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IN GENERAL EQUILIBRIUM

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# INTERGENERATIONAL RISK SHARING, PENSIONS AND ENDOGENOUS LABOR SUPPLY IN GENERAL EQUILIBRIUM

## Abstract

In the context of a two-tier pension system, with a pay-as-you-go first tier and a fully funded second tier, we demonstrate that a system with a defined wage-indexed second tier performs strictly better than one with a defined contribution or defined real benefit second tier. The former completely separates systematic redistribution (confined to the first tier) from intergenerational risk sharing (the role of the second tier). This way labor supply is undistorted.

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Keywords: funded pensions, risk sharing, overlapping generations, endogenous labour supply.

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# 1 Introduction

There is a general trend towards more pension funding. This trend will have important implications for the distribution of risks in society. In this article, we explore the optimal design of two-tier pension systems in an overlapping generations general equilibrium model with endogenous labor supply. While the first tier allows for both systematic redistribution and risk sharing between the young and old generation, the second pillar only allows for intergenerational risk sharing, as it is fully funded. Funded pension benefits can be of the defined contribution (DC) type or of the defined benefit (DB) type. Of the latter type, we shall explore a defined real benefit (DRB) system, where the pension benefit is ex ante determined in real terms and a defined wage-indexed benefit (DWB) system, where the benefit is linked to the realized wage rate. With a DC fund, no risk sharing is possible through the second pension pillar because the entire value of the fund is paid out to the retired. Hence, the social optimum can generally not be replicated. With a DRB fund, optimal risk sharing requires wage risks to be shared via the first pillar. However, this requires a distortionary pension premium to be levied on wages, which, in turn, distorts the labor supply. Hence, also with a DRB fund, the social optimum can generally not be achieved. The only system that enables the market economy to replicate the social optimum is a properly designed DWB system. Such a system allows a complete separation between systematic redistribution, which is the task of the first pillar, and optimal risk sharing, which is the domain of the second pillar. This way, labor supply is undistorted and the first best can be mimicked.

The closest antecedent to this paper is Beetsma and Bovenberg (2007). There, however, the labor supply is exogenous and DRB and DWB are equally desirable arrangements. Other works on intergenerational risk sharing via social security systems, though using different frameworks, are Hassler and Lindbeck (1997), Krueger and Kubler (2002), De Menil and Sheshinski (2003), Matsen and Thogerson (2004), Wagener (2004) and Gottardi and Kubler (2006).

The remainder of this paper is structured as follows. Section 2 lays out the model and solves for the social planner's solution. Section 3 presents the decentralized economy and the different second pillar pension systems. In Section 4 we explore the conditions for a market economy to replicate the social planner solution. Finally, Section 5 provides a short conclusion.

## 2 The command economy

### 2.1 Individuals and preferences

The model represents a closed economy. It incorporates two periods (0 and 1) and two generations. In period 0, a generation of mass 1 is born. This generation lives through periods 0 and 1. We call this generation the "old generation". The representative agent

within this generation features the following utility function:

$$u(c_{y,0}) + \beta E_0[u(c_o)], \quad (1)$$

where  $c_{y,0}$  denotes consumption when this agent is young, while  $c_o$  represents consumption when it is old. Further,  $\beta$  is the discount factor and  $E_0[\cdot]$  is the expectations operator conditional on information before any of the shocks (see below) have occurred. We assume  $u_c > 0$  and  $u_{cc} < 0$ , where subscripts denote partial derivatives.

In period 1, a new generation of mass 1 is born. This generation is termed the “young generation” and it lives just for this period. The lives of the two generations thus overlap in period 1. The representative individual of the young generation features the utility function:

$$u(c_y) - z(N), \quad (2)$$

which is defined over consumption  $c_y$  and endogenous work effort  $N$  in period 1. We assume that  $z_N > 0$  and  $z_{NN} > 0$ .

## 2.2 Production

In period 0 each old generation member receives an exogenous non-storable endowment  $\eta_0$ . Production is endogenous only in period 1, when the two generations co-exist. It is then given by

$$Y = AF(K, N), \quad (3)$$

where  $A$  denotes stochastic total factor productivity,  $K$  represents the aggregate capital stock and  $N$  is the aggregate labor input. The production function exhibits constant returns to scale. In our closed economy, the capital stock  $K$  is the result of investment in the previous period 0.

## 2.3 Resource constraints

The resource constraints in periods 0 and 1 are given by, respectively,

$$c_{y,0} = \eta_0 - K, \quad (4)$$

$$c_y + c_o = AF(K, N) + (1 - \delta)K, \quad (5)$$

where  $0 \leq \delta \leq 1$  is the stochastic depreciation rate of the capital stock. The left-hand sides of these expressions denote aggregate consumption in the economy. The right-hand side of (4) represents total endowment income minus the investment in physical capital. The right-hand side of (5) stands for total production plus what is left over of the capital stock after taking depreciation into account.

## 2.4 The social planner's solution

The vector of the stochastic shocks hitting the command economy is  $\xi \equiv \{A, \delta\}$ . It is unknown in period 0, but becomes known before period 1 variables are determined. As a benchmark, we consider a (utilitarian) social planner who aims to maximize the sum of the discounted expected utilities of all individuals. In period 0, this planner commits to an optimal state-contingent plan. This yields the following optimality conditions:

$$c_y = c_o, \forall \xi, \quad (6)$$

$$z_N(N)/u_c(c_y) = AF_N, \forall \xi, \quad (7)$$

$$u_c(c_{y,0}) = \beta E_0 [(1 + r^{kn}) u_c(c_o)]. \quad (8)$$

where  $r^{kn} \equiv AF_K - \delta$ , which in the sequel we will refer to as the *net-of-depreciation return on capital*, and  $F_K$  and  $F_N$  are the marginal products of capital and labor, respectively. (In the following we drop function arguments whenever this does not create ambiguities.) If a decentralized equilibrium is to replicate the planner's solution, conditions (6) – (8) need to be met in addition to the resource constraints (4) and (5). Condition (6) equalizes the marginal utilities of the two generations, while condition (7) sets the marginal rate of substitution between consumption and work equal to minus the marginal product of labor. Finally, expression (8) is an Euler equation.

## 3 The decentralized economy

This section describes the decentralized market economy in which individuals and firms maximize their objective functions under the relevant constraints. A key question will be which pension system can replicate the command optimum. We can interpret (6) as the condition for ex-ante trade in risks between the young and old generations in complete financial markets. However, in a decentralized economy, the two generations cannot trade risk in financial markets, because the young generation is born only after the shocks have materialized. Other institutions thus have to replace this missing market and we shall explore to what extent the pension system can perform that role.

The timing of events in the market economy will be as follows. In period 0, the old generation take their investment and consumption decisions. Beginning of period 1, the shocks  $A$  and  $\delta$  materialize. Finally, firms take hiring and production decisions, while young individuals decide on their labour supply and consumption.

### 3.1 The pension systems

The first pillar in the pension system is a pay-as-you-go (PAYG) part composed of a lump-sum part and a wage-indexed part. The young pay a (possibly negative) lump-sum amount  $\theta^p$  and a fraction  $\theta^w$  of their wage income to the old. The total systematic transfer from the young to the old via the first pillar is  $\theta^p + \theta^w wN$ .

The second pillar of the pension system consists of a pension fund that collects contributions  $\theta^f$  from old-generation members in period 0, invests these contributions and pays out benefits to the old generation in period 1. If we denote the rate of return that the old generation receives on their contribution by  $r^f$ , then the total payout is  $(1 + r^f)\theta^f$ .

The pension fund can invest the old generation's contribution in real bonds ( $b^f$ ) or in physical capital ( $k^f$ )

$$\theta^f = b^f + k^f. \quad (9)$$

The real bonds provide a non-stochastic return of  $r$  and physical capital provides the net-of-depreciation rate of return  $r^{kn}$ . The actual or net return on the assets held by the fund is  $r^a$  so the value of the fund before the payout is

$$(1 + r^a)\theta^f = (1 + r)b^f + (1 + r^{kn})k^f. \quad (10)$$

Depending on the pension scheme and the fund's investment scheme, there can be a difference between the payout and the value of the fund equal to  $(r^a - r^f)\theta^f$ . The young are the residual claimants of the fund and receive this difference.

Since the second pillar is fully funded, the contributions equal the expected value to the old generation of the benefits they collect in period 1. This implies that there is no redistribution of wealth between the generations. We analyze risksharing opportunities for the cases where the second pillar is of the defined contribution, defined benefit in real terms or defined benefit indexed to wages type.

We will summarise the net flows from the young generation to the old generation by the generational accounts. In a closed economy the generational accounts always add up to zero. If we denote the generational account of the old, respectively young, by  $G^o$  and  $G^y$  we have

$$G^o = -G^y = \theta^p + \theta^w wN + (r^f - r^a)\theta^f. \quad (11)$$

### 3.1.1 Defined contribution (DC)

If the second pillar pension is of the DC type, assets and liabilities are always equal, not only *ex ante* in period 0 when contributions and pension fund investments are made, but also *ex post* when benefits are paid out. Hence,  $r^f = r^a$  and the second pillar provides no intergenerational risk-sharing opportunities in addition to those already provided by the capital market. Since  $r^f = r^a$ , the generational account of the old under a DC system reduces to:

$$G^o = \theta^p + \theta^w wN. \quad (12)$$

### 3.1.2 Defined real benefit (DRB)

In case the system is of the DRB type, the old receive a safe real benefit  $B^{drb}$  in period 1. The system is fully funded if it does not produce *ex ante* redistribution between the two

generations. Because the pension benefit is safe, the return on the pension contribution must be equal to the return on a safe bond:

$$r^f = r. \quad (13)$$

The young receive what is left over of the pension fund after the old have been paid their safe real benefit. Hence, the young absorb the mismatch risk of the pension fund and each one of them receives

$$\begin{aligned} (r^a - r)\theta^f &= (1 + r)b^f + (1 + r^{kn})k^f - (1 + r)\theta^f \\ &= (r^{kn} - r)k^f, \end{aligned}$$

where the second equality has employed (9) to eliminate  $\theta^f$ . As the first line shows, the young *in effect* issue an amount of debt  $\theta^f$  to the old and invest the resources for their own risk according to the portfolio of the pension fund. Hence, depending on the equity investment of the pension fund  $k^f$ , the young effectively participate in the stock market, thereby sharing in equity (productivity and depreciation) risks with the old generation.

Now, the generational account of the old is equal to:

$$G^o = (\theta^p + \theta^w Nw) + (r - r^{kn})k^f. \quad (14)$$

### 3.1.3 Defined wage-indexed benefit (DWB)

Under a DWB system, the pension fund benefit is indexed to the wage rate. This way, the second pillar of the pension system is able to share wage risks. As we shall see, this allows the PAYG pillar to focus purely on redistribution and avoids the need to levy a distortionary pension premium on wage income.

The pension fund benefit of the old in period 1 is:

$$(1 + r^f)\theta^f = Nw\theta^{dwb}, \quad (15)$$

where  $\theta^{dwb}$  captures the fixed (non-stochastic) factor that links the benefit to the wage rate  $w$ . It is important to notice that the individual pension benefit is linked to the aggregate wage income. Hence, a change in  $\theta^{dwb}$  leaves work effort unaffected, because an individual person's labor supply decision has a negligible effect on aggregate wage income. The return the old generation receives is stochastic since the wage rate is determined by market forces only after the shocks are known. Using (15) we have

$$r^f - E_0 r^f = N \frac{\theta^{dwb}}{\theta^f} (w - E_0 w). \quad (16)$$

This clearly shows the stochastic nature of  $r^f$ . If the realised wage rate exceeds the expected wage rate, the payout of the pension fund is higher than expected. Full funding implies that  $u_c(c_{y,0}) = \beta E_0 [u_c(c_o)(1 + r^f)]$ , which by using (15) links  $\theta^f$  to  $\theta^{dwb}$

$$\theta^f = \theta^{dwb} \frac{\beta E_0 [Nw u_c(c_o)]}{u_c(c_{y,0})}. \quad (17)$$



As residual claimant of the pension fund, each young receives:

$$(r^a - r^f)\theta^f = (1 + r)b^f + (1 + r^{kn})k^f - (1 + r^f)\theta^f.$$

The young in effect issue a wage-indexed bond to the old and invest the borrowed resources in debt and physical capital, conform the portfolio decisions of the pension fund. Because wage risk is not traded in financial markets, the DWB pension fund always suffers from mismatch risk. It thereby creates new possibilities for risk trading. Specifically, the DWB fund allows the young generation not only to trade in equity, but also to diversify away wage risk.

Under a DWB system, the generational account of the old becomes

$$G^o = \theta^p + Nw(\theta^w + \theta^{dwb}) - (1 + r)b^f - (1 + r^{kn})k^f. \quad (18)$$

### 3.2 Individual budget constraints and generational accounts

The old receive their deterministic period 0 endowment  $\eta_0$  and spend this on current consumption, pay the mandatory pension fund contribution  $\theta^f$ , and invest the rest in physical capital and real bonds.

$$c_{y,0} = \eta_0 - (b + k + \theta^f), \quad (19)$$

Consumption of the young and old generations in period 1 is:

$$c_y = Nw + G^y, \quad (20)$$

$$c_o = (1 + r^{kn})k + (1 + r)b + (1 + r^a)\theta^f + G^o, \quad (21)$$

where  $b$  denotes real debt directly held by the old and  $k$  is the direct claim of the old on the capital stock in period 1.

### 3.3 Individual and firm optimization

We solve the model by backwards induction. A young person in period 1 chooses labor effort  $N$  and consumption  $c_y$  to maximize its utility, (2), subject to its budget constraint (20). The first-order conditions yield:

$$z_N(N) / u_c(c_y) = (1 - \theta^w)w. \quad (22)$$

A continuum of perfectly competitive representative firms, with mass normalized to unity, produce according to (3) and maximize profits  $AF(K, N) - wN - r^kK$  over  $N$  and  $K$ , taking the wage rate and rental rate of capital as given. The first-order conditions are:

$$AF_N = w, \quad (23)$$

$$AF_K = r^k. \quad (24)$$

In period 0, the old generation decides on the allocation of its savings over the various assets. They maximize (1) over  $b$  and  $k$ , where  $c_{y,0}$  and  $c_o$  are given by (19) and (21), respectively. The first-order conditions are:

$$\beta (1 + r) E_0 [u_c(c_o)] = u_c(c_{y,0}), \quad (25)$$

$$\beta E_0 [(1 + r^k - \delta) u_c(c_o)] = u_c(c_{y,0}). \quad (26)$$

### 3.4 Market equilibrium conditions

The goods market equilibrium conditions in periods 0 and 1 are (4) and (5), respectively. Given that the mass of the old generation is 1, equilibrium in the capital market is:

$$K = k + k^f. \quad (27)$$

Combining (22) and (23) yields the labor market equilibrium condition:

$$z_N(N) / u_c(c_y) = (1 - \theta^w) AF_N. \quad (28)$$

Finally, with zero net outstanding debt, debt market equilibrium in period 0 requires that:

$$b + b^f = 0. \quad (29)$$

## 4 The optimality of pension systems

We study how the decentralized market economy can replicate the social optimum with an appropriate choice of the pension system. We design the appropriate two pillar pension system (a PAYG first pillar and a funded second pillar) to ensure that the planner's solution derived in Section 2 is replicated in a decentralized market economy. We do this using the optimality and market equilibrium conditions derived above.

The following lemma, which is easy to prove, provides the necessary and sufficient conditions for the replication of the social optimum:

**Lemma 1** *When a policy produces  $c_o = c_y$  and  $z_N(N) / u_c(c_y) = AF_N$  for all possible realizations of the shock vector  $\xi$ , then the competitive equilibrium reproduces the socially-optimal allocation under all types of funded pension systems.*

For the remainder of the analysis, we assume that the pension system parameters  $\theta^p$ ,  $\theta^w$ ,  $\theta^f$ ,  $\theta^{dwb}$ ,  $B^{drb}$ ,  $k^f$  and  $b^f$  are not contingent on the shocks. Although the potential objections to making these parameters shock-contingent are not modelled explicitly, frequent changes in the pension parameters would inevitably lead to political struggles and introduce additional uncertainty not directly linked to the fundamental economic shocks themselves.

First, consider the condition  $c_y = c_o$ . Combining this with (20) and (21), replication of the social optimum requires the generational accounts to vary such that:

$$\frac{1}{2}AF_NN - \frac{1}{2}(1 + r^{kn})K = G^o = -G^y, \quad (30)$$

Hence, if individual profit income plus the scrap value of capital per old individual,  $(1 + r^{kn})K$ , exceeds individual wage income per young individual,  $AF_NN$ , the old would have more per-capita resources for consumption in period 1 than the young. Intergenerational equality of period 1 consumption requires the generational accounts to offset these income differences.

Reproduction for all shock realizations of (7) by the market condition (22) is possible if and only if

$$\theta^w = 0. \quad (31)$$

In other words, replication of the social optimum requires the elimination of the wage-linked part of the first pillar.

## 4.1 A DC system

We want to establish whether the DC system is able to replicate the social optimum. The available instruments are the investment composition  $(k^f, b^f)$  of the fund and the parameter combination  $(\theta^p, \theta^w)$  characterizing the PAYG system. However, because individual investment decisions exactly offset the pension fund investments, only  $\theta^p$  and  $\theta^w$  remain as potential instruments to affect allocations.

We have the following proposition:

**Proposition 2** *The DC system is generally unable to replicate the social optimum when there is at least one source of shocks.*

**Proof.** Under DC,  $G^o = \theta^p + \theta^w AF_NN$ . Substitution into (30) yields as a necessary condition for reproducing the social optimum that  $\theta^p + \theta^w AF_NN = \frac{1}{2}AF_NN - \frac{1}{2}[AF_K + (1 - \delta)]K$ . We see immediately that with depreciation shocks, this expression cannot hold for all possible shock realizations for constant  $(\theta^p, \theta^w)$ . With productivity shocks only, the condition requires that  $\theta^w$  differs from 0, which contradicts (31). ■

With productivity shocks only, a DC system fails to reproduce the planner's solution because it would require setting  $\theta^w > 0$ , which introduces inefficient distortions in the labor market. With depreciation shocks, the DC system also fails on account of the fact that there is no way that the elderly can share the depreciation risk with the younger generation.

## 4.2 A DRB system

The design of the DRB pension fund requires the choice of four policy parameters:  $\theta^f, B^{drb}, b^f$  and  $k^f$ . Restriction (9) and the full-funding requirement  $\theta^f = B^{drb}/(1 + r)$

leave only two parameters to be freely selected. We take these to be  $b^f$  and  $k^f$ . Further, the fund's investment in debt is not a source of mismatch between the assets and liabilities of the pension fund and therefore does not affect the equilibrium. Hence, only  $k^f$  is left as the key parameter to be chosen for the design of the second pillar.

Even though the young now share in equity risks, we have:

**Proposition 3** *The DRB system is generally unable to replicate the social optimum.*

**Proof.** Now  $G^o = \theta^p + \theta^w AF_N N + (r - r^{kn}) k^f$ . Substitution into (30) and rewriting yields as a necessary condition for reproduction of the social optimum:

$$\theta^p + \theta^w AF_N N + (r - r^{kn}) k^f = \frac{1}{2} AF_N N - \frac{1}{2} [AF_K + (1 - \delta)] K.$$

We have three instruments  $(\theta^p, \theta^w, k^f)$  to produce equality of the constant terms and the shock coefficients on both sides of this expression. The solution is  $k^f = k$ ,  $\theta^p = -(1 + r) k^f$  and  $\theta^w = \frac{1}{2}$ . The solution for  $\theta^w$  contradicts (31). ■

Hence, replication of the social optimum requirement (6) would require risk sharing via the wage-linked part of the first pension pillar. However, this would distort the labor supply, implying that the first-best cannot be achieved with a DRB second pillar.

### 4.3 A DWB system

The optimal design of the DWB pension fund requires the choice of  $\theta^f, \theta^{dwb}, b^f$  and  $k^f$ . With restriction (9) and the full-funding requirement (17), only two parameters remain to be chosen. For this, we take  $\theta^{dwb}$  and  $k^f$ , which determine, respectively, the size of the second pillar and the composition of the pension fund's investment portfolio.

The key result of this paper is:

**Proposition 4** *The combination of a properly designed first pillar and a second DWB pillar replicates the social optimum for all possible shock combinations. The appropriate parameters of the pension arrangement are  $\theta^p = (1 + r) (\theta^f - k^f)$ ,  $\theta^w = 0$ ,  $\theta^{dwb} = \frac{1}{2}$  and  $k^f = k$ , where  $\theta^f$  follows from (17).*

**Proof.** Under a DWB fund,  $G^o = \theta^p + ANF_N (\theta^w + \theta^{dwb}) - (1 + r) b^f - (1 + r^{kn}) k^f$ . Substitute this into (30). It is easy to check that the proposed solution ensures that the resulting expression holds for all possible shock combinations. Because  $\theta^w = 0$  is part of the proposed solution, also (7) is fulfilled for all possible shock combinations. ■

The appropriate pension arrangement *completely separates the respective roles of both pension system pillars*, where the role of the PAYG first pillar is to provide the right amount of systematic redistribution and that of the DWB second pillar is to provide for optimal risk sharing. This contrasts with the DRB system where a distortionary pension premium was needed to share wage risks between the two generations.

## 5 Conclusion

This paper has shown that a two-tier pension arrangement with a DWB second tier is able to combine optimal intergenerational redistribution (via the first pillar) with optimal intergenerational risk sharing (via the second pillar), without distorting the labor market. In this regard, a DWB second pillar is preferable to a DRB second pillar. In practice, the separation of risk sharing from redistribution under DWB is likely to yield additional benefits, such as providing more transparency about the consequences of a specific pension system design for different generations and facilitating a potential reform of the first pillar if there is a need to change the amount of systematic redistribution between generations.

## References

- [1] Beetsma, R.M.W.J. and A.L. Bovenberg, 2007, Pensions and intergenerational risk sharing in general equilibrium, *Economica*, forthcoming.
- [2] De Menil, G. and E. Sheshinski, 2003, The Optimal Balance of Intergenerational Transfers and Funded Pensions in the Presence of Risk, *mimeo*, EHESS/Hebrew University of Jerusalem.
- [3] Gottardi, P. and F. Kubler, 2006, Social Security and Risk Sharing, *mimeo*, Universities of Venice/Mannheim.
- [4] Hassler, J. and A. Lindbeck, 1997, Intergenerational Risk Sharing, Stability and Optimality of Alternative Pension Systems, *mimeo*, Institute for International Economic Studies, Stockholm.
- [5] Krueger, D. and F. Kubler, 2002, Intergenerational Risk-Sharing via Social Security when Financial Markets are Incomplete, *American Economic Review, Papers and Proceedings* 92, 2, 407-410.
- [6] Matsen, E. and Ø. Thogerson, 2004, Designing Social Security – a Portfolio Choice Approach, *European Economic Review* 48, 883-904.
- [7] Wagener, A., 2004, On Intergenerational Risk Sharing within Social Security Schemes, *European Journal of Political Economy* 20, 181-206.

# Appendix

## A Derivation of the planner's solution

In period 0, this planner commits to a state-contingent plan. Hence, the consumption levels and the labor supply in period 1 are functions of the shocks, so that we write  $c_o = c_o(\xi)$ ,  $c_y = c_y(\xi)$  and  $N = N(\xi)$ . We can write the planner's problem as:

$$\begin{aligned} \mathcal{L} = & \int \left[ \begin{aligned} & [u(c_{y,0}) + \beta u(c_o(\xi))] + \beta [u(c_y(\xi)) - z(N(\xi))] \\ & + \beta \lambda_1(\xi) [AF(K, N(\xi)) + (1 - \delta)K - c_y(\xi) - c_o(\xi)] \end{aligned} \right] f(\xi) d\xi \\ + & \lambda_0 [\eta_0 - K - c_{y,0}]. \end{aligned}$$

Here,  $f(\xi)$  stands for the probability density function of the vector of stochastic shocks  $\xi$ . The Lagrange multipliers on the resource constraints in period 0 and 1 are denoted by  $\lambda_0$  and  $\lambda_1(\xi)$ , respectively. Maximization of the planner's program with respect to  $c_{y,0}$ ,  $K$ ,  $c_y(\xi)$ ,  $c_o(\xi)$  and  $N(\xi)$  for all  $\xi$  yields the following first-order conditions:

$$\begin{aligned} u_c(c_{y,0}) &= \lambda_0, \\ \lambda_0 &= \int \beta \lambda_1(\xi) (1 + r^{kn}) f(\xi) d\xi, \\ u_c(c_y(\xi)) &= \lambda_1(\xi), \forall \xi, \\ u_c(c_o(\xi)) &= \lambda_1(\xi), \forall \xi, \\ z_N(N(\xi)) &= \lambda_1(\xi) AF_L, \forall \xi. \end{aligned}$$

By eliminating the Lagrange multipliers from these first-order conditions, we obtain

$$u_c(c_y) = u_c(c_o), \forall \xi,$$

(7) and (8). This reduces to (6) - (8).

## B Proof of Lemma 1

Add equation (20) and equation (21). Using  $r^{kn} = AF_K - \delta$ , the resulting equation can be simplified to (5), which for given  $K$  coincides with the planner's resource constraint. The combination of (a) expressions  $c_o = c_y$  and  $z_N(N)/u_c(c_y) = AF_N$ , which in the proposition hold by assumption, (b) expression (5), (c) equation (26), and (d) equation (19) exactly coincides with the system (4) - (8) to be solved under the planner. Hence, the decentralized economy is solved for the same combination(s)  $\{c_{y,0}, c_y, c_o, N, K\}$  as in the social planner's problem.

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