A note on the neglect of the doppler effect in the modelling of traffic flow as a line of stationary point sources

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A NOTE ON THE NEGLECT OF THE DOPPLER EFFECT
IN THE MODELLING OF TRAFFIC FLOW AS A LINE
OF STATIONARY POINT SOURCES

In many traffic noise prediction models the traffic flow is considered to be a line of
stationary point sources, emitting the standard spectrum of a vehicle moving in a direction
perpendicular to the line between observer and the vehicle. In practice, however, most
vehicles will have a velocity component towards the observer which will result in a
changed source spectrum because of the Doppler effect. In this article a formula is
derived that makes it possible to predict the influence of the steepness of a spectrum on
the $L_{eq}$ of a traffic flow. It is shown that at a source speed of 100 km/h the Doppler
effect can indeed be neglected.

Figure 1. An observer $O$ at a distance $a$ from a line on which a source $S$ moves with speed $v$. The projection
of $O$ on the x-axis is the zero reference point.

The situation under consideration is shown in Figure 1. A point source $S$ is moving
on a line with speed $v$. An observer $O$ on a perpendicular distance $a$ to the line and a
distance $r$ to $S$ is present near the line. The frequency shift from $f_{0}$, the source frequency,
to $f$, the frequency observed at $O$, can be written as

$$f = f_{0}/ \left(1 + v x / c (x^2 + a^2)^{1/2}\right)$$

(1)

with $a$ as indicated in Figure 1 and $c$ the sound speed. According to reference [1] the
equivalent sound pressure level $L_{eq}$ can be written as

$$L_{eq} = 10 \log \left\{ \frac{1}{T p_{0}^2} \int_{t_{1}}^{t_{2}} p^2 \, dt \right\},$$

(2)

with $T = t_{2} - t_{1}$ and $p_{0} = 2 \times 10^{-5}$ Pa. As $p(f)$ is related to the source power spectrum
$W(f)$ by

$$p^2(f) = W(f) Z_{0}/4 \pi r^2,$$

(3)

with $Z_{0}$ the impedance in air, $L_{eq}$ can be written as

$$L_{eq}(f) = 10 \log \left\{ \frac{1}{T p_{0}^2} \int_{t_{1}}^{t_{2}} \frac{W(f) Z_{0}}{4 \pi r^2} \, dt \right\}.$$

(4)

This can be rewritten as an integration over distance (see Figure 1) as

$$L_{eq} = 10 \log \left\{ \frac{Z_{0}}{4 \pi v p_{0}^2} \int_{x_{a}}^{x_{b}} \frac{W(f)}{x^2 + a^2} \, dx \right\},$$

(5)
where \(x_1\) and \(x_2\) are the positions of the source at times \(t_1\) and \(t_2\). Because the apparent source spectrum \(W(f)\) depends on the magnitude of the velocity component towards the observer, it is also dependent upon the position \(x\) along the line and can therefore not be removed from under the integral. To solve the integral in equation (5) some sort of approximation of \(W\) has to be made. As a first order approximation one can assume that \(W\) depends linearly on the frequency over a large interval around \(f\):

\[
W(f) = \alpha f + \beta. 
\]  

By using equations (1), (5) and (6) the following expression can be given for the equivalent sound pressure level for frequency \(f\) (perceived by the observer) as a function of the frequency \(f_0\), emitted by the moving source, the velocity of the source and the frequency characteristics of the source:

\[
L_{eq}(f) = 10 \log \left[ \frac{Z_0}{4 \pi \rho p_0^2 T} \left\{ \int_{x_1}^{x_2} \frac{\beta}{x^2 + a^2} \, dx + \int_{x_1}^{x_2} \frac{\alpha f_0}{x^2 + a^2 + vx(x^2 + a^2)^{1/2}} \, dx \right\} \right]. 
\]  

According to reference [2] the solution of this integral is

\[
L_{eq}(f) = 10 \log \left[ \frac{Z_0}{4 \pi \rho p_0^2 T} \left\{ \beta \left( \arctan \frac{x_2}{a} - \arctan \frac{x_1}{a} \right) \right. \right.
\]

\[
+ \left. \frac{\alpha f_0}{1} \left( \arctan \left( \frac{x_2}{(1 - v^2/c^2)^{1/2}} \right) - \arctan \left( \frac{x_1}{(1 - v^2/c^2)^{1/2}} \right) \right) \right. 
\]

\[
+ \left. \arctan \left( \left( \frac{x_2^2}{a^2} + 1 \right) \left( \frac{c^2}{v^2} - 1 \right) \right)^{1/2} \right) \right. 
\]

\[
- \left. \arctan \left( \left( \frac{x_1^2}{a^2} + 1 \right) \left( \frac{c^2}{v^2} - 1 \right) \right)^{1/2} \right) \right. \right]. 
\]  

Because in most situations the position of the observer is thus that \(x_2 = -x_1\), the last part of this equation is non-zero only for non-symmetrical cases. The part with \(\beta\) represents the solution as derived by Rathe [3] for a white noise spectrum, and the middle part with \(\alpha f_0\) takes into account the influence of the source speed and the steepness of the spectrum. In most cases the source starts at \(x_1 = -\infty\) and continues to \(x_2 = +\infty\) and equation (8) reduces to

\[
L_{eq}(f) = 10 \log \left[ \frac{Z_0}{4 \pi T \rho p_0^2} \left\{ \beta + \alpha f_0/(1 - v^2/c^2)^{1/2} \right\} \right]. 
\]  

This formula for the \(L_{eq}\) of a pure tone \(f\) can now be used to estimate, for example, the influence of the neglect of the Doppler term on the \(L_{eq}\) of a 1/3-octave band,

\[
L_{eq,\text{terts}} = 10 \log_{f_1}^{f_2} 10^{L_{eq}(f)/10} \, df, 
\]  

with \(f_1\) and \(f_2\) the lower and upper cut-off frequencies, respectively. Combination of equations (9) and (10) gives

\[
L_{eq,\text{terts}} = 10 \log \left[ \frac{[Z_0/4 \pi T \rho p_0^2]}{\beta(f_2 - f_1) + \frac{1}{2} \alpha (f_2^2 - f_1^2)/(1 - v^2/c^2)^{1/2}} \right]. 
\]  

The error \(\varepsilon\), caused by neglecting the \((1 - v^2/c^2)^{1/2}\) term, can be written as

\[
\varepsilon = 10 \log \left\{ 1 + \frac{\left[ 1/(1 - v^2/c^2)^{1/2} \right] - 1}{\frac{2 \beta}{\alpha (f_2 - f_1)} + 1} \right\}. 
\]  

For \(v = 100\) km/h and \(\beta/\alpha = 0\), \(\varepsilon\) amounts to only 0.015 dB. Hence modelling a traffic flow as a line of stationary point sources does not introduce important errors in the calculated \(L_{eq}\) of that traffic flow. With regard to the spectrum shape it can be seen that the error vanishes for a flat spectrum \((\alpha = 0)\) and for steep spectra with high levels
It seems that the Doppler effect can safely be neglected in traffic noise modelling. Also the so-called convective amplification, i.e., the change in directionality of the sound source as a result of the velocity of the source, can be neglected [4]. The difference in sound pressure level (calculated in front of the source) between a moving source and an immobile source is, according to reference [4], 1.8 dB for $v/c = 0.1$.

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