Quantum Noise of a Power Recycled Interferometer

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I. Strain power spectral density

The single side strain power spectral density for a power recycled Michelson interferometer with optical losses is [1]

$$S_{hh}(f) = \frac{h^2_{SQL}}{2}\left\{\frac{\tilde{\epsilon}+\epsilon_d}{\mathcal{K}} + \frac{\epsilon_c}{2}\mathcal{K} + \left(1-\frac{\tilde{\epsilon}}{2}\right)\left[\mathcal{K}+\frac{1}{\mathcal{K}}\right]\right\} \tag{1}$$

where

$$h_{SQL} = \sqrt{\frac{2\hbar}{\pi^2 M f^2 L^2}} \tag{2}$$

and

$$\mathcal{K} = \frac{16 P_{bs}\mathcal{F}^2}{\pi^3 \lambda f^2 M c}\left[1+\left(\frac{f}{f_p}\right)^2\right]^{-1} \tag{3}$$

L is the arm length, M the mirror mass, λ the carrier wavelength, $\mathcal{F}$ the arm cavity finesse, P_{bs} the power on the beamsplitter and $f_p = c/4\mathcal{F}L$ the cavity pole frequency.

The fractional losses of the interferometer output (beam splitter losses, output faraday isolator losses, mismatch and misalignment of the output mode cleaner, detection photodiodes quantum efficiency, ..) are quantified by the coefficient ϵ_d. The fractional frequency dependent optical loss in the arm cavities $\tilde{\epsilon}$ is

$$\tilde{\epsilon} = \frac{2\epsilon_c}{1+(f/f_p)^2} \tag{4}$$

where $\epsilon_c = 2\epsilon_{rtl}/T$ and ϵ_{rtl} is the fractional arm round trip loss coefficient.

If squeezed light is injected on the interferometer antisymmetric port equation (1) becomes [1]

$$S_{hh} = \frac{h^2_{SQL}}{2}\left\{\frac{\tilde{\epsilon}+\epsilon_d}{\mathcal{K}} + \frac{\epsilon_c}{2}\mathcal{K} + \left(1-\frac{\tilde{\epsilon}}{2}\right)\left[\mathcal{K}+\frac{1}{\mathcal{K}}\right]\cdot\left[Cosh(2\xi) - \cos(2(\Phi+\theta))Sinh(2\xi)\right]\right\} \tag{5}$$

where θ is the squeezing ellipse rotation angle, $\Phi = ArcCot(\mathcal{K})$, $e^{2\xi}$ and $e^{-2\xi}$ the quadrature noise variance along the major and minor squeezing ellipse axis normalized to the unsqueezed case.

After its production, the squeezed beam is injected in to the interferometer dark port. In this operation the squezeed

*Full Virgo author list given at the end of the article beam suffers from optical losses (OPO escape efficiency, mismatch and misalignment with the output mode cleaner, ..) which are quantified by the fractional injection loss coefficient $\epsilon_i \equiv 1 - \eta_i$. Consequently the input vacuum filed at the interferometer beam splitter is not a pure state and equation (5) becomes

$$S_{hh} = \frac{h_{SQL}^2}{2}\left\{\frac{\tilde{\epsilon}+\epsilon_d}{\mathcal{K}} + \frac{\epsilon_c}{2}\mathcal{K} + \left(1-\frac{\tilde{\epsilon}}{2}\right)\left[\mathcal{K}+\frac{1}{\mathcal{K}}\right]\cdot\left[\epsilon_i + (1-\epsilon_i)[Cosh(2\xi) - \cos(2(\Phi+\theta))Sinh(2\xi)]\right]\right\} \tag{6}$$

given that the lossless degree of amplification $e^{2\xi}$ and attenuation $e^{-2\xi}$ of the quadratures can be expressed in terms of the non-linear strenght x of the OPA as [see for instance [2]]

$$e^{-2\xi} = 1 - \frac{4x}{(1+x)^2} \qquad e^{2\xi} = 1 + \frac{4x}{(1-x)^2} \tag{7}$$

equation (6) can be written as *

$$S_{hh}(f,\theta,x) = \alpha_S S_{QSN}\left\{1 - \frac{4x\beta}{\alpha_S}\left[\frac{\cos^2(\theta)}{(1+x)^2} - \frac{\sin^2(\theta)}{(1-x)^2}\right]\right\} + \alpha_R S_{QRPN}\left\{1 - \frac{4x\beta}{\alpha_R}\left[\frac{\sin^2(\theta)}{(1+x)^2} - \frac{\cos^2(\theta)}{(1-x)^2}\right]\right\} + \\ - 8x\sin(2\theta)\beta\sqrt{S_{QRPN}S_{QSN}}\,\frac{1+x^2}{(1-x^2)^2} \tag{8}$$

where

$$S_{QRPN} = \frac{1}{L^2}\left(\frac{4\mathcal{F}}{M}\right)^2 \frac{\hbar P_{bs}}{\pi^5 \lambda c f^4}\left(1+\left(\frac{f}{f_p}\right)^2\right)^{-1} \quad , \quad S_{QSN} = \frac{\pi\lambda\hbar c}{(4\mathcal{F})^2 P_{bs} L^2}\left(1+\left(\frac{f}{f_p}\right)^2\right) \tag{9}$$

and

$$\alpha_R = 1 + \frac{\epsilon_c}{2} - \frac{\tilde{\epsilon}}{2} \qquad \beta = \left(1-\frac{\tilde{\epsilon}}{2}\right)\eta_i \qquad \alpha_S = 1 + \epsilon_d + \frac{\tilde{\epsilon}}{2} \tag{10}$$

The terms in brace of equation (8) represent the squeezed light enhancement factor of the shot and the radiation pressure noise respectively while the last term is not relevant to our analysis as only data sets with $\theta = 0$ and $\theta = \pi/2$ have been considered.

The change in quantum noise with squeezed light injection

$$S_{hh}(f,\theta,x) - S_{hh}(f,0,0) = \tag{11}$$

$$= 4x\beta(f)\left\{S_{\mathrm{QSN}}(f)\left[\frac{\sin^2(\theta)}{(1-x)^2} - \frac{\cos^2(\theta)}{(1+x)^2}\right] + S_{\mathrm{QRPN}}(f)\left[\frac{\cos^2(\theta)}{(1-x)^2} - \frac{\sin^2(\theta)}{(1+x)^2}\right]\right\} \tag{12}$$

does not depend on α_S and α_R, i.e. it is independent on detection losses. Moreover, the ratio of quantum noise

*Here we have redefined the angle θ as $\theta \to \theta + \pi/2$. This is not substantial but allows us to be consistent with the notation of the article in which phase squeezing occurs for $\theta = 0$.

change in the low frequency limit (QRPN dominated) with phase squeezing and in the high frequency limit (QSN dominated) with amplitude squeezing

$$\frac{\lim_{f\to 0}[S_{hh}(f,\pi/2,x)-S_{hh}(f,0,0)]}{\lim_{f\to\infty}[S_{hh}(f,0,x)-S_{hh}(f,0,0)]}=(1-\epsilon_c)\frac{S_{\mathrm{QRPN}}(f)}{S_{\mathrm{QSN}}(f)} \tag{13}$$

does not depend on injection efficiency either.

References

[1] Kimble, H. J., Levin, Y., Matsko, A. B., Thorne, K. S., and Vyatchanin, S. P., *Phys. Rev. D*, Vol. 65, 2001, p. 022002.

[2] Dwyer, S. E., "Quantum noise reduction using squeezed state in LIGO," Ph.D. thesis, Massachusets Institute of Technology, 2013.