Constraints on Cosmic Strings Using Data from the Third Advanced LIGO–Virgo Observing Run

R. Abbott et al.*

(LIGO Scientific Collaboration, Virgo Collaboration, and KAGRA Collaboration)

(Received 30 January 2021; revised 31 March 2021; accepted 23 May 2021; published 16 June 2021)

We search for gravitational-wave signals produced by cosmic strings in the Advanced LIGO and Virgo full O3 dataset. Search results are presented for gravitational waves produced by cosmic string loop features such as cusps, kinks, and, for the first time, kink-kink collisions. A template-based search for short-duration transient signals does not yield a detection. We also use the stochastic gravitational-wave background energy density upper limits derived from the O3 data to constrain the cosmic string tension $G\mu$ as a function of the number of kinks, or the number of cusps, for two cosmic string loop distribution models. Additionally, we develop and test a third model that interpolates between these two models. Our results improve upon the previous LIGO–Virgo constraints on $G\mu$ by 1 to 2 orders of magnitude depending on the model that is tested. In particular, for the one-loop distribution model, we set the most competitive constraints to date: $G\mu \lesssim 4 \times 10^{-15}$. In the case of cosmic strings formed at the end of inflation in the context of grand unified theories, these results challenge simple inflationary models.

DOI: 10.1103/PhysRevLett.126.241102

Introduction.—The Advanced LIGO [1] and Advanced Virgo [2] detectors have opened a new channel to observe the Universe through the detection of gravitational waves. In their first three observing runs (O1, O2, and the first half of O3), the LIGO Scientific Collaboration and the Virgo Collaboration have reported the detection of 50 candidate gravitational-wave events from compact binary coalescences [3]. These detections have yielded important information on the population properties of these compact binary sources [4]. In the future, ground-based detectors may discover new sources of gravitational waves [5], some of which could probe the physics of the early Universe. Cosmic strings [6] belong to this category of sources. The third observing run (O3) started on April 1, 2019, and ended on March 27, 2020, and we use the data from the LIGO-Hanford (H1), LIGO-Livingston (L1), and Virgo (V1) interferometers to place constraints on cosmic strings. These constraints are reported in this Letter.

Cosmic strings are linelike topological defects—analogs of vortices in different condensed matter systems—that are formed from spontaneous symmetry breaking phase transitions (with the additional condition that the vacuum manifold has noncontractible closed curves [6–9]). In cosmology, such phase transitions may have occurred at grand unifications [10] corresponding to an energy scale of about $10^{16}$ GeV and more generally at lower energy scales. Thus, cosmic strings, through their different observational predictions, offer a tool to probe particle physics beyond the standard model at energy scales much above the ones reached by accelerators. In particular, the production of gravitational waves by cosmic strings [11,12] is one of the most promising observational signatures accessible by ground-based detectors.

The width of the string, of the order of the energy scale of the transition, is generally negligible compared to the cosmological scales over which it extends. This limit is well described by the Nambu-Goto action. Nambu-Goto strings [7] are parameterized by a dimensionless quantity: the string tension $G\mu$ related to the string formation energy scale, $\eta$, $G\mu \sim (\eta/M_{Pl})^2$, where $G$ is Newton’s constant, $M_{Pl}$ is the Planck mass, and $\mu$ denotes the string linear mass density [13]. We set the speed of light at $c = 1$. In an expanding background, such as a radiation or dominated era, a cosmic string network relaxes toward a scaling solution—a self-similar, attractor solution in which all typical loop lengths are proportional to cosmic time, or equivalently they scale with the Hubble radius. Superhorizon (also called infinite) strings reach this scaling solution [16–18], being stretched by the expansion of the Universe and by losing energy through the formation of subhorizon (loop) strings, which consequently lead to a cascade of smaller loops eventually decaying through the emission of gravitational waves [12,19,20]. In this Letter, we focus on the gravitational waves emitted by the network of loops. The length distribution of loops will therefore be crucial in determining the gravitational-wave signatures. We consider different loop distribution models that have been studied in the literature; they differ in the way they model the production and cascade of loops from the infinite string network.

—*

*Full author list given at the end of the article.
Cosmic string loops oscillate periodically in time, emitting gravitational waves with power \[ P_{gw} = \Gamma_d G\mu^2 \] and decay in a lifetime \( \ell/\gamma_d \), where \( \gamma_d \) is a numerical factor \((\Gamma_d \sim 50 \text{ [21]})\), \( \ell \) is the invariant loop length, and \( \gamma_d = \Gamma_d G\mu L \) is the gravitational-wave-length scale measured in units of time [22]. The high-frequency \((f\ell \gg 1, \text{ where } f \text{ denotes frequency})\) gravitational-wave spectrum of an oscillating loop is dominated by bursts emitted by string features called cusps and kinks [25–27]. Cusps [28] are points on the string that briefly travel at the speed of light; they are generic features for smooth loops. Kinks are discontinuities in the tangent vector of the string that propagate at the speed of light. They appear in pairs as the result of collisions between two cosmic strings and are chopped off when a loop forms; hence, a loop can contain any integer number of kinks. Numerical simulations of Nambu-Goto strings have shown that kinks accumulate over the cosmological evolution [16–18], while the number of cusps per loop is yet undetermined.

Cusps are short-lived and produce beamed gravitational waves in the forward direction of the cusp, while left-moving (right-moving) kinks propagate around the string, creating gravitational waves with a fanlike emission (like a lighthouse) in the directions generated by right-moving (left-moving) waves. Additionally, the collision of two kinks is expected to radiate gravitational waves isotropically. We report here searches for gravitational waves produced by cusps, kinks, and kink-kink collisions using O3 LIGO–Virgo data. In addition to distinct individual bursts, the incoherent superposition of weaker gravitational-wave bursts from cosmic strings produced over the history of the Universe would create a stochastic gravitational-wave background [27,30].

Cosmic strings emit gravitational waves with a wide range of frequencies that can be searched by other means, including the cosmic microwave background [31], Big Bang nucleosynthesis [32], and pulsar timing arrays [33–35]; see also, e.g., [36–38].

The gravitational-wave emission from cosmic string loops is introduced in the next section. We consider two simulation-based models [39,40] (labeled \( \text{A and B} \)) for the loop distribution. We further develop a third model (labeled \( \text{C} \)) that interpolates between the other two models. We also derive the burst rates and the dimensionless energy density in that section. Individual gravitational-wave bursts are searched in O3 data with a dedicated analysis presented in the “Burst search” section. The incoherent superposition of bursts from cusps, kinks, and kink-kink collisions produces a stationary and nearly Gaussian stochastic background of gravitational waves. We search O3 data for this background, and the results, detailed in [41], are summarized in the “Stochastic search” section. Both the burst and stochastic background searches yield no detections. Combining their sensitivities, we constrain two cosmic string parameters in the “Constraints” section: the string tension \( G\mu \) and the number of kinks per loop. We provide a table listing the meanings of symbols used in this study in the Supplemental Material [42].

**Gravitational waves from cosmic string loops.**—Gravitational waves are produced by cusps, kinks, and kink-kink collisions on cosmic string loops. The strain waveforms are linearly polarized and have been calculated in [25–27]. For a loop of length \( \ell \) at redshift \( z \), they are power-law functions in the frequency domain for the star in [44]

\[
h_i(\ell, z, f) = A_i(\ell, z)f^{-q_i},
\]

where \( i = \{c, k, kk\} \) identifies the cusp, kink, and kink-kink collision cases. The power-law indices are \( q_c = 4/3 \), \( q_k = 5/3 \), and \( q_{kk} = 2 \), and the amplitude \( A_i \) is [26]

\[
A_i(\ell, z) = g_{1,i}\frac{G\mu\ell^{2-q_i}}{(1+z)^{q_i-1}r(z)},
\]

where \( r(z) \) is the comoving distance to the loop. We adopt the cosmological model used in [44]; it is encoded in three functions: \( \varphi_c(z) \), \( \varphi_k(z) \), and \( \varphi_{kk}(z) \) (see Appendix A of [44]). The proper distance, the proper volume element, and the proper time are \( r(z) = \varphi_i(z)/H_0 \), \( dV(z) = \varphi_k(z)/H_0^2 dz \), and \( t(z) = \varphi_i(z)/H_0 \), respectively, where \( H_0 = 67.9 \text{ km s}^{-1} \text{ Mpc}^{-1} \) [45]. The prefactor \( g_{1,i} \) is [46]

\[
g_{1,c} = 8/\Gamma^2(1/3) \times (2/3)^{2/3} \approx 0.85, \quad g_{1,k} = 2\sqrt{2/\pi}/\Gamma(1/3) \times (2/3)^{2/3} \approx 0.29, \quad \text{and} \quad g_{1,kk} = 1/\pi^2 \approx 0.10, \quad \text{where} \quad \Gamma \text{ is the Gamma function [47].}
\]

Cusps and kinks emit gravitational waves in highly concentrated beams. Cusps are transient and produce a beam along a single direction, while kinks propagate around the loop, beaming over a fanlike range of directions. The beam opening angle is

\[
\theta_m = [g_2f(1+z)\ell]^{-1/3},
\]

where \( g_2 = \sqrt{3}/4 \) [46]. To guarantee self-consistency (validity of the waveform), we require that \( \theta_m < 1 \text{ rad} \), which is equivalent to setting a lower limit on the frequency for a fixed loop length. For kink-kink collisions, the gravitational-wave emission is isotropic [48].

The burst rate of type \( i \) per unit loop size and per unit volume can be decomposed into four factors:

\[
\frac{dN_i}{d\ell dV} = 2\frac{\ell}{\ell} N_i \times n(\ell, t) \times \Delta_i \times (1+z)^{-1}.
\]

The first factor accounts for an average of \( N_i \) gravitational-wave burst events of type \( i \) produced per loop oscillation time periodicity \( \ell/2 \). The second factor stands for the number of loops per unit loop size and per unit volume at cosmic time \( t \):
\begin{equation}
n(\ell', t) = \frac{d^2 N}{d\ell' dV}(\ell', t).
\end{equation}

The third factor, \(\Delta_i\), reflects that only a fraction of burst events can be effectively detected due to the beamed emission of gravitational waves with respect to the 4\(\pi\) solid angle. The gravitational-wave emission within a cone for cusps, a fanlike range of directions for kinks, and all directions for kink-kink collisions can be conveniently absorbed into a single beaming fraction expression: 

\[ \Delta_i = (\theta_m/2)^{3(2-q_i)}. \]

Finally, the last factor shows that the burst emission rate is redshifted by \((1 + z)^{-1}\).

The burst rate at redshift \(z\) is then obtained by integrating over all loop sizes:

\begin{equation}
\frac{dR_i}{dz} = \frac{q_i(z)}{H_0^2(1 + z)} \int_{\ell_{\min}}^{\ell_{\max}} d\ell' \frac{2N_i}{\ell'} n(\ell', t) \Delta_i.
\end{equation}

Introducing the dimensionless loop size parameter \(\gamma \equiv \ell'/t\), Eq. (6) reads

\begin{equation}
\frac{dR_i}{dz}(z, f) = \frac{\phi_i(z)}{H_0^2(1 + z)} \int_{\gamma_{\min}(z, f)}^{\gamma_{\max}(z)} d\gamma \frac{2N_i}{\gamma} n(\gamma, z) \Delta_i(\gamma, z, f).
\end{equation}

The upper bound of the integral \(\gamma_{\max}(z)\) is derived by requiring the loop size to be smaller than the horizon size, i.e., \(\gamma_{\max} = 2\) and \(3\) for radiation and matter dominated universes, respectively [44]. The lower bound \(\gamma_{\min}\) corresponds to the fundamental frequency of a loop, i.e., \(2/\ell'\), leading to \(\gamma_{\min}(z, f) = 2/[f(1 + z)\phi_i(z)/H_0]\).

We consider two analytical models, labeled A [39] and B [40], to describe the distribution of cosmic string loops \(n(\gamma, z)\) in a scaling regime within a Friedmann-Lemaître-Robertson-Walker metric. These models were respectively dubbed \(M = 2\) and \(M = 3\) in [44]. In model A, the number of long-lived non-self-intersecting loops of invariant length \(\ell'\) per unit volume per unit time formed at cosmic time \(t\) is directly inferred from Nambu-Goto simulations of cosmic string networks in the radiation and matter eras. Model B is based on a different Nambu-Goto string simulation [49]. In this model, the distribution of non-self-intersecting scaling loops is the extracted quantity. Within model B, loops are formed at all sizes following a power law specified by a parameter taking different values in the radiation and matter eras, while the scaling loop distribution is cut off on small scales by the gravitational backreaction scale. There is a qualitative difference between these two models since in the latter, tiny loops are produced in a much larger amount than in the former. In addition, we will use a new model, based on [50] and labeled C, that extends and encompasses both models A and B. Like model B, model C assumes that the scaling loop distribution is a power law but leaves its slope unspecified. Given the wide parameter space opened by model C, we will select two samples: models C-1 and C-2. Model C-1 (respectively, C-2) reproduces qualitatively the loop production function of model A (B) in the radiation era and the loop production of model B (A) in the matter era. We expect the addition of these two models to showcase intermediate situations in between the two simulation-inferred models A and B. The loop distribution functions \(n(\gamma, z)\) for the three models are given in the Supplemental Material [42].

For models A, B, and C, the contributions from cusps, kinks, and kink-kink collisions to the gravitational-wave emission must be considered all together. Indeed, the dimensionless decay constant \(\Gamma_d\) of a cosmic string, driving the loop size evolution, can be decomposed into three contributions:

\begin{equation}
\Gamma_d \equiv \frac{P_{gw}}{G\mu^2} = \sum_i \frac{P_{gw,i}}{G\mu^2}
= N_c \frac{3\pi^2 g_i^2 c_i}{(2\delta)^{3/2} g_2^{3/2}} + N_k \frac{3\pi^2 g_k^2 c_k}{(2\delta)^{3/2} g_2^{3/2}} + N_{kk} \frac{2\pi^2 g_{kk}^2 c_{kk}}{g_2},
\end{equation}

where \(\delta = \max[1, 1/(2g_2)]\) since the gravitational-wave frequency cannot be smaller than the fundamental frequency of the loop \(2/\ell'\), while the condition \(\theta_m < 1\) for cusps and kinks imposes \(f > 1/(\ell g_2)\). Parameters \(N_c, N_k\) are, respectively, the average number of cusps and kinks per oscilation. The number of kink-kink collisions per oscillation \(N_{kk}\) is \(N_{kk} \approx N_k^2/4\) for large \(N_k\). While this equation is only an approximation when \(N_k\) is order unity, the kink-kink contribution is very small in this case and the error would hardly affect our results. On the other hand, it is clear that the kink-kink collision quickly dominates the gravitational-wave production when the number of kinks increases, as was also shown in [51]. Here we fix \(N_k\) to be 1 and comment later on the effects of increasing \(N_c\). The only free parameter is \(N_k\); we consider \(N_k = 1, \ldots, 200\), with the upper limit motivated by numerical simulations of string loops that favor \(\Gamma_d \sim 0.021\).

The incoherent superposition of bursts from loops with all possible sizes through the history of the Universe produces a stochastic gravitational wave background (SGWB) [52]; its normalized energy density is defined as

\begin{equation}
\frac{\Omega_{gw}(f)}{\rho_c} = \frac{f}{d\rho_{gw}/df},
\end{equation}

where \(\rho_c = 3H_0^2\) is the critical density. The spectrum of the SGWB is [53]

\begin{equation}
\frac{\Omega_{gw}(f)}{\rho_c} = \frac{4\pi^2}{3H_0^2} f^3 \sum_i \int dz \int d\ell' h_0^2 \times \frac{d^2 R_i}{dz d\ell'}. \end{equation}
Second, the frequency has to be larger than the low-frequency cutoff $f_l(1+z) > \delta$. In Fig. 1, we show examples of gravitational-wave spectra calculated with Eq. (10). The two plots at the top are derived from models A and B with $N_k \gg 1$. The dominant contribution comes from kink-kink collisions. The lower plots show gravitational-wave spectra for $N_k = 1$ (left) and $N_k = 100$ (right) and are derived from model C with a given set of parameters (see the Supplemental Material [42]): $\chi_{\text{rad}} = 0.45$, $\chi_{\text{mat}} = 0.295$, $c_{\text{rad}} = 0.15$, and $c_{\text{mat}} = 0.019$; the subscripts refer to radiation and matter eras, respectively. We also show the energy density spectra of the three different components and 2-$\sigma$ power-law integrated (PI) curves [55] for the O3 isotropic stochastic search [41], and projections for the Hanford, Livingston, and Virgo network at design sensitivity, and the A + detectors [56].

FIG. 1. Predictions of the gravitational-wave energy density spectra using different models for the loop distribution function $n(\gamma, z)$ and for two values of the number of kinks per loop oscillation $N_k$: 1 and 100. The string tension $G\mu$ is fixed to $10^{-8}$. Top left: model A, $N_k = 100$. Top right: model B, $N_k = 100$. Bottom left: model C-1, $N_k = 1$. Bottom right: model C-1, $N_k = 100$. For model C-1, we use the following model parameters (see the Supplemental Material [42]): $\chi_{\text{rad}} = 0.45$, $\chi_{\text{mat}} = 0.295$, $c_{\text{rad}} = 0.15$, and $c_{\text{mat}} = 0.019$; the subscripts refer to radiation and matter eras, respectively. We also show the energy density spectra of the three different components and 2-$\sigma$ power-law integrated (PI) curves [55] for the O3 isotropic stochastic search [41], and projections for the Hanford, Livingston, and Virgo network at design sensitivity, and the A + detectors [56].

Second, the frequency has to be larger than the low-frequency cutoff $f/(1+z) > \delta$. In Fig. 1, we show examples of gravitational-wave spectra calculated with Eq. (10). The two plots at the top are derived from models A and B with $N_k \gg 1$. The dominant contribution comes from kink-kink collisions. The lower plots show gravitational-wave spectra taking $N_k = 1$ (left) and $N_k = 100$ (right) and are derived from model C with a given set of parameters (see the Supplemental Material [42]), i.e., $\chi_{\text{rad}} = 0.45$, $\chi_{\text{mat}} = 0.295$, $c_{\text{rad}} = 0.15$, and $c_{\text{mat}} = 0.019$; the subscripts refer to matter and radiation eras, respectively. When $N_k$ is large, the dominant contribution depends on the frequency band, which is a unique feature in this model. In this study, we ignore the suppression of the gravitational waves from cusps due to the primordial black hole production as pointed out in [54]. Including such an effect leads to lower spectrum amplitudes for small $N_k$, thus reducing the sensitivity to cosmic string signals. In Fig. 1, we also show the 2$\sigma$ power-law integrated (PI) curves [55] indicating the integrated sensitivity of the O3 search [41], along with projections for two years of the Advanced LIGO–Virgo network at design sensitivity, and the envisioned upgrade of Advanced LIGO, A+ [56], sensitivity after two years, assuming a 50% duty cycle.

Burst search.—The O3 dataset is analyzed with a dedicated burst search algorithm previously used to produce LIGO–Virgo results [44,57,58]. The burst analysis pipeline, as well as its O3 configuration, is described in the Supplemental Material [42]. The search can be summarized into three analysis steps. First, we carry out a matched-filter search using the cosmic string waveform in Eq. (1). Then, resulting candidates are filtered to retain only those detected in more than one detector within a time window accounting for the difference in the gravitational-wave arrival time between detectors. Finally, double- and triple-coincident events are ranked using an approximated likelihood ratio $\Lambda(x)$, where $x$ is a set of parameters used to discriminate true cosmic string signals from noise [59]. The burst search is performed separately for cusps, kinks, and kink-kink collision waveforms, integrating $T_{\text{obs}} = 273.5$ days of data when at least two detectors are operating simultaneously.
The left panel of Fig. 2 presents the cumulative distribution of coincident O3 burst events as a function of the likelihood ratio $\Lambda$ for the cusp, kink, and kink-kink collision searches. To estimate the background noise associated with each search, time shifts are applied to each detector strain data such that no real gravitational-wave event can be found in coincidence. For this study, we use 300 time shifts, totaling $T_{\text{bkg}} = 225$ years of data containing only noise coincident events, the distribution of which is represented in the left panel of Fig. 2 with a ±1σ shaded band. The candidate events, obtained with no time shift, are all compatible with the noise distribution within ±2σ. The cusp, kink, and kink-kink collision waveforms are very similar, resulting in the loudest events being the same for the three searches. The ten loudest events were carefully scrutinized. They all originate from a well-known category of transient noise affecting all detectors that are broadband and very short-duration noise events of unknown instrumental origin [60,61].

From the nondetection result, we measure our search sensitivity to cosmic string signals by performing the burst search analysis over O3 data with injections of simulated cusp, kink, and kink-kink collision waveforms. The amplitudes of injected signals comfortably cover the range where none to almost all the signals are detected. Other parameters (sky location, polarization angle, high-frequency cutoff) are randomly distributed. To recover injected signals, we use the loudest-event method described in [62], where the detection threshold is set to the level of the highest-ranked event found in the search: $\log_{10}(\Lambda) \approx 15.0, 15.1, \text{and } 15.1$ for cusps, kinks, and kink-kink collisions, respectively. The resulting efficiencies $e_i(A_i)$ as a function of the signal amplitude are presented in the right panel of Fig. 2. Cusp events directed at Earth with $A_i > 2 \times 10^{-20}$ s$^{-1/3}$ would have produced a result more significant than any of the ones obtained by our search with ~90% confidence. In terms of loop proper lengths, this corresponds, for example, to loops larger than $1.7 \times 10^6 (G\mu/10^{-10})^{-3/2}$ light years at redshift 100. The expected detection burst rate is calculated from the detection efficiency

$$ R_i = \int \frac{dR_i}{dA_i}(A_i, f_s; G\mu, N_k)e_i(A_i)dA_i. \quad (11) $$

The detectable burst rate $dR_i/dA_i$ is obtained from Eq. (7), which can be expressed in terms of amplitude using Eq. (2) and calculated for the lowest value of the high-frequency cutoff $f_s$ that can be most abundantly observed (see the Supplemental Material [42] for details). We assume that the occurrence of a detectable burst of gravitational waves follows a Poisson distribution with mean given by the estimated detection rate. For a set of parameters $(G\mu, N_k)$, models that predict a detection rate larger than $2.996/T_{\text{obs}}$ are excluded at 95%, i.e., we
exclude models that predict a > 95% confidence level detection.

**Stochastic search.**—A search for a stochastic gravitational wave background [52] is carried out using the LIGO and Virgo O3 data [41] in which a correlated background in different interferometer pairs is sought. These results are combined with those from the previous two observing runs: O1 and O2 [44,63,64]. The results reported in [41] assume the normalized energy density of the stochastic background, Eq. (9), is a power law $\alpha$ of the frequency

$$\Omega_{GW}(f) = \Omega_{\text{ref}} \left(\frac{f}{f_{\text{ref}}}\right)^\alpha,$$

\[ \tag{12} \]

where $f_{\text{ref}}$ denotes a reference frequency fixed to 25 Hz, a convenient choice in the sensitive part of the frequency band. The search reported in [41] does not detect a stochastic background and so sets upper limits depending on the value of $\alpha$. The stochastic background from cosmic strings in the LIGO–Virgo frequency band is predicted to be approximately flat, setting the upper bound $\Omega_{GW} \lesssim 5.8 \times 10^{-9}$ at the 95% credible level for a flat $\alpha = 0$ background and using a log-uniform prior in $\Omega_{GW}$; the 20–76.6 Hz band is responsible for 99% of this sensitivity.

Here, we perform a Bayesian analysis taking into account the precise shape of the background (see Fig. 1) instead of a power law and use it to derive upper limits on the cosmic string parameters. We first calculate the log-likelihood function assuming a Gaussian distributed noise, which up to a constant is

$$\ln \mathcal{L}(\hat{C}_{ij} | G\mu, N_k) = -\frac{1}{2} \sum_{ij,a} \left[ \frac{\hat{C}_{ij}(f_a) - \Omega_{GW}(f_a, G\mu, N_k)}{\sigma_{ij}(f_a)} \right]^2,$$

\[ \tag{13} \]

Here, $\hat{C}_{ij} \equiv C_{ij}(f_a)$ with $IJ$ as the detector pairs L1-H1, L1-V1, and H1-V1. $C_{ij}(f_a)$ and $\sigma^2(f_a)$ are, respectively, a cross-correlation estimator for the pair $IJ$ and its variance at $f_a$ [65]. Following the same approach as in the O1 stochastic analysis, we use the frequency bins from 20 to 86 Hz [44]; higher frequencies do not contribute to the sensitivity. The spectrum, $\Omega_{GW}(f_a, G\mu, N_k)$ at $f_a$ is predicted by the model $M = \{A, B, C\}$ through Eq. (10).

We specify priors for the parameters in the cosmic string model, i.e., $p(G\mu | I_{Ga})$ and $p(N_k | I_{Nk})$. The variables $I_{Ga}$ and $I_{Nk}$ denote the information on the distributions of $G\mu$ and $N_k$, which are determined by theory predictions. For $p(G\mu | I_{Ga})$, we choose a log-uniform prior for $10^{-18} \lesssim G\mu \lesssim 10^{-6}$. The upper bound is set by the cosmic microwave background measurements [66–69]. The lower bound is arbitrary, chosen for consistency with the study in [70]; we note that our results remain almost unchanged if we choose a smaller value for the lower bound on $G\mu$.

For $p(N_k | I_{Nk})$, we constrain $G\mu$ for each choice of $N_k$. Therefore, the prior $p(N_k | I_{Nk})$ is taken to be a $\delta$ function for each value of $N_k$. The number of kinks per loop oscillation $N_k$ being fixed, the posterior for $G\mu$ is calculated from Bayes’ theorem:

$$p(G\mu | N_k) \propto \mathcal{L}(\hat{C}_{ij} | G\mu, N_k) p(G\mu | I_{Ga}) p(N_k | I_{Nk}).$$

\[ \tag{14} \]

We calculate 95% credible intervals for $G\mu$.

**Constraints.**—We show in Fig. 3 the region of the $G\mu$ and $N_k$ parameter space excluded at the 95% confidence level by the burst and stochastic searches where $N_c = 1$. For the stochastic search, we present constraints from the combined O1 + O2 + O3 data; for the burst search, we derive constraints from the nondetection result using O3 data for models A, B, and C. For model C, we choose two sets of benchmark numbers: C-1, where $(\chi_{\text{rad}}, \chi_{\text{mat}}) = (0.45, 0.295)$, and C-2, where $(\chi_{\text{rad}}, \chi_{\text{mat}}) = (0.2, 0.45)$ (see the Supplemental Material [42]).

For model A, the gravitational-wave signal is much weaker than the other models, leading to weaker constraints. Model C-2 mimics the loop production function of model A in the matter era and of model B in the radiation era. In the frequency band of LIGO–Virgo, the stochastic background is dominated by the contribution from loops in the radiation era, hence models B and C-2 give similar results. Conversely, the spectrum from model C-1, which mimics the loop production function of model A in the radiation era and of model B in the matter era, presents more subtle features. Larger values of $G\mu$ do not necessarily produce larger signals, creating structures in this figure. For an analytical understanding of these findings, see [71]. For a better understanding of the loop visibility domain in terms of redshift; see Fig. 2 of [51].

From the stochastic analysis, the following regions, depending on $N_k$, are excluded: $G\mu \gtrsim (9.6 \times 10^{-9}–10^{-6})$ for model A, $G\mu \gtrsim (4.0–6.3) \times 10^{-15}$ for model B, and $G\mu \gtrsim (2.1–4.5) \times 10^{-15}$ aside from a small region where $N_k \gtrsim 180$ for model C-1 and $G\mu \gtrsim (4.2–7.0) \times 10^{-15}$ for model C-2.

The burst search upper limits are not as stringent as those from the stochastic search. The constraints on $G\mu$ for model A are too weak to be represented in the figure. The only case where the burst analysis leads to tighter constraints is for model C-1 and for $N_k > 70$.

Here $N_c$ has been set to 1. It was shown that $N_c$ scales with the number of harmonics on the loop [72]. For large $N_c$, the decay constant $G\delta$ is enhanced, leading to a reduced lifetime of the loop. Consequently, a large $N_c$ gives qualitatively the same result as increasing $N_k$: for model A, the constraints are weakened, whereas for models B and C, the bounds are insensitive to $N_c$; this has been confirmed by our numerical study.

One can also compare these results with limits obtained from pulsar timing array measurements, indirect limits
from Big Bang nucleosynthesis, and cosmic microwave background data [33]. Note: here, we do not investigate nonstandard thermal history; see, however, e.g., [73,74]. Repeating the analysis done in [44] with $N_k$ up to 200, we find that for model A, the strongest limit comes from pulsar timing measurements, with $G \mu \gtrsim 10^{-10}$ excluded. For models B, C-1, and C-2, the strongest upper limits are derived from this search.

**Conclusions.**—Using data from the third observing run of Advanced LIGO and Virgo, we have performed a burst and a stochastic gravitational-wave background search to constrain the tension of Nambu-Goto strings, as a function of the number of kinks per oscillation, for four loop distributions. We have tested models A and B, already considered in the O1 and O2 analyses [64]. The current constraints on $G \mu$ are stronger by 2 orders of magnitude for model A and 1 order of magnitude for model B when fixing $N_k = 1$. In addition, we have used two variants of a new model, dubbed model C, that interpolates between models A and B. For the first time, we have studied the effect of kink-kink collision interactions, which is relevant for large numbers of kinks, and investigated the effect of a large number of cusps, as both effects are favored by cosmic string simulations. In the context of cosmic strings formed at the end of an inflationary era, these results raise questions about the validity of simple inflationary models (which occurred between $10^{16}$ and $10^{11}$ GeV) in the context of grand unified theories [10], unless one invokes extra fields in order to avoid cosmic string formation [75].

Given the current experimental results, it would seem important to intensify numerical and theoretical studies on cosmic strings. From a numerical point of view, the number of kinks and cusps should be determined. Concerning phenomenological aspects, new models, like model C that interpolates between models A and B, should be further explored, as well as models including particle physics leading to cosmic string formation in the early Universe. On the experimental side, the sensitivity of Advanced LIGO and Virgo detectors will continue to improve [56], and a fourth interferometer, KAGRA [76], will join the network.

The authors gratefully acknowledge the support of the United States National Science Foundation (NSF) for the construction and operation of the LIGO Laboratory and

FIG. 3. Exclusion regions at 95% C.L. on the cosmic string parameter space ($N_k, G \mu$) derived from the stochastic search (pink) and the burst search (turquoise). Four models are considered to describe the distribution of cosmic string loops: model A (top left), model B (top right), model C-1 (bottom left), and model C-2 (bottom right). Note that the stochastic result combines the data of O1, O2, and O3, while the burst search only includes O3 data. We also report limits from other experiments: pulsar timing arrays (PTA) [33,34], cosmic microwave background (CMB) [31], and Big Bang nucleosynthesis [32]. The notch in the SGWB constraint for model C-1 is explained in the Supplemental Material [42].
Advanced LIGO as well as the Science and Technology Facilities Council (STFC) of the United Kingdom, the Max Planck Society (MPS), and the State of Niedersachsen/Germany for support of the construction of Advanced LIGO and construction and operation of the GEO600 detector. Additional support for Advanced LIGO was provided by the Australian Research Council. The authors gratefully acknowledge the Italian Istituto Nazionale di Fisica Nucleare (INFN), the French Centre National de la Recherche Scientifique (CNRS), and the Netherlands Organization for Scientific Research for the construction and operation of the Virgo detector and the creation and support of the EGO consortium. The authors also gratefully acknowledge research support from these agencies as well as by the Council of Scientific and Industrial Research of India, the Department of Science and Technology, India, the Science and Engineering Research Board (SERB), India, the Ministry of Human Resource Development, India, the Spanish Agencia Estatal de Investigación, the Vicepresidencia i Conselleria d’Innovació, Recerca i Turisme and the Conselleria d’Educació i Universitat del Govern de les Illes Balears, the Conselleria d’Innovació, Universitats, Ciència i Societat Digital de la Generalitat Valenciana and the CERCA Programme Generalitat de Catalunya, Spain, the National Science Centre of Poland and the Foundation for Polish Science (FNP), the Swiss National Science Foundation (SNF), the Russian Foundation for Basic Research, the Russian Science Foundation, the European Commission, the European Regional Development Funds (ERDF), the Royal Society, the Scottish Funding Council, the Scottish Universities Physics Alliance, the Hungarian Scientific Research Fund (OTKA), the Belgian Fonds de la Recherche Scientifique (FRS-FNRS), Actions de Recherche Concertées (ARC) and Fonds Wetenschappelijk Onderzoek—Vlaanderen (FWO), Belgium, the Paris Île-de-France Region, the National Research, Development and Innovation Office Hungary (NKFIH), the National Research Foundation of Korea, the Natural Science and Engineering Research Council Canada, the Canadian Foundation for Innovation (CFI), the Brazilian Ministry of Science, Technology, and Innovations, the International Center for Theoretical Physics South American Institute for Fundamental Research (ICTP-SAIFR), the Research Grants Council of Hong Kong, the National Natural Science Foundation of China (NSFC), the Leverhulme Trust, the Research Corporation, the Ministry of Science and Technology (MOST), Taiwan, the United States Department of Energy, and the Kavli Foundation. The authors gratefully acknowledge the support of the NSF, STFC, INFN, and CNRS for provision of computational resources. This work was supported by MEXT, JSPS Leading-edge Research Infrastructure Program, JSPS Grant-in-Aid for Specially Promoted Research 26000005, JSPS Grant-in-Aid for Scientific Research on Innovative Areas 2905: JP17H06358, JP17H06361 and JP17H06364, JSPS Core-to-Core Program A. Advanced Research Networks, JSPS Grant-in-Aid for Scientific Research (S) 17H06133, the joint research program of the Institute for Cosmic Ray Research, University of Tokyo, National Research Foundation (NRF) and Computing Infrastructure Project of KISTI-GSDC in Korea, Academia Sinica (AS), AS Grid Center (ASGC) and the Ministry of Science and Technology (MoST) in Taiwan under grants including AS-CDA-105-M06, Advanced Technology Center (ATC) of NAOJ, and Mechanical Engineering Center of KEK. This document has been assigned the LIGO document number LIGO-P2000506.

[13] Cosmic superstrings [14], the analogs of cosmic strings arising from string theory, are characterized also by the intercommutation probability, which can take values between $10^{-3}$ and 1 for fundamental superstrings (F strings) and between 0 and 1 for D branes extended in one macroscopic dimension (D strings). In our present study, we concentrate on field theoretical objects [15] and in particular Nambu-Goto strings with an intercommutation probability of order 1.
Superhorizon cosmic strings also emit gravitational waves due to their small-scale structure resulting from string intercommutations [23,24].

[22] B. Abbott


[28] We also include the so-called pseudacusps [29], defined as cuspy features moving with a velocity close to the speed of light.


264 Faculty of Science, Toho University, Funabashi City, Chiba 274-8510, Japan
265 Faculty of Information Science and Technology, Osaka Institute of Technology, Hirakata City, Osaka 573-0196, Japan
266 Indian Institute of Technology Hyderabad, Sangareddy, Khandi, Telangana 502285, India
267 iTHEMS (Interdisciplinary Theoretical and Mathematical Sciences Program), The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan
268 INAF, Osservatorio di Astrofisica e Scienza dello Spazio, I-40129 Bologna, Italy
269 Department of Space and Astronautical Science, The Graduate University for Advanced Studies (SOKENDAI), Sagamihara, Kanagawa 252-5210, Japan
270 Andrews University, Berrien Springs, Michigan 49104, USA
271 Department of Information and Management Systems Engineering, Nagaoka University of Technology, Nagaoka City, Niigata 940-2188, Japan
272 Institute for Cosmic Ray Research (ICRR), Research Center for Cosmic Neutrinos (RCCN), The University of Tokyo, Kashiwa City, Chiba 277-8582, Japan
273 National Metrology Institute of Japan, National Institute of Advanced Industrial Science and Technology, Tsukuba City, Ibaraki 305-8568, Japan
274 Dipartimento di Scienze Aziendali—Management and Innovation Systems (DISA-MIS), Università di Salerno, I-84084 Fisciano, Salerno, Italy
275 Van Swinderen Institute for Particle Physics and Gravity, University of Groningen, Nijenborgh 4, 9747 AG Groningen, Netherlands
276 Department of Communications Engineering, National Defense Academy of Japan, Yokosuka City, Kanagawa 239-8686, Japan
277 Department of Physics, University of Florida, Gainesville, Florida 32611, USA
278 Trinity University, San Antonio, Texas 78212, USA
279 Department of Physics and Astronomy, Sejong University, Gwangjin-gu, Seoul 143-747, Korea
280 Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan
281 Department of Physics, Rikkyo University, Toshima-ku, Tokyo 171-8501, Japan

† Deceased, August 2020.