SUPPLEMENTAL MATERIAL

In this *Letter*, we presented the results for the NLO evolution of the first three moments of the track functions under the simplified assumption that $T_q = T_{\bar{q}}$, and that the track functions are equal for all quark flavors. This simplified case was sufficient to illustrate the structure of the equations, without excessive notation. In this *Supplemental Material*, we present the complete results in several different forms. These different forms may be useful for different users, depending on their particular application.

Although the results can be drastically simplified by working in terms of shift-invariant central moments, we begin by presenting the results for the evolution of the standard moments of the track functions. Using the following notation

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} T_i(n) = D_{T_i(n)}\,, \quad D_{T_i}(n) = \sum_{L=0}^{\infty} a_s^{L+1} D^{(L)}_{T_i(n)}\,, \tag{S-1}$$

we find that the evolution for the first three moments of the gluon track function is given by

$$D^{(0)}_{T_g(1)} = -\gamma^{(0)}_{gg}(2) T_g(1) - \sum_i \gamma^{(0)}_{qg}(2)(T_{q_i}(1) + T_{\bar{q}_i}(1))\,, \tag{S-2}$$

$$D^{(0)}_{T_g(2)} = -\gamma^{(0)}_{gg}(3) T_g(2) - \sum_i \gamma^{(0)}_{qg}(3)(T_{q_i}(2) + T_{\bar{q}_i}(2)) + \frac{14}{5} C_A\, T_g(1) T_g(1) + \sum_i \frac{2}{5} T_F\, T_{q_i}(1) T_{\bar{q}_i}(1)\,,$$

$$D^{(0)}_{T_g(3)} = -\gamma^{(0)}_{gg}(4) T_g(3) - \sum_i \gamma^{(0)}_{qg}(4)(T_{q_i}(3) + T_{\bar{q}_i}(3)) + \frac{21}{5} C_A\, T_g(2) T_g(1) + \sum_i \frac{3}{10} T_F (T_{q_i}(2) T_{\bar{q}_i}(1) + T_{\bar{q}_i}(2) T_{q_i}(1))\,,$$

$$D^{(1)}_{T_g(1)} = -\gamma^{(1)}_{gg}(2) T_g(1) - \sum_i \gamma^{(1)}_{qg}(2)(T_{q_i}(1) + T_{\bar{q}_i}(1))\,, \tag{S-3}$$

$$D^{(1)}_{T_g(2)} = -\gamma^{(1)}_{gg}(3) T_g(2) - \sum_i \gamma^{(1)}_{qg}(3)(T_{q_i}(2) + T_{\bar{q}_i}(2)) + \left[C_A^2 \left(-8\zeta_3 + \frac{26}{45}\pi^2 + \frac{2158}{675}\right) - \frac{4}{9} C_A n_f T_F\right] T_g(1) T_g(1)$$
$$+ \sum_i \left[T_F \left(-\frac{299}{225} C_A - \frac{4387}{900} C_F\right)\right] T_g(1)(T_{q_i}(1) + T_{\bar{q}_i}(1)) + \sum_i T_F \left[\left(\frac{12413}{1350} - \frac{52}{45}\pi^2\right) C_A + \frac{1528}{225} C_F - \frac{16}{25} n_f T_F\right] T_{q_i}(1) T_{\bar{q}_i}(1)\,,$$

$$D^{(1)}_{T_g(3)} = -\gamma^{(1)}_{gg}(4) T_g(3) - \sum_i \gamma^{(1)}_{qg}(4)(T_{q_i}(3) + T_{\bar{q}_i}(3)) + \left[C_A^2 \left(24\zeta_3 - \frac{278}{15}\pi^2 + \frac{767263}{4500}\right) - \frac{2}{3} C_A n_f T_F\right] T_g(2) T_g(1)$$
$$+ \sum_i \left[T_F \left(-\frac{46}{15} C_A - \frac{1727}{2250} C_F\right)\right] T_g(2)(T_{q_i}(1) + T_{\bar{q}_i}(1)) + \sum_i T_F \left[\left(\frac{14}{15}\pi^2 - \frac{10318}{1125}\right) C_A - \frac{4544}{1125} C_F\right] (T_{q_i}(2) + T_{\bar{q}_i}(2)) T_g(1)$$
$$+ \sum_i T_F \left[\left(\frac{5321}{3000} - \frac{2}{5}\pi^2\right) C_A + \frac{1523}{240} C_F - \frac{12}{25} n_f T_F\right] (T_{q_i}(2) T_{\bar{q}_i}(1) + T_{q_i}(1) T_{\bar{q}_i}(2))$$
$$+ C_A^2 \left(-\frac{248561}{2250} + \frac{194}{15}\pi^2 - 24\zeta_3\right) T_g(1) T_g(1) T_g(1) + \sum_i \left[C_A T_f \left(\frac{23051}{1125} - \frac{28}{15}\pi^2\right) - C_F T_f \frac{501}{100}\right] T_g(1) T_{q_i}(1) T_{\bar{q}_i}(1)\,,$$

while for quarks,

$$D^{(0)}_{T_q(1)} = -\gamma^{(0)}_{gq}(2) T_g(1) - \gamma^{(0)}_{qq}(2) T_q(1)\,, \tag{S-4}$$

$$D^{(0)}_{T_q(2)} = -\gamma^{(0)}_{gq}(3) T_g(2) - \gamma^{(0)}_{qq}(3) T_q(2) + 3 C_F\, T_g(1) T_q(1)\,,$$

$$D^{(0)}_{T_q(3)} = -\gamma^{(0)}_{gq}(4) T_g(3) - \gamma^{(0)}_{qq}(4) T_q(3) + \frac{13}{10} C_F\, T_g(2) T_q(1) + \frac{16}{5} C_F\, T_g(1) T_q(2)\,,$$

$$D^{(1)}_{T_q(1)} = -\gamma^{(1)}_{gq}(2) T_g(1) - \gamma^{(1)}_{qq}(2) T_q(1) - \gamma^{(1)}_{\bar{q}q}(2) T_{\bar{q}}(1) - \sum_i \gamma^{(1)}_{Qq}(2)(T_{Q_i}(1) + T_{\bar{Q}_i}(1))\,,$$

$$D^{(1)}_{T_q(2)} = -\gamma^{(1)}_{gq}(3) T_g(2) - \gamma^{(1)}_{qq}(3) T_q(2) - \gamma^{(1)}_{\bar{q}q}(3) T_{\bar{q}}(2) - \sum_i \gamma^{(1)}_{Qq}(3)(T_{Q_i}(2) + T_{\bar{Q}_i}(2))$$
$$+ \left[\left(\frac{1399}{5400} - \frac{7}{9}\pi^2\right) C_A C_F - \frac{67}{18} C_F^2\right] T_g(1) T_g(1)$$
$$+ \left[\left(-\frac{3023}{108} + \frac{34}{9}\pi^2 - 8\zeta_3\right) C_A C_F + \left(\frac{3023}{54} - \frac{68}{9}\pi^2 + 16\zeta_3\right) C_F^2 - \frac{53}{18} C_F T_F\right] T_q(1) T_q(1)$$

$$+\left[\left(\frac{14057}{216}-\frac{77}{9}\pi^2+16\zeta_3\right)C_AC_F+\left(-\frac{14057}{108}+\frac{154}{9}\pi^2-32\zeta_3\right)C_F^2-\frac{2803}{900}C_FT_F\right]T_q(1)T_{\bar{q}}(1)$$
$$+\left[\frac{229}{18}C_AC_F+\left(\frac{2573}{72}-4\pi^2\right)C_F^2\right]T_g(1)T_q(1)-\sum_i\frac{17}{100}C_FT_FT_{Q_i}(1)T_{\bar{Q}_i}(1)-\sum_i\frac{53}{18}C_FT_FT_q(1)(T_{Q_i}(1)+T_{\bar{Q}_i}(1)),$$
$$D^{(1)}_{T_q(3)}=-\gamma^{(1)}_{gq}(4)T_g(3)-\gamma^{(1)}_{qq}(4)T_q(3)-\gamma^{(1)}_{\bar{q}q}(4)T_{\bar{q}}(3)-\sum_i\gamma^{(1)}_{Qq}(4)(T_{Q_i}(3)+T_{\bar{Q}_i}(3))$$
$$+\left[-\frac{3787}{750}C_AC_F-\frac{249}{50}C_F^2\right]T_g(2)T_g(1)+\left[\left(\frac{7}{3}\pi^2-\frac{14161}{3000}\right)C_AC_F+\left(\frac{84329}{6000}-\frac{26}{15}\pi^2\right)C_F^2\right]T_g(2)T_q(1)$$
$$+\left[\frac{2327}{180}C_AC_F+\left(\frac{10189}{250}-\frac{64}{15}\pi^2\right)C_F^2\right]T_g(1)T_q(2)-\sum_i\frac{724}{225}C_FT_FT_q(2)(T_{Q_i}(1)+T_{\bar{Q}_i}(1))$$
$$-\sum_i\frac{9557}{9000}C_FT_FT_q(1)(T_{Q_i}(2)+T_{\bar{Q}_i}(2))-\sum_i\frac{59}{1000}C_FT_F\left(T_{Q_i}(2)T_{\bar{Q}_i}(1)+T_{Q_i}(1)T_{\bar{Q}_i}(2)\right)$$
$$+\left[\left(-\frac{353801}{3600}+\frac{77}{6}\pi^2-24\zeta_3\right)C_AC_F+\left(\frac{353801}{1800}-\frac{77}{3}\pi^2+48\zeta_3\right)C_F^2-\frac{12839}{3000}C_FT_F\right]T_q(2)T_q(1)$$
$$+\left[\left(-\frac{369503}{3000}+\frac{77}{5}\pi^2-24\zeta_3\right)C_AC_F+\left(\frac{369503}{1500}-\frac{154}{5}\pi^2+48\zeta_3\right)C_F^2-\frac{1261}{1125}C_FT_F\right]T_{\bar{q}}(2)T_q(1)$$
$$+\left[\left(\frac{649211}{6000}-\frac{139}{10}\pi^2+24\zeta_3\right)C_AC_F+\left(-\frac{649211}{3000}+\frac{139}{5}\pi^2-48\zeta_3\right)C_F^2-\frac{29491}{9000}C_FT_F\right]T_{\bar{q}}(1)T_q(2)$$
$$+\left[\left(\frac{97883}{9000}-\frac{7}{3}\pi^2\right)C_AC_F-\frac{181}{150}C_F^2\right]T_q(1)T_g(1)T_g(1)-\sum_i\frac{137}{500}C_FT_FT_q(1)T_{Q_i}(1)T_{\bar{Q}_i}(1)$$
$$+\left[\left(\frac{202651}{1800}-\frac{43}{3}\pi^2+24\zeta_3\right)C_AC_F+\left(-\frac{202651}{900}+\frac{86}{3}\pi^2-48\zeta_3\right)C_F^2-\frac{137}{500}C_FT_F\right]T_q(1)T_q(1)T_{\bar{q}}(1).$$

Here $\gamma^{(0)}_{ij}(n)$ ($\gamma^{(1)}_{ij}(n)$) are the (N)LO moments of the timelike splitting function, and $Q\neq q$ is used to denote the distinct quark flavors. The expressions for anti-quarks can be obtained by charge conjugation.

As described in the *Letter*, when written in terms of standard moments these equations are highly redundant, due to the presence of the underlying shift symmetry. To present the evolution equations in terms of central moments for the general case with different track functions for each quark flavor, we must extend the situation discussed in the *Letter*, by introducing $\Delta_{q_i}=T_{q_i}(1)-T_g(1)$, in addition to σ_j for each flavor.

For gluons, we find that the evolution of the second and third central moments, can be written as

$$D^{(0)}_{\sigma_g(2)}=-\gamma^{(0)}_{gg}(3)\sigma_g(2)+\sum_i\left\{-\gamma^{(0)}_{qg}(3)\left(\sigma_{q_i}(2)+\sigma_{\bar{q}_i}(2)+\Delta^2_{q_i}+\Delta^2_{\bar{q}_i}\right)+\frac{2}{5}T_F\,\Delta_{q_i}\Delta_{\bar{q}_i}\right\}\,, \tag{S-5}$$
$$D^{(0)}_{\sigma_g(3)}=-\gamma^{(0)}_{gg}(4)\sigma_g(3)+\sum_i\Big\{-\gamma^{(0)}_{qg}(4)\left(\sigma_{q_i}(3)+\sigma_{\bar{q}_i}(3)+3\sigma_{q_i}(2)\Delta_{q_i}+3\sigma_{\bar{q}_i}(2)\Delta_{\bar{q}_i}+\Delta^3_{q_i}+\Delta^3_{\bar{q}_i}\right)$$
$$-2T_F\,\sigma_g(2)(\Delta_{q_i}+\Delta_{\bar{q}_i})+\frac{3}{10}T_F\left(\sigma_{q_i}(2)\Delta_{\bar{q}_i}+\sigma_{\bar{q}_i}(2)\Delta_{q_i}+\Delta^2_{q_i}\Delta_{\bar{q}_i}+\Delta^2_{\bar{q}_i}\Delta_{q_i}\right)\Big\}\,,$$
$$D^{(1)}_{\sigma_g(2)}=-\gamma^{(1)}_{gg}(3)\sigma_g(2)+\sum_i\Big\{-\gamma^{(1)}_{qg}(3)(\sigma_{q_i}(2)+\sigma_{\bar{q}_i}(2)+\Delta^2_{q_i}+\Delta^2_{\bar{q}_i})$$
$$+T_F\Big[\Big(\frac{12413}{1350}-\frac{52}{45}\pi^2\Big)C_A+\frac{1528}{225}C_F-\frac{16}{25}n_fT_F\Big]\Delta_{q_i}\Delta_{\bar{q}_i}\Big\}\,,$$
$$D^{(1)}_{\sigma_g(3)}=-\gamma^{(1)}_{gg}(4)\sigma_g(3)+\sum_i\Big\{-\gamma^{(1)}_{qg}(4)(\sigma_{q_i}(3)+\sigma_{\bar{q}_i}(3)+3\sigma_{q_i}(2)\Delta_{q_i}+3\sigma_{\bar{q}_i}(2)\Delta_{\bar{q}_i}+\Delta^3_{q_i}+\Delta^3_{\bar{q}_i})$$
$$+T_F\Big[\Big(-\frac{638}{45}+\frac{8}{3}\pi^2\Big)C_A-\frac{3803}{250}C_F\Big]\sigma_g(2)(\Delta_{q_i}+\Delta_{\bar{q}_i})$$
$$+T_F\Big[\Big(\frac{5321}{3000}-\frac{2}{5}\pi^2\Big)C_A+\frac{1523}{240}C_F-\frac{12}{25}n_fT_F\Big](\sigma_{q_i}(2)\Delta_{\bar{q}_i}+\sigma_{\bar{q}_i}(2)\Delta_{q_i}+\Delta^2_{q_i}\Delta_{\bar{q}_i}+\Delta^2_{\bar{q}_i}\Delta_{q_i})\Big\}\,.$$

This form emphasizes the large redundancy present in the expressions given in Eq. (S-2). We emphasize that while it is true that the mixing into $\sigma_{q_i}(2)$ and $\sigma_{q_i}(3)$ is governed to all loop order by $\gamma^{(1)}_{qg}$, the fact that the mixing into the products $\sigma_{q_i}(2)\Delta_{q_i}$ and $\Delta^3_{q_i}$ is also governed by this same anomalous dimension is a coincidence at this order in perturbation theory.

Finally, for the evolution of the quark track functions in terms of central moments, we have

$$
\begin{aligned}
D^{(0)}_{\sigma_q(2)} &= -\gamma^{(0)}_{gq}(3)\big(\sigma_g(2)+\Delta_q^2\big) - \gamma^{(0)}_{qq}(3)\sigma_q(2)\,, \\
D^{(0)}_{\sigma_q(3)} &= -\gamma^{(0)}_{gq}(4)\big(\sigma_g(3)-3\sigma_g(2)\Delta_q-\Delta_q^3\big) - \gamma^{(0)}_{qq}(4)\sigma_q(3) + \frac{24}{5}C_F\,\sigma_q(2)\Delta_q\,, \\
D^{(1)}_{\sigma_q(2)} &= -\gamma^{(1)}_{gq}(3)\,\sigma_g(2) - \gamma^{(1)}_{qq}(3)(\sigma_q(2)+\Delta_q^2) - \sum_j \gamma^{(1)}_{Qq}(3)(\sigma_{Q_j}(2)+\sigma_{\bar{Q}_j}(2)+\Delta^2_{Q_j}+\Delta^2_{\bar{Q}_j}) \\
&\quad - \gamma^{(1)}_{\bar{q}q}(\sigma_{\bar{q}}(2)+\Delta^2_{\bar{q}}-2\Delta_q\Delta_{\bar{q}}) + \frac{97}{54}C_FT_F\sum_j \Delta_q(\Delta_{Q_j}+\Delta_{\bar{Q}_j}) \\
&\quad + \left[\frac{2957}{108}C_AC_F + \left(\frac{2323}{54}-\frac{64\pi^2}{9}\right)C_F^2 + \left(\frac{97}{54}-\frac{256}{27}n_f\right)C_FT_F\right]\Delta_q^2 \\
&\quad - \sum_j \frac{17}{100}C_FT_F\Delta_{Q_j}\Delta_{\bar{Q}_j}\,, \\
D^{(1)}_{\sigma_q(3)} &= -\gamma^{(1)}_{gq}(4)(\sigma_g(3)-2\sigma_g(2)\Delta_q) - \gamma^{(1)}_{qq}(4)(\sigma_q(3)-2\sigma_g(2)\Delta_q+3\sigma_q(2)\Delta_q-2\Delta_q^3) \\
&\quad - \gamma^{(1)}_{\bar{q}q}(4)(\sigma_{\bar{q}}(3)+\sigma_g(2)\Delta_q+3\sigma_{\bar{q}}(2)\Delta_{\bar{q}}+3\sigma_q(2)\Delta_{\bar{q}}-3\sigma_{\bar{q}}(2)\Delta_q+3\Delta_q^2\Delta_{\bar{q}} \\
&\qquad -3\Delta_q\Delta^2_{\bar{q}}+\Delta^3_{\bar{q}}) \\
&\quad - \gamma^{(1)}_{Qq}(4)\sum_{j\neq i}(\sigma_{Q_j}(3)+\sigma_{\bar{Q}_j}(3)-\sigma_g(2)\Delta_q+3\sigma_{Q_j}(2)\Delta_{Q_j}+3\sigma_{\bar{Q}_j}(2)\Delta_{\bar{Q}_j} \\
&\qquad -3(\sigma_{Q_j}(2)+\sigma_{\bar{Q}_j}(2))\Delta_q+\Delta^3_{Q_j}+\Delta^3_{\bar{Q}_j}-3\Delta_q(\Delta^2_{Q_j}+\Delta^2_{\bar{Q}_j}-\Delta_{Q_j}\Delta_{\bar{Q}_j})) \\
&\quad - \frac{59}{1000}C_FT_F\sum_j(\sigma_{Q_j}(2)\Delta_{\bar{Q}_j}+\sigma_{\bar{Q}_j}(2)\Delta_{Q_j}-(\sigma_{\bar{Q}_j}(2)+\sigma_{Q_j}(2))\Delta_q+\Delta^2_{Q_j}\Delta_{\bar{Q}_j} \\
&\qquad +\Delta_{Q_j}\Delta^2_{\bar{Q}_j}-\Delta_q(\Delta^2_{\bar{Q}_j}+\Delta^2_{Q_j}+\Delta_{Q_j}\Delta_{\bar{Q}_j})) \\
&\quad + \frac{292}{75}C_FT_F\sum_j(\sigma_q(2)(\Delta_{Q_j}+\Delta_{\bar{Q}_j})+\Delta_q^2(\Delta_{Q_j}+\Delta_{\bar{Q}_j})-\Delta_q\Delta_{Q_j}\Delta_{\bar{Q}_j}) \\
&\quad - \frac{97}{18}C_FT_F\sum_j(\Delta_q^2(\Delta_{Q_j}+\Delta_{\bar{Q}_j})-\Delta_q\Delta_{Q_j}\Delta_{\bar{Q}_j}) - \frac{12929}{9000}(n_f-1)C_FT_F\,\sigma_g(2)\Delta_q \\
&\quad + \left[\frac{29}{300}C_AC_F-\frac{29}{150}C_F^2+\frac{5797}{1125}C_FT_F\right]\sigma_q(2)\Delta_{\bar{q}} \\
&\quad + \left[\left(-\frac{12929}{9000}C_F+\frac{4648}{225}C_Fn_f\right)T_F+\left(-\frac{2163833}{18000}+\frac{247}{30}\pi^2-12\zeta_3\right)C_AC_F\right. \\
&\qquad \left.+\left(\frac{81443}{3000}-\frac{23}{15}\pi^2+24\zeta_3\right)C_F^2\right](\sigma_g(2)\Delta_q+\Delta_q^3) \\
&\quad + \left[\frac{45253}{450}C_AC_F+C_F^2\left(\frac{662327}{3600}-\frac{82}{3}\pi^2\right)+\left(\frac{23719}{4500}C_F-\frac{671}{18}C_Fn_f\right)T_F\right]\sigma_q(2)\Delta_q\,.
\end{aligned}
\tag{S-6}
$$

This case is notationally more cumbersome than for the gluon evolution due to the contributions from different quark flavors. As with Eq. (S-5), this result exhibits a number of coincidences in the evolution, that will not persist at higher orders in perturbation theory.

For completeness, we also provide results for the timelike anomalous dimensions appearing in the evolution equations for the track functions. Expanding the timelike splitting functions perturbatively in $a_s = \alpha_s/(4\pi)$ as

$$
P_{ij}(z) = \sum_{L=0}^{\infty} a_s^{L+1} P^{(L)}_{ij}(z)\,, \tag{S-7}
$$

we define the Mellin moments of the timelike splitting functions as

$$
\gamma^{(L)}_{ij}(k) = -\int_0^1 \mathrm{d}z\; z^{k-1}P^{(L)}_{ij}(z)\,. \tag{S-8}
$$

This definition is chosen so that for the case of the spacelike splitting function, one obtains the standard twist-2 spin-k anomalous dimensions. We obtained our results by directly integrating the z-space results of [**?**]. This has the advantage that it works for both even and odd k.

At LO we have,

$$\begin{aligned}
&\gamma_{gg}^{(0)}(2) = \frac{4}{3} n_f T_F\,, \quad \gamma_{gg}^{(0)}(3) = \frac{14}{5} C_A + \frac{4}{3} n_f T_F\,, \quad \gamma_{gg}^{(0)}(4) = \frac{21}{5} C_A + \frac{4}{3} n_f T_F\,,\\
&\gamma_{gq}^{(0)}(2) = -\frac{8}{3} C_F\,, \quad \gamma_{gq}^{(0)}(3) = -\frac{7}{6} C_F\,, \quad \gamma_{gq}^{(0)}(4) = -\frac{11}{15} C_F\,,\\
&\gamma_{qg}^{(0)}(2) = -\frac{2}{3} T_F\,, \quad \gamma_{qg}^{(0)}(3) = -\frac{7}{15} T_F\,, \quad \gamma_{qg}^{(0)}(4) = -\frac{11}{30} T_F\,,\\
&\gamma_{qq}^{(0)}(2) = \frac{8}{3} C_F\,, \quad \gamma_{qq}^{(0)}(3) = \frac{25}{6} C_F\,, \quad \gamma_{qq}^{(0)}(4) = \frac{157}{30} C_F\,,\\
&\gamma_{\bar{q}q}^{(0)}(2) = \gamma_{\bar{q}q}^{(0)}(3) = \gamma_{\bar{q}q}^{(0)}(4) = 0\,,\\
&\gamma_{Qq}^{(0)}(2) = \gamma_{Qq}^{(0)}(3) = \gamma_{Qq}^{(0)}(4) = 0\,,\\
&\gamma_{\bar{Q}q}^{(0)}(2) = \gamma_{\bar{Q}q}^{(0)}(3) = \gamma_{\bar{Q}q}^{(0)}(4) = 0\,.
\end{aligned} \tag{S-9}$$

At NLO we have,

$$\begin{aligned}
\gamma_{gg}^{(1)}(2) &= n_f T_F \left[\left(\frac{200}{27} - \frac{16\pi^2}{9}\right) C_A + \frac{260}{27} C_F\right],\\
\gamma_{gq}^{(1)}(2) &= \left(\frac{32\pi^2}{9} - \frac{568}{27}\right) C_F^2 - \frac{376}{27} C_A C_F\,,\\
\gamma_{qg}^{(1)}(2) &= T_F \left[\left(\frac{8\pi^2}{9} - \frac{100}{27}\right) C_A - \frac{130}{27} C_F\right],\\
\gamma_{qq}^{(1)}(2) &= C_A C_F \left(4\zeta_3 + \frac{1495}{54} - \frac{17\pi^2}{9}\right) + C_F^2 \left(-8\zeta_3 - \frac{175}{27} + \frac{2\pi^2}{9}\right) - \frac{128}{27} C_F n_f T_F + \frac{64}{27} C_F T_F\,,\\
\gamma_{\bar{q}q}^{(1)}(2) &= C_A C_F \left(-4\zeta_3 - \frac{743}{54} + \frac{17\pi^2}{9}\right) + C_F^2 \left(8\zeta_3 + \frac{743}{27} - \frac{34\pi^2}{9}\right) + \frac{64}{27} C_F T_F\,,\\
\gamma_{Qq}^{(1)}(2) &= \frac{64}{27} C_F T_F\,,\\
\gamma_{\bar{Q}q}^{(1)}(2) &= \frac{64}{27} C_F T_F\,,\\
\gamma_{gg}^{(1)}(3) &= C_A^2 \left(-8\zeta_3 + \frac{2158}{675} + \frac{26\pi^2}{45}\right) + n_f T_F \left[\left(\frac{3803}{675} - \frac{16\pi^2}{9}\right) C_A + \frac{12839}{2700} C_F\right],\\
\gamma_{gq}^{(1)}(3) &= \left(-\frac{39451}{5400} - \frac{7\pi^2}{9}\right) C_A C_F + \left(\frac{14\pi^2}{9} - \frac{2977}{432}\right) C_F^2\,,\\
\gamma_{qg}^{(1)}(3) &= T_F \left[\left(\frac{619}{2700} + \frac{14\pi^2}{45}\right) C_A - \frac{833}{216} C_F\right] - \frac{8}{25} n_f T_F^2\,,\\
\gamma_{qq}^{(1)}(3) &= C_A C_F \left(4\zeta_3 + \frac{16673}{432} - \frac{43\pi^2}{18}\right) + C_F^2 \left(-8\zeta_3 + \frac{989}{432} - \frac{7\pi^2}{9}\right) - \frac{415}{54} C_F n_f T_F + \frac{4391}{5400} C_F T_F\,,\\
\gamma_{\bar{q}q}^{(1)}(3) &= C_A C_F \left(4\zeta_3 + \frac{8113}{432} - \frac{43\pi^2}{18}\right) + C_F^2 \left(-8\zeta_3 - \frac{8113}{216} + \frac{43\pi^2}{9}\right) + \frac{4391}{5400} C_F T_F\,,\\
\gamma_{Qq}^{(1)}(3) &= \frac{4391}{5400} C_F T_F\,,\\
\gamma_{\bar{Q}q}^{(1)}(3) &= \frac{4391}{5400} C_F T_F\,,\\
\gamma_{gg}^{(1)}(4) &= \left(\frac{90047}{1500} - \frac{28\pi^2}{5}\right) C_A^2 + n_f T_F \left[\left(\frac{2273}{675} - \frac{16\pi^2}{9}\right) C_A + \frac{57287}{13500} C_F\right],\\
\gamma_{qg}^{(1)}(4) &= T_F \left[\left(\frac{22\pi^2}{45} - \frac{60391}{27000}\right) C_A - \frac{166729}{54000} C_F\right] - \frac{12}{25} n_f T_F^2\,,
\end{aligned}$$

$$\begin{aligned}
\gamma_{gq}^{(1)}(4) &= \left(\frac{44\pi^2}{45} - \frac{104389}{27000}\right) C_F^2 - \frac{142591}{13500} C_A C_F\,, \\
\gamma_{qq}^{(1)}(4) &= C_A C_F \left(4\zeta_3 + \frac{2495453}{54000} - \frac{247\pi^2}{90}\right) + C_F^2 \left(-8\zeta_3 + \frac{55553}{6000} - \frac{67\pi^2}{45}\right) - \frac{13271}{1350} C_F n_f T_F \\
&\quad + \frac{11867}{27000} C_F T_F\,, \\
\gamma_{\bar{q}q}^{(1)}(4) &= C_A C_F \left(-4\zeta_3 - \frac{1202893}{54000} + \frac{247\pi^2}{90}\right) + C_F^2 \left(8\zeta_3 + \frac{1202893}{27000} - \frac{247\pi^2}{45}\right) + \frac{11867}{27000} C_F T_F\,, \\
\gamma_{Qq}^{(1)}(4) &= \frac{11867}{27000} C_F T_F\,, \\
\gamma_{\bar{Q}q}^{(1)}(4) &= \frac{11867}{27000} C_F T_F\,.
\end{aligned} \tag{S-10}$$