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Essays on nonlinear evolutionary game dynamics

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Chapter 6

Summary

The aim of this thesis is to study the dynamics of evolutionary games. Initially, evolutionary game theory has been viewed as an evolutionary "repair" of rational actor game theory in the hope that a population of boundedly rational players may attain convergence to classic "rational" solutions such as the Nash Equilibrium through some learning or evolutionary process occurring at the population level. However, there is still no class of evolutionary dynamics, neither simple, imitative learning (Replicator Dynamics) nor involved, belief-based learning (Fictitious Play or Best Reply Dynamics), guaranteeing convergence to Nash equilibrium, beyond certain restricted classes of games, usually with a small number of strategies.

In this thesis the model of boundedly rational players is a perturbed version of the best-reply choice, the so-called Logit rule. The strategic context to which this evolutionary process is applied varies from simple models of cyclical competition (Rock-Paper-Scissors), through industrial organization (Cournot game) and to collective-action choice (repeated Prisoner's Dilemma) games. The logit choice rule has the appealing property of capturing a continuum of degrees of (myopic) rational behavior through differential responsiveness to payoff differences between alternative choices. The connecting line through the chapters lies in the question whether a steady state is obtained as long run outcome of the Logit evolutionary dynamics or

whether multiple steady states (path-dependence) or more complicated, cyclical or perhaps chaotic attractors, may emerge as well.

Chapter 2 contrasts the long-run behavior of Replicator and Logit Dynamics in standard Rock-Paper-Scissors and Coordination games. It is well known that Replicator Dynamics does not give rise to a stable limit cycle but to heteroclinic cycles in 3×3 circulant RSP games (Zeeman (1980)). We obtain an alternative proof of this classical result based on the computation of the first Lyapunov coefficient in the normal form of the vector field induced by the Replicator Dynamics, and show that all Hopf bifurcations are degenerate. Via the same technique, generic Hopf bifurcations are shown to occur under the Logit Dynamics and, moreover, all these bifurcations are supercritical, i.e. the system exhibits stable limit cycles. This result is extended, through continuation analysis with the Matcont software package to the class of 3×3 non-circulant RSP games. Besides generic Hopf bifurcation, fold singularities (i.e. creation of multiple interior steady states) also do not occur in 3×3 games under Replicator Dynamics (Zeeman (1980)). With the help of a simple 3×3 Coordination game we detect numerically a scenario with three consecutive fold bifurcations under the Logit Dynamics giving rise to multiple interior steady states. Both the Hopf and fold codimension I singularities are continued in the game payoffs parameters and a behavioral parameter such as the intensity of choice, in order to obtain bifurcation curves in the parameter space along with certain codimension II singularities along these curves. A frequency-weighted version of the Logit dynamics (iLogit) is put at work on the circulant 3×3 RSP game and shown to generate stable limit cycles with a large amplitude reminiscent of the heteroclinic cycle connecting the three monomorphic steady states in the Replicator Dynamics. Unlike Logit Dynamics, iLogit runs into chaotic behavior in an example of 4×4 symmetric, biologically-inspired game.

While Chapter 2 focuses on the actions chosen in a particular strategic situation, Chapter 3 investigates a deeper level of evolution of learning rules, in the context of a

linear inverse demand-quadratic costs Cournot duopoly. Besides choosing an action in the stage quantity-setting game as a response to expectations about the choice of the opponent (i.e. her quantity produced), players also have to make a decision about the learning heuristic employed to form their expectations. This higher-order competition between the expectation-formation rules has an impact on the stability property of the underlying stage game Cournot-Nash equilibrium. Various pairs of ecologies are drawn from a toolbox consisting of naive, adaptive, (weighted) fictitious play or rational (Nash) strategy. For instance, the evolutionary competition between a free adaptive and a costly weighted fictitious play rule destabilizes the Cournot-Nash equilibrium through a primary period-doubling bifurcation and a secondary Neimark-Sacker bifurcation and eventually enters a chaotic regime. This analysis is extended in Chapter 4 to linear inverse demand linear costs Cournot games with an arbitrary number of players in the spirit of the Theocharis (1960) "two is stable, three is unstable" result. In a heterogenous learning rules environment consisting of adaptive and rational players we show that the Cournot-Nash equilibrium destabilizes through a period-doubling route to chaos as the number of players increases. Theocharis (1960) unstable triopoly is derived as the limit of a system with homogenous (no switching) naive expectation. Interestingly, it could be stabilized by adjusting the model parameters, for instance, by making the expectations more adaptive (placing higher weights on remote past observations).

Bounded rationality models with heterogenous players endowed with smart and simple heuristics (Gigerenzer and Todd (1999)) emerges as a promising way out of the conundrum of observed cooperation in collective action dilemmas. An ecology of simple rules in an iterated Prisoner's Dilemma framework is constructed in Chapter 5 with the selection of rules driven by a logistic-type of evolutionary dynamics. It consists of reciprocators, unconditional defectors and cooperators, stimulus-response (Pavlov) and generous reciprocators all subject to small implementation errors in strategy execution and small mistakes in selecting the best-performing heuristic

available in the population. The evolutionary selection dynamics in the resulting subecologies of rules are investigated by a mixture of analytical and numerical methods, with rich dynamical behaviour unfolding. The surrounding ecology appears critical for the success or failure of certain heuristics and there is no "undisputed" winner. Pavlov (stimulus-response) players do well in an environment without unconditional cooperators, but poorly when such indiscriminatingly "nice" players are around. Rock-Paper-Scissors cycles arise in the 3×3 ecologies while some 4×4 ecologies easily display path-dependence and co-existence of periodic and chaotic attractors.

A general conclusion of this thesis is that the evolutionary selection among boundedly rational strategies does not necessarily ensure convergence to Nash equilibrium and a richer dynamical behavior may be the rule rather than the exception.