On the functioning of markets with frictions

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CHAPTER 4

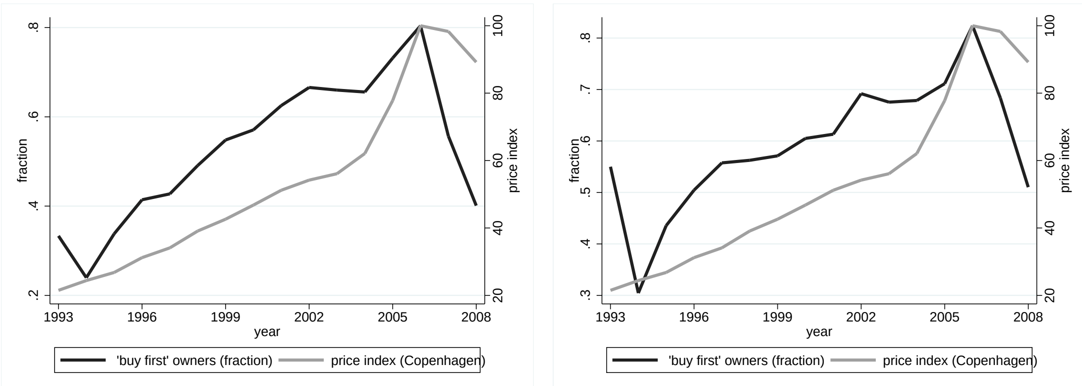
Learning to buy first or sell first in housing markets

4.1 Introduction

Moving between owner-occupied houses requires both buying and selling, and households can choose the order of these transactions. This chapter explains the dynamics of the fraction of households that buy first as the balance of two forces under informational frictions: households' strategic complementarities to buy first whenever other owner-occupiers buy first, and profit opportunities for speculators when houses are relatively inexpensive.

As in the previous chapter, the strategic complementarity is the result of the desire to shorten the costly period between the two transactions. Households that buy first suffer from double housing expenses, while households that sell first have to rent temporary housing. To reduce these costs, moving owner-occupiers would like to be on the short side of the market at their second transaction, and therefore want to join the long side of the market at their first transaction. As joining the long side at the first transaction makes the number of buyers and sellers even more *un*equal, households tend to buy first when others are buying first and sell first when others are selling first. However, although the number of sellers can usually be observed, for instance via commonly used websites with listings, the number of buyers cannot. As a result, market tightness is imperfectly observed and households are not perfectly able to coordinate on the same strategy. This chapter presents a first exploration of households that learn about other moving owner-occupiers in a contagion-like process, in order to choose between buying first and selling first.

Figure 17, reprinted from the previous chapter and based on Moen et al. (2015), shows the dynamics of the fraction of owners that buy first in Copenhagen between 1993 and 2008, together with a co-moving price index. The figure shows that housing prices are low when moving owner-occupiers in majority sell first. Such episodes offer profit opportunities to speculators that can buy houses cheap, rent them out, and bring them on the market later. I assume that moving owner-occupiers take the presence of such speculators into account. For that reason, they try to keep track of speculators' profit opportunities. Because more acquisition by speculators implies that houses can be sold more easily, households tend to buy first when they believe profit opportunities are favorable. Again, however, profit opportunities are observed imperfectly and households need to learn about them.



(a) (b)

Figure 17: Housing prices and fraction of moving owner-occupiers that buy first in Copenhagen (1993-2008). Panel a is based on agreement dates, and Panel b is based on closing dates.

If moving owner-occupiers learn from each other fast enough and speculators react sluggishly to profit opportunities, a limit cycle in profit opportunities and the fraction of households that buy first exists. The time series simulated from this limit cycle are broadly consistent with stylized facts of the housing market, featuring a slowly moving fraction of owners that buy first, moving in tandem with the number of transactions and housing prices, and in the opposite direction of the time-on-market and the stock of houses for sale. Growth rates of prices show momentum, but mean reversion at longer horizons.

The strategic complementarity in the transaction sequence decision of existing home-owners was first identified by Moen et al. (2015), and is presented in Chapter 3 of this thesis. The interplay between this strategic complementarity and the buyer-seller composition of the housing market results in multiple steady state equilibria: one in which all moving owner-occupiers buy first and another in which all moving owner-occupiers sell first. In addition, there can exist sunspot equilibria in which households switch between these steady states. This model therefore predicts that all moving owner-occupiers use the same strategy, and that equilibrium switches consist of a coordinated shift from all of them buying first to selling first, or the reverse. However, the data on moving owner-occupiers presented in Figure 17 show that the fraction of them that buys first is not zero or one hundred percent, and, in addition, moves slowly over time. Although the fraction of households that buy first varies considerably - from as low as 25 to as high as 80 percent - it never reaches zero or one hundred percent, and takes twelve years to switch from the bottom to the top. In this chapter this lack of coordination is explained by the introduction of informational frictions. Moving owner-occupiers still try to mimic the majority because it is in their interest to do so, but are not perfectly able to, because market tightness is unobserved. Only if households mimic each other infinitely fast, the limiting case without informational frictions can be obtained. In this case, all households coordinate on the same strategy. Otherwise, heterogeneity in strategies persists.

The introduction of informational frictions and sluggish speculative behavior results in dynamics that are very similar to Lux (1995). Lux offers an elegant formalization of contagion in speculative financial markets where optimistic and pessimistic traders may infect each other with their beliefs. Moreover, he expands on this idea and allows speculators to change their beliefs based on price dynamics or "the mood of the market". Unlike the strategic complementarity present in my model, however, in Lux (1995) agents have no incentives to mimic each other or to condition their behavior on prices or market conditions, other than to learn from equally ignorant agents.[52] By introducing a transaction sequence decision, I add a reason why people would like to behave

[52] See Banerjee (1992) and Bikhchandani et al. (1992) for conditions under which herd behavior can be rational.

as others. In addition, the presence of speculators does affect households' optimal behavior, so that it in their interest to monitor profitability of real estate agents. Finally, the introduction of search frictions allows for additional predictions on quantities and time-on-market, which are particularly relevant for housing markets. Dieci and Westerhoff (2012) and Bolt et al. (2014) also feature heterogeneous expectations and speculation in housing markets, but similarly lack search frictions and therefore cannot account for liquidity.

The literature on search frictions in housing markets starts with the seminal paper of Wheaton (1990). Krainer (2001) focuses on liquidity in particular. This chapter is especially related to papers that introduce heterogeneous expectations and/or speculation in housing markets with search frictions. Piazzesi and Schneider (2009) provide evidence of momentum traders that buy houses in expectation of rising prices, and include such traders in a search model of the housing market. However, they do not consider changes in liquidity and how it moves together with the fraction of households that buy first, as they assume that all moving owner-occupiers sell first and that the time-on-market is equal for buyers and sellers. Burnside et al. (2011) propose a search model with heterogeneous expectations to explain housing booms and busts. Again, as in Piazzesi and Schneider (2009) but in contrast to the focus of this chapter, they assume that moving owner-occupiers always sell first and that the time-on-market is equal for buyers and sellers. In addition, although households learn from each other, they do not learn about each other but about a future fundamental. Kashiwagi (2014) uses self-fulfilling beliefs in sharing the surplus of trade to explain a boom in housing prices while rents remain stable. However, moving owner-occupiers always sell first and do so without frictions, and as a result changes in beliefs do not affect housing market quantities.

Finally, Anenberg and Bayer (2015) is closest to Moen et al. (2015) and this chapter. It features the joint buyer-seller problem to explain volatility of transaction volume, but buying first is only a stochastic outcome. In addition, their model does not feature a strategic complementarity, so that there is no need for informational frictions to explain a lack of coordination either.

4.2 Model

Time is continuous and agents discount the future at rate $r > 0$. The economy consists of a unit mass of households and a unit mass of houses that do not depreciate. I abstract from construction as well as from heterogeneity in houses, apart from idiosyncratic characteristics that give rise to search frictions. The population consists of five types: households satisfied with the characteristics of the house they own (matched owners), households unsatisfied with the characteristics of the house they own that enter the market as buyers (mismatched owners that buy first), mismatched owners that sell first, households that own two houses (double owners), and households that own no house, but rent temporary housing (non-owners).

All types exit the economy at an exogenous and constant rate $g > 0$, and the houses of those that own upon exit are transferred to real estate firms. New households enter the economy as non-owners at the same rate g. Define the measure of non-owners in the population at time t as $N(t)$, the measure of mismatched owners that buy first as $B_1(t)$, the measure of mismatched owners that sell first as $S_1(t)$, the measure of matched owners as $O(t)$, the measure of double owners as $S_2(t)$, and the measure of houses of real estate firms as $A(t)$. Houses are owned by real-estate firms, matched owners, mismatched owners, and double owners, where the latter own two houses, so that $A(t) + O(t) + B_1(t) + S_1(t) + 2S_2(t) = 1$. Real estate firms rent out their houses receiving an exogenous flow payment of R, just as double owners rent out one of their properties at the same price. Non-owners therefore live in the $A(t) + S_2(t)$ unoccupied houses.

Individual households make transitions between types. At some exogenous and constant rate $\gamma > 0$, a matched owner receives a preference shock, turning her into a mismatched owner. The preference shock captures exogenous reasons for a desire to move, such as finding a new job elsewhere. A mismatched owner is no longer satisfied with her house, but still owns it. Preferences are such that she would like to move to a new house that she is satisfied with, but moving requires finding a new house, and finding a buyer for her old house. Both are time-consuming due to the presence of search frictions.

Search is random, so that all sellers find interested buyers at the same endogenous rate $\mu(t)$, and all buyers find houses to their liking at the same endogenous rate $q(t)$. Define the measure of buyers at time t as $B(t)$, the measure of sellers as $S(t)$, and market tightness as $\theta(t) \equiv B(t)/S(t)$. The number of matches at time t, $m(t)$, follows from a Cobb-Douglas matching function, taking the measure of buyers and sellers as inputs: $m(t) = \mu_0 B(t)^{\alpha} S(t)^{1-\alpha}$. The number of matches per seller, or the rate of finding a potential buyer for one's house, is thus given by $m(t)/S(t) = \mu_0 \theta(t)^{\alpha} = \mu(\theta(t))$. Since the number of sales must always equal the number of purchases, we have that the rate of buying a house is $q(\theta(t)) = \mu(\theta(t))/\theta(t)$.

Immediately upon mismatch, an owner chooses to enter the market either as a buyer or as a seller. For simplicity, mismatched owners commit to their choice until they transact, e.g. because they hire a realtor to carry out the transaction of their choice. From the strategic complementarity in the transactions sequence decision as presented in the previous chapter, mismatched owners would like to enter as sellers whenever $\theta(t)$ is low and as buyers whenever $\theta(t)$ is high. However, I assume that households do not know market tightness, and do not perfectly observe the behavior of mismatched owners either. For that reason, even though newly mismatched owners may prefer to be on the same side of the market as the majority of mismatched owners first, they are not necessarily able to do so. Instead, all households learn about the actions of newly mismatched owners, the households that actually make the choice to buy first or sell first at time t. Because the matching function implies that the outflow rate is inversely related to the stock, this inflow of newly mismatched owners buying first (selling first) also tends to move the measure of mismatched owners that buy first (sell first) in the same direction.

The outcome of the learning process is a belief about the actions of the majority of newly mismatched owners. Learning is modeled as a contagion-like Poisson process. A household believing that the majority of newly mismatched owners buy first may switch to believe that the majority sells first, which happens at a rate that is increasing in the fraction of newly mismatched owners selling first. Similarly, a household that believes newly mismatched owners predominantly sell first comes to believe that the majority buys first at a rate that increases in the fraction of newly mismatched owners buying first. Switching only depends on the average action, and does not allow for local interactions or some newly mismatched owners to be more influential than others. Learning can thus be understood as the result of the observation of the actions of a randomly drawn subset of newly mismatched owners.

Let $x_b(t) \in [0,1]$ denote the fraction of households that believe the majority of newly mismatched owners buy first at time t, so that $1 - x_b(t)$ denotes the fraction of households that believe newly mismatched owners join in majority the measure of those selling first. Even though all households learn about the actions of newly mismatched owners, the beliefs of matched owners are particularly important, because they act upon their beliefs as soon as they become mismatched. Because of the strategic complementarity, a matched owner who believes the majority of newly mismatched owners buy first (sell first), has the strategy to buy first (sell first) herself if she were to become mismatched. Consequently, since preference shocks hit matched owners randomly, the fraction of newly mismatched owners that buy first is also described by $x_b(t)$.

Define the index $z(t) \in [-1,1]$ according to $z(t) \equiv 2x_b(t) - 1$, summarizing the actions of newly mismatched owners and the beliefs of households about them. Consequently, $z = -1$ indicates that all households believe that newly mismatched owners predominantly first sell (and as a result all newly mismatched owners actually sell first), $z = 1$ indicates the shared belief that the majority first buys (and all newly mismatched owners buy first), and $z = 0$ indicates that households are split equally in their beliefs (and half of the newly mismatched owners buy first). The rate at which households that believe that the majority of newly mismatched owners buy first switch to believe that the majority sells first, is given by $\pi_{BS}(z(t))$ with $\pi'_{BS} < 0$. Similarly, $\pi_{SB}(z(t))$ with $\pi'_{BS} > 0$ denotes the rate at which households who believe newly mismatched owners predominantly sell first come to believe that the majority buys first. In addition, I assume that $\pi_{SB}(z(t)) = \pi_{BS}(z(t)) = 0$ for $z(t) \in \{-1,1\}$, since without heterogeneity no social learning can occur. The fraction of newly mismatched owners buying first then evolves as the beliefs of the unit measure of households about them:

$$\dot{x}_b = (1 - x_b(t))\,\pi_{SB}\,(z(t)) - x_b(t)\pi_{BS}\,(z(t))\,,$$

so that the dynamics of index $z(t)$ is given by

$$\dot{z} = (1 - z(t))\,\pi_{SB}\,(z(t)) - (1 + z(t))\,\pi_{BS}\,(z(t))\,. \tag{4.1}$$

Both buying first and selling first constitute a first step out of mismatch. The mismatched owner becomes a double owner if she first buys a new house, whereas she becomes a non-owner if she first sells her old house. A double owner cannot buy more than two houses, and is no longer subject to preference shocks. Preferences are such that she always enters the market to sell the house she doesn't like anymore. After selling she becomes a matched owner, making her subject to preference shocks again. Similarly, non-owners have a preference for owning a house and therefore always enter the market as buyers. If they buy a new house, this also results in matched ownership. Finally, preferences are such that matched owners do not enter the market.[53]

[53] I refer the reader to the previous chapter for a detailed exposition of the preferences of households and the conditions under which the actions described here result.

Real estate firms always put their houses up for sale, searching for a buyer that likes the house. However, in addition to receiving houses from households that exit, they are able to buy houses for speculative purposes without search frictions. Because these speculators have no interest in living in the houses they buy, they don't have to search for a match. Instead, speculative investment in the housing market attempts to obtain a return on buying that comes in the form of rent payments or house price appreciations. When house prices are low, the return on investment is high compared to the risk-free rate of return, and speculators would like to buy houses. Conversely, when prices are high, returns are low, and speculators tend to withdraw from the housing market. They no longer buy houses, and only try to find buyers for their existing stock (facing search frictions).

However, real estate firms look for "margins of safety" in the spirit of Minsky. Because housing units are illiquid as a consequence of search frictions, they need to acquire enough trust in the housing market, and only invest when returns have been favorable for some time. Specifically, the total number of houses bought by real estate firms at time t is given by $\psi S(t)\max[0, y(t)]$, where $\psi \geq 0$ captures the extent of market penetration of real estate firms, and where $y(t)$ is an index of profitability that evolves according to

$$\dot{y} = \tau\left(\frac{R + \dot{p}/\tau}{p(t)} - r\right), \tag{4.2}$$

where $p(t)$ denotes the housing price at time t, and where τ determines the speed of adjustment. As in Lux (1995), dividing by τ is necessary to capture the price change that occurs during the period that profitability changes.

The index of profitability $y(t)$ thus increases when the return on housing in the form of rental payments and house price appreciation exceeds the discount rate, and decreases when net returns are negative. Because real estate firms look for "margins of safety", real estate firms buy houses when the index of profitability is positive. However, as a result they may miss profit opportunities when the returns on housing exceed the discount rate but the index $y(t)$ is (still) negative, and make losses when the index is (still) positive even though net returns are negative.

I assume that prices fluctuate around the discounted sum of rental payments, via an exogenous function of the index describing the actions of newly mismatched owners $z(t)$:

$$p(t) = \frac{R}{r} + z(t)\Sigma, \tag{4.3}$$

where $\Sigma \geq 0$ is a parameter that captures the extent to which prices fluctuate with market conditions as described by $z(t)$. This function for the price level therefore describes in a reduced form the effect of the behavior of mismatched owners on prices, in line with the evidence as presented in Figure 17. Since $z(t)$ will also be shown to move roughly with market tightness, (4.3) also provides a specification for prices depending on market tightness as in Section 3.5.1 of the previous chapter.

That same chapter also justifies a price centered around the discounted sum of rental payments. Appendix 3.8.F shows that under certain conditions, $p = R/r$ can be microfounded as the price resulting from bargaining under private information about agent types, with full bargaining power for buyers. Moreover, Appendix 3.8.B shows that a price equal to the discounted stream of rental payments lies within the bargaining set of all agents, just as its right-sided neighborhood. For prices smaller than R/r, real estate firms would rather not sell.

In this chapter, however, real estate firms can buy houses simultaneously and without frictions, offsetting any frictional sales, so that real estate firms can be indifferent to sell for $p(t) < R/r$. Price movements into the left-sided neighborhood of R/r are therefore not at odds with the incentives of agents. At prices higher than R/r, real estate firms make current losses from investment in housing, but nevertheless they might buy if $y(t)$ is positive. Consequently, also $p(t) > R/r$ is compatible with the actions of real estate firms. With R, r, and Σ fixed, note that price changes are simply given by $\dot{p} = \dot{z}\Sigma$.

Summing up, the measure of buyers is given by

$$B(t) = B_1(t) + N(t),$$

and the measure of sellers by

$$S(t) = S_1(t) + S_2(t) + A(t),$$

so that market tightness is given by

$$\theta(t) = \frac{B_1(t) + N(t)}{S_1(t) + S_2(t) + A(t)}. \tag{4.4}$$

Note that the speculative buying of real estate firms does not enter the measure of buyers, since these firms only face search frictions when they try to sell. Combining the frictional matching rates with the speculative behavior of real estate firms and the choice of newly mismatched owners to buy first or sell first, results in the following differential equations (suppressing time dependence):

$$\dot{O} = q(\theta)N + \left(\mu(\theta) + \psi \max[0, y]\right) S_2 - (\gamma + g)O, \tag{4.5}$$

$$\dot{N} = g + \left(\mu(\theta) + \psi \max[0, y]\right) S_1 - \left(g + q(\theta)\right) N, \tag{4.6}$$

$$\dot{S_1} = \gamma (1 - x_b) O - \left(\mu(\theta) + \psi \max[0, y]\right) S_1 - g S_1, \tag{4.7}$$

$$\dot{B_1} = \gamma x_b O - q(\theta) B_1 - g B_1, \tag{4.8}$$

$$\dot{S_2} = q(\theta) B_1 - \left(g + \mu(\theta) + \psi \max[0, y]\right) S_2, \tag{4.9}$$

$$\dot{A} = g\,(O + S_1 + B_1 + 2S_2) + \psi \max[0, y]\,(S_1 + S_2) - \mu(\theta) A. \tag{4.10}$$

A learning equilibrium with speculation can now be defined as in:

Definition 4.1. A learning equilibrium with speculation is a path $\{y(t), z(t), O(t), N(t), U(t), S_2(t), A(t), p(t), x_b(t), \theta(t)\}$ such that:

1. For all $t \geq 0$, the index of households' beliefs $z(t)$ evolves according to (4.1);
2. For all $t \geq 0$, the index of profitability $y(t)$ evolves according to (4.2);
3. For all $t \geq 0$, housing prices $p(t)$ are given by (4.3), the fraction of newly mismatched owners that buy first by $x_b(t) = (z(t) + 1)/2$, and market tightness $\theta(t)$ by (4.4);
4. For all $t \geq 0$, the stocks of agents $O(t), N(t), U(t), S_2(t), A(t)$ evolve according to (4.5)-(4.10);
5. $z(0) \in [-1, 1], y(0), O(0) \geq 0, N(0) \geq 0, U(0) \geq 0, S_2(0) \geq 0, A(0) \geq 0$ are given such that $N(0) + O(0) + U(0) + S_2(0) = 1$ and $A(0) + O(0) + U(0) + 2S_2(0) = 1$.

For future reference, I also define an equilibrium without speculation.

Definition 4.2. A pure learning equilibrium is equivalent to a learning equilibrium with speculation, except that $\psi = 0$ and that $y(t)$ is not part of the equilibrium.

Finally, a steady state is defined as a stationary path of the equilibrium variables.

4.3 Equilibrium

I follow Lux (1995) in investigating two different specifications for the switching rates $\pi_{SB}(z(t))$ and $\pi_{BS}(z(t))$. The first focuses purely on the behavior of households to learn from each other. The second incorporates in addition that newly mismatched owners take the presence of speculators into account.

4.3.1 Steady state equilibria

First, I use

$$\pi_{SB}(z(t)) = \nu e^{az(t)} \text{ and } \pi_{BS}(z(t)) = \nu e^{az(t)}, \tag{4.11}$$

for $z(t) \in (-1,1)$, where $\upsilon > 0$ determines the speed of switching and $a > 0$ the strength of infection, so that the relative change in the probability to switch is linear in $z(t)$. Note from (4.1) that the evolution of z only depends on the variable $z(t)$ itself. As Lux (1995) shows, there exists a unique steady state in (4.1) at $z = 0$ if $a \leq 1$, but two additional steady states in z exist if households mimic each other fast enough, specifically if $a > 1$. Interestingly, for $1 < a < \infty$ the steady state at $z = 0$ is unstable, while two stable steady states exist at $z_+ \in (0,1)$ and $z_- \in (-1,0)$ with $z_+ = -z_-$, as in Figure 18.

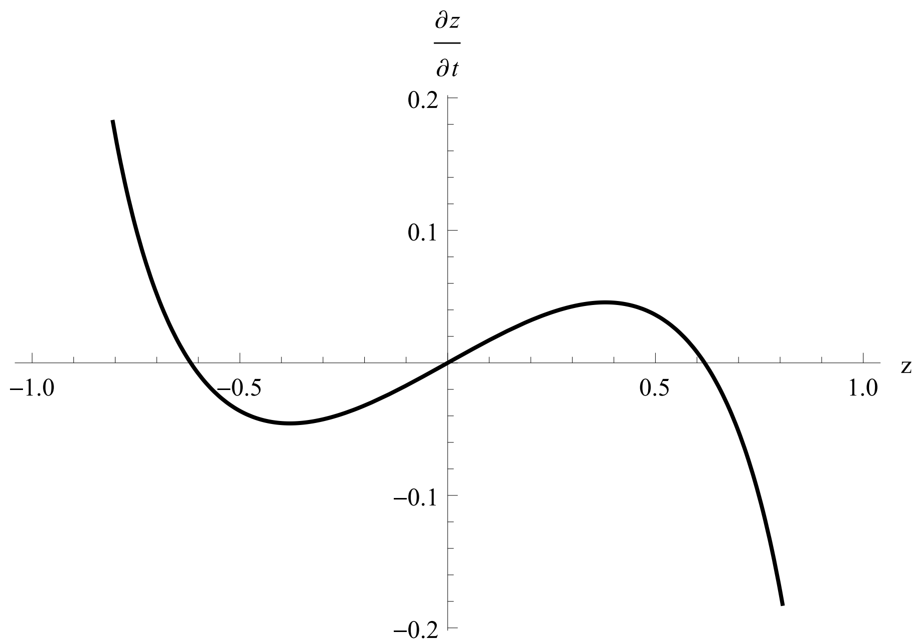


Figure 18: Dynamics of z as a function of z, giving rise to three steady states

Consequently, if contagion is relatively weak, deviations from balanced beliefs will eventually die out because the stock of households that can switch to majority beliefs depletes. However, if contagion is strong, deviations from balanced beliefs will strongly be followed upon, to such an extent that it can exactly balance the depletion of households with minority beliefs. Moreover, the stronger the contagion, the closer to unanimous beliefs these forces balance out. The perfect coordination of the previous chapter is only obtained for $a \to \infty$.

Does constancy of z also imply constant measures of each agent? In steady state, the stocks of agents should satisfy

$$q(\theta)N + \big(\mu(\theta) + \psi \max[0, y]\big) S_2 = (\gamma + g)O, \tag{4.12}$$

$$g + \big(\mu(\theta) + \psi \max[0, y]\big) S_1 = \big(q(\theta) + g\big) N, \tag{4.13}$$

$$\big(\mu(\theta) + \psi \max[0, y] + g\big) S_1 = \gamma (1 - x_b) O, \tag{4.14}$$

$$\big(q(\theta) + g\big) B1 = \gamma x_b O, \tag{4.15}$$

$$\big(\mu(\theta) + \psi \max[0, y] + g\big) S_2 = q(\theta) B_1, \tag{4.16}$$

$$g (O + S_1 + B_1 + 2S_2) + \psi \max[0, y] (S_1 + S_2) = \mu(\theta) A. \tag{4.17}$$

To see under which conditions a constant z implies constant measures of agents, first suppose that real estate firms never buy houses, i.e. $\psi = 0$. In that case, (3.34)-(3.40) in the previous chapter are equivalent to (4.12)-(4.17) for $N \equiv B_0 + B_n$. As a result, the proof of Proposition 3.3 therein applies. The following lemma combines the results from Lux (1995) and the previous chapter.

Lemma 4.3. *Suppose that* $\pi_{SB}(z(t)) = v\mathrm{e}^{az(t)}$ *and* $\pi_{BS}(z(t)) = v\mathrm{e}^{-az(t)}$ *for* $z(t) \in (-1, 1)$. *Then there exists a unique steady state pure learning equilibrium for* $0 < a \leq 1$, *characterized by* $z = 0$, $\theta = 1$, *and* $p = R/r$. *For* $1 < a < \infty$, *two additional steady state pure learning equilibria exist, characterized by* $z_+ \in (0, 1)$ *and* $z_- \in (-1, 0)$ *with* $z_+ = -z_-$.

Proof. Following Lux (1995), for $\pi_{SB}(z(t)) = v\mathrm{e}^{az(t)}$ and $\pi_{BS}(z(t)) = v\mathrm{e}^{-az(t)}$ for $z(t) \in (-1, 1)$, (4.1) can be written as

$$\dot{z} = 2v [\sinh(az(t)) - z(t) \cosh(az(t))] = 2v [\tanh(az(t)) - z(t)] \cosh(az(t)),$$

so that $z(t)$ is stationary for $\tanh(az(t)) = z(t)$. In particular, $\tanh(az(t)) = z(t)$ for $z = 0$, and in case $1 < a < \infty$, also for $z_+ \in (0, 1)$ and $z_- \in (-1, 0)$ with $z_+ = -z_-$. As a result, the fraction of newly mismatched owners buying first, $x_b(t)$, is also stationary. From (4.3), the same goes for housing prices $p(t)$.

To see that a stationary $z \in (-1, 1)$ implies (4.12)-(4.17) for $\psi = 0$, note that any stationary $x_b \in (0, 1)$ corresponds to a steady state $\theta \in (\underline{\theta}, \overline{\theta})$ as defined in the previous chapter. First, from their definitions in Lemma 3.2, if $x_b = 1$, then in steady state $\theta = \overline{\theta}$, and if $x_b = 0$, then in steady state $\theta = \underline{\theta}$. Second, as in the proof of Proposition 3.3, equations (4.12)-(4.17) and the condition that $N(t) = A(t) + S_2(t)$ result in a continuous relation between the fraction of households buying first, x_b, and θ, given by

$$x_b = \frac{\left(1+\frac{g}{\gamma}\right)\frac{(\theta-1)}{\mu(\theta)} + \frac{1}{\mu(\theta)+g}}{\frac{\frac{1}{\theta}+1}{q(\theta)+g}}$$

Consequently, $\theta = \overline{\theta}$ in steady state only if $x_b = 1$, and $\theta = \underline{\theta}$ in steady state only if $x_b = 0$. Because of continuity and because $\theta = \underline{\theta}$ if and only if $x_b = 0$ and $\theta = \overline{\theta}$ if and only if $x_b = 1$, $x_b \in (0,1)$ implies $\theta \in (\underline{\theta}, \overline{\theta})$. Finally, a stationary x_b and θ imply stationary measures of agents from (4.12)-(4.17), and three steady state pure learning equilibria exist for $1 < a < \infty$.

To see that a unique steady state exists for $0 < a \leq 1$, combine the result of Lux (1995) with the proof of Corollary 3.4. The latter shows that (4.12)-(4.17) imply $\theta = 1$ if $x_b = 1/2$, which corresponds to the unique stationary $z = 0$ for $a \leq 1$. Finally, from (4.3) $p = R/r$ if $z = 0$. □

In the steady states with $z_+ \in (0,1)$ or $z_- \in (-1,0)$, one strategy is therefore more frequent than the other, but the fraction of newly mismatched owners that buy first is unequal to one hundred or zero percent. For that reason, switching between beliefs based on limited information about the actions of newly mismatched owners can explain situations in which for example 25 or 80 percent of the newly mismatched owners chooses to buy first. The lemma also shows that a stationary $z \in (-1,1)$ implies stationary market tightness and prices, and constant measures of agents.

Now suppose that $\psi > 0$, so that speculation does affect the rate at which sellers sell their houses. As before, from (4.3) a stationary z implies stationary prices p. The following proposition shows that there exists a steady state learning equilibrium with speculation, but that it is unstable if $a > 1$. However, there exists another stationary value for the index of households beliefs, z, for which market tightness, prices, and measures of agents are constant, but the index of profitability, $y(t)$, is not.

Proposition 4.4. *Suppose that $\pi_{SB}(z(t)) = v\mathrm{e}^{az(t)}$ and $\pi_{BS}(z(t)) = v\mathrm{e}^{-az(t)}$ for $z(t) \in (-1,1)$. Then a necessary condition for a steady state learning equilibrium with speculation is $z = 0$. One such steady state exists with $\theta = 1$, $p = R/r$ and $y \leq 0$. For $1 < a < \infty$, this steady state is unstable, but there exist, in addition,*

1. *a stationary path for $\{z(t), O(t), N(t), U(t), S_2(t), A(t), p(t), x_b(t), \theta(t)\}$, provided that $y(0) \leq 0$. On this path, $z = z_+ \in (0,1)$, $p = R/r + \Sigma z_+$, and $y(t) \to -\infty$ as $t \to \infty$;*

2. *a stationary path for* $\{z(t), p(t), x_b(t)\}$, *with* $z = z_- \in (-1, 0)$, $p = R/r + \Sigma z_-$, *and* $y(t) \to \infty$ *as* $t \to \infty$.

Proof. From Lemma 4.3 we know that there exists a steady state pure learning equilibrium that features $z = 0$, $\theta = 1$, and $p = R/r$. Then there exists a steady state learning equilibrium with speculation if (1) y is stationary and (2) speculation does not affect (4.12)-(4.17). The first follows from (4.2) with $z = 0$, which implies $\dot{p} = 0$, $p = R/r$, and thus $\dot{y} = 0$. For $\psi > 0$, the second is then the case if $y \le 0$. For $a > 1$, however, $z = 0$ is unstable.

Similarly, we know for $1 < a < \infty$ that there exist two additional steady state pure learning equilibrium that feature $z_+ \in (0, 1)$ and $z_- \in (-1, 0)$, respectively. On the one hand, if $z \neq 0$, then $p \neq R/r$ and $\dot{y} \neq 0$, so that there exist no steady state learning equilibria with speculation for $z_+ \in (0, 1)$ or $z_- \in (-1, 0)$. On the other hand, if $z_+ \in (0, 1)$, then $p = R/r + \Sigma z_+$, and $\dot{y} < 0$ for all t. Consequently, for a sufficiently low starting value $y(0)$, speculation does not affect (4.12)-(4.17). Therefore there exists a stationary path for $\{z(t), O(t), N(t), U(t), S_2(t), A(t), p(t), x_b(t), \theta(t)\}$ if $y(0) \le 0$. Similarly, if $z_- \in (-1, 0)$, then $p = R/r + \Sigma z_-$, and $\dot{y} > 0$ for all t. Consequently, there always exists some t for which speculation matters for (4.12)-(4.17), and increasingly so. However, a stationary path for $\{z(t), p(t), x_b(t)\}$ exists. □

Proposition 4.4 shows that there can only exist steady state learning equilibria with speculation if there are as many households that believe that the majority of newly mismatched owners buy first as there are households that believe that the majority of newly mismatched owners sell first. Otherwise, housing prices differ from the discounted stream of rental payments, and the index of profitability diverges. When housing prices are above the discounted stream of rental payments, there is no role for profitable speculation, and a path for all variables except the index of profitability can be stationary. However, when housing prices are below the discounted stream of rental payments, speculators would like to buy houses. In this case, houses can be rented out with profits, and, if prices were to rise, houses can also be sold for profit. As a result, the presence of real estate firms in the housing market increases, and selling houses becomes easier and easier.

Under such circumstances, in order to reduce the costly time period in between transactions, newly mismatched owners would not like to sell first, but to buy first.

However, the learning dynamics as specified in (4.11) do not take the presence of speculators into account. As a result, there exist a stationary path for $\{z(t), p(t), x_b(t)\}$ only because newly mismatched owners do not act in their best interests. For that reason, I consider a second specification for the rates to switch.

4.3.2 Housing cycles

To incorporate that newly mismatched owners take the presence of speculators into account, I allow households to condition their learning on the returns from speculative investment in the housing market. By their behavior, real estate firms affect the time on the market for sellers. Acknowledging the presence of these forces, newly mismatched owners should also condition their transaction sequence decision on their estimate of the activity of real estate firms, and all households should accordingly adjust their beliefs about the actions of newly mismatched owners. Following Lux (1995), I assume the Poisson rates are given by

$$\pi_{SB}(z(t), y(t)) = \nu e^{y(t)+az(t)} \text{ and } \pi_{BS}(z(t), y(t)) = \nu e^{-y(t)-az(t)}. \tag{4.18}$$

Now the driving differential equations of the dynamic system are (4.2) and (4.1) with the switching rates specified above. The stocks of different household types and houses owned by real estate firms as described in (4.5)-(4.10) only follow from the behavior of $z(t)$ and $y(t)$ and do not affect them. For that reason, the analysis of Lux (1995) applies, and at least one stable limit cycle in $\{z(t), y(t)\}$ exists if owners mimic each other fast enough, specifically if $1 - r\Sigma/R < a < \infty$. In that case all trajectories for the fraction of newly mismatched owners that buy first converge to a periodic orbit. A limit cycle exists because speculators act upon an index of profitability, which is a stock that increases in profit opportunities. Because speculators buy houses when $y(t) > 0$, they are slow to respond to profit opportunities, and similarly continue buying houses for too long. Such sluggish behavior may be a characterization of bubbles and crashes.

If there exists a limit cycle in $\{z(t), y(t)\}$, then Figure 19 shows that the number of transactions, the time-on-market, the stock of houses for sale, and (by construction) housing prices can also converge to a periodic orbit (legend below). This numerical example suggests the existence of a limit cycle learning equilibrium with speculation. The parameters of the numerical example are presented in Table 11.

The model-generated fluctuations in the fraction of newly mismatched owners that buy first, the number of transactions, time-on-market, the stock of houses for sale, and

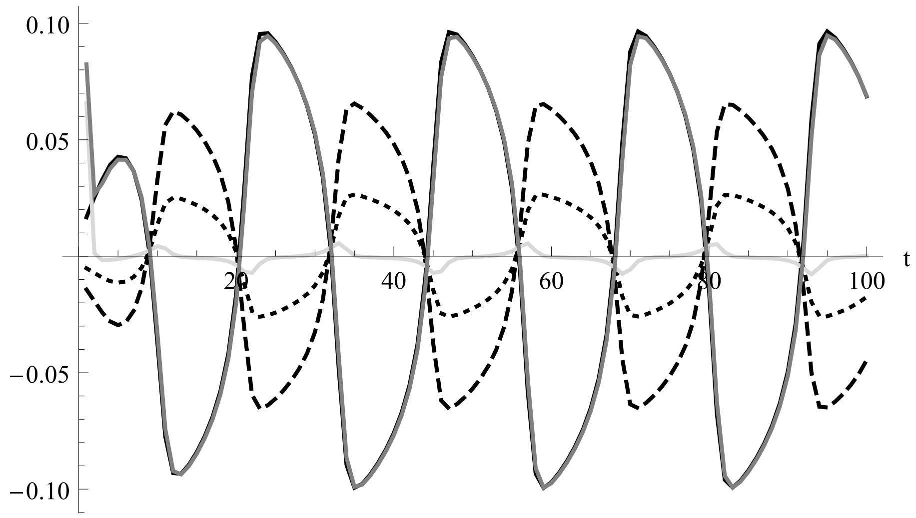


Figure 19: Simulated dynamics of housing market variables (in log deviations), and market index z (scaled by 0.1).

Parameter		Value	Target
Matching efficiency	μ_0	3.6	Average TOM
Matching elasticity	α	0.84	Genesove & Han (2012)
Entry/exit rate	g	0.05	Moen *et al.* (2015)
Mismatch rate	γ	0.01	Moen *et al.* (2015)
Discount rate	r	0.05	Standard
Rental rate	R	0.025	Price level
Market penetration	ψ	0.001	Size real estate firms
Amplitude prices	Σ	0.02	Amplitude and period
Speed profitability	τ	13	Period
Speed switching	v	1	Period
Strength contagion	a	1.1	Amplitude fraction

Table 11: Parameter values

housing prices are broadly consistent with the stylized facts of the labor market. Figure 20, reprinted from the previous chapter and based on Moen et al. (2015), shows the behavior of these variables for Copenhagen between 2004 and 2008 (including legend).

Now zooming in on one cycle of the simulated dynamics as presented in Figure 21, a numerical example of the limit cycle captures the dynamics of the same variables to a large extent. Both the structure of the time lags and the relative size of the fluctuations do roughly correspond across these figures, except that time-on-market in the simulation fluctuates a bit too much. Possibly, an endogenous moving decision as in Ngai and Sheedy (2015) could reduce the amplitude of time-on-market a bit, as mismatched owners would withdraw from the market when housing market conditions become too unfavorable.

On the one hand, the good performance with respect to prices is not too surprising, given that housing prices are an exogenous function of the fraction of newly mismatched owners that buy first. On the other hand, the growth rates of prices show momentum in the short run but mean reversion in the long run, which the literature finds hard to explain. In this chapter, growth rates of prices are positively autocorrelated at high frequencies and negatively autocorrelated at low frequencies, because prices move with the behavior of moving owner-occupiers.

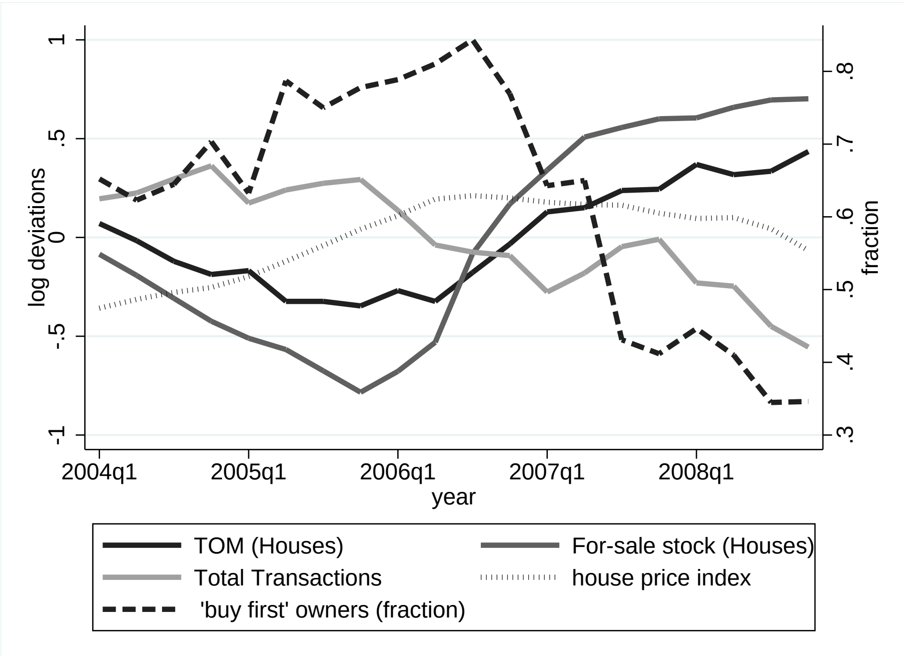


Figure 20: Dynamics of housing market variables (in log deviations) and the fraction of owners that buy first in Copenhagen between 2004 and 2008.

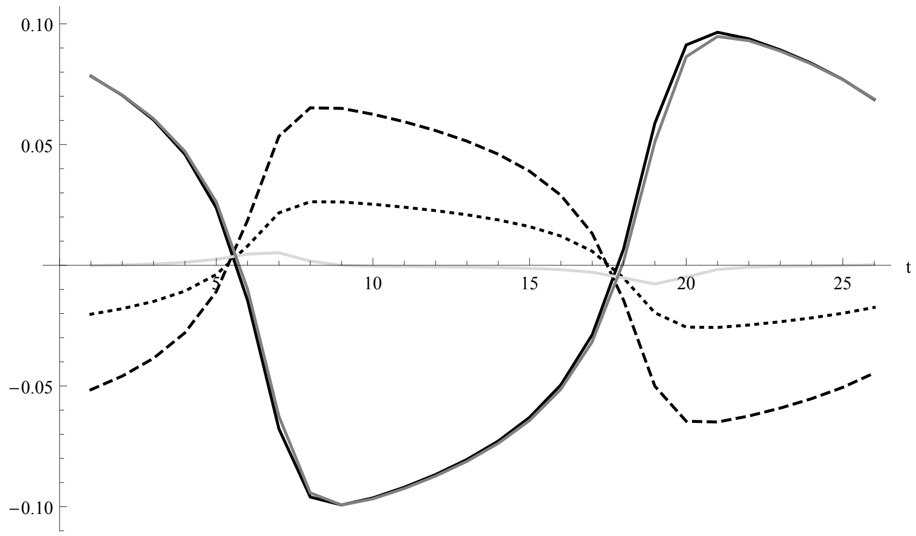


Figure 21: Simulated dynamics of housing market variables (in log deviations), and market index z (scaled by 0.1).

4.4 Conclusion

This chapter offers a reinterpretation of Lux (1995) that explains the lack of coordination in the transaction sequence decision of moving owner-occupiers. The strong strategic complementarities identified in Moen et al. (2015) to buy first when other households are also buying first still exist, but because market tightness is unobserved, households have to learn what others are doing. When households learn fast enough and on top of that condition their strategy on the sluggish behavior of speculators, a stable limit cycle exists. The time series simulated from this limit cycle qualitatively match actual housing market data.