Small steps in dynamics of information
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Citation for published version (APA):

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By focussing on finer notions of information, the framework developed in this dissertation allows us to represent small steps in dynamics of information.

More precisely, by zooming in on the omniscient notions of knowledge and belief, we have identified the notions of awareness, implicit/explicit knowledge, and implicit/explicit beliefs that have been discussed through this work, as well as many others that have been just sketched. Technically, this has been achieved by extending the possible worlds model with functions that associate to each possible world a set of formulas, a set of atomic propositions and a set of rules. This merge of semantic and syntactic machineries has allowed us to represent finer notions of information and therefore non-omniscient agents. Table 8.1 shows the most important discussed notions.

Then we have studied different informational acts that transform these finer notions, focusing not only on non-omniscient versions of acts already studied, like observation and upgrade, but also on the actions that become meaningful in this non-omniscient setting: changes in awareness and different kinds of inferences. Technically, these actions have been defined as operations that modify not only the semantic part but also the syntactic component of our models. Table 8.2 shows the most important defined actions.

Let us review in more detail what each particular chapter has achieved.

8.1 Summary of the chapters

In Chapter 2 we have put together ideas from frameworks representing syntactic inference in a modal style (Duc 1997; Jago 2009) with the key ideas of the semantic-based Epistemic Logic (Hintikka 1962; Pagin et al. 1995). The result is a setting in which we can represent an agent’s implicit and explicit information. Thus, the agent does not need to be omniscient anymore, because her implicit information does not need to be explicit. An important observation here is that our agent has explicit information not only about the way the world can be (i.e.,
### Notion

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<th>Notion</th>
<th>Definition</th>
<th>Model requirements</th>
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<td>Chap. 4: $\square \forall \varphi$</td>
<td>——</td>
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<tr>
<td><strong>Awareness of rules</strong></td>
<td>Chap. 4: $\square \mathcal{U}(\rho)$</td>
<td>——</td>
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<tr>
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<td></td>
<td>Chap. 4: $\square (\forall \varphi \land \varphi)$</td>
<td>——</td>
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<tr>
<td><strong>Implicit information about rules</strong></td>
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<td>——</td>
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<tr>
<td><strong>Explicit information about formulas</strong></td>
<td>Chap. 2: $\forall \gamma$</td>
<td>Coherence</td>
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<tr>
<td><strong>Explicit information about rules</strong></td>
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<tr>
<td><strong>Implicit knowledge about formulas</strong></td>
<td>Chap. 2: $\square \varphi$</td>
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<td>Chap. 4: $\square (\mathcal{U}(\varphi) \land \varphi)$</td>
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<td>Chap. 5: $\square \lnot \varphi$</td>
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<tr>
<td><strong>Explicit knowledge about formulas</strong></td>
<td>Chap. 2: $\forall \gamma$</td>
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<td></td>
<td>Chap. 4: $\square (\mathcal{U}(\varphi) \land \varphi \land \forall \varphi)$</td>
<td>Equivalence relation</td>
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<td><strong>Implicit belief about formulas</strong></td>
<td>Chap. 5: $\diamond [\leq] \varphi$</td>
<td>Locally well-preorder</td>
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Table 8.1: Static notions of information.
8.1. Summary of the chapters

<table>
<thead>
<tr>
<th>Action</th>
<th>Description</th>
<th>Studied in Chapters</th>
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<tbody>
<tr>
<td>Increasing awareness</td>
<td>The agent increases her awareness. Studied in Chapters 5 (public and private versions) and 4</td>
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</tr>
<tr>
<td>Dropping awareness</td>
<td>The agent increases her awareness. Studied in Chapter 3 (public and private versions).</td>
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<tr>
<td>Knowledge-based (i.e., truth-preserving) inference</td>
<td>Inference with explicitly known premises and explicitly known rule. Studied in Chapters 2, 4 and 5</td>
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</tr>
<tr>
<td>Belief-based inference</td>
<td>Inference that involve believed premises and/or believed rules. Studied in Chapter 5</td>
<td></td>
</tr>
<tr>
<td>Structural operation</td>
<td>Extend the rules the agent knows. Studied in Chapter 2</td>
<td></td>
</tr>
<tr>
<td>Implicit observation</td>
<td>An observation that does not affect the agent’s explicit information. Studied in Chapter 3</td>
<td></td>
</tr>
<tr>
<td>Explicit observation</td>
<td>An observation that affects the agent’s explicit information. Studied in Chapters 2, 3 and 4</td>
<td></td>
</tr>
<tr>
<td>Upgrade (revision)</td>
<td>Reordering (revision) of the agent’s beliefs (omniscient and non-omniscient versions). Studied in Chapter 5</td>
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</table>

Table 8.2: Actions and their effects.

her explicit information is not only about formulas), but also about procedures that allow her to extract more explicit information from what she already has (i.e., she also has information about rules).

Then we have turned our attention to the actions that modify the agent’s information. We have presented an explicit version of the known observation (public announcement) action from Plaza (1989); Gerbrandy (1999). More interestingly, we have provided a model operation representing the act of inference, an act not considered in standard DEL due to the omniscient nature of the represented agents. This action allows the agent to extend her explicit information by making explicit the information that was only implicit before. We have also observed that, just like the agent can increase the formulas she explicitly knows, she can also extend the rules she explicitly have. We have provided model operations representing the application of structural rules (reflexivity, monotonicity, cut); their effect is to increase the rules the agent can apply.

Chapter 3 is devoted to the notion of awareness. We have observed that another reason for which an agent may not have explicit information about something that is true in all the worlds she considers possible is because she may not be aware of it. Then, we have looked at the Awareness Logic of Fagin and Halpern (1988), and discussed the different possibilities it offer us for defining a notion of explicit information.
Chapter 8. Conclusions

On the dynamic side, we have examined different actions that modify the primitive notions of the framework, awareness and implicit information, and therefore modify the defined notion of explicit information. For the notion of awareness, we have reviewed actions that increase and decrease what the agent is aware of; for the notion of implicit information, we have recalled the notion of observation (public announcement). In all cases we reviewed the effect of these actions in the agent’s explicit information. When going to a multi-agent scenario, we have observed that the actions we defined are ‘public’; then, we have presented an extension of the action models of Baltag et al. (1999) that can deal with the syntactic component of our model, and therefore allows us to represent private and even unconscious versions of the mentioned actions.

In Chapter 4 we have put together the ingredients that have helped us to defined explicit information in the two previous chapters: in order to have explicit information about a certain formula, the agent needs to be aware of it, have implicit information about it, and acknowledge it as true. In particular, we have worked with a language-based notion of awareness that is given not by an arbitrary formulas as in Chapter 3, but by those generated from a set of atomic propositions. We have reviewed properties of these notions, focusing on the particular case of true information, that is, implicit and explicit knowledge.

We have then reviewed the actions of awareness raising, inference and explicit observations defined in the previous chapters, adapting them to the new richer setting. The new awareness raising operation works now by adding atomic propositions to the proper set (and the related act of awareness dropping can be represented by removing atoms from it); the inference action takes advantage of the language-based definition of the awareness notion; the explicit observation action becomes now an explicit announcement action that extends its previous behaviour by producing not only implicit and explicit information, but also awareness about the announced formula.

Chapter 5 has focused on a fine representation of the notion of beliefs. By combining ideas for representing beliefs in a possible worlds framework (van Benthem 2007) with ideas from the previous chapters for representing non-omniscient information, we have introduced a semantic model that allows us to represents an agent’s implicit/explicit knowledge/beliefs about formulas/rules.

Several actions can be defined over our new model. We first explored a notion of belief revision already existing in the DEL literature, and we adapted it to our non-omniscient setting. But just like our agent can perform inference based on knowledge (that is, deduction), she can also perform inference that involve beliefs. By combining the plausibility action models of Baltag and Smets (2008) with the action models introduced in Chapter 3, we provided a setting that allows us to express a large variety of inferences. In particular, we defined inference with known premises and believed rule, believed premises and known rule, and even weak and strong forms of local inference.
Finally, Chapters 6 and 7 provide connections and applications of the developed framework in other fields. The first discusses the relations with diverse known forms of reasoning, focusing on deductive reasoning, default reasoning, abductive reasoning, the relation of belief bases with our explicit beliefs and the relation with purely inferential belief revision. The second discusses on connections with Linguistics, Cognitive Science and Game Theory.

While looking for answers for our original questions, the present work has also shed some light on some other areas.

First, we have shown how a definition of information that merges semantic and syntactic ideas allows us to get the best of both worlds. We still have some level of the abstract structure a semantic approach give us, but we also have the fine granularity that syntactic approaches provide us. More importantly, in our framework we can define the ‘external’ semantic actions that represent the agent’s interaction with her environment as well as the ‘internal’ introspective acts that represent the agent’s own reasoning.

Second, we have shown that there is a harmony between the external and the internal actions that change our information. Just as in standard DEL we have acts of ‘hard’ information that produce knowledge (observations) and ‘soft’ acts that produce beliefs (upgrade), in our non-omniscient setting we have ‘hard’ acts of knowledge-based inference that produce explicit knowledge as well as ‘soft’ acts of belief-based inference that produce explicit beliefs.

Third, though the presented framework for belief-based inference, PA action models, was developed for representing inferences that combine known/believed premises with known/believed rules, these ideas have produced a rich framework in which we can represent deductive reasoning as well as certain forms of default and abductive reasoning.

Finally, our whole approach has been based in defining explicit information in terms of other notions, like awareness, acceptance of formulas and implicit information. Thus, we have shown that, in a setting with multiple notions of information, the reductionist approach that defines some of them in terms of combinations of the others is feasible and interesting in its own right.

8.2 Further work

Like most research works, ours has provided some answers, but has also raised interesting questions. Here are the ones that we consider most appealing.

Long-term We have defined the effect of a single execution of several informational actions, but the result of their iterative application is also important. More precisely, fixed-point operators would allow us express the effect of iterative application of the defined actions, analogous to the Kleene star operator in Propositional Dynamic Logic.
How are fixed-point operators useful? Consider first the case of knowledge. Restricting ourselves to the agent’s purely propositional knowledge, one would expect that, when provided with a ‘complete’ set of rules, the result of an agent’s iterative applications of truth-preserving inferences over her explicit knowledge would be her implicit knowledge. This situation can be expressed with a formula of the form

$$K_{Im} \gamma \rightarrow [(\leftrightarrow_{\cup})^*] K_{Ex} \gamma$$

where the modality $$[(\leftrightarrow_{\cup})^*]$$ stands for the reflexive transitive closure of the application of the union of all truth-preserving inferences the agent can perform. But recall that explicit knowledge is already implicit knowledge, and that in the case of purely propositional facts, this explicit knowledge is not affected by inference operations (that is, if the agent knows explicitly some purely propositional fact, then she will still know it after any truth-preserving inference). Then what we have in fact is the full equivalence

$$K_{Im} \gamma \leftrightarrow [(\leftrightarrow_{\cup})^*] K_{Ex} \gamma$$

This formula shows how the omniscient epistemic notion of knowledge can be seen as the fixed point of a sequence of actions over a non-omniscient but dynamic notion. More generally, the use of fixed points suggest that ideal states can be seen not as a static property, but as the fixed point of the application of finer actions, thus highlighting the actions that are needed to reach such optimal point.

Now, the equivalence does not need to extend to the case of epistemic knowledge, since part of the agent’s explicit knowledge can be invalidated by further inferences (consider Moore-type sentences “I do not know $$\phi$$ explicitly”). But then fixed points would provide us a way to study which pieces of epistemic information will be eventually overthrown and which ones will not.

Expressing the result of long-term iterative application of actions is also interesting when we look at the agent’s beliefs. It would allow us to talk about information that, though it might never become proper knowledge, can nevertheless become a ‘stable’ belief that will not be affected by further steps. There are already some results on the effect of iterated belief revision (e.g., Baltag and Smets (2009)), and a fixed-point extension of our setting would allow us to include the described forms of default and abductive reasoning in the iteration. Some specific fields in which fixed points of our finer informational acts can be useful are

- **Learning Theory**, in particular, when dealing with the **identification in the limit** paradigm in which learning is seen as an infinite process that is successful if there is a finite number of steps after which the agent’s hypothesis about the real language becomes stable,
• Game Theory, in particular, in the Evolutionary Game Theory approach that focuses not on properties of an ideal strategy but rather on the way strategies arise and evolve until they become optimal, and hence stable.

Multi-agent notions of information We have dealt with the single-agent notions of awareness, implicit/explicit knowledge, and implicit/explicit beliefs. And though we have dealt with multi-agent situations, we have not dealt multi-agent notions of information, like group knowledge/beliefs and, more interestingly, common knowledge/beliefs. In our fine-notions-of-information setting, this amounts to the study of implicit and explicit forms of group and common knowledge/beliefs.

The combination of implicit/implicit forms information gives us several cases. For example, in the case of group knowledge, while the very implicit form can be defined as “everybody in the group knows \( \varphi \) implicitly” (coinciding with the standard omniscient notion) and the very explicit form can be defined as “everybody in the group knows \( \varphi \) explicitly”, there are now intermediate points in which while some agents know \( \varphi \) explicitly, the rest know it only implicitly.

The case of common knowledge is more interesting. Different from group knowledge, the omniscient version of common knowledge is not defined by a finite conjunction (assuming the group of agents is finite), but as an infinite one. There is more room for intermediate forms: explicit and full common knowledge corresponds to “everybody knows it explicitly and everybody knows that everybody knows it explicitly and . . . ”, and we can find several situations in which some levels of the knowledge of some agent is only implicit.

Now for the dynamics. Once that implicit group knowledge has been reached (everybody knows implicitly that \( \varphi \) holds), then just actions of awareness raising (in the case of Chapter 3) or even acts of inference (in the case of Chapter 4) are needed to reach explicit group knowledge. But for reaching common explicit knowledge from its implicit form we need group introspective acts in which the agent recognizes that everybody in the group knows something explicitly. Moreover, being an infinitary notion, we do need fixed-points operations to define explicit common knowledge, since our language can only deal with finite formulas. But then we can look at the finite versions and verify how many levels of group introspection are actually needed for real agents to behave in a proper way (cf. Flobbe et al. (2008)).

Justifications By comparing our framework with the Logic of Justifications of Artemov and Nogina (2005), we can observe that our static framework can be seen as a special case in which each formula can have one and only one justification: the formula itself. Under this interpretation, our definitions of explicit information get a different reading. For example, our awareness-less definition of explicit knowledge, \( \Box (\varphi \land A \varphi) \), read as “\( \varphi \) is true and the agent has
acknowledged it in all the worlds she considers possible”, can be read now as “\( \varphi \) is true and the agent has the justification for it in all the worlds she considers possible”. But clearly there may be more than one justification (evidence) for every true formula. In fact, some authors (e.g., van Benthem and Martínez (2008)) have proposed a definition of explicit knowledge that involves a universal quantification over the possible situations and existential quantification over evidence: the agent knows explicitly \( \varphi \) if and only if she has a justification for it in all the worlds she considers possible.

This suggests a further classification of justification. For example, while some facts are justified by deductive reasoning, some others are justified by their observation. But then we should look more formally at a calculus of justifications that involve our inferential steps. With this idea in mind, the act of deductive inference can be seen as an act of building a justification for the conclusion from the justifications of the premises.

**Further dimension of syntactic structure** When dealing with knowledge, existing semantic and syntactic approaches consider plain sets: of possible worlds in the first case and of formulas in the second. Our approach for pure knowledge of Chapters 2 and 4 simply combines them by assigning a set of formulas (and of rules) to each possible world.

When dealing with beliefs, existing semantic and syntactic approaches consider ordered sets: of possible worlds in the first case and of formulas in the second. Our approach for beliefs of Chapter 5 provides only an ordering for the worlds the agent considers possible, and the collection of formulas attached to each world does not have any further structure. The proposed framework can be extended in order to be fair to both semantic and syntactic approaches by considering, besides an ordering among the possible worlds, an ordering among the formulas (and rules) accepted in each one of them. This would allow us to deal with internal syntactic contradictions, as we briefly discussed in Section 6.3, but would also provide us with tools for more refined inference processes. In particular, it would give us tools to choose ‘the best’ explanation in abductive reasoning (cf. Section 6.3).