \frac{\mu(\theta)\left(1-\frac{1}{\theta}\right)\left(u-\chi-c+\lambda\left(\overline{V}_N - v^{B0}\right)\right)}{\left(\rho+q(\theta)+\lambda\right)\left(\rho+\mu(\theta)+\lambda\right)} + \frac{\frac{\lambda\mu(\theta)\left(1-\frac{1}{\theta}\right)q(\theta)}{\left(r+\mu(\theta)\right)\left(r+q(\theta)\right)}\left[\rho V - c\right] + \mu(\theta)\left(1+\frac{1}{\theta}\right)\lambda\left(p_N - p\right)}{\left(\rho+q(\theta)+\lambda\right)\left(\rho+\mu(\theta)+\lambda\right)}. \tag{3.45}$$

Consider the case of $1 < \theta \leq \overline{\theta}$, so $\overline{V}_N = V_N^{B1}$. If $\overline{V}_N = V_N^{B1}$, where V_N^{B1} denotes the value of buying first after the price change, this difference simplifies further to

$$D(\theta) = \frac{\mu(\theta)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+q(\theta)}\right)\left(u-\chi-c\right)+\left(1+\frac{1}{\theta}\right)\lambda\left(p_N-p\right)\right]}{\left(\rho+q(\theta)+\lambda\right)\left(\rho+\mu(\theta)+\lambda\right)}. \tag{3.46}$$

Suppose that $p_N < p$ and define θ_{B1}^{PR} as the solution to

$$\frac{\theta_{B1}^{PR}-1}{\theta_{B1}^{PR}+1}\left(1+\frac{\lambda}{\rho+q\left(\theta_{B1}^{PR}\right)}\right) = \frac{\lambda\left(p-p_N\right)}{\left(u-\chi-c\right)}. \tag{3.47}$$

Therefore, θ_{B1}^{PR} is the value of θ that leaves a mismatched owner indifferent between buying first and selling first he anticipates a price change of $p_N - p$ and a market tightness of $\theta > 1$ after the price change. Note that θ_{B1}^{PR} is increasing in $p - p_N$ if $\theta_{B1}^{PR} \geq 1$. Therefore, a sufficient condition for mismatched owners to prefer to sell first, given $1 < \theta \leq \overline{\theta}$, is that $\theta_{B1}^{PR} > \overline{\theta}$.

Similarly, consider the case of $\underline{\theta} \leq \theta < 1$, so $\overline{V}_N = V_N^{S1}$, where V_N^{S1} denotes the value of selling first after the price change. In that case the difference in value functions can be written as

$$D(\theta) = \frac{\mu(\theta)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+\mu(\theta)}\right)\left(u-\chi-c\right)+\left(1+\frac{1}{\theta}\right)\lambda\left(p_N-p\right)\right]}{\left(\rho+q(\theta)+\lambda\right)\left(\rho+\mu(\theta)+\lambda\right)}. \tag{3.48}$$

Suppose that $p_N > p$ and define θ_{S1}^{PR} as the solution to

$$\frac{\theta_{S1}^{PR}-1}{\theta_{S1}^{PR}+1}\left(1+\frac{\lambda}{\rho+\mu\left(\theta_{S1}^{PR}\right)}\right) = \frac{\lambda\left(p-p_N\right)}{\left(u-\chi-c\right)}. \tag{3.49}$$

Similarly, to the case of θ_{B1}^{PR}, θ_{S1}^{PR} is increasing in $p - p_N$ if $\theta_{S1}^{PR} \leq 1$. Then, a sufficient condition for mismatched owner to prefer to buy first, given $\underline{\theta} \leq \theta < 1$ is that $\theta_{S1}^{PR} < \underline{\theta}$. □

3.8.D A model with prices determined by Nash bargaining

We show our main analytical result for the model with Nash bargaining under the following parametric assumptions:

Assumption 3.B1: $u_2 - u_0 = u - u_n$.

Assumption 3.B2: $r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0\chi > 0$.

Assumption 3.B3: $r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0\chi\right]$.

Assumption 3.B4: $\frac{\gamma}{g} \leq \kappa^*$,

where $\kappa^* > 0$ is determined below. The first assumption ensures that buying first and selling first are equally attractive at $\theta = 1$. Assumptions 3.B2-3.B4 ensure that the trading pattern described in the main text emerges in equilibrium.

Proof of Proposition 3.5

Below, we use the notation Σ_{ij} to denote the surplus from trade between agents of type i and type j.

"Sell first" equilibrium existence. We first show that a "Sell first" equilibrium exists with $\theta = \underline{\theta} < 1$. We proceed in two steps. First, we show that no mismatched owner has an incentive to deviate and buy first when $\theta = \underline{\theta} < 1$. This is verified under the conjecture that $\Sigma_{ij} \geq 0$ for all buyer-seller pairs except for Σ_{S1B1}. Second, we verify the conjecture on the different surpluses.

Step 1. In the limit economy with small flows of a "Sell first" equilibrium candidate, the faction of buyers who are forced renters is given by

$$\lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{B_0}{B} = \lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{q\left(\underline{\theta}\right) - \mu\left(\underline{\theta}\right)}{g + q\left(\underline{\theta}\right)} = 1 - \underline{\theta},$$

where $\underline{\theta} = \frac{1}{1+\kappa}$. Similarly,

$$\lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{A}{S} = \underline{\theta}.$$

Thus,

$$\begin{aligned} rV^{B0} &= u_0 - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\frac{A}{S}\Sigma_{AB0} + \frac{S_1}{S}\Sigma_{S1B0}\right) \\ &= u_0 - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{AB0} + \left(1-\underline{\theta}\right)\Sigma_{S1B0}\right), \end{aligned}$$

and similarly,

$$rV^{Bn} = u_n - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{ABn} + \left(1-\underline{\theta}\right)\Sigma_{S1Bn}\right),$$

so

$$V^{Bn} - V^{B0} = \frac{u_n - u_0}{r + \frac{1}{2} q\left(\underline{\theta}\right)}. \tag{3.50}$$

Also,

$$\begin{aligned} rV^{A} &= R + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(\frac{B_n}{B}\left(V - V^{Bn} - V^{A}\right) + \frac{B_0}{B}\left(V - V^{B0} - V^{A}\right)\right) \\ &= R + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta}\left(V - V^{Bn} - V^{A}\right) + \left(1-\underline{\theta}\right)\left(V - V^{B0} - V^{A}\right)\right), \end{aligned}$$

or

$$V^{A} = \frac{R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} + \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V - V^{B0} - \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2} q\left(\underline{\theta}\right)}\right).$$

Analogous to Equation 3.50

$$V^{S2} - V^{A} = \frac{u_2 + \frac{1}{2}\mu\left(\underline{\theta}\right) V}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}.$$

This in turn implies that

$$V - V^{S2} = \frac{rV - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A} = \frac{u - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A}.$$

Turning to the value functions of mismatched owners, a mismatched seller has a value function given by

$$rV^{S1} = u - \chi + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(V - V^{S1} - \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2} q\left(\underline{\theta}\right)}\right),$$

which can be re-written as

$$V^{S1} = \frac{u-\chi}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)} + \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}V - \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\theta\right)}.$$

For the value function of a deviating mismatched buyer, assuming that trade takes place when he meets a real-estate firm but not when he meets a mismatched seller, writes

$$rV^{B1} = u-\chi+\frac{1}{2}q\left(\underline{\theta}\right)\underline{\theta}\Sigma_{AB1}.$$

Or

$$\left(r+\frac{1}{2}\mu\left(\underline{\theta}\right)\right)V^{B1} = u-\chi+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2}-V^{A}\right).$$

Consider the difference between the utilities from buying first compared to selling first. In the limit we consider, we have that

$$\left(r+\frac{1}{2}\mu\left(\underline{\theta}\right)\right)\left(V^{B1}-V^{S1}\right) = \frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2}-V^{A}-V+\underline{\theta}\frac{u_n-u_0}{\rho+\frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Substituting for $V^{S2}-V^{A}-V$, we get that

$$V^{B1}-V^{S1} = \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(\frac{u_2-u}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Note that at $\underline{\theta}=1$ (i.e. for $\kappa=0$),

$$\frac{u_2-u}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}=0,$$

given Assumption 3.B1. As $\underline{\theta}$ moves away from 1 toward 0 (κ increases), we have that $\frac{u_2-u}{r+\frac{1}{2}\mu(\underline{\theta})}+\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q(\underline{\theta})}$ decreases, so $V^{B1}<V^{S1}$ for $\underline{\theta}<1$. Therefore, it is not optimal for a mismatched owner to deviate and buy first in an equilibrium in which mismatched owners sell first and $\theta<1$.

Step 2. We verify that our conjectures for the surpluses are correct. It is clear given our assumptions that $\Sigma_{S2B1}=V-V^{B1}>0$ and $\Sigma_{S1B0}=V-V^{S1}>0$. Also, $\Sigma_{ABn}\geq 0$. Next, we show that $\Sigma_{S1Bn}>0$. In the limit we consider,

$$\begin{aligned}
\Sigma_{S1Bn} &= V - V^{Bn} + V^{B0} - V^{S1} = V - V^{S1} - \frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)} \\
&= \frac{\chi}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} + \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta} - 1\right) - r}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} \frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)} \\
&= \frac{r\left(\chi + u_0 - u_n\right) + \frac{1}{2}q\left(\underline{\theta}\right)\chi + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta} - 1\right)\left(u_n - u_0\right)}{\left(r + \frac{1}{2}q\left(\underline{\theta}\right)\right)\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right)}.
\end{aligned}$$

Therefore, at $\underline{\theta} = 1$, $\Sigma_{S1Bn} > 1$ if

$$r\left(\chi + u_0 - u_n\right) + \frac{1}{2}\mu_0\chi > 0.$$

Note that given Assumption 3.B1, this is equivalent to

$$r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0\chi > 0,$$

which holds by Assumption 3.B2. Therefore, by continuity of the value functions with respect to θ, it follows that there is a $\kappa_1 > 0$, such that for $\kappa < \kappa_1$, $\Sigma_{S1Bn} > 0$. Next, we show that $\Sigma_{ABn} > 0$. To show, this suppose, toward a contradiction, that $\Sigma_{ABn} < 0$. Then

$$rV^{Bn} + rV^A \leq u_n + \frac{1}{2}q\left(\underline{\theta}\right)\Sigma_{ABn} + \frac{1}{2}\mu\left(\underline{\theta}\right)\Sigma_{ABn},$$

where the inequality comes from $\Sigma_{ABn} < 0 < \Sigma_{S1Bn}$ and $\Sigma_{ABn} < \Sigma_{AB0}$, since $V^{Bn} > V^{B0}$. Therefore,

$$\Sigma_{ABn} \geq \frac{rV - u_n}{r + \frac{1}{2}q\left(\underline{\theta}\right) + \frac{1}{2}\mu\left(\underline{\theta}\right)} > 0,$$

so we arrive at a contradiction. $\Sigma_{ABn} > 0$ also implies that $\Sigma_{AB0} > 0$, since $V^{Bn} > V^{B0}$. Next notice that

$$\Sigma_{S2Bn} = \Sigma_{ABn} + V - V^{S2} + V^A = \Sigma_{ABn} + \frac{rV - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} > 0.$$

Again, this also implies that $\Sigma_{S2B0} > 0$. Next, we show that $\Sigma_{AB1} > 0$. In the limit we consider,

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2} - V^{B1} - V^A = V^{S2} - V^A - \frac{u - \chi}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V^{S2} - V^A\right) \\
&= \frac{r\left(V^{S2} - V^A\right) - \left(u - \chi\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} = \frac{\frac{r}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}u_2 + \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}u - \left(u - \chi\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} \\
&= \frac{r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu\left(\underline{\theta}\right)\chi}{\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right)^2}.
\end{aligned}$$

At $\underline{\theta} = 1$, $\Sigma_{AB1} > 0$ if $r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0\chi > 0$, which is our parametric Assumption 3.B2. Therefore, by continuity of the value functions with respect to $\underline{\theta}$, it follows that there is a $\kappa_2 > 0$, such that for $\kappa < \kappa_2$, $\Sigma_{AB1} > 0$. Finally, in the limit we consider

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - \left(u - \chi\right) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A} \\
&= \Sigma_{AB1} + \frac{rV^{B0} - \left(u - \chi\right) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}.
\end{aligned}$$

At $\underline{\theta} = 1$,

$$\begin{aligned}
\frac{rV^{B0} - \left(u - \chi\right) + R}{r + \frac{1}{2}\mu_0} &= \frac{\frac{r}{r+\frac{1}{2}\mu_0}\left(u_0 - R\right) + \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}u - \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}rV^{A} - \left(u - \chi\right) + R}{r + \frac{1}{2}\mu_0} \\
&= \frac{ru_0 + \frac{1}{2}\mu_0 u - \frac{1}{2}\mu_0\left(rV^{A} - R\right) - \left(r + \frac{1}{2}\mu_0\right)\left(u - \chi\right)}{\left(r + \frac{1}{2}\mu_0\right)^2}.
\end{aligned}$$

Substituting for Σ_{AB1}, we get

$$\Sigma_{S1B1} = \frac{r\left(u_0 + u_2 - 2\left(u - \chi\right)\right) + \mu\chi - \frac{1}{2}\mu_0\left(rV^{A} - R\right)}{\left(r + \frac{1}{2}\mu_0\right)^2}.$$

Therefore, a sufficient condition for $\Sigma_{S1B1} < 0$ at $\underline{\theta} = 1$ is

$$r\left(u_0 + u_2 - 2\left(u - \chi\right)\right) + \mu_0\chi \leq 0,$$

or

$$r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0\chi\right],$$

which is our parametric assumption 3.B3. Again by continuity of the value functions with respect to $\underline{\theta}$, we have that there is a $\kappa_3 > 0$, s.t. for $\kappa < \kappa_3$, $\Sigma_{S1B1} < 0$. Taking $\underline{\kappa} = \min\{\kappa_1, \kappa_2, \kappa_3\}$, we have that for $\kappa < \underline{\kappa}$, there is a "Sell first" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$.

"Buy first" equilibrium existence. We follow the same two steps to show the existence of a "Buy first" equilibrium with $\theta = \overline{\theta} > 1$. Again, we make the same conjectures on the different surpluses as in the case of the "Sell first" equilibrium. In the limit economy, the fraction of buyers who are new entrants is

$$\lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{B_n}{B} = \frac{1}{\overline{\theta}},$$

where $\overline{\theta} = 1+\kappa$. Also,

$$\lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{A}{S} = \frac{1}{\overline{\theta}}$$

as well. Therefore, similarly to the value functions in the "Sell first" equilibrium, we have that

$$\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^{A} = R+\frac{1}{2}\mu\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\left(V-V^{Bn}\right)+\frac{\overline{\theta}-1}{\overline{\theta}}\left(V^{S2}-V^{B1}\right)\right),$$

and

$$\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^{S2} = u_2+\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^{A}+\frac{1}{2}\mu\left(\overline{\theta}\right)V.$$

Therefore, as in the "Sell first" equilibrium,

$$V-V^{S2} = \frac{rV-u_2}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}-V^{A}.$$

Also, as in the "Sell first" equilibrium,

$$V^{Bn}-V^{B0} = \frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}.$$

Turning to the value functions of a mismatched buyer, we have that

$$rV^{B1} = u-\chi+\frac{1}{2}q\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\left(V^{S2}-V^{B1}-V^{A}\right)+\left(1-\frac{1}{\overline{\theta}}\right)\left(V-V^{B1}\right)\right),$$

For the value function of a deviating agent who chooses to sell first, we have that

$$rV^{S1} = u-\chi+\frac{1}{2}\mu\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\Sigma_{S1Bn}\right),$$

since $\Sigma_{S1B1}<0$. Then,

$$\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)V^{S1} = u-\chi+\frac{1}{2}q\left(\overline{\theta}\right)V+\frac{1}{2}q\left(\overline{\theta}\right)\frac{u_0-u_n}{r+\frac{1}{2}q\left(\overline{\theta}\right)}.$$

Therefore, the difference between $V^{B1}-V^{S1}$ satisfies

$$\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)\left(V^{B1}-V^{S1}\right)=\frac{1}{2}q\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\right).$$

At $\overline{\theta}=1$, we have that

$$\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}=0,$$

by Assumption 3.B1. As $\overline{\theta}$ increases, we have that $\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}$ increases, so $V^{B1}>V^{S1}$ for $\overline{\theta}>1$. Therefore, it is not optimal for a mismatched owner to deviate and sell first in an equilibrium in which mismatched owners buy first and $\theta>1$.

Finally, we verify that our conjectures for the surpluses are correct. As in the "Sell first" case, $\Sigma_{S1B0}>0$ and $\Sigma_{S2B1}>0$. Also, as in the "Sell first" case, in the limit we consider,

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2}-V^{B1}-V^{A}\\
&= V^{S2}-V^{A}-V+V-\frac{u-\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)}-\frac{\frac{1}{2}q\left(\overline{\theta}\right)}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\left[\frac{1}{\overline{\theta}}\left(V^{S2}-V^{A}-V\right)+V\right]\\
&= \frac{\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\frac{\overline{\theta}-1}{\overline{\theta}}\right)}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\\
&= \frac{r\left(u_2-\left(u-\chi\right)\right)+\frac{1}{2}\mu\left(\overline{\theta}\right)\chi+\frac{1}{2}q\left(\overline{\theta}\right)\frac{\overline{\theta}-1}{\overline{\theta}}\left(u_2-u\right)}{\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)}.
\end{aligned}$$

Note that at $\overline{\theta}=1$, Σ_{AB1} in the "Buy first" case is the same as the "Sell first" case. Therefore, there is a $\kappa_4>0$, such that for $\kappa<\kappa_4$ and $\overline{\theta}=1+\kappa$, $\Sigma_{AB1}>0$. Similarly,

$$\begin{aligned}
\Sigma_{S1Bn} &= V-V^{Bn}+V^{B0}-V^{S1}=V-V^{S1}-\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\\
&= \frac{\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)}-\frac{r}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\\
&= \frac{r\left(\chi+u_0-u_n\right)+\frac{1}{2}q\left(\overline{\theta}\right)\chi}{\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)^2},
\end{aligned}$$

which at $\overline{\theta}=1$ is again the same as for the "Sell first" case. Therefore, there is a $\kappa_5>0$, such that for $\kappa<\kappa_5$, $\Sigma_{S1Bn}>0$. A similar argument to the one for the "Sell first" case also confirms that $\Sigma_{ABn}>0$, $\Sigma_{AB0}>0$, $\Sigma_{S2Bn}>0$ and $\Sigma_{S2B0}>0$. Finally,

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R + \frac{1}{2}q\left(\overline{\theta}\right)\left(\overline{\theta}-1\right)\left(V^{S2} - V^{B1} - V^{A}\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)} - V^{A} \\
&= \left(1 + \frac{\frac{1}{2}q\left(\overline{\theta}\right)\left(\overline{\theta}-1\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)}\right)\Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}q\left(\overline{\theta}\right)}.
\end{aligned}$$

At $\overline{\theta} = 1$, showing that $\Sigma_{S1B1} < 0$ in the "Buy first" case therefore follows the "Sell first" case, so that $\Sigma_{S1B1} < 0$ for $\kappa < \kappa_6$, for some $\kappa_6 > 0$. Taking $\overline{\kappa} = \min\{\kappa_4, \kappa_5, \kappa_6\}$, we have that for $\kappa < \overline{\kappa}$, there is a "Buy first" equilibrium with a market tightness given by $\overline{\theta} = 1 + \kappa$. Finally, taking $\kappa^* = \min\{\overline{\kappa}, \underline{\kappa}\}$, we arrive at the desired result. □

3.8.E A model with competitive search

We define a competitive search equilibrium for the economy described in Section 3.5.3. Let $(\mathscr{P}, \Theta)$ denote the active market segments in the economy, i.e. segments that attract a positive measure of buyers and sellers. The following equations describe the steady state value functions of agents. For new entrants we have:

$$\rho V^{Bn} = u_n - R + \max_{(p,\theta)\in(\mathscr{P},\Theta)} \left\{q(\theta)\left(-p + V - V^{Bn}\right)\right\}. \tag{3.51}$$

Similarly, for a real estate firm, we have

$$\rho V^{A} = R + \max_{(p,\theta)\in(\mathscr{P},\Theta)} \left\{\mu(\theta)\left(p - V^{A}\right)\right\}. \tag{3.52}$$

For mismatched owners that buy first, we have

$$\rho V^{B1} = u - \chi + \max\left\{0, \max_{(p,\theta)\in(\mathscr{P},\Theta)} \left\{q(\theta)\left(-p + V^{S2} - V^{B1}\right)\right\}\right\}, \tag{3.53}$$

where the value function takes into account the possibility that a mismatched buyer may be better off not searching. Similarly, if the mismatched owner sells first, we have

$$\rho V^{S1} = u - \chi + \max\left\{0, \max_{(p,\theta)\in(\mathscr{P},\Theta)} \left\{\mu(\theta)\left(p + V^{B0} - V^{S1}\right)\right\}\right\}. \tag{3.54}$$

A double owner solves

$$\rho V^{S2} = u_2 + R + \max_{(p,\theta)\in(\mathscr{P},\Theta)} \left\{\mu(\theta)\left(p + V - V^{S2}\right)\right\}, \tag{3.55}$$

while a forced renter solves

$$\rho V^{B0} = u_0 - R + \max_{(p,\theta)\in(\mathscr{P},\Theta)} \left\{q(\theta)\left(-p + V - V^{B0}\right)\right\}. \tag{3.56}$$

Finally, for a matched owner we have

$$\rho V = u + \gamma\left(\max\left\{V^{B1}, V^{S1}\right\} - V\right). \tag{3.57}$$

Next, we describe the steady state stock-flow conditions. Let

$$\left(p^{Bn}, \theta^{Bn}\right) \in \left(\mathscr{P}^{Bn}, \Theta^{Bn}\right) \equiv \arg\max_{(p,\theta)} \left\{q(\theta)\left(-p + V - V^{Bn}\right)\right\} \subset (\mathscr{P}, \Theta) \tag{3.58}$$

denote a market segment that maximizes the value of searching for a new entrant. We define $\left(p^j, \theta^j\right)$ and $\left(\mathscr{P}^j, \Theta^j\right)$ analogously for an agent type $j \in \{A, B1, S1, B0, S2\}$. For agents $j \in \{B1, S1\}$, we adopt the convention that $\Theta^j = \emptyset$ if they choose not to search.

We have the following stock-flow conditions

$$g = \left(\sum_{\theta\in\Theta} x^{Bn}(\theta)\, q(\theta) + g\right) B_n, \tag{3.59}$$

$$\sum_{\theta\in\Theta} x^{S1}(\theta)\, \mu(\theta)\, S_1 = \left(\sum_{\theta\in\Theta} x^{B0}(\theta)\, q(\theta) + g\right) B_0, \tag{3.60}$$

$$\gamma x_b O = \left(\sum_{\theta\in\Theta} x^{B1}(\theta)\, q(\theta) + g\right) B_1, \tag{3.61}$$

$$\gamma x_s O = \left(\sum_{\theta\in\Theta} x^{S1}(\theta)\, \mu(\theta) + g\right) S_1, \tag{3.62}$$

$$\sum_{\theta\in\Theta} x^{B1}(\theta)\, q(\theta)\, B_1 = \left(\sum_{\theta\in\Theta} x^{S2}(\theta)\, \mu(\theta) + g\right) S_2, \tag{3.63}$$

$$g\left(O + B_1 + S_1 + 2S_2\right) = \sum_{\theta\in\Theta} x^A(\theta)\, \mu(\theta)\, A, \tag{3.64}$$

$$x_b + x_s = 1, \tag{3.65}$$

with

$$\sum_{\theta \in \Theta} x^j(\theta) = 1 \; \forall j \in \{Bn, A, B0, S2\}, \tag{3.66}$$

where $x^j(\theta) = 0$ if $\theta \notin \Theta^j$ and, if a mismatched buyer/seller chooses to search,

$$\sum_{\theta \in \Theta} x^j(\theta) = 1 \text{ for } j \in \{B1, S1\}, \tag{3.67}$$

with $x^j(\theta) = 0$ if $\theta \notin \Theta^j$. In the above expressions $\mathbf{x}^j(\theta) \geq 0$ is the vector of mixing probabilities over segments in Θ for an agent $j \in \{Bn, A, B1, S1, B0, S2\}$. Market tightnesses in each segment are given by

$$\theta = \frac{x^{Bn}(\theta) B_n + x^{B1}(\theta) B_1 + x^{B0}(\theta) B_0}{x^A(\theta) A + x^{S2}(\theta) S_2 + x^{S1}(\theta) S_1}, \tag{3.68}$$

where $x^j(\theta) = 0$ if $\theta \notin \Theta^j$.

Finally, we have the population constancy and housing ownership conditions

$$B_n + B_0 + B_1 + S_1 + S_2 + O = 1, \tag{3.69}$$

and

$$O + B_1 + S_1 + A + 2S_2 = 1. \tag{3.70}$$

Following Moen (1997), we additionally require that the active market segments $(\mathscr{P}, \Theta)$ are such that the equilibrium allocation is a "no-surplus allocation". Formally, we make the following requirement.

No-surplus allocation. Let $\mathscr{B} \subset \{Bn, B1, B0\}$ and $\mathscr{S} \subset \{A, S1, S2\}$ denote the sets of *active* buyers and sellers in a steady state equilibrium, that is agents that have a strictly positive measure in steady state. Given the set of active segments $(\mathscr{P}, \Theta)$ and agents' steady state value functions $\{V^{Bn}, V^{B1}, V^{B0}, V^A, V^{S1}, V^{S2}\}$, there exists no pair $(p, \theta) \notin (\mathscr{P}, \Theta)$, such that $V^i(p, \theta) > V^i$, for some $i \in \{Bn, B1, B0\}$, and $V^j(p, \theta) \geq V^j$ for some $j \in \mathscr{S}$, or $V^i(p, \theta) > V^i$, for some $i \in \{A, S1, S2\}$, and $V^j(p, \theta) \geq V^j$ for some $j \in \mathscr{B}$, where $V^i(p, \theta)$ denotes the steady state value function of an agent that trades in segment (p, θ), for $i \in \{Bn, B1, B0, A, S1, S2\}$.

Informally, the no-surplus allocation condition requires that in equilibrium there are no agents that would be strictly better off from deviating and opening a new market segment that would be at least as attractive for some active agents (buyers or sellers) compared to their equilibrium values.

We can now define a symmetric steady state competitive search equilibrium of this economy as follows

Definition 3.10. A symmetric steady state competitive search equilibrium of this economy consists of a set of active market segments $(\mathscr{P},\Theta)$, steady state value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^A, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A, distributions of agent types over active market segments $\{\mathbf{x}^j\}_{j\in\{Bn,A,B1,S1,B0,S2\}}$, and set of active buyers and sellers, $\mathscr{B}$ and $\mathscr{S}$, such that

1. The value functions satisfy equations (3.51) - (3.57) and the mixing distributions $\{\mathbf{x}^j\}_j$ are consistent with the agents' optimization problems.
2. Mismatched owners choose to buy first or sell first, in order to maximize $\overline{V} = \max\{V^{B1}, V^{S1}\}$ and the fractions x_b, and x_s reflect that choice, i.e.

$$x_b = \int_i I\{x_i = b\}\, di,$$

   where $i \in [0,1]$ indexes the i-th mismatched owner, and similarly for x_s;
3. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (3.59)-(3.64) and (3.69)-(3.70) given Θ, $\{\mathbf{x}^j\}_j$ and mismatched owners' optimal decisions, reflected in x_b and x_s.
4. Every $\theta \in \Theta$ satisfies equation (3.68) given B_n, B_0, B_1, S_1, S_2, O, A, and $\{\mathbf{x}^j\}_j$;
5. The set of active buyers and sellers, $\mathscr{B}$ and $\mathscr{S}$, is consistent with mismatched owners' optimal decisions;
6. $(\mathscr{P},\Theta)$ and agents' steady state value functions satisfy the no-surplus allocation condition.

Proof of Proposition 3.6

Consider first the "Buy first" equilibrium as described in section 3.5.3. In this "Buy first" equilibrium there are three active market segments characterized by prices $p_1^{B1} > p_2^{B1} > p_3^{B1}$ and market tightnesses $\theta_1^{B1} < \theta_2^{B1} < \theta_3^{B1}$. New entrants trade with real estate agents in market 1 and with double owners in market 2, while the latter also trade with mismatched buyers in market 3. Let x^{Bn} denote the probability with which a new entrant visits segment $\left(p_1^{B1}, \theta_1^{B1}\right)$, and x^{S2} the probability with which a double owner visits segment $\left(p_2^{B1}, \theta_2^{B1}\right)$. The stock-flow conditions for this equilibrium are

$$B_n = \frac{g}{x^{Bn} q\left(\theta_1^{B1}\right) + \left(1 - x^{Bn}\right) q\left(\theta_2^{B1}\right) + g}, \tag{3.71}$$

$$A = \frac{g}{\mu\left(\theta_1^{B1}\right) + g}, \tag{3.72}$$

$$B_1 = \frac{\gamma O}{q\left(\theta_3^{B1}\right) + g}, \tag{3.73}$$

$$S_2 = \frac{q\left(\theta_3^{B1}\right) B_1}{x^{S2} \mu\left(\theta_2^{B1}\right) + \left(1 - x^{S2}\right) \mu\left(\theta_3^{B1}\right) + g}, \tag{3.74}$$

$$B_n + B_1 + S_2 + O = 1, \tag{3.75}$$

and

$$B_n = A + S_2. \tag{3.76}$$

The market tightnesses in each active segment satisfy

$$\theta_1^{B1} = \frac{x^{Bn} B_n}{A}, \tag{3.77}$$

$$\theta_2^{B1} = \frac{\left(1 - x^{Bn}\right) B_n}{x^{S2} S_2}, \tag{3.78}$$

and

$$\theta_3^{B1} = \frac{B_1}{\left(1 - x^{S2}\right) S_2}. \tag{3.79}$$

Observe that (3.71), (3.72), (3.76), and (3.77) imply that $x^{Bn} < 1$, as otherwise, (3.71), (3.72) and (3.77) give

$$\theta_1^{B1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{B1}\right) + g}{q\left(\theta_1^{B1}\right) + g}, \tag{3.80}$$

which has a unique solution at $\theta_1^{B1} = 1$. However, this is inconsistent with (3.76).

Let Σ_{ij}, for $i \in \{B_n, B_0, B_1\}$ and $j \in \{A, S_1, S_2\}$ denote the match surplus from trading between a buyer i and seller j. The no-surplus allocation condition determines the equilibrium prices in each segment as a function of the steady state values of agents. Define

$$\begin{aligned}\overline{V}^{Bn} &= q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right)\\ &= q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right),\end{aligned}$$

$$\overline{V}^{A} = \mu\left(\theta_1^{B1}\right)\left(p_1^{B1} - V^{A}\right),$$

$$\overline{V}^{B1} = q\left(\theta_3^{B1}\right)\left(-p_3^{B1} + V^{S2} - V^{B1}\right),$$

and

$$\begin{aligned}\overline{V}^{S2} &= \mu\left(\theta_2^{B1}\right)\left(p_2^{B1} + V - V^{S2}\right)\\ &= \mu\left(\theta_3^{B1}\right)\left(p_3^{B1} + V - V^{S2}\right),\end{aligned}$$

as the maximized value of searching for each trader. The no-surplus allocation condition implies that

$$\begin{aligned}\left(p_1^{B1}, \theta_1^{B1}\right) &= \arg\max_{p,\theta} \mu(\theta)\left(p - V^{A}\right),\\ \text{s.t. } & q(\theta)\left(-p + V - V^{Bn}\right) \geq \overline{V}^{Bn}.\end{aligned}$$

Denote the elasticity of the matching function with respect to buyers by α (which may depend on θ). Solving for p_1^{B1} and θ_1^{B1} gives the well-known Hosios rule (Hosios (1990)),

$$p_1^{B1} - V^{A} = (1-\alpha)\,\Sigma_{BnA},$$

or equivalently,

$$p_1^{B1} = (1-\alpha)\left(V - V^{Bn}\right) + \alpha V^{A}.$$

Therefore,

$$\overline{V}^{Bn} = \alpha q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2},$$

or

$$q\left(\theta_1^{B1}\right)\Sigma_{BnA} = q\left(\theta_2^{B1}\right)\Sigma_{BnS2}. \tag{3.81}$$

We have similar surplus sharing rules between the other trading pairs, which determine p_2^{B1} and p_3^{B1}. There is one more indifference condition for a double owner that relates θ_2^{B1} and θ_3^{B1}. Specifically,

$$\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \mu\left(\theta_3^{B1}\right)\Sigma_{B1S2}. \tag{3.82}$$

These surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2}, \tag{3.83}$$

$$\rho V^A = R + (1-\alpha)\,\mu\left(\theta_1^{B1}\right)\Sigma_{BnA}, \tag{3.84}$$

$$\rho V^{B1} = u - \chi + \alpha q\left(\theta_3^{B1}\right)\Sigma_{B1S2}, \tag{3.85}$$

$$\rho V^{S2} = u_2 + R + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2}, \tag{3.86}$$

and

$$\rho V = u + \gamma\left(V^{B1} - V\right). \tag{3.87}$$

Finally, use V^{Bn} and V^{S2} from (3.83) and (3.86) to solve for

$$\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\rho + \alpha q\left(\theta_2^{B1}\right) + (1-\alpha)\,\mu\left(\theta_2^{B1}\right)}. \tag{3.88}$$

Similarly, using V^{Bn} and V^A from (3.83) and (3.52), combined with indifference condition (3.81), to solve for

$$\Sigma_{BnA} = V - V^{Bn} - V^A = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{B1}\right) + (1-\alpha)\,\mu\left(\theta_1^{B1}\right)}. \tag{3.89}$$

Solving for V^{B1} from equation (3.85), we get

$$V^{B1} = \frac{u-\chi}{\rho + \alpha q\left(\theta_3^{B1}\right)} + \frac{\alpha q\left(\theta_3^{B1}\right)}{\rho + \alpha q\left(\theta_3^{B1}\right)} V,$$

so

$$\Sigma_{B1S2} = V - V^{B1} = \frac{\rho V - (u-\chi)}{\rho + \alpha q\left(\theta_3^{B1}\right)}. \tag{3.90}$$

Therefore, equations (3.71)-(3.79), combined with the two indifference conditions (3.81) and (3.82), and the value function equations (3.83)-(3.87) with surpluses (3.88)-(3.90) jointly determine the equilibrium stocks of agents, market tightnesses, mixing probabilities x^{Bn} and x^{S2}, and active agent value functions in a "Buy first" equilibrium.

We now prove existence of this equilibrium when χ and u_2 are small, and u_0 is strictly smaller than u_n. Note that $\Sigma^{S2B1} = V - V^{B1} > 0$ for any u_2, but that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{B1} = \frac{u}{\rho}$, so that $\lim_{\chi\to 0}\Sigma_{B1S2} = 0$. This in turn implies that $\lim_{\chi\to 0}\theta_3^{B1} = \infty$ and $\lim_{\chi\to 0} x^{S2} = 1$. To see this, suppose to the contrary that as $\chi \to 0$, θ_3^{B1} remains bounded and thus x^{S2} is strictly below one. Therefore, $\mu\left(\theta_3^{B1}\right)\Sigma_{B1S2} \to 0$, so indifference condition (3.82) implies that $\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} \to 0$. Given (3.88), this in turn means that $\theta_2^{B1} \to 0$ and thus $x^{Bn} \to 1$. However,

$$\lim_{\theta_2^{B1}\to 0} q\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\alpha},$$

which is inconsistent with $x^{Bn} \to 1$. To see this, remember from (3.80) that $\theta_1^{B1} \to 1$ as $x^{Bn} \to 1$. Because

$$\lim_{\theta_1^{B1}\to 1} q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \frac{\rho V - u_n}{\frac{\rho}{q(1)} + 1} < \rho V - u_n < \frac{2\rho V - u_n - u_2}{\alpha},$$

in this case new entrants would be strictly better off participating in the second market segment. Thus, we arrive at a contradiction.

As $\theta_3^{B1} \to \infty$, $q\left(\theta_3^{B1}\right) \to 0$, and mismatched owners do not buy to become double owners: $S_2 \to 0$. Without trading partners in market 2, all new entrants visit market 1: $x^{Bn} \to 1$ and thus $\theta_1^{B1} \to \frac{B_n}{A} \to 1$. In this case, V^{Bn} from (3.83) is given by

$$\lim_{\chi\to 0} \rho V^{Bn} = u_n + \frac{\alpha q(1)}{\rho + q(1)}(u - u_n) - R, \tag{3.91}$$

which is strictly between 0 and ρV, as long as R is not too large. Similarly, using that $\mu(1) = q(1)$, V^A is given by

$$\lim_{\chi\to 0} \rho V^A = R + \frac{(1-\alpha)q(1)}{\rho + q(1)}(u - u_n), \tag{3.92}$$

which is also strictly between 0 and ρV if R is not too large. As a result,

$$\lim_{\chi\to 0}\left(V^A + V^{Bn}\right) = \frac{u_n}{\rho} + \frac{q(1)}{\rho + q(1)}\frac{u - u_n}{\rho},$$

which is strictly between 0 and V. By continuity, there exists a $\overline{\chi}_1 > 0$ such that for $\chi \in \left(0, \overline{\chi}_1\right)$, it is the case that $\Sigma_{BnA} > 0$, $x^{Bn} \in (0,1)$ and $\theta_1^{B1} \in (0,1)$, but also $\Sigma_{B1S2} > 0$.

With V^{Bn} as defined in (3.91) above, it follows that V^{S2} is uniquely determined as

$$\rho V^{S2} = \max_{p,\theta}\left\{u_2 + R + \mu(\theta)(V + p - V^{S2})\right\},$$

subject to $u_n - R + q(\theta)(V - p - V^{Bn}) = \rho V^{Bn}$. Note that V^{S2} goes to negative infinity for any $\chi > 0$ when u_2 does. To see this, suppose to the contrary that V^{S2} remains bounded when u_2 goes to negative infinity. Then $\mu(\theta)$ must go to infinity, and hence θ must go to infinity. But then $q(\theta)$ goes to zero, and for the new entrants to get their outside option, p must go to $-\infty$. In this case V^{S2} still goes to negative infinity, so that we arrive at a contradiction. Consequently, for any $\chi > 0$ there exists a $\overline{u}_2^{B1}$ such that for $u_2 < \overline{u}_2^{B1}$, V^{S2} is sufficiently low such that both $\Sigma_{B1A} = V^{S2} - V^A - V^{B1} < 0$ and $\Sigma_{BnS2} = 2V - V^{Bn} - V^{S2} > \Sigma_{BnA} > 0$. We can then conclude that there is trade in markets 1, 2, and 3, but that real estate agents and mismatched buyers do not open a fourth market.[48]

Furthermore, it follows that for any $u_2 < \overline{u}_2^{B1}$ there exists a $\overline{\chi}_2 > 0$ such that for $\chi \in \left(0, \overline{\chi}_2\right)$, it is the case that $\Sigma_{BnS2} > \Sigma_{B1S2}$. Given this ranking and the fact that $\Sigma_{BnA} < \Sigma_{BnS2}$, the ranking of tightnesses across segments then follows from the indifference conditions (3.81) and (3.82). Having established the ranking of tightnesses, the ranking of prices across segments immediately follows from the indifference conditions as well. Specifically, (3.81) implies that

$$q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right) = q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right),$$

or

$$\frac{q\left(\theta_1^{B1}\right)}{q\left(\theta_2^{B1}\right)} = \frac{-p_2^{B1} + V - V^{Bn}}{-p_1^{B1} + V - V^{Bn}}.$$

$\theta_1^{B1} < \theta_2^{B1}$ and $q(.)$ decreasing imply that $p_1^{B1} > p_2^{B1}$. Similarly, (3.82) implies that $p_2^{B1} > p_3^{B1}$. Finally, note that $\Sigma_{BnS2} > \Sigma_{B1S2}$ implies that $V - V^{Bn} > V^{S2} - V^{B1}$, so a new entrant is more impatient than a mismatched buyer in the sense that the direct utility gain from transacting is higher for a new entrant compared to a mismatched buyer.[49]

Consider now a mismatched owner that deviates and sells first, and upon trade becomes a forced renter. We allow both a mismatched seller and a forced renter to open new market segments with active agents as counterparties. First, observe that $V^{Bn} > V^{B0}$ for any $\chi > 0$, that is, a new entrant is always better off than a forced renter. This ranking comes from the assumption that $u_0 < u_n$ and from a revealed preference argument. Specifically, suppose to the contrary that $V^{B0} > V^{Bn}$. Suppose also that it is optimal for a forced renter to trade with a real estate firm (the argument for the

[48]For market 2 to be active, it is sufficient for u_2 to be low enough to ensure $\Sigma_{BnS2} > 0$, even if $\Sigma_{BnS2} < \Sigma_{BnA}$. However, $u_2 < \overline{u}_2^{B1}$ ensures the ranking of tightnesses and prices proven next.

[49]This also implies that a new entrant has steeper sloped indifference curves in the $\theta - p$ space, so he is willing to trade-off a higher price for the same decrease in market tightness compared to a mismatched buyer.

case where the forced renter trades with a double owner is analogous). The no-surplus allocation condition again implies that the Hosios condition holds, so

$$\rho V^{B0} = u_0 - R + \alpha q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^{A}\right),$$

where $\tilde{\theta}$ is such that a real estate firm is indifferent between trading in this new segment and trading in the segment with a tightness of θ_1^{B1} and a price of p_1^{B1}. In contrast, we have that

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right).$$

Since $u_0 < u_n$ but $V^{B0} > V^{Bn}$, we have $q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^{A}\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right)$ and so $\tilde{\theta} < \theta_1^{B1}$. But then a new entrant is better off deviating and trading in the segment with tightness $\tilde{\theta}$, since $q\left(\tilde{\theta}\right)\left(V - V^{Bn} - V^{A}\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right)$. Furthermore, given that $V^{B0} > V^{Bn}$, $\Sigma_{BnA} > \Sigma_{B0A}$, so a real estate firm is in fact also strictly better off trading with a new entrant in the segment with tightness $\tilde{\theta}$. However, this is not consistent with $\left(p_1^{B1}, \theta_1^{B1}\right)$ not violating the no-surplus allocation condition. Therefore, in an equilibrium where $\left(p_1^{B1}, \theta_1^{B1}\right)$ are consistent with the no-surplus allocation, we must have $q\left(\tilde{\theta}\right) < q\left(\theta_1^{B1}\right)$. However, this means that $V^{B0} < V^{Bn}$, and we arrive at a contradiction.

We conclude that $V^{B0} > V^{Bn}$ and $\Sigma_{BnA} < \Sigma_{B0A}$, so that a forced renter is the most impatient of the buyers. The forced renter will therefore trade with a real estate agent, the most patient of the sellers. A new submarket opens up, and real estate firms flow into this submarket up to the point where they are indifferent between selling to the deviator and to a new agent. Now suppose the deviating mismatched owner sells to a new entrant. Then the match surplus reads

$$\Sigma_{BnS1} = V - V^{Bn} + V^{B0} - V^{S1} \leq V - V^{Bn} + V^{B0} - \frac{u - \chi}{\rho}.$$

Given that $V^{Bn} - V^{B0}$ is bounded away from zero for any $\chi > 0$, there exists a $\overline{\chi}_3 > 0$ such that for $\chi \in \left(0, \overline{\chi}_3\right)$, it is the case that $\Sigma_{BnS1} < 0$. Note, however, that $\Sigma_{BnS1} > \Sigma_{B1S1}$ for $\chi < \overline{\chi}_2$, since, as shown above, in that case $V - V^{Bn} > V^{S2} - V^{B1}$, meaning that a new entrant is more impatient than a mismatched buyer. Therefore, for $\chi < \min\left\{\overline{\chi}_1, \overline{\chi}_2, \overline{\chi}_3\right\}$, $\Sigma_{B1S1} < \Sigma_{BnS1} < 0$ and $\Sigma_{B1S2} > 0$. In that case a mismatched owner that deviates and sells first is better off not trading. However, not trading is dominated by buying first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from buying first in a "Buy first" equilibrium.

Constructing a "Sell first" equilibrium follows similar steps. In this equilibrium there are three active market segments characterized by prices $p_1^{S1} < p_2^{S1} < p_3^{S1}$ and market tightnesses $\theta_1^{S1} > \theta_2^{S1} > \theta_3^{S1}$. Real estate agents trade with new entrants in market 1 and with forced renters in market 2, while the latter also trade with mismatched sellers in market 3. Let x^A denote the probability with which a real estate firm visits segment $\left(p_1^{S1}, \theta_1^{S1}\right)$, and x^{B0} the probability with which a forced renter visits segment $\left(p_2^{S1}, \theta_2^{S1}\right)$. The stock-flow conditions in this case become

$$B_n = \frac{g}{q\left(\theta_1^{S1}\right) + g}, \tag{3.93}$$

$$A = \frac{g}{x^A \mu\left(\theta_1^{S1}\right) + \left(1 - x^A\right)\mu\left(\theta_2^{S1}\right) + g}, \tag{3.94}$$

$$S_1 = \frac{\gamma O}{\mu\left(\theta_3^{S1}\right) + g}, \tag{3.95}$$

$$B_0 = \frac{\mu\left(\theta_3^{S1}\right) S_1}{x^{B0} q\left(\theta_2^{S1}\right) + \left(1 - x^{B0}\right) q\left(\theta_3^{S1}\right) + g}, \tag{3.96}$$

$$B_n + B_0 + S_1 + O = 1, \tag{3.97}$$

and

$$B_n + B_0 = A. \tag{3.98}$$

The market tightnesses in each active segment satisfy

$$\theta_1^{S1} = \frac{B_n}{x^A A}, \tag{3.99}$$

$$\theta_2^{S1} = \frac{x^{B0} B_0}{\left(1 - x^A\right) A}. \tag{3.100}$$

and

$$\theta_3^{S1} = \frac{\left(1 - x^{B0}\right) B_0}{S_1}, \tag{3.101}$$

Similarly to before, observe that (3.93), (3.94), (3.98), and (3.101) imply that $x^A < 1$, as otherwise, (3.93), (3.94) and (3.101) give

$$\theta_1^{S1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{S1}\right) + g}{q\left(\theta_1^{S1}\right) + g},$$

which has a unique solution at $\theta_1^{S1} = 1$. However, this is inconsistent with (3.98). As before, the no-surplus allocation implies that the match surpluses between trading pairs are split according to the Hosios rule. Consequently, there are two indifference conditions for real estate firms and forced renters given by

$$\mu\left(\theta_1^{S1}\right)\Sigma_{BnA} = \mu\left(\theta_2^{S1}\right)\Sigma_{B0A}, \tag{3.102}$$

and

$$q\left(\theta_2^{S1}\right)\Sigma_{B0A} = q\left(\theta_3^{S1}\right)\Sigma_{B0S1}, \tag{3.103}$$

respectively. In addition, the surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{S1}\right)\Sigma_{BnA}, \tag{3.104}$$

$$\rho V^{A} = R + (1-\alpha)\,\mu\left(\theta_1^{S1}\right)\Sigma_{BnA}, \tag{3.105}$$

$$\rho V^{S1} = u - \chi + (1-\alpha)\,q\left(\theta_3^{S1}\right)\Sigma_{B0S1}, \tag{3.106}$$

$$\rho V^{B0} = u_0 - R + \alpha\mu\left(\theta_3^{S1}\right)\Sigma_{B0S1}, \tag{3.107}$$

and

$$\rho V = u + \gamma\left(V^{S1} - V\right). \tag{3.108}$$

Finally, the above value functions allow us to solve for the surpluses as follows:

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{S1}\right) + (1-\alpha)\,\mu\left(\theta_1^{S1}\right)}. \tag{3.109}$$

$$\Sigma_{B0S1} = V - V^{S1} = \frac{\rho V - \left(u-\chi\right)}{\rho + (1-\alpha)\,\mu\left(\theta_3^{S1}\right)}, \tag{3.110}$$

and

$$\Sigma_{B0A} = \frac{\rho V - u_0}{\rho + \alpha q\left(\theta_2^{S1}\right) + (1-\alpha)\,\mu\left(\theta_2^{S1}\right)}. \tag{3.111}$$

The stock-flow and market tightness equations (3.93)-(3.101), combined with the two indifference conditions (3.102) and (3.103), and value functions and surpluses (3.104)-(3.111) fully characterize the equilibrium stocks of agents, market tightnesses, mixing probabilities x^A and x^{B0}, and active agent value functions in a "Sell first" equilibrium. We now prove existence of this equilibrium when χ and u_2 are small, and u_0 is strictly smaller than u_n.

Note that $\Sigma_{B0S1} = V - V^{S1} > 0$ for any u_0, but that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{S1} = \frac{u}{\rho}$, so that $\lim_{\chi\to 0}\Sigma_{B0S1} = 0$. Then, a set of arguments similar to the case of the "Buy first" equilibrium shows that $\lim_{\chi\to 0}\theta_3^{B1} = 0$ and $\lim_{\chi\to 0} x^{B0} = 1$, so that $\lim_{\chi\to 0}\mu\left(\theta_3^{B1}\right) = 0$ and $\lim_{\chi\to 0} B_0 = 0$, implying that $\lim_{\chi\to 0} x^A = 1$ and $\lim_{\chi\to 0}\theta_1^{B1} = 1$. As a result, $\lim_{\chi\to 0} V^{Bn}$ and $\lim_{\chi\to 0} V^A$ are the same as in the "Buy first" equilibrium, and there exists a $\overline{\chi}_4 > 0$ such that for $\chi \in \left(0, \overline{\chi}_4\right)$, it is the case that $\Sigma_{BnA} > 0$, $x^A \in (0,1)$ and $\theta_1^{S1} > 1$ but remains bounded, while $\Sigma_{B0S1} > 0$. As a result, markets 1 and 3 are active.

Following the same arguments as in the "Buy first" equilibrium, it is then the case that $V^{Bn} > V^{B0}$, also as $\chi \to 0$, so that $0 < \Sigma_{BnA} < \Sigma_{B0A}$ and market 2 is active. Furthermore, there must exist a $\overline{\chi}_5 > 0$ such that for $\chi \in \left(0, \overline{\chi}_5\right)$, it is the case that $\Sigma_{B0A} > \Sigma_{B0S1}$, since $\lim_{\chi\to 0}\Sigma_{B0S1} = 0$. This ranking implies that $-V^A > V^{B0} - V^{S1}$, so that a real estate firm is more impatient than a mismatched seller in the sense that the direct utility gain from transacting is higher for a real estate firm compared to a mismatched seller. The ranking of tightnesses and prices across segments then follows from the indifference conditions (3.102) and (3.103), similar to the case of a "Buy first" equilibrium. The fact that $V^{Bn} - V^{B0}$ is bounded away from zero also implies that there exists a exist a $\overline{\chi}_6 > 0$ such that for $\chi \in \left(0, \overline{\chi}_6\right)$, a mismatched owner and a new entrant will not open a fourth market, because $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{S1} = \frac{u}{\rho}$ and thus $\Sigma_{BnS1} = V - V^{S1} + V^{B0} - V^{Bn} < 0$ for a sufficiently small χ.

Now consider a mismatched owner that deviates and buys first. Potential sellers are real estate firms and mismatched homeowners, and upon trade the deviator becomes a double owner, who can open up new market segments with new entrants and forced renters. Note that V^{S2} falls without bounds as u_2 does, because a deviating double owner has to offer new entrants or forced renters their market value, following a similar argument as in the "Buy first" equilibrium. Then there exists a $\overline{u}_2^{S1}$ such that for all $u_2 < \overline{u}_2^{S1}$ it is the case that $\Sigma_{B1A} = V^{S2} - V^A - V^m < 0$, so that a deviating mismatched owner does not buy from a real estate agent. Note, however, that $\Sigma_{B1S1} < \Sigma_{B1A} < 0$ for $\chi < \overline{\chi}_5$ since, as shown above, in that case $-V^A > V^{B0} - V^{S1}$, meaning that a new entrant is more impatient than a mismatched buyer. As a result, a mismatched owner that deviates and buys first is better off not trading. However, not trading is dominated by selling first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from selling first in a "Sell first" equilibrium.

Finally, setting $\overline{\chi} = \min\left\{\overline{\chi}_1, \overline{\chi}_2, \overline{\chi}_3, \overline{\chi}_4, \overline{\chi}_5, \overline{\chi}_6\right\}$ and $\overline{u}_2 = \min\left\{\overline{u}_2^{B1}, \overline{u}_2^{S1}\right\}$, we arrive at our result. $\square$

3.8.F Additional extensions

Simultaneous Entry as Buyer and Seller

We assume that a mismatched owner can allocate a fixed amount of time (normalized to 1 unit) to search in the housing market as a buyer or a seller. A mismatched owner that chooses to enter as a buyer or seller only allocates all of his time to one activity. Otherwise, a mismatched owner that enters as both a buyer and a seller can allocate a fraction $\phi \in (0,1)$ of his time to searching as buyer, and searches the remaining $1-\phi$ of his time as seller. For a given market tightness θ, the value function V^{SB} for a mismatched owner that enters as both buyer and seller satisfies the following equation in a steady state equilibrium:

$$\rho V^{SB} = u - \chi + (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} + \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}.$$

We then show the following

Proposition 3.11. *For $\theta \in \left(0,\tilde{\theta}\right)$, $V^{S1} > V^{SB}$, for any $\phi \in (0,1)$. Also, for $\theta \in \left(\tilde{\theta},\infty\right)$, $V^{B1} > V^{SB}$, for any $\phi \in (0,1)$.*

Proof. To show the first part, suppose the opposite, so $V^{S1} \leq V^{SB}$. Then

$$\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} \leq$$
$$(1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} + \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}.$$

Under the assumption that $V^{S1} \leq V^{SB}$, and since we know from Lemma 3.1 that $V^{B1} < V^{S1}$ for $\theta \in \left(0,\tilde{\theta}\right)$, it must then be the case that

$$\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} \leq$$
$$(1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} + \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{B1}\right\},$$

which does not hold because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > 0$ for $\theta \in \left(0,\tilde{\theta}\right)$ by Assumption 3.A2, and because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > q(\theta)\left(-p + V^{S2} - V^{B1}\right)$ for $\theta \in \left(0,\tilde{\theta}\right)$, as in Lemma 3.1.

To show the second part, suppose the opposite, so $V^{B1} \leq V^{SB}$. Then

$$q(\theta)\max\left\{0, p+V^{S2}-V^{B1}\right\} \leq$$
$$(1-\phi)\mu(\theta)\max\left\{0, p+V^{B0}-V^{SB}\right\}+\phi q(\theta)\max\left\{0,-p+V^{S2}-V^{SB}\right\}.$$

Under the assumption that $V^{B1} \leq V^{SB}$, and since we know from Lemma 3.1 that $V^{S1} < V^{B1}$ for $\theta \in \left(\tilde{\theta}, \infty\right)$, it must then be the case that

$$q(\theta)\max\left\{0, p+V^{S2}-V^{B1}\right\} \leq$$
$$(1-\phi)\mu(\theta)\max\left\{0, p+V^{B0}-V^{S1}\right\}+\phi q(\theta)\max\left\{0,-p+V^{S2}-V^{B1}\right\},$$

which does not hold because $q(\theta)\left(-p+V^{S2}-V^{B1}\right)>0$ for $\theta \in \left(\tilde{\theta}, \infty\right)$ by Assumption 3.A2, and because $\mu(\theta)\left(p+V^{B0}-V^{S1}\right) < q(\theta)\left(-p+V^{S2}-V^{B1}\right)$ for $\theta \in \left(\tilde{\theta}, \infty\right)$, as in Lemma 3.1.[50] □

Finally, note that under payoff symmetry (i.e. $\tilde{u}_0 = \tilde{u}_2 = c$) the possibility to enter as both buyer and seller while allocating each an equal amount of time can result in an equilibrium with a market tightness of $\theta = 1$. Specifically, at $\theta = 1$, $\mu(\theta) = q(\theta) = \mu(1)$. At these flow rates it can easily be seen that if $\tilde{u}_0 = \tilde{u}_2 = c$, then $V^{B1} = V^{S1} = V^{SB}$ for any ϕ. Finally, a tightness of $\theta = 1$ can result from mismatched owners entering as buyers and sellers simultaneously and allocating each an equal amount of time (so $\phi = 0.5$).

This is analogous to the equilibrium described in Proposition 3.4, with the only difference that now agents follow symmetric strategies compared to asymmetric strategies with one half of mismatched owners buying first and the other half selling first.

Homeowners compensated for their housing unit upon exit

Suppose that upon exit homeowners receive bids for their housing unit(s) from a set of competitive real estate firms. Therefore, given that the value of a housing unit to a real estate firm is $V^A(\theta)$, homeowners receive $V^A(\theta)$ for each housing unit that they own. Again, we consider a steady state equilibrium with a fixed market tightness θ. We define $\tilde{u}_0\left(\theta, g\right) \equiv u_0 + \triangle - gV^A(\theta)$ and $\tilde{u}_2\left(\theta, g\right) = u_2 - \triangle + gV^A(\theta)$. Note that $V^A(\theta)$ is (weakly) increasing in θ, so $\tilde{u}_2$ is increasing in θ and $\tilde{u}_0$ is decreasing in θ;

[50]Note also that for $\theta = 0$ and $\theta \to \infty$, mismatched owners are indifferent between remaining mismatched and any search strategy, because $V^{B1} = V^{S1} = V^{SB} = \frac{u-\chi}{\rho}$, but that such tightnesses cannot occur in steady state by Lemma 2.

Given this definition, the difference between the values from buying first and selling first (assuming a mismatched owner transacts in both cases), $D(\theta) \equiv V^{B1} - V^{S1}$, can be written as

$$D(\theta) = \frac{\mu(\theta)}{\left(\rho + q(\theta)\right)\left(\rho + \mu(\theta)\right)} \left[\left(1 - \frac{1}{\theta}\right)\left(u - \chi - \tilde{u}_2(\theta, g)\right) - \tilde{u}_0(\theta, g) + \tilde{u}_2(\theta, g) \right].$$

Let $\tilde{\theta}$ be defined implicitly by

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2\left(\tilde{\theta}, g\right)}{u - \chi - \tilde{u}_0\left(\tilde{\theta}, g\right)},$$

whenever that equation has a solution.[51] Note that in the limit as $g \to 0$, assumption A1 will hold. Therefore, for g sufficiently close to zero, we will have that $u - \chi > \max\left\{\tilde{u}_0(\theta, g), \tilde{u}_2(\theta, g)\right\}$, for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, and so a version of Lemma 3.1 will hold in this case as well. Given this result one can then easily construct multiple steady state equilibria as in Proposition 3.3.

A fixed price as the outcome of take-it-or-leave-it offers under private information

In this section we show that a fixed price equal to the present discounted value of rental income can be microfounded as the outcome of bargaining under private information about types, with full bargaining power for buyers. Suppose therefore in this section that buyers make take-it-or-leave-it offers, but do not know the type of the seller. However, buyers do know the fractions of the types in the economy. Because of heterogeneity among sellers, their reservation prices vary. Matching is still random, so that buyers cannot direct their search to the seller type with the lowest reservation price but meet a particular seller type with a probability equal to their proportion in the population of sellers. The question is then whether buyers, upon meeting a seller, make an offer that only sellers with a low reservation price would accept (and thus trade only if they have met a seller of this type), or make an offer that all sellers would accept (and therefore trade for sure).

[51] Note that the above equation for $\tilde{\theta}$, whenever it has a solution, has a unique solution for any $g \geq 0$, since given the properties of $\tilde{u}_0$ and $\tilde{u}_2$, it follows that the right hand side of this expression is (weakly) decreasing in θ. Furthermore, the right hand side is strictly decreasing in g for any $\theta > 0$, so by the implicit function theorem, $\tilde{\theta}$ is decreasing in g.

We consider the symmetric case with $\tilde{u}_0 = \tilde{u}_2 = c$ (which for $p = \frac{R}{\rho}$ amounts to $u_0 = u_2 = c$), so that $\tilde{\theta} = 1$. In addition, we maintain Assumptions A1 and A2 and assume that $u_n < u - \chi$, so that both mismatched owners and new entrants are strictly better off to enter the market. As in the model with symmetric Nash bargaining, we focus on steady state equilibria with value functions, market tightness θ, and the stocks of different agent types constant over time. Moreover, although results hold more generally, we again consider a limit economy with small flows where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is kept constant in the limit. Remember that in this case $\overline{\theta} \to 1 + \kappa$ and $\underline{\theta} \to \frac{1}{1+\kappa}$. We will show that under these conditions both in a “Buy first” and in a “Sell first” equilibrium no buyer has an incentive to deviate from targeting both types of sellers by demanding a lower price than the unique prevailing price $p = \frac{R}{\rho}$.

Still denoting the value of a matched owner that remains passive upon mismatch by $\tilde{V}$, note first that at $\tilde{\theta} = 1$ Assumption A2 can be simplified to

$$\frac{u-\chi}{\rho} < \frac{u-\chi}{\rho+\mu_0} + \frac{\mu_0}{\left(\rho+\mu_0\right)^2} c + \frac{{\mu_0}^2}{\left(\rho+\mu_0\right)^2} \tilde{V},$$

$$\Leftrightarrow \frac{u-\chi}{\rho} < \frac{c}{\rho+\mu_0} + \frac{\mu_0}{\rho+\mu_0} \tilde{V},$$

$$\Leftrightarrow 0 < \rho\left(c - \left(u-\chi\right)\right) + \mu_0\left(\rho\tilde{V} - \left(u-\chi\right)\right),$$

which, for future reference, is not greater than $\rho\left(c - \left(u-\chi\right)\right) + \mu_0\left(\rho V - \left(u-\chi\right)\right)$.

Under the unique price to be proven, the value functions are given by equations (3.5)-(3.8) and (3.26)-(3.28), given θ and R. We first show that in an equilibrium in which mismatched owners “Buy first”, buyers have no incentive to demand a lower price than $p = \frac{R}{\rho}$. In such an equilibrium there are two types of sellers: double owners and real estate agents. As before, the lowest price that a real estate agent would be willing to accept is $p^A = V^A = \frac{R}{\rho}$. The lowest price that a double owner would be willing to accept is $p^{S2} = V^{S2} - V$. Substituting these prices in the value functions, in an equilibrium with price dispersion $p^{S2} < p^A$, since

$$\rho\left(V^{S2} - V - V^A\right) = u_2 + R + \mu(\theta)\left(p^{S2} + V - V^{S2}\right) - \rho V - R - \mu(\theta)\left(p^A - V^A\right),$$

$$\Leftrightarrow \rho\left(V^{S2} - V - V^A\right) = u_2 - \rho V < 0,$$

$$\Leftrightarrow V^{S2} - V < V^A.$$

For that reason, under full information buyers would like to buy from a double owner, but the question is whether under private information they will make an offer that only double owners would accept. Note that for any $p \geq p^{S2}$ double owners are willing to sell, while for $p < p^{S2}$ they are not. As a result, since buying a house is preferred to being passive, among all possible deviations no offer is more profitable than demanding $V^{S2} - V$. The proof can therefore be restricted to this deviating offer. Note also that a deviating mismatched seller has zero mass, so that its presence doesn't affect the take-it-or-leave-it offers that buyers make.

First considering new entrants, for them to demand p^A it must be the case that

$$V - V^{Bn} - p^A \geq \frac{S_2}{S}\left(V - V^{Bn} - p^{S2}\right).$$

Substituting prices and using that $S = S_2 + A$ yields

$$\frac{A}{S}\left(V - V^{Bn} - \frac{R}{\rho}\right) \geq \frac{S_2}{S}\left(V - V^{S2} + \frac{R}{\rho}\right). \tag{3.112}$$

From the value functions we have that

$$\rho\left(V - V^{Bn} - \frac{R}{\rho}\right) = \rho V - u_n + R - q(\theta)\left(V - V^{Bn} - \frac{R}{\rho}\right) - R,$$

$$\Leftrightarrow \left(\rho + q(\theta)\right)\left(V - V^{Bn} - \frac{R}{\rho}\right) = \rho V - u_n,$$

and

$$\rho\left(V - V^{S2} + \frac{R}{\rho}\right) = \rho V - u_2 - R - \mu(\theta)\left(V - V^{S2} + \frac{R}{\rho}\right) + R,$$

$$\Leftrightarrow \left(\rho + \mu(\theta)\right)\left(V - V^{S2} + \frac{R}{\rho}\right) = \rho V - u_2. \tag{3.113}$$

Moreover, in the limit we consider, we know from the section on Nash bargaining that $\frac{A}{S} = \frac{1}{\overline{\theta}}$ and $\frac{S_2}{S} = \frac{\overline{\theta}-1}{\overline{\theta}}$, so that (3.112) amounts to

$$\frac{1}{\overline{\theta}}\left(\rho + \mu\left(\overline{\theta}\right)\right)\left(\rho V - u_n\right) \geq \frac{\overline{\theta}-1}{\overline{\theta}}\left(\rho + q\left(\overline{\theta}\right)\right)\left(\rho V - u_2\right).$$

where both sides are positive, but where the right-hand side can be made arbitrarily close to zero by moving closer to $\overline{\theta} = 1$. Therefore, it follows that there is a $\kappa_7 > 0$, such that for $\kappa < \kappa_7$, new entrants in a "Buy first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a seller. Substituting u_0 for u_n, the same condition holds for a deviating mismatched seller, and then becomes a forced renter. Therefore, there is a $\kappa_8 > 0$, such that for $\kappa < \kappa_8$, forced renters in a "Buy first" equilibrium make the same offer.

For mismatched owners that buy first to demand p^A it must be the case that

$$V^{S2} - V^{B1} - p^A \geq \frac{S_2}{S}\left(V^{S2} - V^{B1} - p^{S2}\right),$$

$$\Leftrightarrow \frac{A}{S}\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) \geq \frac{S_2}{S}\left(V - V^{S2} + \frac{R}{\rho}\right). \tag{3.114}$$

Rearranging the value functions yields

$$\rho\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) = u_2 + R + \mu(\theta)\left(V - V^{S2} + \frac{R}{\rho}\right) - R - (u-\chi) - q(\theta)\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right),$$

$$\Leftrightarrow (\rho + q(\theta))\left(V^{S2} - V^{B1} - \frac{R}{\rho}\right) = u_2 - (u-\chi) + \mu(\theta)\left(V - V^{S2} + \frac{R}{\rho}\right). \tag{3.115}$$

Substituting the steady state fractions and (3.115) into (3.114), in the limit we consider we have that

$$\frac{1}{\overline{\theta}}\left(u_2 - (u-\chi) + \mu\left(\overline{\theta}\right)\left(\frac{R}{\rho} + V - V^{S2}\right)\right) \geq \frac{\overline{\theta}-1}{\overline{\theta}}\left(\rho + q\left(\overline{\theta}\right)\right)\left(V - V^{S2} + \frac{R}{\rho}\right),$$

$$u_2 - (u-\chi) \geq \left[\left(\overline{\theta}-1\right)\left(\rho + q\left(\overline{\theta}\right)\right) - \mu\left(\overline{\theta}\right)\right]\left(V - V^{S2} + \frac{R}{\rho}\right).$$

Substituting (3.113) yields

$$\left(\rho + \mu\left(\overline{\theta}\right)\right)\left(u_2 - (u-\chi)\right) \geq \left[\left(\overline{\theta}-1\right)\left(\rho + q\left(\overline{\theta}\right)\right) - \mu\left(\overline{\theta}\right)\right]\left(\rho V - u_2\right),$$

$$\Leftrightarrow \rho\left(u_2 - (u-\chi)\right) + \mu\left(\overline{\theta}\right)\left(\rho V - (u-\chi)\right) \geq \left(\overline{\theta}-1\right)\left(\rho + q\left(\overline{\theta}\right)\right)\left(\rho V - u_2\right).$$

The left-hand side is positive for any $\overline{\theta} \geq 1$ by Assumption 3.A2. Moving $\overline{\theta}$ towards 1 can make the right-hand side arbitrarily close to zero, so that there exists a $\kappa_9 > 0$, such that for $\kappa < \kappa_9$, mismatched owners that buy first in a "Buy first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a seller. Taking $\overline{\kappa}' = \min\{\kappa_7, \kappa_8, \kappa_9\}$, we have that for $\kappa < \overline{\kappa}'$, all buyers demand $p = \frac{R}{\rho}$ upon meeting a seller in a "Buy first" equilibrium with a market tightness given by $\overline{\theta} = 1 + \kappa$.

Secondly, we show that in an equilibrium in which mismatched owners sell first, buyers have no incentive to demand a lower price than $p = \frac{R}{\rho}$. In such an equilibrium there are two types of sellers: mismatched owners that sell first, and real estate agents. The lowest price that a real estate agent would be willing to accept is still $p^A = V^A = \frac{R}{\rho}$. The lowest price that a mismatched owner would be willing to accept is $p^{S1} = V^{S1} - V^{B0}$. It must be the case that $V^{B0} - V^{S1} + p^A \geq 0$, because mismatched owners don't remain passive by Assumption 3.A2. It follows that $p^{S1} \leq p^A$, so that with full information buyers would like to buy from a mismatched owner. Again the question is whether under private information buyers will make an offer that only mismatched owners would accept. Similar to the "Buy first" equilibrium, the proof can be restricted to the deviation of demanding $V^{S1} - V^{B0}$.

First considering forced renters, for them to demand p^A it must be the case that

$$V - V^{B0} - p^A \geq \frac{S_1}{S}\left(V - V^{B0} - p^{S1}\right),$$

$$\Leftrightarrow \frac{A}{S}\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right). \tag{3.116}$$

Rearranging the value functions yields

$$\rho\left(V - V^{B0} - \frac{R}{\rho}\right) = \rho V - u_0 + R - q(\theta)\left(V - V^{B0} - \frac{R}{\rho}\right) - R,$$

$$\Leftrightarrow \left(\rho + q(\theta)\right)\left(V - V^{B0} - \frac{R}{\rho}\right) = \rho V - u_0, \tag{3.117}$$

and

$$\rho\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right) = u_0 - R + q\left(\theta\right)\left(-\frac{R}{\rho} + V - V^{B0}\right) - (u - \chi) - \mu\left(\theta\right)\left(\frac{R}{\rho} + V^{B0} - V^{S1}\right) + R,$$

$$\Leftrightarrow \left(\rho + \mu(\theta)\right)\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right) = u_0 - (u - \chi) + q\left(\theta\right)\left(V - V^{B0} - \frac{R}{\rho}\right). \tag{3.118}$$

Substituting (3.118), (3.116) therefore amounts to

$$\frac{A}{S}\left(\rho + \mu(\theta)\right)\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(u_0 - (u - \chi) + q\left(\theta\right)\left(V - V^{B0} - \frac{R}{\rho}\right)\right),$$

$$\Leftrightarrow \left[\frac{A}{S}\left(\rho + \mu(\theta)\right) - \frac{S_1}{S}q(\theta)\right]\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(u_0 - (u - \chi)\right).$$

Substituting (3.117) yields

$$\left[\frac{A}{S}\left(\rho + \mu(\theta)\right) - \frac{S_1}{S}q(\theta)\right]\left(\rho V - u_0\right) \geq \frac{S_1}{S}\left(\rho + q(\theta)\right)\left(u_0 - (u - \chi)\right),$$

$$\Leftrightarrow \frac{A}{S}\left(\rho + \mu(\theta)\right)\left(\rho V - u_0\right) \geq \frac{S_1}{S}\rho\left(u_0 - (u - \chi)\right) + \frac{S_1}{S}q(\theta)\left(\rho V - (u - \chi)\right).$$

From the section on Nash bargaining we know that $\frac{A}{S} = \underline{\theta}$ and $\frac{S_1}{S} = 1 - \underline{\theta}$. Substituting these steady state fractions, we have that

$$\underline{\theta}\left(\rho+\mu\left(\underline{\theta}\right)\right)\left(\rho V-u_0\right)\geq\left(1-\underline{\theta}\right)\left[\rho\left(u_0-(u-\chi)\right)+q\left(\underline{\theta}\right)\left(\rho V-(u-\chi)\right).\right]$$

Again, by moving towards $\underline{\theta}=1$ the right-hand side can be made arbitrarily close to zero while the left-hand side remains positive. Therefore, there exists a $\kappa_{10}>0$, such that for $\kappa<\kappa_{10}$, forced renters in a "Sell first" equilibrium demand $p=\frac{R}{\rho}$ upon meeting a seller. Substituting u_n for u_0 the same condition holds for a new entrant, so that there is a $\kappa_{11}>0$, such that for $\kappa<\kappa_{11}$, new entrants make the same offer.

Finally, for a deviating mismatched buyer to demand p^A it must be the case that

$$V^{S2}-V^{B1}-p^A\geq\frac{S_1}{S}\left(V^{S2}-V^{B1}-p^{S1}\right),$$

$$\Leftrightarrow\frac{A}{S}\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)\geq\frac{S_1}{S}\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right).\tag{3.119}$$

Rearranging the value functions yields

$$\rho\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)=u_2+R+\mu\left(\theta\right)\left(\frac{R}{\rho}+V-V^{S2}\right)-u-\chi+q\left(\theta\right)\left(-\frac{R}{\rho}+V^{S2}-V^{B1}\right)-R,$$

$$\Leftrightarrow\left(\rho+q(\theta)+\mu(\theta)\right)\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)=u_2-(u-\chi)+\mu\left(\theta\right)\left(V-V^{B1}\right),$$

with

$$\mu\left(\theta\right)\left(V-V^{B1}\right)=\mu\left(\theta\right)\left(V-\frac{u-\chi}{\rho}\right)-\mu\left(\theta\right)q\left(\theta\right)\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right).$$

From (3.118) we know that

$$\left(\rho+\mu(\theta)\right)\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right)=u_0-(u-\chi)+q\left(\theta\right)\left(V-V^{B0}-\frac{R}{\rho}+V^{S1}-V^{S1}\right),$$

$$\Leftrightarrow\left(\rho+q(\theta)+\mu(\theta)\right)\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right)=u_0-(u-\chi)+q\left(\theta\right)\left(V-V^{S1}\right),$$

with

$$q\left(\theta\right)\left(V-V^{S1}\right)=q\left(\theta\right)\left(V-\frac{u-\chi}{\rho}\right)-\mu\left(\theta\right)q\left(\theta\right)\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right).$$

Therefore, (3.119) simply amounts to

$$\frac{A}{S}\left(u_2-(u-\chi)+\mu\left(\theta\right)\left(V-\frac{u-\chi}{\rho}\right)\right)\geq\frac{S_1}{S}\left(u_0-(u-\chi)+q\left(\theta\right)\left(V-\frac{u-\chi}{\rho}\right)\right).$$

Substituting the steady state fractions, we have that

$$\underline{\theta}\left(u_2-(u-\chi)+\mu\left(\underline{\theta}\right)\left(V-\frac{u-\chi}{\rho}\right)\right)\geq\left(1-\underline{\theta}\right)\left(u_0-(u-\chi)+q\left(\underline{\theta}\right)\left(V-\frac{u-\chi}{\rho}\right)\right).$$

The left-hand side is positive for any $0<\underline{\theta}\leq1$ by Assumption 3.A2. Moving $\overline{\theta}$ towards 1 can make the right-hand side arbitrarily close to zero, so that there exists a $\kappa_{12}>0$, such that for $\kappa < \kappa_{12}$, deviating mismatched owners that buy first in a "Sell first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a seller. Taking $\underline{\kappa}' = \min\{\kappa_{10}, \kappa_{11}, \kappa_{12}\}$, we have that for $\kappa < \underline{\kappa}'$, all buyers demand $p = \frac{R}{\rho}$ upon meeting a seller in a "Sell first" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$. Finally, taking $\kappa' = \min\left\{\overline{\kappa}', \underline{\kappa}'\right\}$, we have that both in a "Buy first" and in a "Sell first" equilibrium, the take-it-or-leave-it offer that buyers make is equal to $p = \frac{R}{\rho}$.

3.8.G Institutional details

The information in Tables 9 and 10 is based on:

- http://boligejer.dk/koebsaftale/ for Denmark.
- http://www.eiendomsrettsadvokaten.no/advokathjelp for Norway.
- http://www.eigenhuis.nl/juridisch/ for the Netherlands.
- http://www.realtor.com and https://www.doorsteps.com/ for the U.S.
- http://hoa.org.uk/advice/guides-for-homeowners for England and Wales.

Table 9: Institutional details for buyers

Country	Grace period	Penalty for reneging during grace period	Penalty for reneging after grace period	Possible conditions to dissolve contract (requires proof)	Period to refer to dissolving conditions
Denmark	6 days	1% of price	Liable for full amount	May include as conditions: • Ability to secure financing • Lawyer reservation	As specified in the purchase offer
Norway	None	N/A	Liable for full amount	May include as condition: • Ability to secure financing	As specified in the purchase offer
Netherlands	3 days	None	Standard contract: • End contract: >10% price • Demand fulfillment: >0.3% sale price per day • Court	Standard contract: • Ability to secure financing • Applicability for national mortgage insurance • Structural inspection	As specified in the contract: usually not for more than a few weeks after signing it
United States	None (3 days in NJ only)	N/A	Losing the Earnest Money Deposit, ranging from $500 to 10% of the price	Standard purchase offer: • Ability to secure financing • Appraisal • Structural inspection	Usually not for more than a few weeks after signing the offer, e.g. 17 days in CA
England & Wales	Pull out until exchange of (unconditional) contracts	Occasionally one loses holding deposit, ranging from £500 to £1000	10% price	None	N/A

Table 10: Institutional details for sellers

Country	Grace period	Penalty for reneging during grace period	Penalty for reneging after grace period	Possible conditions to dissolve contract
Denmark	None	N/A	Court	None
Norway	None	N/A	Court	None
Netherlands	None	N/A	Standard contract: • End contract: >10% price • Demand fulfillment: >0.3% sale price per day • Court	None
United States	None (3 days in NJ only)	N/A	Court	None
England & Wales	Pull out until exchange of contracts	None	Court	None

CHAPTER 4

Learning to buy first or sell first in housing markets

4.1 Introduction

Moving between owner-occupied houses requires both buying and selling, and households can choose the order of these transactions. This chapter explains the dynamics of the fraction of households that buy first as the balance of two forces under informational frictions: households' strategic complementarities to buy first whenever other owner-occupiers buy first, and profit opportunities for speculators when houses are relatively inexpensive.

As in the previous chapter, the strategic complementarity is the result of the desire to shorten the costly period between the two transactions. Households that buy first suffer from double housing expenses, while households that sell first have to rent temporary housing. To reduce these costs, moving owner-occupiers would like to be on the short side of the market at their second transaction, and therefore want to join the long side of the market at their first transaction. As joining the long side at the first transaction makes the number of buyers and sellers even more *un*equal, households tend to buy first when others are buying first and sell first when others are selling first. However, although the number of sellers can usually be observed, for instance via commonly used websites with listings, the number of buyers cannot. As a result, market tightness is imperfectly observed and households are not perfectly able to coordinate on the same strategy. This chapter presents a first exploration of households that learn about other moving owner-occupiers in a contagion-like process, in order to choose between buying first and selling first.

Figure 17, reprinted from the previous chapter and based on Moen et al. (2015), shows the dynamics of the fraction of owners that buy first in Copenhagen between 1993 and 2008, together with a co-moving price index. The figure shows that housing prices are low when moving owner-occupiers in majority sell first. Such episodes offer profit opportunities to speculators that can buy houses cheap, rent them out, and bring them on the market later. I assume that moving owner-occupiers take the presence of such speculators into account. For that reason, they try to keep track of speculators' profit opportunities. Because more acquisition by speculators implies that houses can be sold more easily, households tend to buy first when they believe profit opportunities are favorable. Again, however, profit opportunities are observed imperfectly and households need to learn about them.

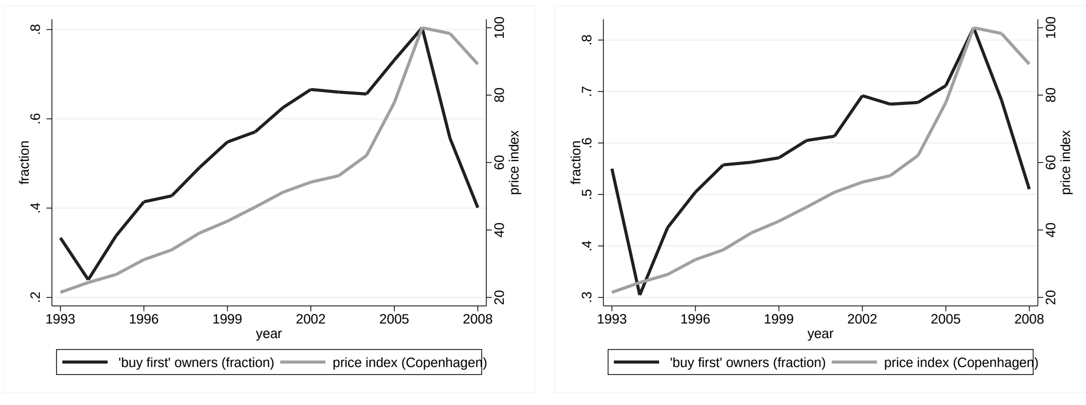

Figure 17: Housing prices and fraction of moving owner-occupiers that buy first in Copenhagen (1993-2008). Panel a is based on agreement dates, and Panel b is based on closing dates.

If moving owner-occupiers learn from each other fast enough and speculators react sluggishly to profit opportunities, a limit cycle in profit opportunities and the fraction of households that buy first exists. The time series simulated from this limit cycle are broadly consistent with stylized facts of the housing market, featuring a slowly moving fraction of owners that buy first, moving in tandem with the number of transactions and housing prices, and in the opposite direction of the time-on-market and the stock of houses for sale. Growth rates of prices show momentum, but mean reversion at longer horizons.

The strategic complementarity in the transaction sequence decision of existing home-owners was first identified by Moen et al. (2015), and is presented in Chapter 3 of this thesis. The interplay between this strategic complementarity and the buyer-seller composition of the housing market results in multiple steady state equilibria: one in which all moving owner-occupiers buy first and another in which all moving owner-occupiers sell first. In addition, there can exist sunspot equilibria in which households switch between these steady states. This model therefore predicts that all moving owner-occupiers use the same strategy, and that equilibrium switches consist of a coordinated shift from all of them buying first to selling first, or the reverse. However, the data on moving owner-occupiers presented in Figure 17 show that the fraction of them that buys first is not zero or one hundred percent, and, in addition, moves slowly over time. Although the fraction of households that buy first varies considerably - from as low as 25 to as high as 80 percent - it never reaches zero or one hundred percent, and takes twelve years to switch from the bottom to the top. In this chapter this lack of coordination is explained by the introduction of informational frictions. Moving owner-occupiers still try to mimic the majority because it is in their interest to do so, but are not perfectly able to, because market tightness is unobserved. Only if households mimic each other infinitely fast, the limiting case without informational frictions can be obtained. In this case, all households coordinate on the same strategy. Otherwise, heterogeneity in strategies persists.

The introduction of informational frictions and sluggish speculative behavior results in dynamics that are very similar to Lux (1995). Lux offers an elegant formalization of contagion in speculative financial markets where optimistic and pessimistic traders may infect each other with their beliefs. Moreover, he expands on this idea and allows speculators to change their beliefs based on price dynamics or "the mood of the market". Unlike the strategic complementarity present in my model, however, in Lux (1995) agents have no incentives to mimic each other or to condition their behavior on prices or market conditions, other than to learn from equally ignorant agents.[52] By introducing a transaction sequence decision, I add a reason why people would like to behave

[52] See Banerjee (1992) and Bikhchandani et al. (1992) for conditions under which herd behavior can be rational.

as others. In addition, the presence of speculators does affect households' optimal behavior, so that it in their interest to monitor profitability of real estate agents. Finally, the introduction of search frictions allows for additional predictions on quantities and time-on-market, which are particularly relevant for housing markets. Dieci and Westerhoff (2012) and Bolt et al. (2014) also feature heterogeneous expectations and speculation in housing markets, but similarly lack search frictions and therefore cannot account for liquidity.

The literature on search frictions in housing markets starts with the seminal paper of Wheaton (1990). Krainer (2001) focuses on liquidity in particular. This chapter is especially related to papers that introduce heterogeneous expectations and/or speculation in housing markets with search frictions. Piazzesi and Schneider (2009) provide evidence of momentum traders that buy houses in expectation of rising prices, and include such traders in a search model of the housing market. However, they do not consider changes in liquidity and how it moves together with the fraction of households that buy first, as they assume that all moving owner-occupiers sell first and that the time-on-market is equal for buyers and sellers. Burnside et al. (2011) propose a search model with heterogeneous expectations to explain housing booms and busts. Again, as in Piazzesi and Schneider (2009) but in contrast to the focus of this chapter, they assume that moving owner-occupiers always sell first and that the time-on-market is equal for buyers and sellers. In addition, although households learn from each other, they do not learn about each other but about a future fundamental. Kashiwagi (2014) uses self-fulfilling beliefs in sharing the surplus of trade to explain a boom in housing prices while rents remain stable. However, moving owner-occupiers always sell first and do so without frictions, and as a result changes in beliefs do not affect housing market quantities.

Finally, Anenberg and Bayer (2015) is closest to Moen et al. (2015) and this chapter. It features the joint buyer-seller problem to explain volatility of transaction volume, but buying first is only a stochastic outcome. In addition, their model does not feature a strategic complementarity, so that there is no need for informational frictions to explain a lack of coordination either.

4.2 Model

Time is continuous and agents discount the future at rate $r > 0$. The economy consists of a unit mass of households and a unit mass of houses that do not depreciate. I abstract from construction as well as from heterogeneity in houses, apart from idiosyncratic characteristics that give rise to search frictions. The population consists of five types: households satisfied with the characteristics of the house they own (matched owners), households unsatisfied with the characteristics of the house they own that enter the market as buyers (mismatched owners that buy first), mismatched owners that sell first, households that own two houses (double owners), and households that own no house, but rent temporary housing (non-owners).

All types exit the economy at an exogenous and constant rate $g > 0$, and the houses of those that own upon exit are transferred to real estate firms. New households enter the economy as non-owners at the same rate g. Define the measure of non-owners in the population at time t as $N(t)$, the measure of mismatched owners that buy first as $B_1(t)$, the measure of mismatched owners that sell first as $S_1(t)$, the measure of matched owners as $O(t)$, the measure of double owners as $S_2(t)$, and the measure of houses of real estate firms as $A(t)$. Houses are owned by real-estate firms, matched owners, mismatched owners, and double owners, where the latter own two houses, so that $A(t) + O(t) + B_1(t) + S_1(t) + 2S_2(t) = 1$. Real estate firms rent out their houses receiving an exogenous flow payment of R, just as double owners rent out one of their properties at the same price. Non-owners therefore live in the $A(t) + S_2(t)$ unoccupied houses.

Individual households make transitions between types. At some exogenous and constant rate $\gamma > 0$, a matched owner receives a preference shock, turning her into a mismatched owner. The preference shock captures exogenous reasons for a desire to move, such as finding a new job elsewhere. A mismatched owner is no longer satisfied with her house, but still owns it. Preferences are such that she would like to move to a new house that she is satisfied with, but moving requires finding a new house, and finding a buyer for her old house. Both are time-consuming due to the presence of search frictions.

Search is random, so that all sellers find interested buyers at the same endogenous rate $\mu(t)$, and all buyers find houses to their liking at the same endogenous rate $q(t)$. Define the measure of buyers at time t as $B(t)$, the measure of sellers as $S(t)$, and market tightness as $\theta(t) \equiv B(t)/S(t)$. The number of matches at time t, $m(t)$, follows from a Cobb-Douglas matching function, taking the measure of buyers and sellers as inputs: $m(t) = \mu_0 B(t)^{\alpha} S(t)^{1-\alpha}$. The number of matches per seller, or the rate of finding a potential buyer for one's house, is thus given by $m(t)/S(t) = \mu_0 \theta(t)^{\alpha} = \mu(\theta(t))$. Since the number of sales must always equal the number of purchases, we have that the rate of buying a house is $q(\theta(t)) = \mu(\theta(t))/\theta(t)$.

Immediately upon mismatch, an owner chooses to enter the market either as a buyer or as a seller. For simplicity, mismatched owners commit to their choice until they transact, e.g. because they hire a realtor to carry out the transaction of their choice. From the strategic complementarity in the transactions sequence decision as presented in the previous chapter, mismatched owners would like to enter as sellers whenever $\theta(t)$ is low and as buyers whenever $\theta(t)$ is high. However, I assume that households do not know market tightness, and do not perfectly observe the behavior of mismatched owners either. For that reason, even though newly mismatched owners may prefer to be on the same side of the market as the majority of mismatched owners first, they are not necessarily able to do so. Instead, all households learn about the actions of newly mismatched owners, the households that actually make the choice to buy first or sell first at time t. Because the matching function implies that the outflow rate is inversely related to the stock, this inflow of newly mismatched owners buying first (selling first) also tends to move the measure of mismatched owners that buy first (sell first) in the same direction.

The outcome of the learning process is a belief about the actions of the majority of newly mismatched owners. Learning is modeled as a contagion-like Poisson process. A household believing that the majority of newly mismatched owners buy first may switch to believe that the majority sells first, which happens at a rate that is increasing in the fraction of newly mismatched owners selling first. Similarly, a household that believes newly mismatched owners predominantly sell first comes to believe that the majority buys first at a rate that increases in the fraction of newly mismatched owners buying first. Switching only depends on the average action, and does not allow for local interactions or some newly mismatched owners to be more influential than others. Learning can thus be understood as the result of the observation of the actions of a randomly drawn subset of newly mismatched owners.

Let $x_b(t) \in [0,1]$ denote the fraction of households that believe the majority of newly mismatched owners buy first at time t, so that $1 - x_b(t)$ denotes the fraction of households that believe newly mismatched owners join in majority the measure of those selling first. Even though all households learn about the actions of newly mismatched owners, the beliefs of matched owners are particularly important, because they act upon their beliefs as soon as they become mismatched. Because of the strategic complementarity, a matched owner who believes the majority of newly mismatched owners buy first (sell first), has the strategy to buy first (sell first) herself if she were to become mismatched. Consequently, since preference shocks hit matched owners randomly, the fraction of newly mismatched owners that buy first is also described by $x_b(t)$.

Define the index $z(t) \in [-1, 1]$ according to $z(t) \equiv 2x_b(t) - 1$, summarizing the actions of newly mismatched owners and the beliefs of households about them. Consequently, $z = -1$ indicates that all households believe that newly mismatched owners predominantly first sell (and as a result all newly mismatched owners actually sell first), $z = 1$ indicates the shared belief that the majority first buys (and all newly mismatched owners buy first), and $z = 0$ indicates that households are split equally in their beliefs (and half of the newly mismatched owners buy first). The rate at which households that believe that the majority of newly mismatched owners buy first switch to believe that the majority sells first, is given by $\pi_{BS}(z(t))$ with $\pi'_{BS} < 0$. Similarly, $\pi_{SB}(z(t))$ with $\pi'_{BS} > 0$ denotes the rate at which households who believe newly mismatched owners predominantly sell first come to believe that the majority buys first. In addition, I assume that $\pi_{SB}(z(t)) = \pi_{BS}(z(t)) = 0$ for $z(t) \in \{-1, 1\}$, since without heterogeneity no social learning can occur. The fraction of newly mismatched owners buying first then evolves as the beliefs of the unit measure of households about them:

$$\dot{x}_b = (1 - x_b(t))\,\pi_{SB}(z(t)) - x_b(t)\pi_{BS}(z(t)),$$

so that the dynamics of index $z(t)$ is given by

$$\dot{z} = (1 - z(t))\,\pi_{SB}(z(t)) - (1 + z(t))\,\pi_{BS}(z(t)). \tag{4.1}$$

Both buying first and selling first constitute a first step out of mismatch. The mismatched owner becomes a double owner if she first buys a new house, whereas she becomes a non-owner if she first sells her old house. A double owner cannot buy more than two houses, and is no longer subject to preference shocks. Preferences are such that she always enters the market to sell the house she doesn't like anymore. After selling she becomes a matched owner, making her subject to preference shocks again. Similarly, non-owners have a preference for owning a house and therefore always enter the market as buyers. If they buy a new house, this also results in matched ownership. Finally, preferences are such that matched owners do not enter the market.[53]

[53] I refer the reader to the previous chapter for a detailed exposition of the preferences of households and the conditions under which the actions described here result.

Real estate firms always put their houses up for sale, searching for a buyer that likes the house. However, in addition to receiving houses from households that exit, they are able to buy houses for speculative purposes without search frictions. Because these speculators have no interest in living in the houses they buy, they don't have to search for a match. Instead, speculative investment in the housing market attempts to obtain a return on buying that comes in the form of rent payments or house price appreciations. When house prices are low, the return on investment is high compared to the risk-free rate of return, and speculators would like to buy houses. Conversely, when prices are high, returns are low, and speculators tend to withdraw from the housing market. They no longer buy houses, and only try to find buyers for their existing stock (facing search frictions).

However, real estate firms look for "margins of safety" in the spirit of Minsky. Because housing units are illiquid as a consequence of search frictions, they need to acquire enough trust in the housing market, and only invest when returns have been favorable for some time. Specifically, the total number of houses bought by real estate firms at time t is given by $\psi S(t)\max[0, y(t)]$, where $\psi \geq 0$ captures the extent of market penetration of real estate firms, and where $y(t)$ is an index of profitability that evolves according to

$$\dot{y} = \tau\left(\frac{R + \dot{p}/\tau}{p(t)} - r\right), \tag{4.2}$$

where $p(t)$ denotes the housing price at time t, and where τ determines the speed of adjustment. As in Lux (1995), dividing by τ is necessary to capture the price change that occurs during the period that profitability changes.

The index of profitability $y(t)$ thus increases when the return on housing in the form of rental payments and house price appreciation exceeds the discount rate, and decreases when net returns are negative. Because real estate firms look for "margins of safety", real estate firms buy houses when the index of profitability is positive. However, as a result they may miss profit opportunities when the returns on housing exceed the discount rate but the index $y(t)$ is (still) negative, and make losses when the index is (still) positive even though net returns are negative.

I assume that prices fluctuate around the discounted sum of rental payments, via an exogenous function of the index describing the actions of newly mismatched owners $z(t)$:

$$p(t) = \frac{R}{r} + z(t)\Sigma, \tag{4.3}$$

where $\Sigma \geq 0$ is a parameter that captures the extent to which prices fluctuate with market conditions as described by $z(t)$. This function for the price level therefore describes in a reduced form the effect of the behavior of mismatched owners on prices, in line with the evidence as presented in Figure 17. Since $z(t)$ will also be shown to move roughly with market tightness, (4.3) also provides a specification for prices depending on market tightness as in Section 3.5.1 of the previous chapter.

That same chapter also justifies a price centered around the discounted sum of rental payments. Appendix 3.8.F shows that under certain conditions, $p = R/r$ can be microfounded as the price resulting from bargaining under private information about agent types, with full bargaining power for buyers. Moreover, Appendix 3.8.B shows that a price equal to the discounted stream of rental payments lies within the bargaining set of all agents, just as its right-sided neighborhood. For prices smaller than R/r, real estate firms would rather not sell.

In this chapter, however, real estate firms can buy houses simultaneously and without frictions, offsetting any frictional sales, so that real estate firms can be indifferent to sell for $p(t) < R/r$. Price movements into the left-sided neighborhood of R/r are therefore not at odds with the incentives of agents. At prices higher than R/r, real estate firms make current losses from investment in housing, but nevertheless they might buy if $y(t)$ is positive. Consequently, also $p(t) > R/r$ is compatible with the actions of real estate firms. With R, r, and Σ fixed, note that price changes are simply given by $\dot{p} = \dot{z}\Sigma$.

Summing up, the measure of buyers is given by

$$B(t) = B_1(t) + N(t),$$

and the measure of sellers by

$$S(t) = S_1(t) + S_2(t) + A(t),$$

so that market tightness is given by

$$\theta(t) = \frac{B_1(t) + N(t)}{S_1(t) + S_2(t) + A(t)}. \tag{4.4}$$

Note that the speculative buying of real estate firms does not enter the measure of buyers, since these firms only face search frictions when they try to sell. Combining the frictional matching rates with the speculative behavior of real estate firms and the choice of newly mismatched owners to buy first or sell first, results in the following differential equations (suppressing time dependence):

$$\dot{O} = q(\theta)N + \left(\mu(\theta) + \psi \max[0, y]\right) S_2 - (\gamma + g)O, \tag{4.5}$$

$$\dot{N} = g + \left(\mu(\theta) + \psi \max[0, y]\right) S_1 - \left(g + q(\theta)\right) N, \tag{4.6}$$

$$\dot{S_1} = \gamma (1 - x_b) O - \left(\mu(\theta) + \psi \max[0, y]\right) S_1 - gS_1, \tag{4.7}$$

$$\dot{B_1} = \gamma x_b O - q(\theta)B_1 - gB_1, \tag{4.8}$$

$$\dot{S_2} = q(\theta)B_1 - \left(g + \mu(\theta) + \psi \max[0, y]\right) S_2, \tag{4.9}$$

$$\dot{A} = g\,(O + S_1 + B_1 + 2S_2) + \psi \max[0, y]\,(S_1 + S_2) - \mu(\theta)A. \tag{4.10}$$

A learning equilibrium with speculation can now be defined as in:

Definition 4.1. A learning equilibrium with speculation is a path $\{y(t), z(t), O(t), N(t), U(t), S_2(t), A(t), p(t), x_b(t), \theta(t)\}$ such that:

1. For all $t \geq 0$, the index of households' beliefs $z(t)$ evolves according to (4.1);
2. For all $t \geq 0$, the index of profitability $y(t)$ evolves according to (4.2);
3. For all $t \geq 0$, housing prices $p(t)$ are given by (4.3), the fraction of newly mismatched owners that buy first by $x_b(t) = (z(t)+1)/2$, and market tightness $\theta(t)$ by (4.4);
4. For all $t \geq 0$, the stocks of agents $O(t), N(t), U(t), S_2(t), A(t)$ evolve according to (4.5)-(4.10);
5. $z(0) \in [-1, 1], y(0), O(0) \geq 0, N(0) \geq 0, U(0) \geq 0, S_2(0) \geq 0, A(0) \geq 0$ are given such that $N(0) + O(0) + U(0) + S_2(0) = 1$ and $A(0) + O(0) + U(0) + 2S_2(0) = 1$.

For future reference, I also define an equilibrium without speculation.

Definition 4.2. A pure learning equilibrium is equivalent to a learning equilibrium with speculation, except that $\psi = 0$ and that $y(t)$ is not part of the equilibrium.

Finally, a steady state is defined as a stationary path of the equilibrium variables.

4.3 Equilibrium

I follow Lux (1995) in investigating two different specifications for the switching rates $\pi_{SB}(z(t))$ and $\pi_{BS}(z(t))$. The first focuses purely on the behavior of households to learn from each other. The second incorporates in addition that newly mismatched owners take the presence of speculators into account.

4.3.1 Steady state equilibria

First, I use

$$\pi_{SB}(z(t)) = \upsilon e^{az(t)} \text{ and } \pi_{BS}(z(t)) = \upsilon e^{az(t)}, \tag{4.11}$$

for $z(t) \in (-1, 1)$, where $\upsilon > 0$ determines the speed of switching and $a > 0$ the strength of infection, so that the relative change in the probability to switch is linear in $z(t)$. Note from (4.1) that the evolution of z only depends on the variable $z(t)$ itself. As Lux (1995) shows, there exists a unique steady state in (4.1) at $z = 0$ if $a \leq 1$, but two additional steady states in z exist if households mimic each other fast enough, specifically if $a > 1$. Interestingly, for $1 < a < \infty$ the steady state at $z = 0$ is unstable, while two stable steady states exist at $z_+ \in (0, 1)$ and $z_- \in (-1, 0)$ with $z_+ = -z_-$, as in Figure 18.

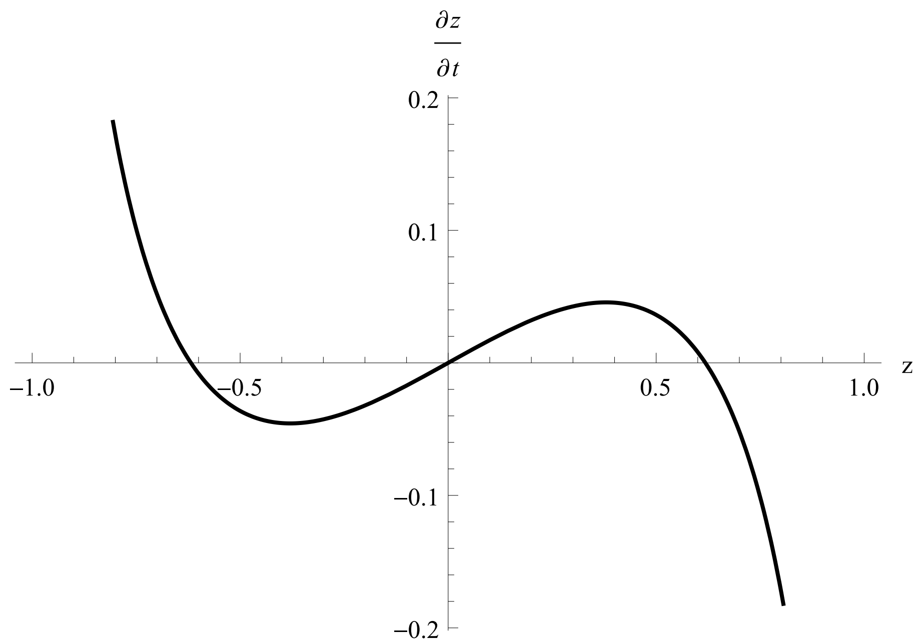

Figure 18: Dynamics of z as a function of z, giving rise to three steady states

Consequently, if contagion is relatively weak, deviations from balanced beliefs will eventually die out because the stock of households that can switch to majority beliefs depletes. However, if contagion is strong, deviations from balanced beliefs will strongly be followed upon, to such an extent that it can exactly balance the depletion of households with minority beliefs. Moreover, the stronger the contagion, the closer to unanimous beliefs these forces balance out. The perfect coordination of the previous chapter is only obtained for $a \to \infty$.

Does constancy of z also imply constant measures of each agent? In steady state, the stocks of agents should satisfy

$$q(\theta)N + \left(\mu(\theta) + \psi \max[0, y]\right) S_2 = (\gamma + g)O, \tag{4.12}$$

$$g + \left(\mu(\theta) + \psi \max[0, y]\right) S_1 = \left(q(\theta) + g\right) N, \tag{4.13}$$

$$\left(\mu(\theta) + \psi \max[0, y] + g\right) S_1 = \gamma (1 - x_b) O, \tag{4.14}$$

$$\left(q(\theta) + g\right) B1 = \gamma x_b O, \tag{4.15}$$

$$\left(\mu(\theta) + \psi \max[0, y] + g\right) S_2 = q(\theta) B_1, \tag{4.16}$$

$$g\,(O + S_1 + B_1 + 2S_2) + \psi \max[0, y]\,(S_1 + S_2) = \mu(\theta) A. \tag{4.17}$$

To see under which conditions a constant z implies constant measures of agents, first suppose that real estate firms never buy houses, i.e. $\psi = 0$. In that case, (3.34)-(3.40) in the previous chapter are equivalent to (4.12)-(4.17) for $N \equiv B_0 + B_n$. As a result, the proof of Proposition 3.3 therein applies. The following lemma combines the results from Lux (1995) and the previous chapter.

Lemma 4.3. *Suppose that* $\pi_{SB}(z(t)) = v e^{az(t)}$ *and* $\pi_{BS}(z(t)) = v e^{-az(t)}$ *for* $z(t) \in (-1, 1)$. *Then there exists a unique steady state pure learning equilibrium for* $0 < a \leq 1$, *characterized by* $z = 0$, $\theta = 1$, *and* $p = R/r$. *For* $1 < a < \infty$, *two additional steady state pure learning equilibria exist, characterized by* $z_+ \in (0, 1)$ *and* $z_- \in (-1, 0)$ *with* $z_+ = -z_-$.

Proof. Following Lux (1995), for $\pi_{SB}(z(t)) = v e^{az(t)}$ and $\pi_{BS}(z(t)) = v e^{-az(t)}$ for $z(t) \in (-1, 1)$, (4.1) can be written as

$$\dot{z} = 2v\,[\sinh(az(t)) - z(t)\cosh(az(t))] = 2v\,[\tanh(az(t)) - z(t)]\cosh(az(t)),$$

so that $z(t)$ is stationary for $\tanh(az(t)) = z(t)$. In particular, $\tanh(az(t)) = z(t)$ for $z = 0$, and in case $1 < a < \infty$, also for $z_+ \in (0, 1)$ and $z_- \in (-1, 0)$ with $z_+ = -z_-$. As a result, the fraction of newly mismatched owners buying first, $x_b(t)$, is also stationary. From (4.3), the same goes for housing prices $p(t)$.

To see that a stationary $z \in (-1, 1)$ implies (4.12)-(4.17) for $\psi = 0$, note that any stationary $x_b \in (0, 1)$ corresponds to a steady state $\theta \in (\underline{\theta}, \overline{\theta})$ as defined in the previous chapter. First, from their definitions in Lemma 3.2, if $x_b = 1$, then in steady state $\theta = \overline{\theta}$, and if $x_b = 0$, then in steady state $\theta = \underline{\theta}$. Second, as in the proof of Proposition 3.3, equations (4.12)-(4.17) and the condition that $N(t) = A(t) + S_2(t)$ result in a continuous relation between the fraction of households buying first, x_b, and θ, given by

$$x_b = \frac{\left(1+\frac{g}{\gamma}\right)\frac{(\theta-1)}{\mu(\theta)} + \frac{1}{\mu(\theta)+g}}{\frac{\frac{1}{\theta}+1}{q(\theta)+g}}$$

Consequently, $\theta = \overline{\theta}$ in steady state only if $x_b = 1$, and $\theta = \underline{\theta}$ in steady state only if $x_b = 0$. Because of continuity and because $\theta = \underline{\theta}$ if and only if $x_b = 0$ and $\theta = \overline{\theta}$ if and only if $x_b = 1$, $x_b \in (0,1)$ implies $\theta \in (\underline{\theta}, \overline{\theta})$. Finally, a stationary x_b and θ imply stationary measures of agents from (4.12)-(4.17), and three steady state pure learning equilibria exist for $1 < a < \infty$.

To see that a unique steady state exists for $0 < a \leq 1$, combine the result of Lux (1995) with the proof of Corollary 3.4. The latter shows that (4.12)-(4.17) imply $\theta = 1$ if $x_b = 1/2$, which corresponds to the unique stationary $z = 0$ for $a \leq 1$. Finally, from (4.3) $p = R/r$ if $z = 0$. □

In the steady states with $z_+ \in (0,1)$ or $z_- \in (-1,0)$, one strategy is therefore more frequent than the other, but the fraction of newly mismatched owners that buy first is unequal to one hundred or zero percent. For that reason, switching between beliefs based on limited information about the actions of newly mismatched owners can explain situations in which for example 25 or 80 percent of the newly mismatched owners chooses to buy first. The lemma also shows that a stationary $z \in (-1,1)$ implies stationary market tightness and prices, and constant measures of agents.

Now suppose that $\psi > 0$, so that speculation does affect the rate at which sellers sell their houses. As before, from (4.3) a stationary z implies stationary prices p. The following proposition shows that there exists a steady state learning equilibrium with speculation, but that it is unstable if $a > 1$. However, there exists another stationary value for the index of households beliefs, z, for which market tightness, prices, and measures of agents are constant, but the index of profitability, $y(t)$, is not.

Proposition 4.4. *Suppose that $\pi_{SB}(z(t)) = v\mathrm{e}^{az(t)}$ and $\pi_{BS}(z(t)) = v\mathrm{e}^{-az(t)}$ for $z(t) \in (-1,1)$. Then a necessary condition for a steady state learning equilibrium with speculation is $z = 0$. One such steady state exists with $\theta = 1$, $p = R/r$ and $y \leq 0$. For $1 < a < \infty$, this steady state is unstable, but there exist, in addition,*

1. *a stationary path for $\{z(t), O(t), N(t), U(t), S_2(t), A(t), p(t), x_b(t), \theta(t)\}$, provided that $y(0) \leq 0$. On this path, $z = z_+ \in (0,1)$, $p = R/r + \Sigma z_+$, and $y(t) \to -\infty$ as $t \to \infty$;*

2. *a stationary path for* $\{z(t), p(t), x_b(t)\}$*, with* $z = z_- \in (-1,0)$*,* $p = R/r + \Sigma z_-$*, and* $y(t) \to \infty$ *as* $t \to \infty$*.*

Proof. From Lemma 4.3 we know that there exists a steady state pure learning equilibrium that features $z = 0$, $\theta = 1$, and $p = R/r$. Then there exists a steady state learning equilibrium with speculation if (1) y is stationary and (2) speculation does not affect (4.12)-(4.17). The first follows from (4.2) with $z = 0$, which implies $\dot{p} = 0$, $p = R/r$, and thus $\dot{y} = 0$. For $\psi > 0$, the second is then the case if $y \leq 0$. For $a > 1$, however, $z = 0$ is unstable.

Similarly, we know for $1 < a < \infty$ that there exist two additional steady state pure learning equilibrium that feature $z_+ \in (0,1)$ and $z_- \in (-1,0)$, respectively. On the one hand, if $z \neq 0$, then $p \neq R/r$ and $\dot{y} \neq 0$, so that there exist no steady state learning equilibria with speculation for $z_+ \in (0,1)$ or $z_- \in (-1,0)$. On the other hand, if $z_+ \in (0,1)$, then $p = R/r + \Sigma z_+$, and $\dot{y} < 0$ for all t. Consequently, for a sufficiently low starting value $y(0)$, speculation does not affect (4.12)-(4.17). Therefore there exists a stationary path for $\{z(t), O(t), N(t), U(t), S_2(t), A(t), p(t), x_b(t), \theta(t)\}$ if $y(0) \leq 0$. Similarly, if $z_- \in (-1,0)$, then $p = R/r + \Sigma z_-$, and $\dot{y} > 0$ for all t. Consequently, there always exists some t for which speculation matters for (4.12)-(4.17), and increasingly so. However, a stationary path for $\{z(t), p(t), x_b(t)\}$ exists. □

Proposition 4.4 shows that there can only exist steady state learning equilibria with speculation if there are as many households that believe that the majority of newly mismatched owners buy first as there are households that believe that the majority of newly mismatched owners sell first. Otherwise, housing prices differ from the discounted stream of rental payments, and the index of profitability diverges. When housing prices are above the discounted stream of rental payments, there is no role for profitable speculation, and a path for all variables except the index of profitability can be stationary. However, when housing prices are below the discounted stream of rental payments, speculators would like to buy houses. In this case, houses can be rented out with profits, and, if prices were to rise, houses can also be sold for profit. As a result, the presence of real estate firms in the housing market increases, and selling houses becomes easier and easier.

Under such circumstances, in order to reduce the costly time period in between transactions, newly mismatched owners would not like to sell first, but to buy first.

However, the learning dynamics as specified in (4.11) do not take the presence of speculators into account. As a result, there exist a stationary path for $\{z(t), p(t), x_b(t)\}$ only because newly mismatched owners do not act in their best interests. For that reason, I consider a second specification for the rates to switch.

4.3.2 Housing cycles

To incorporate that newly mismatched owners take the presence of speculators into account, I allow households to condition their learning on the returns from speculative investment in the housing market. By their behavior, real estate firms affect the time on the market for sellers. Acknowledging the presence of these forces, newly mismatched owners should also condition their transaction sequence decision on their estimate of the activity of real estate firms, and all households should accordingly adjust their beliefs about the actions of newly mismatched owners. Following Lux (1995), I assume the Poisson rates are given by

$$\pi_{SB}(z(t), y(t)) = \nu \mathrm{e}^{y(t)+az(t)} \text{ and } \pi_{BS}(z(t), y(t)) = \nu \mathrm{e}^{-y(t)-az(t)}. \tag{4.18}$$

Now the driving differential equations of the dynamic system are (4.2) and (4.1) with the switching rates specified above. The stocks of different household types and houses owned by real estate firms as described in (4.5)-(4.10) only follow from the behavior of $z(t)$ and $y(t)$ and do not affect them. For that reason, the analysis of Lux (1995) applies, and at least one stable limit cycle in $\{z(t), y(t)\}$ exists if owners mimic each other fast enough, specifically if $1 - r\Sigma/R < a < \infty$. In that case all trajectories for the fraction of newly mismatched owners that buy first converge to a periodic orbit. A limit cycle exists because speculators act upon an index of profitability, which is a stock that increases in profit opportunities. Because speculators buy houses when $y(t) > 0$, they are slow to respond to profit opportunities, and similarly continue buying houses for too long. Such sluggish behavior may be a characterization of bubbles and crashes.

If there exists a limit cycle in $\{z(t), y(t)\}$, then Figure 19 shows that the number of transactions, the time-on-market, the stock of houses for sale, and (by construction) housing prices can also converge to a periodic orbit (legend below). This numerical example suggests the existence of a limit cycle learning equilibrium with speculation. The parameters of the numerical example are presented in Table 11.

The model-generated fluctuations in the fraction of newly mismatched owners that buy first, the number of transactions, time-on-market, the stock of houses for sale, and

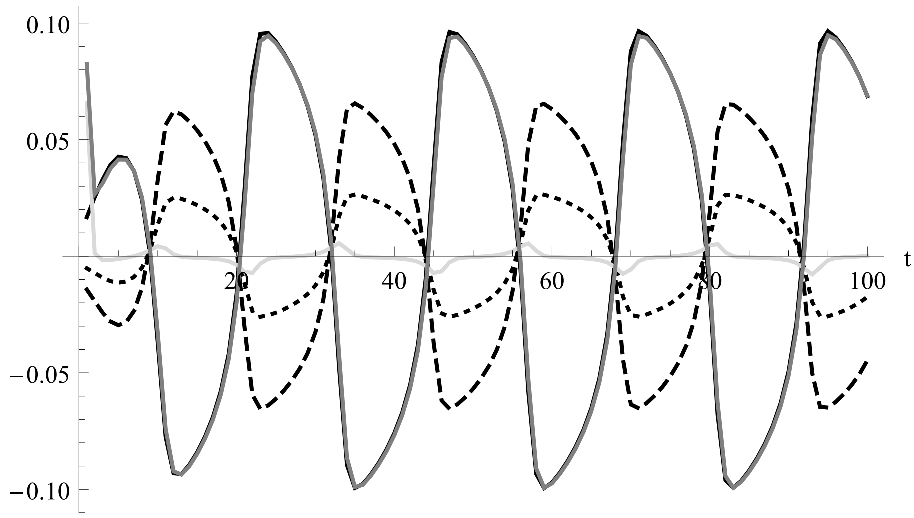

Figure 19: Simulated dynamics of housing market variables (in log deviations), and market index z (scaled by 0.1).

Parameter		Value	Target
Matching efficiency	μ_0	3.6	Average TOM
Matching elasticity	α	0.84	Genesove & Han (2012)
Entry/exit rate	g	0.05	Moen *et al.* (2015)
Mismatch rate	γ	0.01	Moen *et al.* (2015)
Discount rate	r	0.05	Standard
Rental rate	R	0.025	Price level
Market penetration	ψ	0.001	Size real estate firms
Amplitude prices	Σ	0.02	Amplitude and period
Speed profitability	τ	13	Period
Speed switching	v	1	Period
Strength contagion	a	1.1	Amplitude fraction

Table 11: Parameter values

housing prices are broadly consistent with the stylized facts of the labor market. Figure 20, reprinted from the previous chapter and based on Moen et al. (2015), shows the behavior of these variables for Copenhagen between 2004 and 2008 (including legend).

Now zooming in on one cycle of the simulated dynamics as presented in Figure 21, a numerical example of the limit cycle captures the dynamics of the same variables to a large extent. Both the structure of the time lags and the relative size of the fluctuations do roughly correspond across these figures, except that time-on-market in the simulation fluctuates a bit too much. Possibly, an endogenous moving decision as in Ngai and Sheedy (2015) could reduce the amplitude of time-on-market a bit, as mismatched owners would withdraw from the market when housing market conditions become too unfavorable.

On the one hand, the good performance with respect to prices is not too surprising, given that housing prices are an exogenous function of the fraction of newly mismatched owners that buy first. On the other hand, the growth rates of prices show momentum in the short run but mean reversion in the long run, which the literature finds hard to explain. In this chapter, growth rates of prices are positively autocorrelated at high frequencies and negatively autocorrelated at low frequencies, because prices move with the behavior of moving owner-occupiers.

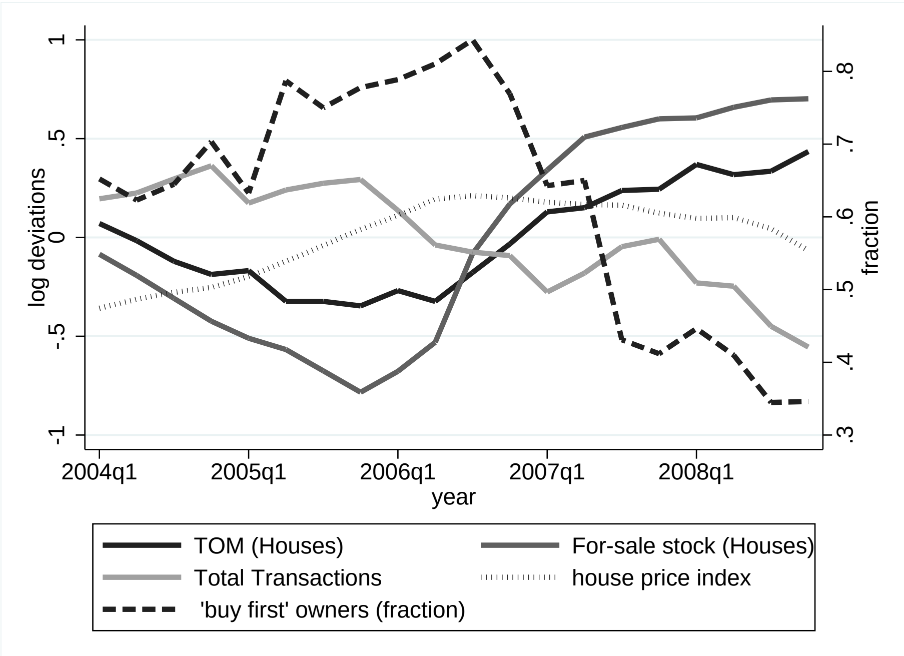

Figure 20: Dynamics of housing market variables (in log deviations) and the fraction of owners that buy first in Copenhagen between 2004 and 2008.

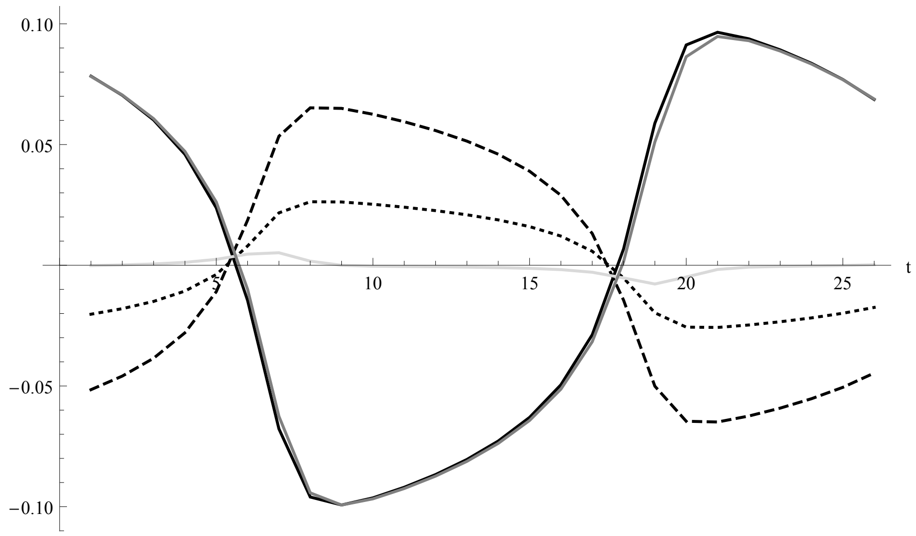

Figure 21: Simulated dynamics of housing market variables (in log deviations), and market index z (scaled by 0.1).

4.4 Conclusion

This chapter offers a reinterpretation of Lux (1995) that explains the lack of coordination in the transaction sequence decision of moving owner-occupiers. The strong strategic complementarities identified in Moen et al. (2015) to buy first when other households are also buying first still exist, but because market tightness is unobserved, households have to learn what others are doing. When households learn fast enough and on top of that condition their strategy on the sluggish behavior of speculators, a stable limit cycle exists. The time series simulated from this limit cycle qualitatively match actual housing market data.

CHAPTER 5

Efficient insurance in a market theory of payroll and self-employment[54]

5.1 Introduction

Over the last two decades, the majority of the developed economies have experienced a shift in the composition of employment towards more own-account work and freelancing. For example, about half of the new jobs taken up in the United Kingdom in the last decade were self-employment jobs. As a result of the shift in the composition of employment, the self-employed now constitute a group larger than the unemployed in many countries. Furthermore, there is significant variation of the self-employment rate between developed economies. These facts, illustrated in Table 12, are difficult to reconcile with existing theories of self-employment that emphasize individual heterogeneity of skills, preferences, or cognitive biases (Parker, 2012), which we don't expect to vary much over time and across countries.

In this paper we propose a theory of the composition of employment that focuses on the key distinction between self- and payroll employment: exposure to the risk of not selling output versus exposure to the risk of not finding a job. The self-employed are the sole claimants of the fruits of their labor, but bear the risk of not selling their products

[54] This chapter is based on Denderski and Sniekers (2016)

Country	1993	2013
UK	9.1	11.7
Netherlands	6.9	11.8
Germany	4.0	6.0
Italy	11.8	16.4
Sweden	7.8	6.4
Denmark	4.7	5.4

Table 12: Share of own account workers in total employment. Source: Key Indicators of the Labor Market, ILO.

or services fully on their own. The wage contract limits this risk for the employee, but requires sharing any surplus of a match with the firm. Moreover, not all of those looking for a wage contract are able to find one, so that some become unemployed. In order to highlight these differences between payroll and self-employment, we explicitly and jointly model the problems of finding a job in the labor market and selling output in the goods market.

All developed countries offer some form of unemployment insurance for those that do not find a job. Denmark and Sweden have recently also introduced unemployment insurance schemes that are designed specifically for the self-employed. However, in the majority of developed countries this type of insurance is still absent. On the one hand, it is widely believed that being self-employed is riskier than being an employee. On the other hand, if self-employment is driven by the desire to be one's own boss or by higher tolerance for risks, insurance for the self-employed is hard to justify.

We aim to make two contributions. First, we propose a novel parsimonious theory of self-employment that does not rely on individual-level differences between people but focuses on trade-offs between labor and goods market frictions. Changes in these markets, possibly resulting from the spread of modern communication technologies, provide a natural explanation for the long run behavior of self-employment rates.

Second, we show that there should be insurance benefits for the self-employed that fail to sell, solely for the purpose of maximizing the volume of goods traded with customers. Insurance for the self-employed eliminates business stealing by risk averse self-employed who have an incentive to set their prices too low from the societal point of view, in order to reduce the risk of not selling their output. This paper therefore provides a rationale for the UI policies of Denmark and Sweden that is not only based on risk sharing, but also on efficiency.

We consider an economy represented in Figure 22. It is inhabited by homogenous individuals producing an indivisible good. The good cannot be consumed by these individuals but can be exchanged with buyers for a divisible endowment from which the individuals do derive utility. The individuals face a one-shot career choice problem. They can either become self-employed, which implies producing and trying to sell their output by themselves, or seek a job at a firm, which tries to sell the goods produced by the individual it employs.

Individuals entering the labor market cannot coordinate their job applications to firms that post wages, which results in involuntary unemployment. Similarly, buyers cannot coordinate their visits to firms and self-employed that post prices, resulting in unsold inventories. An employee is guaranteed the wage even if the firm fails to sell the goods. However, an employee has to share the expected surplus with the firm. The self-employed face the risk of not selling, but they forego the risk of unemployment.

Unlike in the standard Mortensen-Pissarides framework, in our model a match with a firm is thus not necessary to generate income. Our model endogenously determines a vacancy and self-employment rate depending on, among others, search technology and the existence of some advantageous business conditions for firms. One can think of these advantageous business conditions as productivity or quality gains to firm formation, resulting from additional capital, training or knowledge that the firm has at its disposal after the investment of an entry cost k. In a richer framework with intermediate inputs, such gains may result from economizing on internal transaction costs as in Coase (1937). Alternatively, advantageous business conditions for firms can result from a superior visibility in the goods market after the investment of cost k. This is the interpretation used in this chapter. Firms in our model are both an intermediary between the employee and the buyers, and a vehicle of production and marketing that cannot exist without any form of competitive advantage over independent production by the self-employed. Yet, the trade-off between the frictions in the goods and labor market leads to the coexistence of firm employment and self-employment in equilibrium.

The presence of firms and self-employed implies the goods market consists of two different types of sellers. If individuals are risk averse, these two different types of

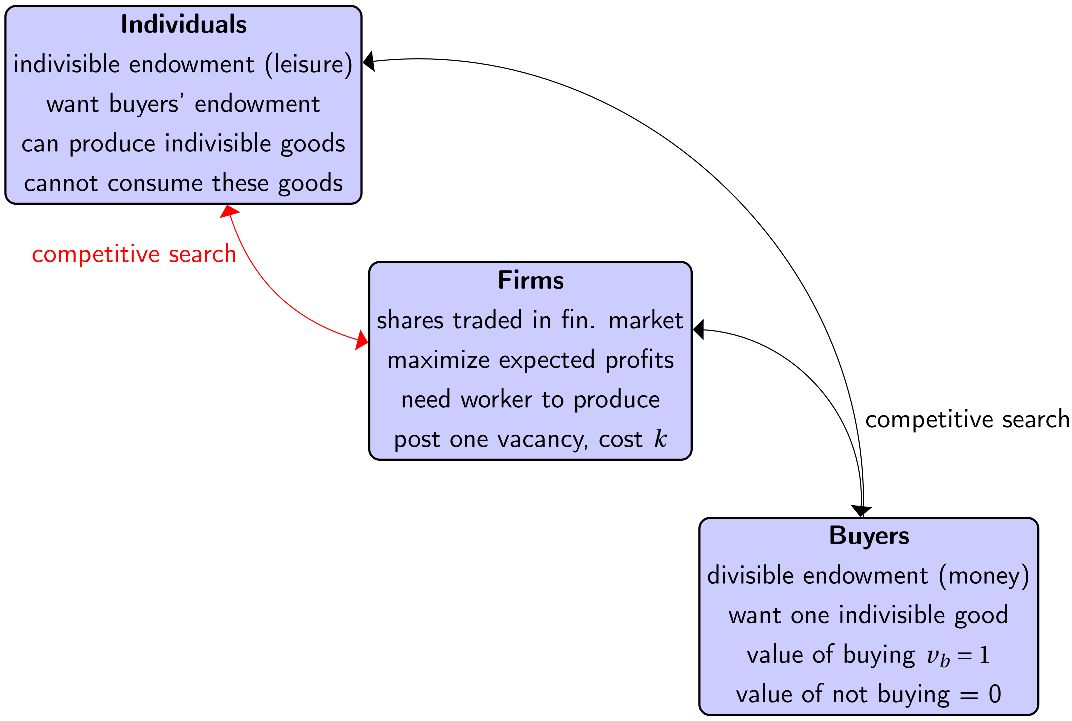

Figure 22: A snapshot of the model economy

sellers have different objectives, creating inefficiencies that can be potentially corrected by policy. Risk averse self-employed, unlike risk neutral firms, have an incentive to self-insure via their pricing decision. By decreasing prices they attract on average more buyers so that their selling probability increases. However, the lowering of prices by the self-employed steals business away from firms. Other things equal, firms' expected profits fall. Consequently, fewer firms enter and the economy benefits to a lesser extent from their advantageous business conditions. As a result, the volume of the goods traded drops.

The ability to self-insure makes a career in self-employment relatively more attractive than entering the labor market. This effect is countered by the conventional effect that firms post lower wages to increase the job finding rate of risk averse job seekers. The latter, however, comes at the cost of excessive vacancy creation. Generally, the employment composition is different than the composition that would maximize the volume of goods traded net of firm entry costs. For some parameter values, the ability to self-insure in self-employment can dominate the market insurance offered by firms. As a result, the self-employment rate may *increase* in risk aversion.

We find that the combination of type-of-employment dependent lump-sum taxes and income support benefits under a balanced budget can maximize the output sold net of entry costs, while offering insurance to risk averse individuals. This optimal policy mix consists of differentiated taxes and unemployment insurance benefits for both workers and self-employed. Optimal UI benefits for the self-employed eliminate business stealing, while optimal UI benefits for job seekers raise wages and stop excessive firm entry. Differentiated taxes then balance the budget while ensuring an optimal employment allocation via the career choice of individuals. We show that whenever the job finding probability exceeds the selling probability (so that the self-employment income can be considered riskier), the UI benefits for the self-employed should be more generous than the UI benefits for employees.

Related literature. Our paper is related to three strands of the literature: on the drivers of self-employment, on frictional goods markets and intermediation, and on optimal unemployment insurance. Below we describe our contribution to those papers.

There is a variety of theories explaining self-selection into self-employment. A large fraction of this literature puts individual characteristics and heterogeneity as a reason for self-employment. We list a limited selection of those papers, whereas Parker (2004) offers an extensive survey. Lucas (1978), Jovanovic (1982) and Poschke (2013) assume that being an entrepreneur/self-employed requires a separate skill, potentially different than a skill needed to be an employee. Kihlstrom and Laffont (1979) postulate differences in risk aversion that lead to undertaking entrepreneurial activities. De Meza and Southey (1996) find that self-employed entrepreneurs are significantly more optimistic than employees. Lindquist et al. (2015) document the importance of family background. We complement this literature, because in our model self-employment is an equilibrium outcome that does not require any ex ante individual heterogeneity. Besides, Rissman (2003, 2007) assumes returns to self-employment are drawn from an exogenous distribution riskier than the wage distribution. We offer a model that endogenously generates those risk differentials. Finally, our model is complementary to papers that explain self-employment from financing frictions (Evans and Jovanovic, 1989; Buera, 2009), because a frictional financial market could be introduced as an additional stage in the career choice game for those who choose self-employment.

To the best of our knowledge, self-employment has not been introduced into models with frictions in the goods market. Existing papers study the macroeconomic consequences of goods market frictions (See e.g. Michaillat and Saez (2015); Petrosky-Nadeau and Wasmer (2015); Kaplan and Menzio (2016)), or characteristics of firms that operate in a frictional goods market. Most closely related are Shi (2002), who explains the size-wage differential in the labor market by a sufficiently large size-revenue differential in the goods market, and Godenhielm and Kultti (2015), who allow for endogenous capacity choice and study the resulting firm size distribution.

Our model can also be framed as a choice of producers to trade with buyers via a middleman (firm) or to trade with buyers directly. The papers in the literature on intermediation that are most closely related are Watanabe (2010, 2013). Unlike in those papers, the choice that producers (individuals) face in our model is exclusive. Also, the meetings with the middlemen are subject to a friction. Wright and Wong (2014) offer a general model of middlemen with search and bargaining problems. We employ posting, allow for bypassing of the middlemen, and discuss labor policy implications.

Papers on efficient unemployment insurance for risk averse individuals either do not take self-employment (e.g. Acemoglu and Shimer (1999)) or market frictions into account (e.g. Parker (1999)). Our paper shows that the interaction of risk averse self-employed and goods market frictions is crucial for understanding efficient unemployment insurance.

The rest of the paper is organized as follows. In Section 5.2 we outline the structure of the model. Then, in Section 5.3 we characterize the market equilibrium. In Section 5.4 we present and characterize the conditions under which a unique mixed strategy equilibrium exists, and prove that it maximizes net output sold for risk neutral preferences. In Section 5.5 we show that the decentralized allocation is not efficient for risk averse preferences, but that introducing a type-of-employment dependent tax and unemployment insurance policy can restore efficiency. Section 5.6 presents the steady state of a dynamic version of the model in which jobs in expectation last for multiple periods, and shows how the composition of employment depends on the key parameters of the model. The final section concludes.

5.2 Model

We consider a one-shot game of an economy populated by firms, buyers, and individuals interacting in two markets. Individuals face a career choice between self-employment and entering the labor market. In the goods market, firms and the self-employed exchange with buyers an indivisible produced consumption good (a coconut) for a divisible endowment (money). In the labor market, individuals exchange with firms indivisible labor for money. There are coordination frictions in both markets.

The measure of individuals is normalized to one. Individuals value consumption according to a weakly concave utility function $u(c)$, suffer no disutility from labor, but cannot consume coconuts. Instead, they derive utility from money.[55] For convenience, we also postulate that $u(0) = 0$. There is a unit mass of buyers B in the goods market that can consume money and exactly one coconut. They enjoy a utility v from consuming the coconut, and a smaller, linear utility from their endowment of money, which individuals can also consume. We normalize buyers' utility from not buying to zero and the utility from consuming the coconut to one. We consider a partial equilibrium setup with buyers and individuals living in separate households. Finally, there is an endogenously determined mass V of vacancies opened by profit-maximizing firms upon paying a cost $k > 0$.

The career choice of individuals results in an endogenous measure SE of self-employed and $LM = 1 - SE$ of agents entering the labor market. A match of a single worker and vacancy results in an active firm. The measure of active firms is denoted by F. Because of search frictions in the labor market we have $F \leq V$. Both a self-employed and an active firm produce one coconut. Each opens one outlet to sell its coconut in the goods market and posts a price with commitment. However, an active firm's outlet and price is visible in the goods market with probability $A \in (0,1]$, whereas a self-employed individual's outlet and price is visible with probability $a \in (0,1]$.[56]

Buyers observe prices of visible outlets, can only visit one of these, but cannot coordinate which one to visit. For that reason, the goods market is subject to urn-ball frictions. As a result, some visible sellers face more customers than they can serve and others are not able to sell, while some buyers fail to buy the good. If there is a mass of buyers B_{SE} at the visible outlets opened by the self-employed, then the average queue length at each outlet is $x_{SE} = B_{SE}/(aSE)$. The average queue length of buyers at a visible

[55] For instance, because money can be exchanged in a perfectly competitive third market for another divisible consumption good (which is thus not a coconut).

[56] Here we allow for differences in the visibility of self-employed and active firms. This formulation is not restrictive. A model in which a and A denote the number of coconuts produced by the self-employed and active firms, respectively, is almost identical (exactly identical under risk neutral preferences) as long as every coconut is sold in a separate outlet. If multiple coconuts are sold at one outlet, there exists a selling advantage to larger inventories, see Watanabe (2010). Alternatively, one can write a very similar model with differences in qualities of coconuts, which scale up the utility of buyers.

active firm x_F is defined analogously. The corresponding service probabilities for a buyer are denoted by $\eta(x_{SE})$ and $\eta(x_F)$, at self-employed and active firms respectively. The selling probabilities of visible outlets $\lambda(x_{SE}) = x_{SE}\eta(x_{SE})$ and $\lambda(x_F) = x_F\eta(x_F)$ are the complementary probabilities of having no buyers visiting a visible outlet at all. Using the large market assumption to characterize these probabilities, $\lambda(x) = 1 - \mathrm{e}^{-x}$.

Upon paying an entry cost to open a vacancy, a firm posts a wage and commits to it. Workers observe all wages but can apply to one vacancy only, while a vacancy can be filled by only one worker. As standard in the literature, to capture the coordination frictions we restrict our attention to symmetric and anonymous strategies. We denote the average queue length by $x_{LM} = LM/V$ where V is the measure of open vacancies. Due to coordination frictions, some firms fail to fill their vacancy and do not become active, while some workers remain unemployed. The probability of filling a vacancy is denoted by $q(x_{LM})$, and by the large market assumption $q(x_{LM}) = 1 - \mathrm{e}^{-x_{LM}}$. The job finding probability is simply $\mu(x_{LM}) = q(x_{LM})/x_{LM}$. Finally, we assume that the firms can insure in a competitive market against the risk of not being able to pay the wage, so that the worker is guaranteed a wage once matched. The shares of firms are traded by financial investors (not modeled explicitly) who can buy a market portfolio of those shares so that the firms maximize expected profits upon entry.

The timing of the game is displayed in Figure 23. First, a measure of firms enter the labor market by opening vacancies. In the career choice stage the unit mass of individuals parts into self-employed and prospective workers. In the third stage, the frictional labor market matches vacancies and prospective workers, resulting in a measure $F = q(x_{LM})V$ of active firms and a measure $U = \left(1 - \mu(x_{LM})\right)LM$ of unemployed workers. In the fourth stage, all active firms and self-employed individuals produce and become sellers in the goods market. In the fifth stage, a fraction of the sellers become visible and buyers direct their search to them such that the following accounting identity is satisfied:

$$a x_{SE} SE + A x_F F = 1, \tag{5.1}$$

which means that no buyers stay at home not trying to visit any seller. Now we are in position to define the market equilibrium of this economy in the following section.

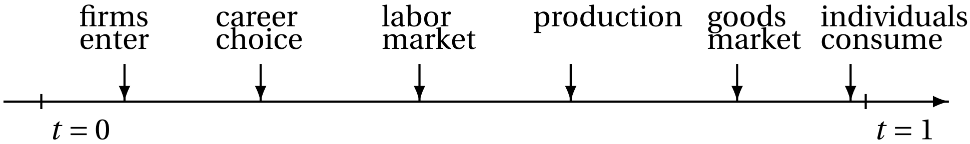

Figure 23: Timing of events

5.3 Equilibrium definition

We decompose the one-shot game into two stages: the career choice and the labor market as the first stage, and the goods market as the second stage. We solve the game backwards, starting from the goods market. We focus on equilibria that feature both self-employment and payroll employment. The existence conditions for such a mixed strategy equilibrium of the career choice game are presented in the next section.

5.3.1 Goods market

As is standard in competitive search models, separate submarkets open and buyers choose between visiting each of the submarkets such that in equilibrium they are indifferent between all active submarkets and obtain value V^B. Given the specification of buyers preferences, this value reads

$$V^B = \eta(x_F)\left(1 - p_F\right) = \eta(x_{SE})\left(1 - p_{SE}\right). \tag{5.2}$$

Given that the wage is sunk, active firms maximize expected revenue: $A\lambda(x_F)\, p_F$. The self-employed maximize expected utility. The expected value of self-employed sellers, dropping zero utility of not receiving any income, is then

$$V^{SE} = a\lambda(x_{SE})\, u\left(p_{SE}\right). \tag{5.3}$$

The goods market outcomes and payoffs are depicted in Figure 24.

Sellers post prices to maximize their expected payoffs subject to the constraint that buyers must receive their market utility V^B. The optimal prices and an associated goods market sub-equilibrium characterization are presented below.

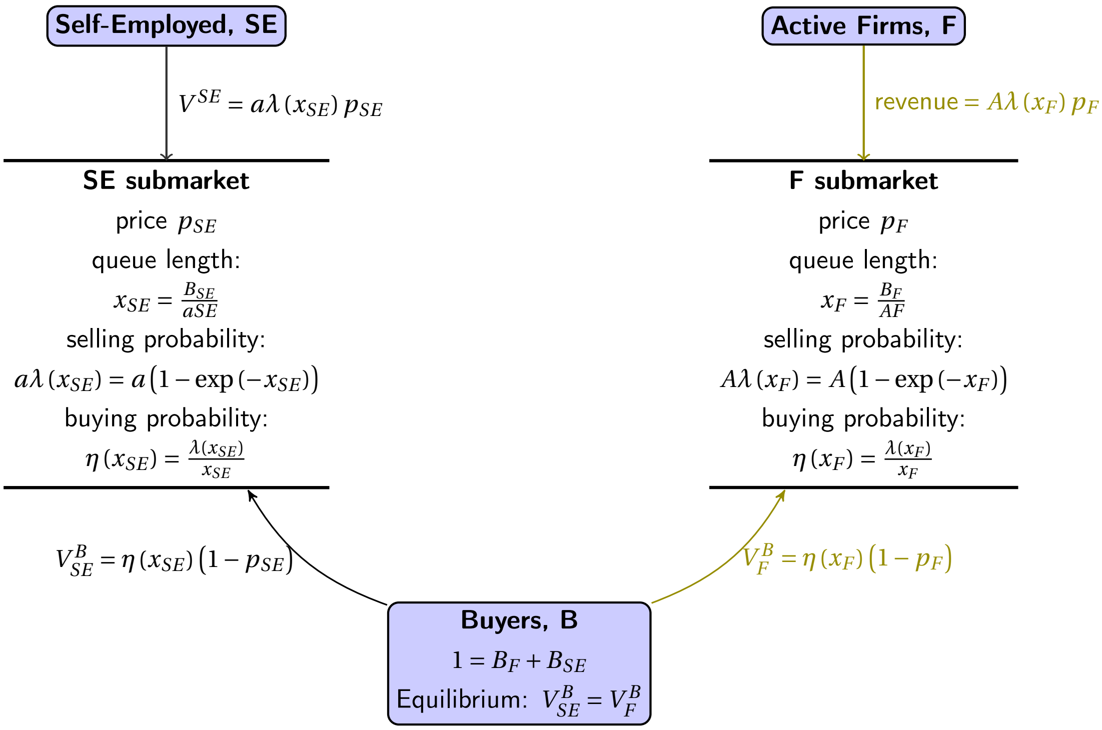

Figure 24: The goods market.

Lemma 5.1. *Assume $SE > 0$, $F > 0$, $B_F > 0$, $B_{SE} > 0$ fixed. Let $\phi(x) = -\frac{x\partial\eta(x)}{\eta(x)\partial x}$ be the elasticity of the buying probability with respect to the queue length x. Given queue lengths $x_{SE} = \frac{B_{SE}}{aSE}$, $x_F = \frac{B_F}{AF}$ the optimal price posting conditions are:*

$$\frac{\phi(x_{SE})(1-p_{SE})}{1-\phi(x_{SE})} = \frac{u(p_{SE})}{u'(p_{SE})}, \tag{5.4}$$

$$p_F = \phi(x_F). \tag{5.5}$$

Proof. See Appendix 5.8.A. □

Definition 5.2. Let $SE > 0$, $F > 0$ be fixed. A goods market sub-equilibrium is a tuple $\{x_{SE}, x_F, p_{SE}, p_F\}$ such that given x_{SE}, x_F the sellers optimally post prices according to (5.4) and (5.5), and the buyers' indifference condition (5.2) and accounting identity (5.1) hold.

5.3.2 Labor market

Now we consider a non-zero mass of prospective workers $LM > 0$ and analyze labor market outcomes. We do that in two steps. First, we fix the measure of vacancies $V > 0$; then we allow for free entry in posting vacancies by prospective firms. Given that the entry cost k is sunk, potential firms post a wage in the labor market to maximize expected profits, taking into account equilibrium outcomes in the goods market. They compete with other potential firms for workers, under the constraint that they must at least offer the market utility level of workers searching for jobs:

$$V^{LM} = \mu\left(x_{LM}\right) u\left(w\right). \tag{5.6}$$

Lemma 5.3. *Assume $LM > 0$ and $V > 0$ fixed. Given queue length $x_{LM} = LM/V$, the optimal wage w that maximizes firms profits subject to workers' market utility* (5.6) *solves the following equation:*

$$\frac{\phi\left(x_{LM}\right)\left[A\lambda\left(x_F\right)p_F - w\right]}{1-\phi\left(x_{LM}\right)} = \frac{u\left(w\right)}{u'\left(w\right)}, \tag{5.7}$$

where p_F and x_F come from the goods market sub-equilibrium $\left\{x_{SE}, x_F, p_{SE}, p_F\right\}$ induced by $SE = 1 - LM$ and $F = q\left(x_{LM}\right)V$ and where $\phi\left(x_{LM}\right) = \frac{x_{LM}\partial q(x_{LM})}{q(x_{LM})\partial x_{LM}}$ is the elasticity of the job filling probability with respect to the queue length.

Proof. See Appendix 5.8.A. □

Expected profits are therefore shared with workers according to the elasticity of the matching function in the labor market, and are decreasing in risk aversion. By posting lower wages when workers are risk averse, firms offer market insurance in the form of higher job finding probabilities from larger firm entry. The free-entry condition drives the value of posting a vacancy net of the entry cost k down to zero. Firms' entry takes into account the resulting goods-market sub-equilibrium where $SE = 1 - LM$. Formally, we can define the labor market sub-equilibrium as follows.

Definition 5.4. Assume $LM, SE > 0$. The labor market sub-equilibrium is a pair $\{x_{LM}, w\}$ such that given x_{LM}, firms optimally post wages according to (5.7) and the following free-entry condition holds:

$$q\left(x_{LM}\right)\left[A\lambda(x_F)p_F - w\right] - k = 0, \tag{5.8}$$

with p_F and x_F from the goods market sub-equilibrium $\left\{x_{SE}, x_F, p_{SE}, p_F\right\}$ induced by SE and $F = q\left(x_{LM}\right)V$ with $V = LM/x_{LM}$.

The labor market equilibrium is represented in Figure 25. Now we are in the position to define a mixed strategy equilibrium for our career choice game.

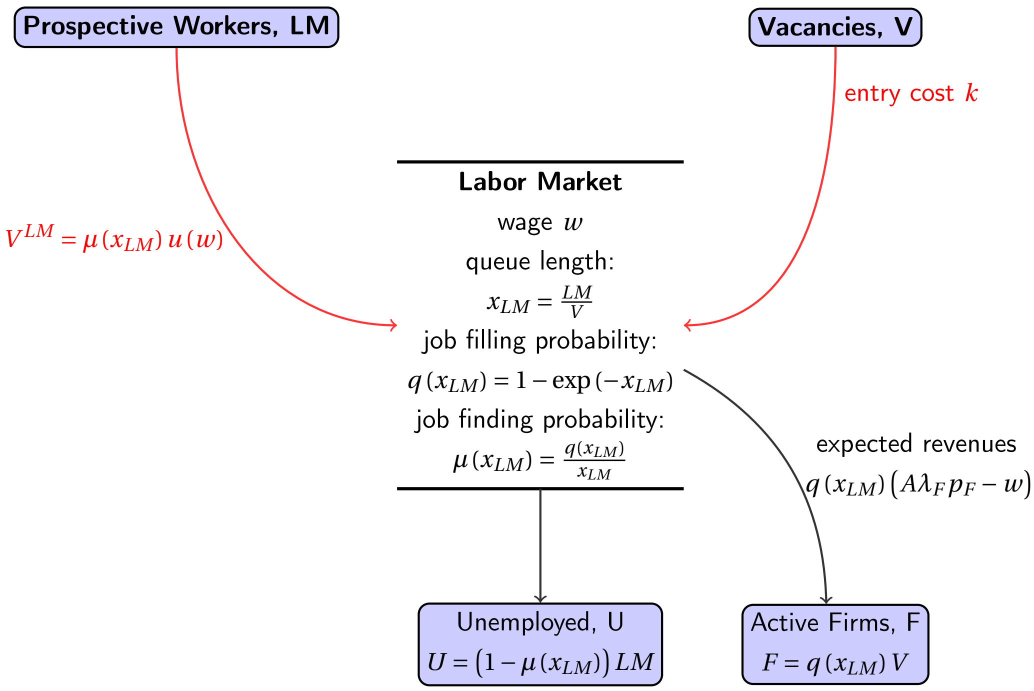

Figure 25: The labor market

Definition 5.5. A mixed strategy career choice equilibrium is a tuple $\{SE^* > 0, LM^* > 0, x^*_{LM}, w^*, x^*_{SE}, x^*_F, p^*_F, p^*_{SE}\}$ such that:

1. all individuals either become self-employed or enter the labor market: $SE^* + LM^* = 1$;
2. given SE^* and LM^*, $\{x^*_{LM}, w^*\}$ is a labor market sub-equilibrium, and $\{x^*_{SE}, x^*_F, p^*_{SE}, p^*_F\}$ is a corresponding goods market sub-equilibrium;
3. individuals are indifferent between self-employment and entering the labor market, i.e. $V^{SE^*} = V^{LM^*}$ as defined in (5.3) and (5.6), respectively.

Observe that the indifference condition requires that whenever the job finding probability μ exceeds the selling probability λ the income from self-employment is higher, conditional on selling, than the wage (and the opposite holds when $\lambda > \mu$).

An interesting special case of the mixed strategy equilibrium is the case of risk neutral individuals. If the self-employed are risk neutral, they maximize expected revenue $a\lambda(x_{SE})\,p_{SE}$. As follows from Lemma 5.1, risk neutral self-employed post prices according to $p_{SE} = \phi(x_{SE})$. Substituting this price in the buyers' indifference condition (5.2), it follows that firm and self-employed sellers can expect the same queue length and set the same prices.

If workers are risk neutral, the wage posting decision of firms as given in (5.7) implies $w = \phi(x_{LM})\,A\lambda(x_F)\,p_F$. Substituting this wage in (5.8) results in the following free entry condition:

$$q(x_{LM})\left[1 - \phi(x_{LM})\right] A\lambda(x_F)p_F = k. \tag{5.9}$$

Besides, as $\mu(x_{LM})\phi(x_{LM}) = q'(x_{LM})$, the value of being a worker can in this case be written as

$$V^{LM} = q'(x_{LM})\,A\lambda(x_F)\,p_F. \tag{5.10}$$

As a result, indifference in the career choice game simply requires

$$V^{SE} = a\lambda(x_{SE})\,p_{SE} = q'(x_{LM})\,A\lambda(x_F)\,p_F = V^{LM}. \tag{5.11}$$

Because self-employed and firms post the same prices, indifference in career choice implies

$$a/A = q'(x_{LM}) = \mathrm{e}^{-x_{LM}} \rightarrow x_{LM} = \log\left(\frac{A}{a}\right) \tag{5.12}$$

Thus, if the mixed strategy equilibrium exists, the queue length of prospective workers is independent of the vacancy posting cost k and only depends on a and A. The next section shows that the mixed strategy equilibrium can exist, both for risk neutral and for risk averse preferences.

5.4 Existence and efficiency of equilibrium

In this section we state the conditions and explain the reasons for the existence of a mixed strategy equilibrium. Afterwards, we prove that it maximizes net output sold if and only if individuals are risk neutral.

5.4.1 Existence

The equilibrium can be shown to exist, to be unique, and to involve mixing of careers if the exogenous parameters $\{a, A, k\}$ are appropriately chosen. Outside of a certain set of

$\{a, A, k\}$ the equilibrium still exists and is unique but features no mixing of careers. The proof is an application of the Implicit Function Theorem.

As a first step, using the accounting identities in the goods and in the labor market, we arrive at the following *mixing condition*:

$$LM = \frac{1 - ax_{SE}}{Ax_F\mu(x_{LM}) - ax_{SE}}, \quad 0 < LM < 1. \tag{5.13}$$

Then, we need to solve for the queue lengths $\{x_{SE}, x_F, x_{LM}\}$ and corresponding prices of goods and labor $\{p_{SE}, p_F, w\}$ using the remaining six equilibrium conditions. A necessary condition for this set of equations to have a solution is that the ratio $u(c)/u'(c)$ is increasing in c, which holds for any utility function with $u'(c) > 0$ and $u''(c) < 0$. For analytical convenience we make the proof operational under CRRA preferences. However, as the previous remark implies, this is without loss of generality.

Proposition 5.6. *Let $A, a \in (0,1]$ fixed and $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma \in [0,1]$. Then there exist numbers $\underline{k}(A,a,\gamma)$, $\overline{k}(A,a,\gamma)$ such that the mixed strategy equilibrium described in Definition 5.5 exists and is unique if and only if the following inequalities hold:*

$$A > a, \tag{5.14}$$

$$\underline{k}(A,a,\gamma) < k < \overline{k}(A,a,\gamma). \tag{5.15}$$

Furthermore, if $k > \overline{k}(A,a,\gamma)$ then $SE^* = 1$ and if $k < \underline{k}(A,a,\gamma)$ then $SE^* = 0$.

Proof. See Appendix 5.8.A. □

A direct result of the working of the proof under risk neutral preferences is the set of comparative statics encapsulated in Corollary 5.7. Intuitively, the net gain of setting up a vacancy can be neither too small, nor too big for agents to play a mixed strategy in the career choice game.

Corollary 5.7. *Consider risk neutral individuals and a, A and k such that the conditions* (5.14) *and* (5.15) *hold, and let $\{SE^*, LM^*, x^*_{LM}, w^*, x^*_{SE}, x^*_F, p^*_F, p^*_{SE}\}$ be the corresponding mixed strategy career choice game equilibrium. Then, the following inequalities hold:*

$$\frac{\partial SE^*}{\partial k} > 0,$$

$$\frac{\partial \overline{k}}{\partial A} > 0, \quad \frac{\partial \underline{k}}{\partial A} > 0,$$

$$\frac{\partial \overline{k}}{\partial a} < 0, \quad \frac{\partial \underline{k}}{\partial a} < 0.$$

There are two reasons a corner solution in the career choice game could occur. First, there may be no firms willing to enter, so that there is no chance of finding a job on a payroll. This may happen, for example, when the vacancy posting cost k is prohibitively large. Second, the business conditions of firms may be too advantageous to sustain self-employment as a valid alternative to seeking a payroll job.

Hence, the key driving force of the composition of employment in the model is the relative size of the probabilities of becoming visible in the goods market, adjusted for entry costs, which can be loosely described by comparing A/k to a. One can argue that the spread of modern communication technologies has contributed to a relative increase in the visibility of the outlets of the self-employed, a. The theory then predicts a rise in self-employment rates, roughly in line with the recent increases in self-employment rates as documented in Table 12.

More generally, the ratio between A/k and a captures the improvement in business conditions on top of producing on one's own that comes with setting up a firm. Correcting expected utility with binomial probabilities, one can interpret A and a as the number of units produced by active firms and self-employed, respectively, as long as each unit is sold at a separate outlet. One can then think of the ratio of A/k over a as a statistic for substitutability between the "self-employment technology" and "payroll employment technology", or the additional returns to innovation from firm formation. For example, a large-scale production industry like an automotive industry is a sector with a very high A/k and low a. In contrast, one can expect the difference between A/k and a to be low in service industries like hairdressing or taxi-driving. A prediction of the model is therefore that self-employment rates are higher in hairdressing than in the automotive industry. Another prediction is that when the share of low capital intensity services in the economy increases, the share of self-employment goes up as well. Finally, the model can shed some light on cross-country differentials in economic development and the composition of employment. In underdeveloped countries the technologies that are used in firms offer very little gains, or no gains at all, from organizing workers and capital into a firm. As our model predicts, those countries exhibit high self-employment rates.

5.4.2 Efficiency

We move onto investigating the efficiency of the decentralized equilibrium. We use output sold net of entry costs as our measure of efficiency and allow the social planner to choose the measure of vacancies to be opened and the measure of households to enter self-employment (and thus the measure to enter the labor market). Thus, the reference point is the optimal use of available individuals. However, this is not equivalent to maximizing the utility of individuals or buyers.

The social planner faces the same visibility probabilities and coordination frictions within every (sub)market as present in the decentralized equilibrium, but can decide the measure of buyers to go shopping at the visible self-employed (and thus the measure of buyers that visits visible active firms). Finally, note that choosing the latter, given SE and V, amounts to choosing x_{SE}. The problem of the social planner is then to maximize:

$$V^{SP}\left(V, x_{SE}, SE\right) = A\lambda\left(x_F\right) q\left(x_{LM}\right) V + a\lambda\left(x_{SE}\right) SE - Vk,$$

where $x_{LM} = (1 - SE)/V$ and $x_F = (1 - x_{SE} aSE) / \left(Aq\left(x_{LM}\right) V\right)$ from the unit mass of individuals and the accounting identity in (5.1), respectively. Similar to Acemoglu and Shimer (1999), for this measure of efficiency the following result can be shown.

Proposition 5.8. *The decentralized allocation is constrained efficient if and only if individuals are risk neutral.*

Proof. See Appendix 5.8.A. □

Consequently, the equilibrium allocation that is implicitly given by (5.2), (5.9), and (5.11), with $p_F = \phi\left(x_F\right)$ and $p_{SE} = \phi\left(x_{SE}\right)$, maximizes output sold net of entry costs.

From Proposition 5.8 it follows that the outcome of the market interactions under risk aversion does not maximize output sold net of entry costs. The drivers of this result are the price and wage posting decisions. When agents are risk averse, the price p_{SE} that the self-employed charge is lower than the efficient p_{SE}^{SP} for a given queue length x_{SE}^{SP}. The self-employed self-insure by decreasing their price to improve the odds of selling their output. By doing so they generate an inefficient distribution of queues which decreases the total output sold. Furthermore, firms offer market insurance as well. They increase the job finding rate of workers by increasing entry at the expense of lower wages. This distorts the allocation by an inefficient increase in entry costs. On top of that, the underpricing by the self-employed forces the firms to lower their prices as well, which exerts another downward pressure on wages. Consequently, wages and both prices are lower than in the efficient allocation.

Risk aversion tilts the career choice decision towards the safer alternative, so that the composition of employment is distorted as well. The two other sources of inefficiency also affect the career choice decision, however, so that the self-employment rate can be either lower or higher than in the planner equilibrium. Thus, the self-employment rate may *increase* when we make *all* agents *identically* risk averse, a prediction of the model that goes against the conventional wisdom that postulates less risk averse individuals to self-select into self-employment.

As demonstrated in Figure 26, the composition of employment in the market equilibrium can even coincide with the planner equilibrium. Generically, however, the decentralized allocation features too little or too much self-employment, depending on entry cost k. When k is large, firms are reluctant to enter and the scope for labor market insurance is narrow, so that there is too much self-employment. This is in strong contrast to the conventional wisdom that risk aversion decreases self-employment. In fact, when firm entry costs are high and there can be large unemployment, the risk averse agents prefer to self-insure. For lower values of k the firm entry margin dominates and self-employment is below the efficient level. Needless to say, a market equilibrium that features the right composition of employment is still inefficient, since the price and wage posting decisions continue to be distorted by inefficiently long queues at the self-employed.

The next section studies optimal insurance policies under risk averse preferences, when the market is not efficient. We investigate policies that make use of type-dependent insurance and taxes to maximize output net of recruitment costs.

5.5 Efficient insurance policy

Having discussed the inefficiency of a market equilibrium under risk averse preferences, we now move towards an analysis of an efficient insurance policy. We consider type-of-employment-dependent policies that satisfy the following definition:

Definition 5.9. A balanced budget policy is a tuple of taxes and unemployment benefits $\mathscr{P} = \{\tau_{SE}, \tau_{LM}, b_{SE}, b_{LM}\}$ that satisfy the following condition:

$$b_E\left(1-\mu\left(x_{LM}\right)\right)LM + b_{SE}\left(1-a\lambda\left(x_{SE}\right)\right)SE = \tau_{LM}LM + \tau_{SE}SE. \qquad (5.16)$$

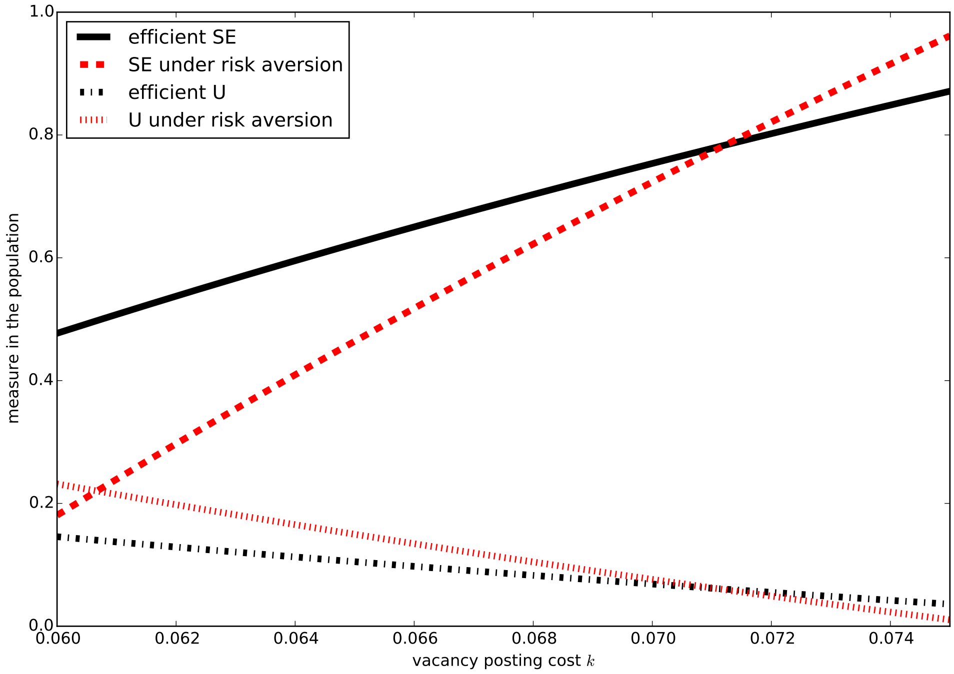

Figure 26: Self-employment and unemployment in the decentralized and planner equilibrium as a function of the vacancy posting cost k.

For analytical tractability, we illustrate the features of the policy under CARA preferences with a risk aversion parameter θ:

$$u(c) = \frac{1 - \mathrm{e}^{-\theta c}}{\theta}.$$

Observe that the introduction of the policy instruments affects the price posting by self-employed, wage posting by firms, and values of workers and self-employed, respectively. These equations now read:

$$\begin{aligned}
\frac{\phi(x_{LM})\left[A\lambda(x_F)\,p_F - w\right]}{1-\phi(x_{LM})} &= \frac{u(w-\tau_{LM}) - u(b_{LM}-\tau_{LM})}{u'(w-\tau_{LM})}, \\
\frac{\phi(x_{SE})\left(1-p_{SE}\right)}{1-\phi(x_{SE})} &= \frac{u\left(p_{SE}-\tau_{SE}\right) - u(b_{SE}-\tau_{SE})}{u'\left(p_{SE}-\tau_{SE}\right)}, \\
V^{LM}(\mathscr{P}) &= \mu(x_{LM})\,u(w-\tau_{LM}) + \left(1-\mu(x_{LM})\right)u(b_{LM}-\tau_{LM}), \\
V^{SE}(\mathscr{P}) &= a\lambda(x_{SE})\,u\left(p_{SE}-\tau_{SE}\right) + (1-a\lambda(x_{SE}))\,u(b_{SE}-\tau_{SE}).
\end{aligned}$$

A natural question unfolds: is it possible to decentralize the planner equilibrium using a balanced budget policy of type-of-employment-dependent taxes and unemployment benefits? The answer, as provided in the following proposition, is positive. More interestingly, there is a clear pattern on how the unemployment benefits and otherwise lump-sum taxes should be conditioned on the type of employment.

Proposition 5.10. *Let agents' preferences be described by a CARA utility function with a risk aversion parameter θ, and let $\{x_{LM}^*, w^*, x_{SE}^*, p_{SE}^*, x_F^*, p_F^*\}$ denote the respective equilibrium variables under risk neutral preferences. Then there exists a balanced budget policy $\mathscr{P}^*$ that for every θ decentralizes a planner equilibrium such that:*

$$\begin{aligned} b_{LM} &= w^* - \frac{1}{\theta}\log\left(1+\theta w^*\right), \\ b_{SE} &= p_{SE}^* - \frac{1}{\theta}\log\left(1+\theta p_{SE}^*\right), \\ V^{LM}\left(\mathscr{P}^*\right) &= V^{SE}\left(\mathscr{P}^*\right). \end{aligned}$$

Moreover, the taxes are characterized by the following inequality:

$$\tau_{LM} \le \tau_{SE} \iff \log\left(1+\left(1-\mu\left(x_{LM}^*\right)\right)\theta w^*\right) - \theta w^* \ge \log\left(1+\left(1-a\lambda\left(x_{SE}^*\right)\right)\theta p_{SE}^*\right) - \theta p_{SE}^*. \tag{5.17}$$

Proof. See Appendix 5.8.A. □

It follows from the proof that the policy instruments separately target the three margins of inefficiency. The unemployment insurance for the self-employed corrects their pricing decision. The unemployment benefits for workers corrects the wage posting decision. Finally, the mix of taxes ensures the correct composition of employment and balances the budget.

Observe that whenever the price that prevails in the decentralized equilibrium under risk neutrality is larger than the wage, the unemployment insurance for the self-employed should be more generous. From the career choice indifference condition that implements the efficient allocation we know that this happens if and only if the selling probability is lower than the job finding probability. Thus, whenever the income from self-employment is riskier, the benefits targeting the self-employed should be higher. This has nothing to do, however, with risk sharing considerations and is solely driven by efficiency.

If θ is small, the ranking of the taxes in (5.17) can be approximated by

$$\tau_{LM} \leq \tau_{SE} \iff \mu\left(x^*_{LM}\right)\theta w^* \leq a\lambda\left(x^*_{SE}\right)\theta p^*_{SE},$$

so that self-employment should be taxed more heavily than labor market participation whenever risk neutral individuals would prefer self-employment over entering the labor market in an environment without taxation, given the unemployment benefits. Whenever we find that $b_{SE} > b_{LM}$, which always happens if $\mu\left(x^*_{LM}\right) > \lambda\left(x^*_{SE}\right)$, we may expect to have $\tau_{SE} > \tau_{LM}$.

5.6 A dynamic model

In this section we describe the steady state of a dynamic version of the model, and perform some comparative statics exercises. The dynamic model captures the idea that employment at a firm is a long-term relationship. In particular, jobs last in expectation for multiple periods and are destructed exogenously with a constant probability δ. Time is discrete, individuals and buyers live forever, and they discount future periods at a factor β. Self-employment's and buyers' outcomes are assumed independent across periods. Therefore, the dynamic version of the model has consequences neither for modeling self-employment and buyers, nor for the firms' pricing decision. The value of being a buyer is thus given by:

$$\left(1-\beta\right)V^B = \eta\left(x_F\right)\left(1-p_F\right) = \eta\left(x_{SE}\right)\left(1-p_{SE}\right),$$

while the value of the self-employed is given by

$$\left(1-\beta\right)V^{SE} = a\lambda\left(x_{SE}\right)u\left(p_{SE}\right).$$

To minimize the differences with the static model, the timing of the model is such that (1) existing jobs are destroyed, (2) vacancies enter, (3) individuals make their career choice, (4) matches form, (5) buyers visit, and (6) individuals consume. To highlight the time spent in unemployment within the model, we allow for an unemployment benefit. As a result, the value of entering the labor market is equal to

$$V^{LM} = \tag{5.18}$$
$$\mu(x_{LM})\,u(w) + \left(1-\mu(x_{LM})\right)u(b) + \beta\left[\left(1-\mu(x_{LM})(1-\delta)\right)V^{LM} + \mu(x_{LM})(1-\delta)\,V^{E}\right],$$

with the value of employment V^E being:

$$V^E = u(w) + \beta\left[\delta V^{LM} + (1-\delta)\,V^E\right].$$

Solving for V^E and substituting the result in (5.18), it can be seen that the value of entering the labor market is still a weighted average of the expected time spent in employment and in unemployment. As a result, the career choice is not so much different in the dynamic version of the model, and individuals are indifferent between self-employment and entering the labor market if and only if

$$a\lambda(x_{SE})\,u\left(p_{SE}\right) = \frac{\mu(x_{LM})\,u(w) + \left(1-\mu(x_{LM})\right)\left(1-\beta(1-\delta)\right)u(b)}{\mu(x_{LM}) + \left(1-\mu(x_{LM})\right)\left(1-\beta(1-\delta)\right)}.$$

The assumption that jobs in expectation last for multiple periods also affects firm entry. The value of opening a vacancy is now

$$V^V = -k + q(x_{LM})\,V^J, \tag{5.19}$$

where V^J is the value of a filled vacancy (the value of a job to the firm):

$$V^J = A\lambda(x_F)\,p_F - w + \beta(1-\delta)\,V^J.$$

Solving for V^J, substituting the result in (5.19), and closing the model by free entry implies:

$$q(x_{LM})\,\frac{A\lambda(x_F)\,p_F - w}{1-\beta(1-\delta)} = k \tag{5.20}$$

As before, firms maximize expected profits by posting a wage, taking into account its effect on x_{LM}, constrained by the requirement to offer at least V^{LM} to workers. As shown in Appendix 5.8.B, this results in the following wage condition:

$$\frac{\phi(x_{LM})}{1-\phi(x_{LM})}\left[A\lambda(x_F)\,p_F - w\right] = \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\right)}\,\frac{u(w)-u(b)}{u'(w)}.$$

Finally, we consider the stocks and flows of the dynamic model. Let F_t now denote the measure of active firms at time t, and V_t the measure of vacancies opened in period t. The flows are such that

$$\begin{aligned}
F_t &= q\left(x_{LM,t}\right) V_t + (1-\delta) F_{t-1}, \\
U_t &= \left(1-\mu\left(x_{LM,t}\right)\right)(1-SE_t-(1-\delta)E_{t-1}), \\
E_t &= \mu\left(x_{LM,t}\right)(1-SE_t-(1-\delta)E_{t-1}) + (1-\delta) E_{t-1},
\end{aligned}$$

with the measure of active jobs $E_t = F_t$, the measure of workers in period t equal to $1-SE_t-(1-\delta)E_{t-1}$, the queue length $x_{LM,t} = (1-SE_t-(1-\delta)E_{t-1})/V_t$, and the measure of labor market matches $\mu\left(x_{LM,t}\right)(1-SE_t-(1-\delta)E_{t-1}) = q\left(x_{LM,t}\right)V_t$. In steady state the measure of self-employed is constant, and by definition $1-SE-(1-\delta)E = U+\delta E$, so that the steady state satisfies:

$$q\left(x_{LM}\right) V = \delta F = \delta E = \mu\left(x_{LM}\right)(U+\delta E).$$

In Appendix 5.8.B, we show that for risk neutral preferences, the decentralized steady state allocation of the dynamic model coincides with the steady state allocation that a social planner would choose, if the planner maximizes the present discounted number of goods sold net of recruiting costs. The social planner then solves the following problem:

$$\max_{\{x_{SE,t},V_t,E_t,SE_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \left[x_{F,t}\eta\left(x_{F,t}\right)AE_t + x_{SE,t}\eta\left(x_{SE,t}\right)aSE_t - V_t k\right],$$

subject to

$$E_t = q\left(x_{LM,t}\right)V_t + (1-\delta)E_{t-1},$$

and an initial condition E_0, where $x_{LM,t} = (1-SE_t-(1-\delta)E_{t-1})/V_t$ from the unit mass of households, the career choice, and job survival, and where $x_{F,t} = (1-x_{SE,t}aSE_t)/(AE_t)$ from the accounting identity in the goods market. In every period we again allow the social planner to choose the measure of vacancies to be opened and the measure of households to enter self-employment (and thus the measure to enter the labor market). The social planner still faces the same visibility probabilities and coordination frictions within every (sub)market as present in the decentralized equilibrium, but can still decide upon the measure of buyers to go shopping at the self-employed (and thus the measure of buyers that visits firms). Finally, note that choosing the latter, given SE_t and

V_t, still amounts to choosing $x_{SE,t}$, because the only state variable E_t is also determined by SE_t and V_t (and E_{t-1}). Since the decentralized and the planner's allocation coincide, the equilibrium of the dynamic model is efficient when individuals are risk neutral, just as the static model.

On the one hand, the steady state of a dynamic version of the model is not so much different from the static model, because the the value of entering the labor market is still a weighted average of the expected time spent in employment and in unemployment. On the other hand, the dynamic model captures the idea that jobs last for multiple periods and introduces two additional parameters: patience, and the expected duration of the wage contract. In Figure 27 we show the response of the equilibrium self-employment rate to changes in the discount factor β and the job destruction probability δ. Not surprisingly, a higher discount factor and lower job separation probability make the long-term nature of a wage contract more valuable to individuals, which decreases the self-employment rate.

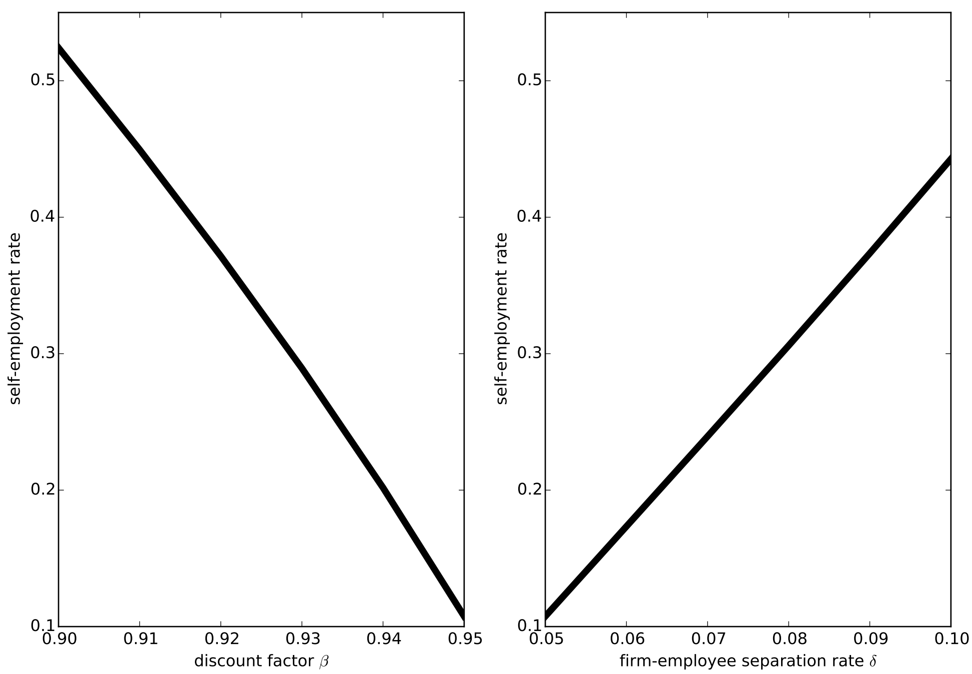

Figure 27: Equilibrium self-employment rate as a function of β and δ.

5.7 Conclusion

We propose a new theory of self-employment that emphasizes the trade-off between the frictions in the goods and in the labor market. Our theory, unlike a vast body of earlier research, does not rely on individual heterogeneity. It also offers microfoundations for the differences in the riskiness of payroll and self-employment incomes. In our model the self-employed forego coordination frictions in the labor market and the sharing of the match surplus with the firm. They are exposed, however, to coordination frictions in the goods market.

We also show that the decentralized equilibrium is inefficient if individuals are risk averse. In this case explicitly modeling trade in the goods market is crucial, as risk averse self-employed steal business from risk neutral firms by charging low prices. This under-pricing is a form of self-insurance, which reduces output sold net of recruiting costs. Interestingly, we show that in this environment unemployment insurance for self-employed individuals can improve efficiency, because it decreases the incentives to self-insure. As a result, firm entry increases and prospects in the labor market improve. Such insurance can therefore increase output sold net of recruiting costs, while simultaneously sharing risks.

5.8 Appendix

5.8.A Proofs for the static model

Proof of Lemma 5.1.

The self-employed post prices to maximize their expected utility as given in (5.3), offering buyers at least their value V^B as given in (5.2). Using the latter, the price that the self-employed post can be written as a function of V^B and x_{SE}:

$$p_{SE} = 1 - \frac{V^B}{\eta\left(x_{SE}\right)}.$$

Substituting out p_{SE}, the self-employed problem can then be written as a choice op the optimal queue length:

$$\max_{x_{SE}} a x_{SE} \eta\left(x_{SE}\right) u\left(1 - \frac{V^B}{\eta\left(x_{SE}\right)}\right).$$

The first-order condition yields:

$$a\eta(x_{SE})\,u(p_{SE}) + ax_{SE}\eta'(x_{SE})\,u(p_{SE}) + ax_{SE}\eta(x_{SE})\,u'(p_{SE})\frac{\partial p_{SE}}{\partial\eta(x_{SE})}\eta'(x_{SE}) = 0.$$

Dividing by $a\eta(x_{SE})$ gives:

$$\left(1-\phi(x_{SE})\right)u(p_{SE}) - \eta(x_{SE})\,\phi(x_{SE})\,u'(p_{SE})\frac{\partial p_{SE}}{\partial\eta(x_{SE})} = 0$$

Using that $\frac{\partial p_{SE}}{\partial\eta(x_{SE})} = \frac{V^B}{\eta^2(x_{SE})}$, and substituting out V^B, we get:

$$\left(1-\phi(x_{SE})\right)u(p_{SE}) = \phi(x_{SE})\,u'(p_{SE})\left(1-p_{SE}\right), \tag{5.21}$$

which is equal to (5.4). The price-posting problem of firms is the same, except that firms maximize expected revenue instead of utility. Replacing $u(p_{SE})$ by p_F and $u'(p_{SE})$ by 1 results in (5.5).

Proof of Lemma 5.3.

After paying an entry cost k, the firm optimally chooses the queue length to maximize profits

$$\Pi = q(x_{LM})\left(A\lambda(x_F)\,p_F - w\right),$$

subject to the market utility of workers V^{LM} as given in (5.6). Via this outside option, the wage to be paid also depends on x_{LM}, so that the first-order condition is

$$q'(x_{LM})\left(A\lambda(x_F)\,p_F - w\right) - q(x_{LM})\frac{dw}{dx_{LM}} = 0.$$

The derivative of the wage is obtained from totally differentiating V^{LM}, which is fixed:

$$0 = \mu'(x_{LM})\,u(w) + \mu(x_{LM})\,u'(w)\frac{dw}{dx_{LM}},$$

so that the optimality condition reads

$$q'(x_{LM})\left(A\lambda(x_F)\,p_F - w\right) = -q(x_{LM})\frac{\mu'(x_{LM})}{\mu(x_{LM})}\frac{u(w)}{u'(w)}.$$

Finally, note that $-\frac{x_{LM}\mu'(x_{LM})}{\mu(x_{LM})} = 1-\phi(x_{LM})$, and (5.7) results.

Proof of Proposition 5.6 and Corollary 5.7.

The mixing condition in (5.13) can be obtained by combining (5.1) with the first condition of Definition 5.5. It follows that $\lim_{ax_{SE}\mapsto 1^-} LM = 0^+$ and $\lim_{Ax_F\mu(x_{LM})\mapsto 1^+} LM = 1^-$ as long as the denominator of (5.13) is not approaching zero and is positive. Then, we can substitute out prices and the wage out of the remaining six equilibrium conditions, so that - together with the mixing condition - we are left with the following three equations in three unknowns $\{x_{SE}, x_F, x_{LM}\}$:

$$\frac{\phi(x_{LM})\left[\frac{k}{q(x_{LM})}\right]}{1-\phi(x_{LM})} = \frac{u\left(A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_{LM})}\right)}{u'\left(A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_{LM})}\right)} \tag{5.22}$$

$$\mu(x_{LM})\, u\left(A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_{LM})}\right) = a\lambda(x_{SE})\, u\left(1 - \frac{\eta(x_F)\left(1-\phi(x_F)\right)}{\eta(x_{SE})}\right) \tag{5.23}$$

$$\frac{\phi(x_{SE})\left(\frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right)}{1-\phi(x_{SE})} = \frac{u\left(1 - \frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right)}{u'\left(1 - \frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right)}. \tag{5.24}$$

Existence and uniqueness follow from invoking the Implicit Function Theorem on this system of equations. To demonstrate the logic of the proof, we prove the risk neutral case first. Afterwards we show that the same argument applies for CRRA preferences.

Risk neutral preferences. For risk neutral preferences, (5.24) boils down to $\mathrm{e}^{-x_{SE}} = \mathrm{e}^{-x_F}$, so that $x_F = x_{SE}$, which echoes the fact that risk neutral self-employed and firms set the same prices. Then, (5.23) implies $x_{LM} = \log(A/a)$, as in (5.12). The mixing condition can now be restated as

$$0 < \frac{1 - a x_{SE}}{\left(\frac{A-a}{\log\left(\frac{A}{a}\right)} - a\right) x_{SE}} < 1,$$

which places bounds on x_{SE}. In particular, for $1 > LM > 0$, it must hold that $1/a > x_{SE} > \frac{\log\left(\frac{A}{a}\right)}{A-a}$, which can only be true if

$$\frac{A}{a} > 1 + \log\left(\frac{A}{a}\right). \tag{5.25}$$

Observe that this condition - the first inequality constraint on the exogenous parameters to have a mixed strategy equilibrium - also guarantees that the share of prospective workers LM is non-negative and finite. Besides, note that condition (5.25) is always satisfied if $A > a$.

Equation 5.22 - the only equilibrium condition that is left - can after manipulation and evaluation at $x_{LM} = \log(A/a)$ be defined as

$$h(x_{SE}, a, A, k) \equiv \frac{A - a - a\log\left(\frac{A}{a}\right)}{A}\left(1 - x_{SE}\mathrm{e}^{-x_{SE}} - \mathrm{e}^{-x_{SE}}\right) - k = 0. \tag{5.26}$$

Differentiating (5.26), we find the following relationships:

$$\begin{aligned}
\frac{\partial h}{\partial x_{SE}} &= \frac{A - a - a\log\left(\frac{A}{a}\right)}{A} x_{SE}\mathrm{e}^{-x_{SE}} > 0, \\
\frac{\partial h}{\partial k} &= -1 < 0, \\
\frac{\partial h}{\partial A} &= \frac{a}{A^2}\log\left(\frac{A}{a}\right)\left(1 - x_{SE}\mathrm{e}^{-x_{SE}} - \mathrm{e}^{-x_{SE}}\right) > 0, \\
\frac{\partial h}{\partial a} &= -\frac{1}{A}\log\left(\frac{A}{a}\right)\left(1 - x_{SE}\mathrm{e}^{-x_{SE}} - \mathrm{e}^{-x_{SE}}\right) < 0.
\end{aligned}$$

Thus, from the Implicit Function Theorem we get, in particular, that $\frac{\partial x_{SE}}{\partial k} > 0$. Then, the bounds for the mixed strategy equilibrium to exist follow from evaluating (5.26) at $x_{SE} \mapsto \frac{1}{a}^{-}$ to get $\overline{k}(A, a)$ and $x_{SE} \mapsto \frac{\log\left(\frac{A}{a}\right)}{A-a}^{+}$ to get $\underline{k}(A, a)$.

The comparative statics of the self-employment rate follow from total differentiation of the accounting identity on individuals:

$$\frac{\partial SE^*}{\partial k} = -\frac{\partial LM^*}{\partial k}.$$

To find the latter, we make use of chain rule: $\frac{\partial LM^*}{\partial k} = \frac{\partial LM^*}{\partial x_{SE}}\frac{\partial x_{SE}}{\partial k}$. From the Implicit Function Theorem applied to (5.26), we have that $\frac{\partial x_{SE}}{\partial k} > 0$. The derivative of LM^* with respect to x_{SE} reads

$$\frac{-\left(\frac{A-a}{\log\left(\frac{A}{a}\right)} - a\right)}{\left[\left(\frac{A-a}{\log\left(\frac{A}{a}\right)} - a\right)x_{SE}\right]^2} < 0,$$

which implies that

$$\frac{\partial SE^*}{\partial k} = -\underbrace{\frac{\partial LM^*}{\partial x_{SE}}}_{<0}\overbrace{\frac{\partial x_{SE}}{\partial k}}^{>0} > 0.$$

The derivatives of bounds on k, $\underline{k}(A,a)$ and $\overline{k}(A,a)$, with respect to A and a also follow from the Implicit Function Theorem:

$$\frac{\partial k}{\partial A} = -\frac{\frac{\partial h}{\partial k}}{\frac{\partial h}{\partial A}} > 0,$$
$$\frac{\partial k}{\partial a} = -\frac{\frac{\partial h}{\partial k}}{\frac{\partial h}{\partial a}} < 0.$$

Preferences with risk aversion. We show that the argument above applies to the more general case case of $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma \in [0,1]$, so that now $\frac{u(c)}{u'(c)} = \frac{c}{1-\gamma}$. From (5.24), the price posting by the self-employed then leads to the following relationship between queue lengths in the mixed strategy equilibrium:

$$x_F = x_{SE} + \log\left(1 - \gamma\phi(x_{SE})\right).$$

Observe that this implies $x_F = x_{SE}$ if and only if $\gamma = 0$. Otherwise, we have that $x_F < x_{SE}$ whenever the mixed strategy equilibrium exists, and this ratio is decreasing in risk aversion parameter γ. Reformulating (5.24) and (5.24), we then have two equations with two unknowns x_{LM} and x_{SE}:

$$\frac{(1-\gamma)\phi(x_{LM})\left[\frac{k}{q(x_{LM})}\right]}{1-\phi(x_{LM})} = A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_{LM})} \tag{5.27}$$

$$\mu(x_{LM})\left(A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_{LM})}\right)^{1-\gamma} = a\lambda(x_{SE})\left(1 - \frac{\eta(x_F)\left(1-\phi(x_F)\right)}{\eta(x_{SE})}\right)^{1-\gamma} \tag{5.28}$$

We can further simplify (5.27) to

$$\frac{k}{q(x_{LM})} = A\lambda(x_F)\phi(x_F)\left[\frac{1-\phi(x_{LM})}{1-\gamma\phi(x_{LM})}\right],$$

so that (5.28) becomes

$$\mu(x_{LM})\left(A\lambda(x_F)\phi(x_F)\frac{(1-\gamma)\phi(x_{LM})}{1-\gamma\phi(x_{LM})}\right)^{1-\gamma} = a\lambda(x_{SE})\left(1 - \frac{\eta(x_F)\left(1-\phi(x_F)\right)}{\eta(x_{SE})}\right)^{1-\gamma}.$$

Now we set this equation to zero and define it as $z(x_{LM}, x_{SE}) = 0$. Collecting terms, taking logs and differentiating, results in:

$$\frac{\partial z}{\partial x_{LM}} = \frac{\mu'(x_{LM})}{\mu(x_{LM})} + (1-\gamma)\frac{\phi'(x_{LM})}{\phi(x_{LM})\left(1-\gamma\phi(x_{LM})\right)} < 0 \quad \text{whenever } x_{LM} \neq 0.$$

The part of $z(x_{LM}, x_{SE})$ that is relevant for computing $\frac{\partial z}{\partial x_{SE}}$ reads:

$$\tilde{z}(x_{LM}, x_{SE}) = \log\left(\frac{\lambda(x_F)}{\lambda(x_{SE})}\right) - \gamma \log(\lambda(x_F)) + \left(1-\gamma\right)\log\left(\frac{p_F}{p_{SE}}\right).$$

Under risk neutrality, $\gamma = 0$ and we have that the derivative of z with respect to x_{SE} is zero which also implies that x_{LM} does not respond to x_{SE}. After some tedious algebra one can show that

$$\frac{\partial z}{\partial x_{SE}} < 0,$$

which also implies that there exists a function $x_{LM}(x_{SE})$, decreasing in its argument. From here we can already establish the existence of equilibrium bounds on k, as the right hand side of identity:

$$k = q(x_{LM})\,A\lambda(x_F)\,\phi(x_F)\left[\frac{1-\phi(x_{LM})}{1-\gamma\phi(x_{LM})}\right],$$

is strictly increasing in x_{SE} so that the logic of the existence proof for the risk neutral preferences carries over.

Proof of Proposition 5.8.

Using the accounting identity in (5.1), the planner's objective can be written as:

$$V^{SP}(V, x_{SE}, SE) = x_{SE}aSE\left(\eta(x_{SE}) - \eta(x_F)\right) + \eta(x_F) - Vk. \tag{5.29}$$

For future reference, note that the partial derivatives of x_F with respect to the choice variables of the social planner are given by:

$$\begin{aligned}
\frac{\partial x_F}{\partial V} &= \frac{x_F}{V}\left(\phi(x_{LM}) - 1\right),\\
\frac{\partial x_F}{\partial x_{SE}} &= -\frac{aSE}{Aq(x_{LM})V},\\
\frac{\partial x_F}{\partial SE} &= \frac{x_F A q'(x_{LM}) - a x_{SE}}{Aq(x_{LM})V}.
\end{aligned}$$

Taking the first order condition with respect to the measure of firms:

$$\begin{aligned}
\frac{\partial V^{SP}}{\partial V} &= \eta'(x_F)\frac{\partial x_F}{\partial V}(1 - x_{SE}aSE) - k = 0,\\
&= \eta'(x_F)(x_F)^2 Aq(x_{LM})\left(\phi(x_{LM}) - 1\right) - k = 0,\\
&= \left(1 - \phi(x_{LM})\right)Aq(x_{LM})\,\lambda(x_F)\,\phi(x_F) - k = 0,
\end{aligned}$$

which is exactly the free entry condition in the decentralized equilibrium if workers are risk neutral as given in (5.9), since firms always post $p_F = \phi(x_F)$.

Let $\Delta\eta \equiv \eta(x_{SE}) - \eta(x_F)$. Taking the first order condition with respect to the queue length at the self-employed:

$$\frac{\partial V^{SP}}{\partial x_{SE}} = SEa\Delta\eta + x_{SE}aSE\eta'(x_{SE}) + (1 - x_{SE}aSE)\eta'(x_F)\frac{\partial x_F}{\partial x_{SE}} = 0,$$
$$= \eta(x_{SE})\big(1 - \phi(x_{SE})\big) - \eta(x_F)\big(1 - \phi(x_F)\big) = 0,$$

which (only) for risk neutral self-employed is exactly the buyer indifference condition in the goods market as in (5.2), since they post $p_{SE} = \phi(x_{SE})$ (and firms post $p_F = \phi(x_F)$).

Finally, the first order condition with respect to the measure of self-employed:

$$\frac{\partial V^{SP}}{\partial SE} = x_{SE}a\Delta\eta + (1 - x_{SE}aSE)\eta'(x_F)\frac{\partial x_F}{\partial SE} = 0,$$
$$= x_{SE}a\Delta\eta + x_F\eta'(x_F)\big(x_F Aq'(x_{LM}) - x_{SE}a\big) = 0,$$
$$= \Delta\eta + \eta(x_F)\phi(x_F)\left(1 - \frac{x_F Aq'(x_{LM})}{x_{SE}a}\right) = 0,$$
$$= \eta(x_{SE})\left(1 - \frac{x_F\eta(x_F)Aq'(x_{LM})\phi(x_F)}{x_{SE}a\eta(x_{SE})}\right) - \eta(x_F)\big(1 - \phi(x_F)\big) = 0.$$

This equation is equal to the buyer indifference condition in the goods market if

$$\phi(x_{SE}) = \frac{A\lambda(x_F)q'(x_{LM})\phi(x_F)}{a\lambda(x_{SE})},$$

which is exactly the condition that makes risk neutral households indifferent between entering the goods market as self-employed on the one hand and entering the labor market on the other. Indeed, it makes V^{LM} as given in (5.10) equal to (5.3) when self-employed post $p_{SE} = \phi(x_{SE})$. Hence, risk neutrality is sufficient for constrained efficiency.

Now, let us assume that in the decentralized allocation we have the measure of firms, the composition of employment, and the queues in the goods market that coincide with the planner solution. In other words, we assume that the decentralized allocation $\{V^*, SE^*, x^*_{SE}\}$ exactly matches its planner counterpart $\{V^{SP}, SE^{SP}, x^{SP}_{SE}\}$. Then, let us assume that individuals are risk averse. Given that $x^{SP}_{SE} = x^*_{SE}$, we can directly compare p^*_{SE} and the $p^{SP}_{SE} = \phi(x_{SE})$ that decentralizes the planner's solution. From the optimal price posting condition we get that under risk aversion $p^{SP}_{SE} \neq p^*_{SE}$, which implies that the buyers' indifference condition is violated: the buying probabilities equal their counterparts from the planner's solution, but the buyers have an incentive to choose visits at the self-employed more often. Thus, one of the planner's solution conditions is violated. This demonstrates the necessity of the risk neutrality assumption.

Consequently, the decentralized allocation is maximizing net output sold if and only if individuals are risk neutral.

Proof of Proposition 5.10.

Formally, we consider policies that have to satisfy the balanced-budget identity:

$$b_{LM}\left(1-\mu\left(x_{LM}\right)\right)LM + b_{SE}\left(1-a\lambda\left(x_{SE}\right)\right)SE = \tau_{LM}LM + SE\tau_{SE}. \tag{5.30}$$

The introduction of taxes and insurance changes the wage posting by firms and the price posting by the self-employed. These equations now read:

$$\begin{aligned}
\frac{\phi\left(x_{LM}\right)\left[A\lambda\left(x_F\right)p_F - w\right]}{1-\phi\left(x_{LM}\right)} &= \frac{u\left(w-\tau_{LM}\right)-u\left(b_{LM}-\tau_{LM}\right)}{u'\left(w-\tau_{LM}\right)},\\
\frac{\phi\left(x_{SE}\right)\left(1-p_{SE}\right)}{1-\phi\left(x_{SE}\right)} &= \frac{u\left(p_{SE}-\tau_{SE}\right)-u\left(b_{SE}-\tau_{SE}\right)}{u'\left(p_{SE}-\tau_{SE}\right)}.
\end{aligned}$$

Observe that the taxes themselves are not relevant in the pricing/wage posting decision, as they affect agents' wealth in all states (employed/unemployed/selling/not selling). Because CARA preferences feature no wealth effect, τ_i drop out. They do matter, however, in making the relative comparison of the career choices available. Starting with pricing by the self-employed:

$$\frac{\phi\left(x_{SE}\right)\left(1-p_{SE}\right)}{1-\phi\left(x_{SE}\right)} = \frac{1}{\theta}\frac{e^{-\theta\left(b_{SE}-\tau_{SE}\right)}-e^{-\theta\left(p_{SE}-\tau_{SE}\right)}}{e^{-\theta\left(p_{SE}-\tau_{SE}\right)}} = \frac{1}{\theta}\left(e^{-\theta\left(b_{SE}-p_{SE}\right)}-1\right).$$

Now, suppose we wish to find b_{SE} that implements $p_{SE} = \phi\left(x_{SE}^*\right)$ with the queue length as in the (efficient) allocation under risk neutral preferences. Then, it has to satisfy the following condition:

$$b_{SE} = \phi\left(x_{SE}^*\right) - \frac{1}{\theta}\log\left(1+\theta\phi\left(x_{SE}^*\right)\right).$$

The wage posting works in an analogous way, namely:

$$\frac{\phi\left(x_{LM}\right)\left[A\lambda\left(x_F\right)p_F - w\right]}{1-\phi\left(x_{LM}\right)} = \frac{1}{\theta}\left(e^{-\theta\left(b_{LM}-w\right)}-1\right).$$

Let's do the same for b_{LM}. The efficient wage satisfies $w = \phi\left(x_{LM}^*\right) A\lambda\left(x_F^*\right) p_F^*$ so that:

$$\phi\left(x_{LM}^*\right) A\lambda\left(x_F^*\right) p_F^* = \frac{1}{\theta}\left(\mathrm{e}^{-\theta\left(b_{LM}^*-w^*\right)}-1\right)$$

$$b_{LM} = \phi\left(x_{LM}^*\right) A\lambda\left(x_F^*\right) p_F^* - \frac{1}{\theta}\log\left(1+\theta\phi\left(x_{LM}^*\right) A\lambda\left(x_F^*\right) p_F^*\right)$$

Observe, that these conditions have an easy interpretation, namely:

$$b_{LM} = w^* - \frac{1}{\theta}\log\left(1+\theta w^*\right)$$

$$b_{SE} = p_{SE}^* - \frac{1}{\theta}\log\left(1+\theta p_{SE}^*\right)$$

The worker indifference condition reads:

$$\begin{aligned} V^{LM} &= V^{SE} \text{ with:} \\ V^{LM} &= \mu\left(x_{LM}\right) u\left(w-\tau_{LM}\right)+\left(1-\mu\left(x_{LM}\right)\right) u\left(b_{LM}-\tau_{LM}\right) \\ V^{SE} &= a\lambda\left(x_{SE}\right) u\left(p_{SE}-\tau_{SE}\right)+\left(1-a\lambda\left(x_{SE}\right)\right) u\left(b_{SE}-\tau_{SE}\right) \end{aligned}$$

so that:

$$\begin{aligned} \mu\left(x_{LM}\right) u\left(w-\tau_{LM}\right)+\left(1-\mu\left(x_{LM}\right)\right) u\left(b_{LM}-\tau_{LM}\right) &= \\ a\lambda\left(x_{SE}\right) u\left(p_{SE}-\tau_{SE}\right)+\left(1-a\lambda\left(x_{SE}\right)\right) u\left(b_{SE}-\tau_{SE}\right). \end{aligned}$$

With $u(c) = \frac{1-\mathrm{e}^{-\theta c}}{\theta}$, we arrive at the following:

$$\begin{aligned} \mu\left(x_{LM}\right) \mathrm{e}^{-\theta\left(w-\tau_{LM}\right)}+\left(1-\mu\left(x_{LM}\right)\right) \mathrm{e}^{-\theta\left(b_{LM}-\tau_{LM}\right)} &= \\ a\lambda\left(x_{SE}\right) \mathrm{e}^{-\theta\left(p_{SE}-\tau_{SE}\right)}+\left(1-a\lambda\left(x_{SE}\right)\right) \mathrm{e}^{-\theta\left(b_{SE}-\tau_{SE}\right)} \end{aligned}$$

so that:

$$\mathrm{e}^{\theta\tau_{LM}}\left(\mu\left(x_{LM}\right) \mathrm{e}^{-\theta w}+\left(1-\mu\left(x_{LM}\right)\right) \mathrm{e}^{-\theta b_{LM}}\right) = \mathrm{e}^{\theta\tau_{SE}}\left(a\lambda\left(x_{SE}\right) \mathrm{e}^{-\theta p_{SE}}+\left(1-a\lambda\left(x_{SE}\right)\right) \mathrm{e}^{-\theta b_{SE}}\right)$$

Using the analytical expressions for b_{LM} and b_{SE} we can rewrite the career choice equation as:

$$\theta\tau_{LM}-\theta w^*+\log\left(1+\left(1-\mu\left(x_{LM}^*\right)\right)\theta w^*\right) = \theta\tau_{se}-\theta p_{SE}^*+\log\left(1+\left(1-a\lambda\left(x_{SE}^*\right)\right)\theta p_{SE}^*\right).$$

The taxes can therefore be ranked such that

$$\tau_{LM} < \tau_{SE} \iff \log\left(1+\left(1-\mu\left(x_{LM}^*\right)\right)\theta w^*\right)-\theta w^* > \log\left(1+\left(1-a\lambda\left(x_{SE}^*\right)\right)\theta p_{SE}^*\right)-\theta p_{SE}^*.$$

5.8.B Derivations of the dynamic model

Optimal wage posting under general preferences

To derive the wage condition under general preferences, let firms maximize the left-hand side of the free-entry condition:

$$\begin{aligned}
\frac{\phi(x_{LM})}{1-\phi(x_{LM})}\left[A\lambda(x_F)\,p_F - w\right] &= \frac{u(w) - \left(1-\beta(1-\delta)\right)u(b) - \beta(1-\delta)\left(1-\beta\right)V^{LM}}{u'(w)},\\
&= \left(1 - \frac{\beta(1-\delta)\,\mu(x_{LM})}{\mu(x_{LM}) + \left(1-\mu(x_{LM})\right)\left(1-\beta(1-\delta)\right)}\right)\frac{u(w)-u(b)}{u'(w)},\\
&= \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\right)}\frac{u(w)-u(b)}{u'(w)}.
\end{aligned}$$

Risk neutral individuals

If workers are risk neutral and $b_{SE} = 0$, then the optimal wage posting simplifies to

$$\frac{\phi(x_{LM})}{1-\phi(x_{LM})}\left[A\lambda(x_F)\,p_F - w\right] = \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\right)}w,$$

$$\phi(x_{LM})\,A\lambda(x_F)\,p_F\left[1-\beta(1-\delta)\left(1-\mu(x_{LM})\right)\right] = \left[1-\beta(1-\delta)\left(1-\mu(x_{LM})\,\phi(x_{LM})\right)\right]w,$$

$$w = \frac{\phi(x_{LM})\,A\lambda(x_F)\,p_F\left[1-\beta(1-\delta)\left(1-\mu(x_{LM})\right)\right]}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\,\phi(x_{LM})\right)}. \tag{5.31}$$

Under these conditions, the value of entering the labor market is given by

$$(1-\beta)V^{LM} = \frac{\mu(x_{LM})\,w}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\right)},$$

so that individuals are indifferent between careers if and only if

$$\left[1-\beta(1-\delta)\left(1-\mu(x_{LM})\right)\right]a\lambda(x_{SE})\,p_{SE} = \mu(x_{LM})\,w. \tag{5.32}$$

Substituting the wage of (5.31) in this individuals' indifference condition, yields:

$$a\lambda(x_{SE})\,p_{SE} = \frac{\mu(x_{LM})\,\phi(x_{LM})\,A\lambda(x_F)\,p_F}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\,\phi(x_{LM})\right)}. \tag{5.33}$$

Finally, substituting the wage of (5.31) in the free entry condition of (5.20), yields:

$$q(x_{LM})\left[A\lambda(x_F)\,p_F - \frac{\phi(x_{LM})\,A\lambda(x_F)\,p_F\left[1-\beta(1-\delta)\left(1-\mu(x_{LM})\right)\right]}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\,\phi(x_{LM})\right)}\right] = k\left[1-\beta(1-\delta)\right],$$

$$q(x_{LM})\,A\lambda(x_F)\,p_F\frac{\left[1-\beta(1-\delta)\right]\left(1-\phi(x_{LM})\right)}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\,\phi(x_{LM})\right)} = k\left[1-\beta(1-\delta)\right],$$

$$\frac{q(x_{LM})\,A\lambda(x_F)\,p_F\left(1-\phi(x_{LM})\right)}{1-\beta(1-\delta)\left(1-\mu(x_{LM})\,\phi(x_{LM})\right)} = k. \tag{5.34}$$

Efficiency

Using $x_{F,t} = \frac{1-x_{SE,t}aSE_t}{AE_t}$, and defining $\Delta\eta_t \equiv \eta\left(x_{SE,t}\right) - \eta\left(x_{F,t}\right)$, simplifies the objective of the social planner to:

$$\max_{\{x_{SE,t},V_t,E_t,SE_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \left[x_{SE,t}aSE_t\Delta\eta_t + \eta\left(x_{F,t}\right) - V_t k\right]. \tag{5.35}$$

Choosing $x_{SE,t}$ is only an intra-temporal problem, but both SE_t and V_t determine future values of employment E. For that reason, we set up a Lagrangian:

$$\mathscr{L} = \sum_{t=1}^{\infty} \left\{\beta^t \left[x_{SE,t}aSE_t\Delta\eta_t + \eta\left(x_{F,t}\right) - V_t k\right] + \nu_t \left[q\left(x_{LM,t}\right)V_t + (1-\delta)E_{t-1} - E_t\right]\right\},$$

where ν_t is the Lagrange multiplier on the law of motion for E_t.

The first-order condition with respect to $x_{SE,t}$ is

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial x_{SE,t}} &= \beta^t \left[aSE_t\Delta\eta_t + x_{SE,t}aSE_t\left(\eta'\left(x_{SE,t}\right) + \eta'\left(x_{F,t}\right)\frac{aSE_t}{AE_t}\right) - \eta'\left(x_{F,t}\right)\frac{aSE_t}{AE_t}\right] = 0,\\
&= aSE_t\Delta\eta_t + aSE_t x_{SE,t}\eta'\left(x_{SE,t}\right) - aSE_t x_{F,t}\eta'\left(x_{F,t}\right) = 0,\\
&= \eta\left(x_{SE,t}\right)\left(1-\phi\left(x_{SE,t}\right)\right) - \eta\left(x_{F,t}\right)\left(1-\phi\left(x_{F,t}\right)\right) = 0,
\end{aligned} \tag{5.36}$$

which is the same intra-temporal condition for the goods market as in the static model. If the self-employed are risk neutral, then $p_{SE,t} = \phi\left(x_{SE,t}\right)$. Together with the price-setting of firms, this condition then coincides with the buyers' indifference condition of the decentralized allocation as given above. Consequently, the decentralized allocation in the goods market is efficient if individuals are risk neutral.

The first-order condition with respect to V_t is

$$\frac{\partial \mathscr{L}}{\partial V_t} = -\beta^t k + \nu_t \left[q\left(x_{LM,t}\right) - q'\left(x_{LM,t}\right)x_{LM,t}\right] = 0, \tag{5.37}$$

$$\frac{\beta^t k}{q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)} = \nu_t. \tag{5.38}$$

The first-order condition with respect to E_t is

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial E_t} &= \beta^t \eta'\left(x_{F,t}\right)\frac{x_{F,t}}{E_t}\left[x_{SE,t}aSE_t - 1\right] - \nu_t + \nu_{t+1}\left[(1-\delta) - q'\left(x_{LM,t}\right)(1-\delta)\right] = 0,\\
&= -\beta^t \eta'\left(x_{F,t}\right)Ax_{F,t}^2 - \nu_t + \nu_{t+1}(1-\delta)\left(1-\phi\left(x_{LM,t}\right)\mu\left(x_{LM,t}\right)\right) = 0,\\
&= \beta^t A\lambda\left(x_{F,t}\right)\phi\left(x_{F,t}\right) - \nu_t + \nu_{t+1}(1-\delta)\left(1-\phi\left(x_{LM,t}\right)\mu\left(x_{LM,t}\right)\right) = 0.
\end{aligned}$$

Substituting (5.38), yields

$$\beta^t A\lambda\left(x_{F,t}\right)\phi\left(x_{F,t}\right) = \frac{\beta^t k}{q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)} - \frac{\beta^{t+1}k\,(1-\delta)\left(1-\phi\left(x_{LM,t}\right)\mu\left(x_{LM,t}\right)\right)}{q\left(x_{LM,t+1}\right)\left(1-\phi\left(x_{LM,t+1}\right)\right)},$$

$$k = \frac{q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)A\lambda\left(x_{F,t}\right)\phi\left(x_{F,t}\right)}{1-\frac{q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)}{q\left(x_{LM,t+1}\right)\left(1-\phi\left(x_{LM,t+1}\right)\right)}\beta\,(1-\delta)\left(1-\phi\left(x_{LM,t}\right)\mu\left(x_{LM,t}\right)\right)},$$

which is a first-order difference equation for optimal entry. In steady state the ratio of today's and tomorrow's matching rates and elasticities is equal to one, and this equation simplifies to

$$k = \frac{q\left(x_{LM}\right)A\lambda\left(x_F\right)\phi\left(x_F\right)\left(1-\phi\left(x_{LM}\right)\right)}{1-\beta\,(1-\delta)\left(1-\mu\left(x_{LM}\right)\phi\left(x_{LM}\right)\right)}, \tag{5.39}$$

which equals the free entry condition in (5.34) given that $p_F = \phi\left(x_F\right)$. Consequently, free entry in the decentralized allocation is optimal if wages are set for the case that workers are risk neutral.

The first-order condition with respect to SE_t is

$$\frac{\partial \mathscr{L}}{\partial SE_t} = \beta^t\left[x_{SE,t}a\Delta\eta_t + \frac{x_{SE,t}^2 aSE_t\eta'\left(x_{F,t}\right)}{AE_t} - \frac{\eta'\left(x_{F,t}\right)x_{SE,t}a}{AE_t}\right] - \nu_t q'\left(x_{LM,t}\right) = 0,$$

$$= \beta^t\left[x_{SE,t}a\Delta\eta_t - x_{SE,t}ax_{F,t}\eta'\left(x_{F,t}\right)\right] - \nu_t\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right) = 0,$$

$$= \beta^t x_{SE,t}a\left[\eta\left(x_{SE,t}\right) - \eta\left(x_{F,t}\right)\left(1-\phi\left(x_{F,t}\right)\right)\right] - \nu_t\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right) = 0.$$

Substituting (5.38) and rearranging, results in

$$\beta^t x_{SE,t}a\left[\eta\left(x_{SE,t}\right) - \eta\left(x_{F,t}\right)\left(1-\phi\left(x_{F,t}\right)\right)\right] = \frac{\beta^t k\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right)}{q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)},$$

$$\eta\left(x_{SE,t}\right) - \eta\left(x_{F,t}\right)\left(1-\phi\left(x_{F,t}\right)\right) = \frac{\eta\left(x_{SE,t}\right)k\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right)}{a\lambda\left(x_{SE,t}\right)q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)},$$

$$\eta\left(x_{SE,t}\right)\left(1-\frac{k\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right)}{a\lambda\left(x_{SE,t}\right)q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)}\right) = \eta\left(x_{F,t}\right)\left(1-\phi\left(x_{F,t}\right)\right),$$

which coincides with the goods market condition in (5.36) if and only if

$$\phi\left(x_{SE,t}\right) = \frac{k\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right)}{a\lambda\left(x_{SE,t}\right)q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)}.$$

Using the steady state optimal firm entry decision in (5.39) to substitute for k,

$$a\lambda\left(x_{SE,t}\right)\phi\left(x_{SE,t}\right) = \frac{\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right)q\left(x_{LM,t}\right)A\lambda\left(x_{F,t}\right)\phi\left(x_{F,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)}{q\left(x_{LM,t}\right)\left(1-\phi\left(x_{LM,t}\right)\right)\left[1-\beta\,(1-\delta)\left(1-\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right)\right)\right]},$$

$$= \frac{\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right)A\lambda\left(x_{F,t}\right)\phi\left(x_{F,t}\right)}{1-\beta\,(1-\delta)\left(1-\mu\left(x_{LM,t}\right)\phi\left(x_{LM,t}\right)\right)}.$$

Given that firms set $p_F = \phi\left(x_{F,t}\right)$ and that risk neutral self-employed set $p_{SE} = \phi\left(x_{SE,t}\right)$, this is exactly the individuals' indifference condition in the career choice game if they are risk neutral, as can be seen in (5.33). Consequently, also the career choice of risk neutral individuals maximizes total output sold. We conclude that $\left\{x_{SE,t}, V_t, E_t, SE_t\right\}_{t=1}^{\infty}$ are chosen efficiently by the market.

CHAPTER 6

Summary

This thesis is titled "On the functioning of markets with frictions". It studies the coordination, dynamics and organization of economic activity in markets with search frictions, with applications in the labor, housing and goods markets. As discussed in the introduction of this thesis, search frictions make the occurrence of trade risky and dependent on the actions of others. Indeed, markets with search frictions are generally characterized by externalities. If there are multiple markets with search frictions, the expected benefits of trade in one market can depend on the presence and behavior of trading partners in another market.

The latter is the case in the second chapter of this thesis in the form of a demand externality. This chapter explains the observed counterclockwise cycles in the unemployment, vacancy rate-plane with a search and matching model that is not driven by shocks, but features endogenous fluctuations. The assumption that the revenue per worker-firm match increases with the level of aggregate economic activity makes firms to open vacancies now if they expect more economic activity in the future. However, if expectations are not sufficiently optimistic, the congestion of the labor market starts to dominate the demand externality before a steady state is reached, and oscillations result. Under plausible parameter values, the equilibrium dynamics include a stable limit cycle that resembles the empirically observed counterclockwise cycles around the Beveridge curve. Calibrated to the duration of the business cycle, these endogenous 'Beveridge cycles' are as persistent as the data, without losing any of the amplification of the standard model, and without the use of any (persistent) stochastic process.

The third chapter of this thesis is based on joint work with Espen Moen and Plamen Nenov. We argue that the search behavior of moving homeowners can be an important source of housing market volatility. By choosing whether to buy or sell first, households who move house affect the ratio of buyers to sellers and market liquidity, with important consequences for time-on-market, transaction volume, and prices. This chapter shows that when moving homeowners want to minimize the delay between transactions, they prefer to buy first whenever there are more buyers than sellers in the market, and to sell first when there are more sellers than buyers. Indeed, a market with more buyers relative to sellers exhibits a short time-on-market for sellers and a longer time-on-market for buyers, and vice versa. However, because the transaction sequence decision of moving homeowners not only depends on, but also affects housing market conditions, multiple steady state equilibria result: one with a high ratio of buyers to sellers and a short seller time-on-market, and one with a low buyer-seller ratio and long seller time-on-market. Equilibrium switches create large fluctuations in the housing market, which are qualitatively consistent with the empirical evidence that we document for Copenhagen.

In the fourth chapter, I build on the strategic complementarities identified in the previous chapter, but assume that the ratio of buyers to sellers is unknown to moving owner-occupiers. For that reason, homeowners that would like to move cannot always buy first when that ratio is high or sell first when it is low. Instead, households learn about the behavior of currently moving owner-occupiers in a contagion-like process. When contagion is fast enough, these dynamics give rise to three steady states, as in the previous chapter. However, not everybody buys first in the steady state with tightness bigger than one, and not everybody sells first in the steady state with tightness smaller than one, consistent with the heterogeneity in the data for Copenhagen. I extend this model by allowing real estate agencies to buy houses for speculation purposes. These boundedly rational agencies compare the return from buying and selling houses to a risk-free rate, and are active in the market when these returns have been positive for some time. When moving owner-occupiers take such speculation into account, the market turns at the troughs and bottoms, but, as the result of the adaptive expectations, does not return the economy to a steady state. Instead, the dynamics are described by a limit cycle, which closely matches the aggregate dynamics of the housing market of Copenhagen.

The fifth chapter, which is joint work with Piotr Denderski, provides a framework to explain the coexistence of firms and self-employment. Homogeneous households face a choice between self-employment and searching for a job at a firm. In the mixed strategy equilibrium, they trade-off the risk of unemployment in a frictional labor market with the risk of not selling in a frictional goods market. We show that risk-averse self-employed can use their pricing decisions to increase their chances of selling, effectively buying insurance against goods market risk. Since firms offer similar insurance against labor market risk to risk-averse job seekers by posting lower wages, the model exhibits two variants of market insurance. Depending on the scope for each, there can be either too much or too little self-employment. In addition, customers face an inefficiently high probability of stock-out at the self-employed when these post low prices, and the wages posted by firms result in excessive firm entry. We show that insurance for the self-employed, as Denmark and Sweden offer, can restore efficiency if it is optimally combined with unemployment insurance and differentiated taxes for the self-employed and labor market participants.

In short, this thesis shows that markets with search frictions can feature excess volatility, inefficiencies, and diversity in market organization, implying that there is a potential role for government intervention.

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Samenvatting (Dutch summary)

Dit proefschrift heeft de titel “Over het functioneren van markten met fricties”. Het bestudeert de coördinatie, dynamiek en organisatie van economische activiteit in markten met zoekfricties, met toepassingen in de arbeids-, goederen- en woningmarkt. Economische activiteit in deze markten varieert sterk over de tijd. Het aantal werklozen, onvervulde vacatures, onverkochte voorraden, of huizen dat te koop staat, kan makkelijk verdubbelen in de loop van een paar jaar. En zo kan het op het ene moment slechts een paar dagen duren om een baan te vinden of een huis te verkopen, terwijl op een ander moment elk van beide meer dan twee jaar in beslag kan nemen. Zoekfricties geven een verklaring voor het naast elkaar bestaan van werklozen en onvervulde vacatures. Omdat zoekfricties de handel vertragen, helpen ze ons daarnaast om te begrijpen dat het tijd kost om te kopen of te verkopen, en waarom dit varieert over de tijd. Ten slotte is het plaatsvinden van handel vanwege zoekfricties onzeker en afhankelijk van het gedrag van anderen. Het functioneren van markten hangt dan af van de strategieën van alle marktdeelnemers, en van de instituties die hun interacties vormgeven. Markten met zoekfricties worden daarom in het algemeen gekenmerkt door externe effecten.

Wanneer de optimale strategie van één economische agent toeneemt met de strategieën van anderen, dan is er sprake van strategische complementariteit (Cooper en John, 1988). In zulke situaties hebben economische agenten prikkels om hun gedrag te coördineren en zich op eenzelfde wijze te gedragen. Veranderingen in de coördinatie van het gedrag van economische agenten kunnen in markten met zoekfricties dus een verklaring geven voor schommelingen in economische activiteit.

Het eerste artikel dat laat zien dat zoekfricties tot een coördinatieprobleem kunnen leiden is Diamond (1982). In een model van een goederenmarkt laat hij zien dat er meerdere niveaus van economische activiteit zijn waarop economische agenten kunnen coördineren, als de kans dat agenten een handelspartner ontmoeten toeneemt in het aantal potentiële handelspartners. De intuïtie is als volgt. Wanneer economische agenten weinig handelspartners verwachten aan te treffen, doen zij alleen aan de paar meest winstgevende projecten, en als gevolg daarvan zijn er daadwerkelijk maar weinig handelspartners. Omgekeerd zijn er veel handelspartners wanneer iedereen verwacht dat er veel handelspartners zullen zijn, waardoor er veel projecten winstgevend zijn en gedaan worden. Diamond en Fudenberg (1989) laten zien dat de dynamiek van dit model leidt tot schommelingen in economische activiteit die gedreven worden door verwachtingen die vervolgens uitkomen. De verwachtingen zijn daarom 'zelf-bevestigend'. Wanneer er meerdere markten met zoekfricties zijn, dan kunnen de verwachte voordelen van handel in één market afhangen van de aanwezigheid en het gedrag van handelspartners in een andere markt. In aanverwante modellen met interacties tussen arbeids- en goederenmarkten met fricties, laten Howitt en McAfee (1992) en Kaplan en Menzio (2016) zien dat exogene veranderingen in verwachtingen over het niveau van economische activiteit ook zelf-bevestigend kunnen zijn en kunnen resulteren in meer onregelmatige schommelingen.

In hoofdstuk 2 van dit proefschrift onderzoek ik of een vergelijkbare interactie tussen arbeids- en goederenmarkt het cyclische gedrag van de werkloosheidsgraad en het aantal vacatures kan verklaren. Ik veronderstel dat de opbrengst van een arbeidsrelatie toeneemt met de totale werkgelegenheid. De voordelen van het aangaan van een relatie in de arbeidsmarkt hangen dus af van het gedrag van anderen op de arbeidsmarkt, omdat veranderingen in de totale werkgelegenheid invloed hebben op de opbrengsten die gegenereerd kunnen worden in de goederenmarkt. Verwachtingen van een hogere werkgelegenheid in de toekomst en daarom een hogere toekomstige opbrengst stimuleren investeringen in het aangaan van arbeidsrelaties en zijn dus zelf-bevestigend. Daarom bestaan er onder rationele verwachtingen toch meerdere evenwichtspaden. Enkele van deze paden convergeren nooit naar een stationaire toestand, maar naar een stabiele limietcyclus. Er bestaat een limietcyclus omdat er niet alleen positieve terugkoppeling is, maar ook congestie. Wanneer de werkgelegenheid hoog is en bedrijven veel vacatures openen, dan kost het meer tijd en geld om één individuele vacature te vervullen. Deze kosten zijn niet langer gerechtvaardigd wanneer bedrijven verwachten dat de hoogconjunctuur zal eindigen, en daarom laten zij hun vacatureaanbod dalen. Als gevolg daarvan daalt de werkgelegenheid, samen met de opbrengst van een arbeidsrelatie, totdat de arbeidsmarkt zo ruim wordt dat bedrijven om die reden wel weer vacatures willen openen. De limietcyclus die hiervan het resultaat is komt overeen met de empirisch geobserveerde cycli rondom de *Beveridge curve* - het negatieve verband tussen de werkloosheidsgraad en het aantal vacatures. Ik kalibreer deze ‘Beveridge cycli’ en laat zien dat zowel werkloosheid als vacatures net zo persistent zijn als in de data, zonder iets aan amplificatie te verliezen vergeleken met het standaard zoekmodel van Pissarides (1985). Persistentie is het resultaat van het kalibreren van de limietcyclus ten opzichte van de gemiddelde duur van de conjunctuurcyclus, en is niet gebaseerd op een (persistent) stochastisch proces. De schommelingen in werkloosheid en vacatures worden gegenereerd door het samenspel van de positieve en negatieve externe effecten, en dit endogene mechanisme vermindert de noodzaak voor exogene schokken in het verklaren van de schommelingen.

Zoals duidelijk is uit de hierboven geciteerde literatuur is het gebruikte positieve externe effect een bekende bron van strategische complementariteit. Hoofdstuk 3 van dit proefschrift, dat gebaseerd is op gezamenlijk werk met Espen Moen en Plamen Nenov, laat zien dat huiseigenaren met een wens om te verhuizen ertoe aangezet worden om hun zoekgedrag te coördineren vanwege een reden die niet eerder in de literatuur naar voren is gebracht. Verhuizende huiseigenaren moeten zowel een nieuwe woning kopen als hun oude woning verkopen. Ze kunnen besluiten om eerst te kopen of eerst te verkopen, maar zoekfricties veroorzaken vertragingen voor zowel kopers als verkopers en resulteren in een periode tussen de twee transacties. In deze periode hebben huiseigenaren dubbele maandlasten (als zij eerst kopen) of wonen zij in een tijdelijk onderkomen (als zij eerst verkopen). Als verhuizende huiseigenaren deze kostbare periode zo kort mogelijk willen laten duren, dan zullen zij geneigd zijn om de transactie die het snelst kan, als laatste af te sluiten. Wanneer er bijvoorbeeld relatief veel kopers in de markt zijn, dan is de verwachte verkooptijd kort, en is het slim om eerst te kopen. Maar omdat huiseigenaren die eerst willen kopen het overschot aan kopers op de markt versterken, wordt de prikkel om eerst te kopen alleen maar groter. Omgekeerd is het aantrekkelijk om eerst te verkopen wanneer er relatief veel verkopers in de markt zijn en woningen relatief lang te koop staan, dus precies wanneer anderen ook eerst verkopen. Als gevolg hiervan bestaan er twee stationaire evenwichten: één waarin iedereen eerst koopt en er relatief veel kopers zijn, en één waarin iedereen eerst verkoopt en er relatief veel woningen te koop staan.

Bovenstaande strategische complementariteit bestaat zonder een expliciete rol voor huizenprijzen. Wanneer huizenprijzen endogeen zijn, als het resultaat van *Nash* onderhandelingen of een *competitive search* evenwicht, dan komt er een extra bron van strategische complementariteit bij. Verhuizende huiseigenaren die eerst gekocht hebben zijn vanwege dubbele maandlasten relatief ongeduldig om hun oude woning te verkopen. Zij zijn daarom bereid om tegen een relatief lage prijs te verkopen. De mogelijkheid om het ongeduld van zo'n eigenaar met dubbele maandlasten uit te buiten maakt het voor andere verhuizende huiseigenaren aantrekkelijk om de markt als koper te betreden. Wanneer zij vervolgens eerst kopen, eindigen zij ook met dubbele maandlasten, ertoe bijdragend dat weer anderen hun voorbeeld zullen volgen. Omgekeerd zorgt de aanwezigheid van ongeduldige kopers ervoor dat ook anderen eerst verkopen en zo zelf ongeduldige kopers worden. Zoals Burdett en Coles (1998) al hebben laten zien maakt de samenstelling van de groep van potentiële handelspartners inderdaad uit voor de verwachte baten van het zoeken naar een partner. Wij tonen ook aan dat de prikkel om eerst te kopen groter is als verhuizende huiseigenaren verwachten dat huizenprijzen gaan stijgen. Wie eerst koopt, profiteert immers van de prijsstijging die plaatsvindt tussen beide transacties, onafhankelijk van de verhouding tussen kopers en verkopers in de markt. Omgekeerd heeft een huiseigenaar die eerst verkoopt baat bij een prijsdaling. Als de prijzen toenemen met de verhouding tussen kopers en verkopers, dan kunnen zelf-bevestigende verwachtingen de woningmarkt daarom destabiliseren en resulteren in plotselinge schommelingen in huizenprijzen en de verhouding tussen kopers en verkopers. Wanneer huiseigenaren immers verwachten dat huizenprijzen gaan dalen, zullen zij geneigd zijn eerst te verkopen, waarmee zij het aantal verkopers in de markt vergroten en huizenprijzen ook daadwerkelijk dalen. Zulke schommelingen zijn kwantitatief relevant, aangezien een sprong van een 'eerst kopen' naar een 'eerst verkopen' evenwicht kan leiden tot prijsdalingen van 30 procent en tot bijna een verdubbeling van de verkooptijd. Ten slotte laten we zien dat deze variabelen, inclusief de fractie van verhuizende huiseigenaren die eerst kopen, zich tot elkaar verhouden zoals in de data voor Kopenhagen.

In deze data voor Kopenhagen bereikt de fractie van verhuizende huiseigenaren die eerst kopen echter nooit nul of honderd procent. Bovendien neemt de transitie van het minimum naar het maximum van deze fractie twaalf jaar in beslag. Daarom bouw ik in hoofdstuk 4 van dit proefschrift voort op de analyse van het voorgaande hoofdstuk maar laat de perfecte coördinatie van verhuizende huiseigenaren los door te veronderstellen dat zij noch de verhouding tussen kopers en verkopers noch de fractie van andere verhuizende huiseigenaren die eerst kopen kennen. Hoewel het aantal verkopers simpelweg afgeleid kan worden uit het aantal woningen dat te koop staat, laat het aantal kopers in de markt zich niet direct zien. Als gevolg daarvan is de verhouding tussen kopers en verkopers niet precies bekend, terwijl dit de cruciale variabele is die het optimale gedrag van verhuizende huiseigenaren bepaalt. In plaats daarvan leren eigenaren het gedrag van anderen kennen via een besmettingsproces *à la* Lux (1995). Als besmetting krachtig genoeg is dan bestaan er meerdere stationaire evenwichten met een fractie van verhuizende huiseigenaren die eerst kopen ongelijk aan nul of honderd procent. Huizenprijzen zijn echter laag wanneer verhuizende huiseigenaren in meerderheid eerst verkopen. Zulke periodes bieden echter kansen aan speculanten om woningen goedkoop op te kopen, te verhuren, en later met winst te verkopen. Daarom breid ik het model uit met speculanten met begrensde rationaliteit die woningen kopen wanneer deze goedkoop zijn. Als verhuizende huiseigenaren rekening houden met de aanwezigheid van zulke speculanten, dan kan de fractie van verhuizende huiseigenaren die eerst kopen voor eeuwig schommelen op een wijze die nauw overeenkomt met de geobserveerde schommelingen in de woningmarkt van Kopenhagen, en waarschijnlijk in andere woningmarkten. De fractie van verhuizende huiseigenaren die eerst kopen beweegt langzaam en evenredig met het aantal transacties en woningprijzen, en omgekeerd evenredig met de verkooptijd en het aantal woningen dat te koop staat.

De interactie tussen de goederen- en arbeidsmarkten met zoekfricties in hoofdstuk 2 is van een herleide vorm. Hoofdstuk 5, gebaseerd op gezamenlijk werk met Piotr Denderski, modelleert beide markten expliciet. Daarmee benadrukt dit hoofdstuk zowel het risico op werkloosheid in de arbeidsmarkt als het risico om met onverkochte voorraden in de goederenmarkt te blijven zitten. Wij poneren een theorie van de graad van zelfstandigen zonder personeel (zzp'ers) en de rol van bedrijven in het functioneren van markten. Deze theorie is gebaseerd op de afruil van de twee risico's en de mogelijkheden om zich ertegen te verzekeren. Omdat elk van beide risico's toeneemt met het aantal individuen dat risico loopt, zijn de keuzes om zzp'er te worden of in loondienst te gaan strategische substituten. Er bestaat daarom een uniek evenwicht waarin zzp'ers en werknemers in loondienst naast elkaar kunnen bestaan. Dit evenwicht is onder risico-averse voorkeuren echter inefficiënt. Eén reden voor deze inefficiëntie, aangedragen door Acemoglu en Shimer (1997), is dat werkgevers de arbeidsmarkt een verzekering bieden door lagere lonen te betalen en de baanvindkans te verhogen. In ons model maakt zo'n verzekering bovendien de keuze voor een carrière in loondienst extra aantrekkelijk. Een andere bron van inefficiëntie, één die niet eerder naar voren is gebracht in de literatuur, wordt veroorzaakt door de mogelijkheid voor een zzp'er om zichzelf te verzekeren door lage prijzen te stellen en zo zijn verkoopkans te verhogen. Ondanks een gedeeltijke reactie in prijsstelling van bedrijven, trekt deze verzekering klanten weg van bedrijven met werknemers waardoor de allocatie van klanten in de goederenmarkt verstoord wordt. Daarmee worden de kansen op en baten van handel voor bedrijven kleiner, waarop zij het aantal vacatures verminderen. Als gevolg daarvan neemt het risico op werkloosheid toe en wordt de keuze voor een carrière als zzp'er relatief aantrekkelijker. De combinatie van deze vormen van verzekeringen resulteert in een zzp-graad die te hoog of te laag kan zijn, maar in het algemeen inefficiënt is. We laten vervolgens zien dat werkloosheidsuitkeringen en uitkeringen voor zzp'ers die hun voorraden niet kunnen verkopen, betaald uit belastingen met verschillende tarieven voor zzp'ers en werknemers, elk van de inefficiënties kan corrigeren terwijl de begroting in balans is. In de aanwezigheid van zoekfricties kunnen herverdeling en efficiëntie dus in dezelfde richting bewegen, en is er een rol voor instituties om het functioneren van markten te verbeteren.

Dit proefschrift laat nogmaals zien dat externe effecten de norm en niet de uitzondering zijn in markten met zoekfricties. Het is cruciaal om rekening te houden met de inefficiënties en positieve terugkoppeling die daarvan het gevolg zijn om de recente ontwikkelingen in arbeids-, goederen- en woningmarkten te begrijpen.

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