Radiating top quarks
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1.1 Physics at the LHC

In March 2010 the Large Hadron Collider (LHC) at CERN in Geneva, Switzerland, accelerated proton beams to 3.5 TeV for the first time. Since then the machine is providing proton-proton collisions at a centre-of-mass energy $\sqrt{s} = 7$ TeV. During the year 2010, the luminosity will be gradually increased to $\mathcal{L} = 10^{32}$ cm$^{-2}$ s$^{-1}$ with 50 ns bunch spacing. According to the latest estimates [15], this first long run will provide the two largest LHC experiments (ATLAS and CMS) with a dataset of collisions until the end of 2011 with an integrated luminosity of $\int \mathcal{L} dt \sim 1$ fb$^{-1}$. Only after an additional maintenance shutdown, operation at the designed centre-of-mass energy $\sqrt{s} = 14$ TeV will be possible. The luminosity will then be further increased in the following years: first up to a ‘low’ luminosity of $\mathcal{L} = 10^{33}$ cm$^{-2}$ s$^{-1}$ with a 25 ns bunch spacing, and eventually towards the maximum ‘high’ luminosity of $\mathcal{L} = 10^{34}$ cm$^{-2}$ s$^{-1}$. At this luminosity, data will be recorded at a rate of 80–120 fb$^{-1}$ per year [16].

![Overview of the CERN accelerator complex in Geneva, Switzerland](figure1.png)

**Figure 1.1:** Overview of the CERN accelerator complex in Geneva, Switzerland [17].
Chapter 1. Top quark physics

The LHC is operated at a larger centre-of-mass energy and will provide more luminosity than the Tevatron accelerator at Fermilab in Chicago, USA. The Tevatron, a proton-antiproton collider, is currently operating at a centre-of-mass energy $\sqrt{s} = 1.96$ TeV with a luminosity of $\mathcal{L} = 10^{32}$ cm$^{-2}$ s$^{-1}$. Since 2001, it has supplied the CDF and DØ experiments with 8 fb$^{-1}$ integrated luminosity of data [18]. The shutdown of the Tevatron is planned for 2011.

Due to the large centre-of-mass energy at the LHC, the colliding protons will be probed deeper and more phase space will be available for the production of particles. In general this leads to larger cross sections than at the Tevatron. In Figure 1.2 cross sections for various processes are shown as function of $\sqrt{s}$. Numerical values for the cross sections at the LHC and the Tevatron are given in Table 1.1 for comparison.

<table>
<thead>
<tr>
<th>Process</th>
<th>Cross section (in pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{bb}$</td>
<td>$9.1 \times 10^7$</td>
</tr>
<tr>
<td>$\sigma_{W^+}$</td>
<td>$1.2 \times 10^4$</td>
</tr>
<tr>
<td>$\sigma_{W^-}$</td>
<td>$1.2 \times 10^4$</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>$7.3 \times 10^3$</td>
</tr>
<tr>
<td>$\sigma_{t\bar{t}}$</td>
<td>$6.8 \times 10^0$</td>
</tr>
<tr>
<td>$\sigma_{H_{120}}$</td>
<td>$7.1 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

Table 1.1: Cross sections (in pb) for various processes calculated at NLO with MCFM [20] for Tevatron ($pp$ at $\sqrt{s} = 1.96$ TeV) and LHC ($pp$ at $\sqrt{s} = 14$ TeV). The MSTW2008nlo PDF [21] was used with $\alpha_s(M_Z^2) = 0.120$. The energy scale $Q^2$ of each process was set equal to mass of the heavy final state particle produced in the process.

First it should be noted that there is a difference in the amount of (anti-)quarks involved in $pp$ collisions with respect to $p\bar{p}$. In the former case, the only source for antiquarks are the sea quarks, whereas in the latter case, the valence quarks are the main source. The cross sections for processes that depend strongly on quark-antiquark annihilation, such as $W^\pm$ boson production, therefore display discontinuities when extrapolated from Tevatron to LHC in Figure 1.2. The difference is also apparent in the ratio of the $W^+$ and $W^-$ boson cross sections in Table 1.1. At the Tevatron the ratio is one since in $p\bar{p}$ there is an equal amount of positively charged quarks ($u$, $\bar{d}$) and negatively charged quarks ($\bar{u}$, $d$). At the LHC this ratio is not one since there are more positively charged quarks ($u$) than negatively charged quarks ($\bar{d}$). Hence there are more $W^+$ than $W^-$ bosons produced at the LHC.

The rise and fall in cross sections can be understood by considering momentum fractions. The momentum fraction $x$ is the fraction of a protons momentum carried by a parton. This parton can be a valence quark, a sea quark, or a gluon inside the proton.
When two incoming partons interact in the hard scattering of a proton collision their momentum fractions can be written in the simple partonic picture as [22]:

\[
\begin{align*}
x_1 &= \frac{M}{\sqrt{s}} e^{+y} \\
x_2 &= \frac{M}{\sqrt{s}} e^{-y}
\end{align*}
\] (1.1)

where \(M\) is the total invariant mass produced in the hard scattering, \(E\) the energy of \(M\), \(p_z\) the momentum component of \(M\) along the beam axis and \(y\) the rapidity of \(M\).
in the lab frame. The production of massive particles, like for example a top quark pair at threshold with \( M_{tt} = 2 \times 175 = 350 \text{ GeV} \) and \( p_z = 0 \), requires smaller momentum fractions at the LHC \( (x_1, x_2 \simeq 0.025) \) than at the Tevatron \( (x_1, x_2 \simeq 0.2) \). Figure 1.3 shows the distributions of partons inside a proton as function of momentum fraction \( x \) and the energy scale \( Q^2 \) at which the proton is probed. For smaller \( x \) values at the same \( Q^2 \) scale, the parton density increases, especially the gluon density. Hence the cross sections, which depend directly on the parton densities, increase with smaller \( x \) values.

\[ x_f(x, Q^2) \]

\[ 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \]

\[ g/10 \]

\[ u, d, s, c, b \]

\[ Q^2 = 10 \text{ GeV}^2 \]

\[ Q^2 = 10^4 \text{ GeV}^2 \]

**Figure 1.3:** Parton distribution function MSTW2008lo [21] of the proton for two different \( Q^2 \) scales.

The production cross sections for jets with a minimum transverse energy \( E_T \) that is a fixed fraction of the centre-of-mass \( \sqrt{s} \) (eg. \( E_T > \sqrt{s}/20 \)) fall with \( \sqrt{s} \). For this type of production cross sections, the momentum fractions involved at the Tevatron and LHC are of equal size \( (x_1, x_2 > E_T/\sqrt{s} = 1/20) \). Because the parton densities remain the same in this case (except for the density of valence quarks), the overall cross section depends mainly\(^1\) on the partonic cross section of the hard scattering process, which behaves in turn like \( \sim 1/E_T^2 \): it is harder to produce high-\( E_T \) jets [23]. As a result, the total production cross section falls with \( \sqrt{s} \).

Another feature, which can also be derived from Eq.(1.1), is that particles produced at the LHC will, on average, have a larger Lorentz boost than at the Tevatron. For example: with a centre-of-mass energy \( \sqrt{s} \) of 1.96 TeV at the Tevatron, centrally produced top quarks at threshold implies \( x_1 = x_2 \approx 0.18 \) while \( y = 0 \) and \( M = 2m_t \). At the LHC however, top quarks produced with \( x_1 = 0.18 \) at threshold corresponds to \( y \approx 2 \) and \( x_2 \approx 0.0034 \) while \( \sqrt{s} \) is now 14 TeV and \( M \) is still \( 2m_t \).

\(^1\)There is still a \( Q^2 \) dependence of the PDF’s.
1.2 Understanding the top quark

The increase in luminosity and the enhancement of cross sections will lead to a larger number of events at the LHC. Top quarks, for example, will be produced abundantly: up to 9 million pairs per year, assuming running at a centre-of-mass energy of 14 TeV and a luminosity of $10^{33}$ cm$^{-2}$ s$^{-1}$ [24]. At these event rates the statistical uncertainties in many top quark measurements will be significantly smaller than at the Tevatron. The LHC will therefore allow to study the production mechanisms of such massive particles in much greater detail. With this greater sensitivity, the search for new phenomena with small cross sections can be pursued further.

1.2 Understanding the top quark

The top ($t$) quark was first observed by both the CDF and DØ collaborations at the Tevatron in 1995 [11, 12]. The signal manifested itself as an excess of events in the reconstructed mass distribution above the $W + \text{jets}$ background. Due to its large mass of almost 175 GeV it was the last quark to be discovered: earlier colliders did not have enough energy to produce it. Although the top quark mass is surprisingly large compared to the other quark masses, the existence of the top quark itself was not a surprise. In fact it was already expected since the discovery of the bottom quark in 1977 [25] which confirmed the existence of a third generation of quarks. This third generation of quarks was proposed earlier in 1973 by Kobayashi and Maskawa to explain CP violation in the Standard Model [13].

The top quark is a spin-$\frac{1}{2}$ particle. Like the other up-type quarks, the up quark ($u$) and the charm quark ($c$), it has electric charge $Q = \frac{2}{3}e$. The left-handed state of the top quark has weak isospin $I_3 = +\frac{1}{2}$ and forms a weak isospin doublet with the left-handed state of the bottom ($b$) quark ($I_3 = -\frac{1}{2}$):

$\begin{pmatrix} t \\ b \end{pmatrix}_L$

The bottom quark and the top quark form together the third, heaviest, generation of quarks. Besides weak isospin and electric charge, a top quark carries colour charge and therefore interacts strongly.

Top quarks are produced at hadron colliders in two ways: either via strong interactions (pair production) or via electroweak interactions (single top production). While top quark pair production was first observed in 1995, single top production has only been observed very recently (2009) by the CDF and DØ collaborations [26, 27]. The reason for this is that the background for single top production is larger than the background for top pair production. It is therefore more difficult to distinguish the single top production signal from other collisions.

1.2.1 Mass

The present world average of all top quark mass measurements is largely dominated by results from direct measurements of the CDF and DØ collaborations and yields
Chapter 1. Top quark physics

\[ m_t = 173.1 \pm 1.3 \text{ GeV} \] [28]. Often, the measured value \( m_t \) is referred to as the top quark pole mass. This definition of the top quark mass has an intrinsic ambiguity of \( \mathcal{O}(\Lambda_{\text{QCD}}) \approx 200 \text{ MeV} \) due to QCD corrections which are not calculable perturbatively [29]. A more precise definition is \( \overline{m}_t(m_t) \), the short-distance \( \overline{\text{MS}} \) mass evaluated at the top mass scale. This definition is also used for light quarks. The value of \( \overline{m}_t(m_t) \) is \( \sim 10 \text{ GeV} \) lower than \( m_t \) [30]. Although the experimentally extracted values are related to the pole mass by default, the choice of a particular mass definition remains an interesting topic of discussion [31, 32].

1.2.2 Decay

The top quark decays to a \( W^\pm \) boson and a \( b \) quark [33]. The decay width of the top quark, calculated at Born level and neglecting the mass of the \( b \) quark, is given by [34]:

\[
\Gamma(t \to Wb) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \left( 1 - \frac{M_W^2}{m_t^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{m_t^2} \right)
\]

with the Fermi coupling constant \( G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \), the \( W^\pm \) boson mass \( M_W = 80.398 \text{ GeV} \), the top mass \( m_t = 171 \text{ GeV} \) and the CKM element \( |V_{tb}| \approx 0.999 \) [33]. For these values the width at leading order is \( \Gamma_t = 1.43 \text{ GeV} \). Effects due to higher order QCD and electroweak corrections plus the finite width of \( M_W \) and \( m_b \neq 0 \) have been investigated. The QCD corrections are dominant and lead to a value of \( \Gamma_t = 1.28 \text{ GeV} \) [34]. The decay width has not been measured yet because at hadron colliders the experimental resolution is limited due to the usage of jets in the top quark event reconstruction. A measurement of the width might be possible by performing a threshold scan at a future lepton collider.

The lifetime of the top quark (\( \tau_t \approx 1/\Gamma_t \approx 5 \times 10^{-25} \text{ s} \)) is shorter than the hadronisation time (\( \tau_{\text{had}} \approx 1/\Lambda_{\text{QCD}} \approx 3 \times 10^{-24} \text{ s} \)). Therefore the top quark does not form any bound states. As a consequence, it is possible to measure top quark polarisation, spin correlations, and \( W^\pm \) boson helicity states by studying various angular distributions of the decay products. Measuring the \( W^\pm \) boson helicity states allows to test the structure of weak interactions. Since positive helicity states are not involved in weak interactions and angular momentum is required to be conserved, the helicity \( \lambda_W \) of the \( W^\pm \) boson in \( t \to Wb \) decays is expected to be \( \lambda_W = -1 \) (transversely polarised) in 30% of the cases and \( \lambda_W = 0 \) (longitudinally polarised) in 70% of the cases [35].

The subsequent decays of the \( W^\pm \) bosons in \( t\bar{t} \to W^+bW^-\bar{b} \) is used for the classification of top pair decay channels. The \( W^\pm \) boson decays \( \sim 1/3^{\text{rd}} \) of the cases leptonically (\( W^\pm \to \ell^\pm \nu_\ell \) with \( \ell = e, \mu \), and \( \tau \)) and \( \sim 2/3^{\text{rd}} \) of the cases hadronically (\( W^\pm \to q\bar{q}' \) with \( q = u, d, s, c \), and \( b \)) [36]. Therefore, the top pairs decay 4/9\(^{\text{th}}\) of the cases ‘fully hadronic’, 4/9\(^{\text{th}}\) ‘semi-leptonic’, and 1/9\(^{\text{th}}\) ‘dileptonic’, as illustrated in Figure 1.4.
1.2. Understanding the top quark

Figure 1.4: Decay modes of top quark pairs in $t\bar{t} \rightarrow W^+W^-b\bar{b}$. Figure taken from [37].

1.2.3 Pair production

The cross section for top quark pair production in hadronic collisions can be written in the following factorised form [36, 38]:

$$
\sigma_{h_1h_2\rightarrow t\bar{t}}(p_1, p_2) = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1/i}(x_1, \mu_r^2) f_{h_2/j}(x_2, \mu_f^2) \\
\times \hat{\sigma}_{ij\rightarrow t\bar{t}} \left( m_t, x_1p_1, x_2p_2, \alpha_s(\mu_r^2), \frac{Q^2}{\mu_r^2}, \frac{Q^2}{\mu_f^2} \right) + O \left( \left( \frac{\Lambda_{QCD}}{Q} \right)^p \right) \tag{1.2}
$$

A schematic representation of this equation is given in Figure 1.5. $h_1$ and $h_2$ are the incoming hadrons (protons at the LHC) with momenta $p_1$ and $p_2$ respectively. The partonic cross section of the hard scattering $\hat{\sigma}_{ij\rightarrow t\bar{t}}$ describes the actual production of top quarks with mass $m_t$ at energy scale $Q^2$ from the partons $i$ and $j$ with momenta $x_1p_1$ and $x_2p_2$. The partons $i$ and $j$ are gluons, valence quarks, or sea quarks.

The partonic cross section is convoluted with the parton distribution functions (PDF’s) $f_i(x_1)$ and $f_j(x_2)$. The PDF’s, shown before in Figure 1.3, parametrise the probability of having parton $i$ and $j$ from hadron $h_1$ and $h_2$ with momentum fraction $x_1$ and $x_2$ respectively in the hard scattering. The scales $\mu_r^2$ and $\mu_f^2$ are the renormalisation and factorisation scales. The former indicates the scale at which $\alpha_s$ is evaluated, the latter indicates the scale at which the PDF’s are evaluated. The term $O((\Lambda_{QCD}/Q)^p)$ denotes non-perturbative contributions.
Chapter 1. Top quark physics

\[ h_2 \hspace{1cm} f_j(x_2) \hspace{1cm} \]
\[ \begin{array}{c}
  x_2 p_2 \\
  t \\
  \bar{t} \\
  \sigma_{ij \rightarrow t\bar{t}} \\
  x_1 p_1
\end{array} \]

\[ h_1 \hspace{1cm} f_i(x_1) \hspace{1cm} \]

Figure 1.5: Schematic representation of a hadron collision: partons \( i \) and \( j \) with momentum fractions \( x_1 \) and \( x_2 \) coming from the colliding protons \( h_1 \) and \( h_2 \) respectively create a top quark pair in the hard scattering \( \sigma_{ij \rightarrow t\bar{t}} \).

The renormalisation and factorisation scale are often chosen \( \mu_r^2 = \mu_f^2 = m_t^2 \) while for top quark production typically \( Q^2 \sim m_t^2 \). In that case \( \alpha_s(m_t^2) \ll 1 \) and the partonic cross section \( \hat{\sigma} \) can be calculated using perturbation theory:

\[
\hat{\sigma} = \alpha_s^2 \sum_{m=0}^{n} c^{(m)} \alpha_s^m
\]  

where \( c^{(m)} \) are functions of the kinematic variables and \( \alpha_s \) the expansion parameter. By choosing \( \mu_r^2 \) and \( \mu_f^2 \) of the order \( \mathcal{O}(Q^2) \) large logarithmic terms of the form \( \log^n(Q^2/\mu^2) \) due to scale differences are prevented from showing up in the calculations which could spoil the perturbation series. All non-perturbative effects which happen at scales below \( \mu_r^2 \) and \( \mu_f^2 \) are absorbed in \( \alpha_s \) and the PDF’s. The values of \( \alpha_s \) and the PDF’s are determined from experiment and can be extrapolated to any desired \( \mu_r^2 \) and \( \mu_f^2 \) scale using the renormalisation equations [39] and DGLAP evolution equations [40, 41, 42, 43] respectively. Note that, although the partonic cross section, the PDF’s, and the strong coupling constant depend on the renormalisation and factorisation scales, the hadronic cross section \( \sigma_{h_1 h_2 \rightarrow t\bar{t}} \) itself should not because it is an observable quantity (and the two scales are not physical).

At leading order (LO) the partonic cross section for \( t\bar{t} \) production is of order \( \mathcal{O}(\alpha_s^2) \). The subprocesses that contribute to the cross section at this level are represented by the Feynman diagrams in Figure 1.6. In Table 1.2 the cross sections and the relative contributions from the initial states \( q\bar{q}, qg \) (and \( \bar{q}g \)), and \( gg \) are given. Top quark pairs are only produced via quark-antiquark annihilation and gluon fusion at LO. At the Tevatron \( q\bar{q} \) annihilation is the main contribution (\( \sim 92 \% \)), while at the LHC \( gg \) fusion is the dominant source (\( \sim 87 \% \)). This difference arises from the distinct initial states (\( p\bar{p} \) versus \( pp \)) and the strongly enhanced gluon density inside the proton for the small momentum fractions \( x \) involved at the LHC.
1.2. Understanding the top quark

\[ \bar{q} \rightarrow g \bar{q} \]
\[ g \rightarrow g \bar{q} \bar{q} \]
\[ \bar{q} g \]

\( \mathcal{O}(\alpha_s^2) \) to the \( t \bar{t} \) production cross section: (a) quark annihilation and (b) gluon fusion.

\[
\begin{array}{cccc}
\mathcal{O}(\alpha_s) & \sigma_{t \bar{t}} (\text{pb}) & q\bar{q} & gg \\
\hline
\text{Tevatron} & \text{LO} & 6.78 & 91.6 \% & - & 8.4 \% \\
& \text{NLO} & 6.85 & 87.3 \% & -1.1 \% & 13.8 \% \\
\text{LHC} & \text{LO} & 692 & 13.4 \% & - & 86.6 \% \\
& \text{NLO} & 903 & 9.7 \% & 1.1 \% & 89.2 \% \\
\end{array}
\]

Table 1.2: \( \text{LO and NLO cross sections for } t \bar{t} \text{ production at the Tevatron (p\bar{p} at } \sqrt{s} = 1.96 \text{ TeV}) \) and the LHC (pp at \( \sqrt{s} = 14 \) TeV). The contributions from \( q\bar{q}, gg \) (and \( \bar{q}g \)) and \( gg \) are shown separately. MCFM was used for the calculations with the MSTW2008(n)lo PDF’s and a top mass of \( m_t = 172.5 \) GeV.

At next-to-leading order (NLO) new subprocesses such as flavour excitation and gluon splitting start contributing to the cross section. As shown in Figure 1.7(a), \( t \bar{t} \) pairs can now also be generated from \( gg \) and \( \bar{q}g \) initial states via these subprocesses. The contributions from \( gg \) initial states are however small \( \sim 1\% \) (Table 1.2). The negative value for the \( gg \) contribution at the Tevatron is due to the fact that part of this contribution is already included in the \( q\bar{q} \) initial state state where one of the quarks originates from a \( g \rightarrow q\bar{q} \) splitting process in the PDF by virtue of factorisation [44]. Besides the new subprocesses, also higher order corrections to the LO subprocesses, like gluon emission and virtual loops, are introduced at \( \mathcal{O}(\alpha_s^3) \). The change of the NLO cross section with respect to the LO cross section is often indicated by a \( K \)-factor, defined as \( K = \sigma_{\text{NLO}}/\sigma_{\text{LO}} \). Values reported in literature [34, 45] are \( K \approx 1.5 \) for the LHC and \( K \approx 1.25 \) for the Tevatron\(^2\). The relative contributions to the cross section from the various initial states slightly change at NLO. At the Tevatron \( gg \) contributions increase a bit, thereby reducing the main \( q\bar{q} \) contribution to 87\%. At the LHC, the same happens, resulting in a somewhat larger dominant \( gg \) source of 89\%.

\(^2\)The ratios of the LO and NLO cross sections in Table 1.2 do not give the same \( K \)-factor, because for the LO cross section calculation a LO PDF was used: \( K' = (\text{PDF}_{\text{NLO}} \times \sigma_{\text{NLO}})/(\text{PDF}_{\text{LO}} \times \sigma_{\text{LO}}) \). For the determination of the \( K \)-factor in the references a NLO PDF was used in that case: \( K = (\text{PDF}_{\text{NLO}} \times \sigma_{\text{NLO}})/(\text{PDF}_{\text{NLO}} \times \sigma_{\text{LO}}) \) [44].
Chapter 1. Top quark physics

Figure 1.7: Feynman diagrams contributing at next-to-leading-order $O(\alpha_s^3)$ to the $t\bar{t}$ production cross section: (a) flavour excitation (b) gluon splitting (c) gluon emission and (d) virtual loops.

It is instructive to rewrite the hadronic cross section of Eq.(1.2) in the following form [46]:

\[
\sigma_{h_1h_2\to t\bar{t}} = \sum_{i,j=q,\bar{q},g} \int_{s_{\text{min}}=(2m_t)^2}^{s_{\text{had}}} ds L_{ij}(\hat{s}, s_{\text{had}}, \mu_f^2) \times \hat{\sigma}_{ij\to t\bar{t}}(\hat{s}, m_t^2, \mu_f^2, \mu_r^2)
\]

with $\sqrt{s_{\text{had}}}$ the collider centre-of-mass energy and $\sqrt{\hat{s}}$ the partonic centre-of-mass energy. The parton luminosity $L_{ij}$ is defined as:

\[
L_{ij}(\hat{s}, s_{\text{had}}, \mu_f^2) = \frac{1}{s_{\text{had}}} \int_{\hat{s}}^{s_{\text{had}}} \frac{ds}{s} f_{h_1/i}(\mu_f^2, \frac{s}{s_{\text{had}}}) f_{h_2/j}(\mu_f^2, \frac{\hat{s}}{\hat{s}})
\]

with $f_{h_1/i}$ and $f_{h_2/j}$ the PDF’s. In Figure 1.8 and 1.9 it is shown graphically how the parton luminosities $L_{ij}$ (top plot), partonic cross sections $\hat{\sigma}_{ij\to t\bar{t}}$ (middle plot) and hadronic cross sections $\sigma_{t\bar{t}}$ (lower plot) behave as function of the partonic centre-of-mass energy $\sqrt{\hat{s}}$. Also shown are the uncertainties in the parton luminosities ($\Delta L_{q\bar{q}}$, $\Delta L_{qg}$ and $\Delta L_{gg}$) due to the PDF’s uncertainties (smaller top plots) and the resulting total uncertainty on the hadronic cross section (lowest plot).

The difference between the Tevatron and the LHC is mainly in the parton luminosities. Note that for the LHC the $qg$ luminosity is higher than the $gg$ luminosity. The $qg$ contribution is however not dominant because the partonic cross section is of $O(\alpha_s^3)$. The dashed line indicates at which value of $\sqrt{s_{95\%}}$ the cross section is 95% saturated. At the Tevatron this is $\sqrt{s_{95\%}} \approx 600$ GeV, while for the LHC $\sqrt{s_{95\%}} \approx 1$ TeV. At the Tevatron, top pairs are thus produced closer to the threshold $\sqrt{\hat{s}} = 2m_t$.

Next-to-next-to-leading order (NNLO) calculations of the $t\bar{t}$ cross section do not exist. However, several approximations to NNLO have been made by resumming large logarithmic terms in the NLO calculation due to soft gluon radiation at production threshold $\hat{s} = 4m_t^2$ [48, 46, 49, 45, 50]. The resulting approximate NNLO cross sections are larger in size and less sensitive for the factorisation and renormalisation scale $\mu_f^2$ and $\mu_r^2$ than the NLO calculation. This can be seen in Figure 1.10, where the error bands due to the PDF uncertainty and scale variation are smaller for the NNLO approximation than for the NLO fixed order calculation.
1.2. Understanding the top quark

Figure 1.8: The parton luminosity $L_{ij}$ with the individual PDF uncertainties (upper plot) and the parton cross sections $\hat{\sigma}_{ij\rightarrow t\bar{t}}$ at NLO in QCD (third plot from below) as a function of the partonic centre-of-mass energy $\sqrt{\hat{s}}$. The lower plot scans the total cross section $\sigma_{t\bar{t}}$ as a function of $\sqrt{\hat{s}}$ for the Tevatron ($p\bar{p}$ at $\sqrt{s} = 1.96$ TeV) with $m_t = 171$ GeV, $\mu = m_t$ and the CTEQ6.5 PDF set [47]. The dashed line indicates the value of $\sqrt{s}_{95\%}$ for which the cross section is saturated to 95%. Figure taken from [46].
Chapter 1. Top quark physics

Figure 1.9: The parton luminosity $L_{ij}$ with the individual PDF uncertainties (upper plot) and the parton cross sections $\hat{\sigma}_{ij \rightarrow t\bar{t}}$ at NLO in QCD (third plot from below) as a function of the partonic centre-of-mass energy $\sqrt{\hat{s}}$. The lower plot scans the total cross section $\sigma_{t\bar{t}}$ as a function of $\sqrt{\hat{s}}$ for the LHC ($pp$ at $\sqrt{s} = 14$ TeV) with $m_t = 171$ GeV, $\mu = m_t$ and the CTEQ6.5 PDF set [47]. The dashed line indicates the value of $\sqrt{\hat{s}_{95\%}}$ for which the cross section is saturated to 95%. Figure taken from [46].
The most recent cross section prediction for the Tevatron and the LHC is from [30]:

\[
\text{Tevatron} \quad \sigma(p\bar{p} \to t\bar{t}) = 7.34^{+0.24}_{-0.38} \text{(scale)} +^{0.41}_{-0.44} \text{(PDF)} \text{ pb at } \sqrt{s} = 1.96 \text{ TeV} \\
\text{LHC} \quad \sigma(pp \to t\bar{t}) = 874^{+9}_{-33} \text{(scale)} +^{28}_{-28} \text{(PDF)} \text{ pb at } \sqrt{s} = 14 \text{ TeV}
\]

For this prediction the CTEQ6.6 PDF set was used and a top (pole) mass of \(m_t = 173\) GeV. The scale and PDF uncertainties are comparable in size. A further reduction of the errors not only requires higher order corrections but also an improvement of the PDF accuracy.

![Graphs showing NLO and approximate NNLO QCD prediction for the t\bar{t} total cross section at (a) Tevatron (p\bar{p} at \sqrt{s} = 1.96 \text{ TeV}) and (b) the LHC (pp at \sqrt{s} = 14 \text{ TeV}). The bands denote the total uncertainty from PDF and scale variations for the MRST06nnlo set. Figures taken from [51].](image)

The CDF and DØ experiments have measured the \(t\bar{t}\) cross section at \(\sqrt{s} = 1.96\) TeV. The latest combined results for summer 2009 are [52, 53]:

\[
\text{CDF} \quad \sigma(p\bar{p} \to t\bar{t}) = 7.50^{+0.48}_{-0.48} \text{ pb using } \mathcal{L} = 4.6 \text{ fb}^{-1}, m_t = 172.5 \text{ GeV} \\
\text{DØ} \quad \sigma(pp \to t\bar{t}) = 8.18^{+0.98}_{-0.87} \text{ pb using } \mathcal{L} = 1.0 \text{ fb}^{-1}, m_t = 170 \text{ GeV}
\]

The statistical uncertainty of the CDF measurement (0.31 pb) is of similar size as the total systematic uncertainty (0.34 pb). The values are smaller than the systematic (\(+0.78\) pb) and statistical (\(+0.47\) pb) uncertainties of the DØ measurement, which is a consequence of the fact that CDF used a more than four times larger dataset than DØ. Theory and experiment seem to be in very good agreement. While error bars are of the same order for theory and experiment, it will be challenging to improve in precision on both.

In Table 1.3 the \(t\bar{t}\) production cross section has been calculated for various centre-of-mass energies at the LHC. Since the first long data taking run in 2010–2011 will be

---

3The scales \(\mu_r\) and \(\mu_f\) were varied independently from each other as suggested by Ref.[45]. Therefore the estimated scale uncertainty is expected to be more reliable than an earlier calculation in Ref.[46] where the scales were fixed with respect to each other as \(\mu_r = \mu_f = \mu\) with \(\mu \in [m_t/2, 2m_t]\).
Chapter 1. Top quark physics

at $\sqrt{s} = 7$ TeV, the expected $t\bar{t}$ production cross section is roughly $1/6^{th}$ of the cross section at the full design centre-of-mass energy $\sqrt{s} = 14$ TeV. At the centre-of-mass energy of 14 TeV, both the ATLAS and the CMS experiment expect to measure the $t\bar{t}$ cross section with an accuracy of $\sim 10\%$ [54, 55]. Assuming an integrated luminosity of 10 fb$^{-1}$ the systematic uncertainties and the uncertainty in the luminosity determination will be dominating the errors of the cross section measurements.

### 1.2.4 Single top production

Single top quarks are produced at LO via three distinct channels: s-channel, t-channel, and $Wt$-channel. The Feynman diagrams associated with these channels are shown in Figure 1.11. Single top production is interesting because it offers the opportunity to directly measure the CKM matrix element $|V_{tb}|$ and because it is an important background to Higgs searches such as the $WH$ production signal.

![Feynman diagrams representing single top production at LO via (a) the s-channel (b) the t-channel and (c) the Wt-channel.](image)

The t-channel is the dominant contribution to the single top cross section. NLO corrections to this channel have been studied [56] and the NLO cross section is known in fully differential form, like for the s-channel [57]. The $Wt$-channel contribution is small at the Tevatron but not at the LHC. For this channel NLO corrections have been determined too [58, 59].
1.3. Top quarks at the LHC

For the Tevatron ($p\bar{p}$ at $\sqrt{s} = 1.96$ TeV), the latest approximations of the NNNLO cross sections have been calculated using the MRST2004nlo PDF's [60] with a top mass $m_t = 171.4 \pm 2.1$ GeV [61]:

<table>
<thead>
<tr>
<th>Channel</th>
<th>Cross Section</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-channel</td>
<td>$\sigma(p\bar{p} \rightarrow tq, \bar{t}q)$</td>
<td>$1.15 \pm 0.07$ pb</td>
</tr>
<tr>
<td>s-channel</td>
<td>$\sigma(p\bar{p} \rightarrow t\bar{b}, \bar{t}b)$</td>
<td>$0.54 \pm 0.05$ pb</td>
</tr>
<tr>
<td>Wt-channel</td>
<td>$\sigma(p\bar{p} \rightarrow tW^-, \bar{t}W^+)$</td>
<td>$0.14 \pm 0.03$ pb</td>
</tr>
</tbody>
</table>

The quoted uncertainties include the uncertainties in scale, PDF and top mass.

The CDF and DØ experiments measured the combined cross sections of the t-channel and s-channel for single top and anti-top production [26, 27]:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Cross Section</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>$\sigma(p\bar{p} \rightarrow tb + X, tqb + X)$</td>
<td>$2.3^{+0.6}_{-0.5}$ pb using $\mathcal{L} = 3.2$ fb$^{-1}$, $m_t = 175$ GeV</td>
</tr>
<tr>
<td>DØ</td>
<td>$\sigma(p\bar{p} \rightarrow tb + X, tqb + X)$</td>
<td>$3.94 \pm 0.88$ pb using $\mathcal{L} = 2.3$ fb$^{-1}$, $m_t = 170$ GeV</td>
</tr>
</tbody>
</table>

From this cross section measurement CDF extracted a value $|V_{tb}|$ of $0.91 \pm 0.11$ (stat + syst) $\pm 0.07$ (theory) with a limit $|V_{tb}| > 0.71$ at 95% C.L. DØ set a slightly higher limit of $|V_{tb}| > 0.78$ at 95% C.L. Hence the measurements are in agreement with the Standard Model predictions.

Approximate NNLO cross sections for the LHC ($pp$ at $\sqrt{s} = 14$ TeV) have also been determined [62]. The situation here is a bit more complicated. Soft gluon corrections in the t-channel do not give a good approximation and therefore the accuracy does not go beyond NLO for this channel. Unlike at the Tevatron, the Wt-channel is more important at the LHC than the s-channel. However, at NLO the Wt-channel diagrams interfere with the decay of the top quark in the LO $t\bar{t}$ production diagrams. By using proper selection cuts to distinguish between final states, it is possible to isolate the two processes and their interference can be neglected [63]. Predictions for the single top cross section using the MRST2004nlo PDF's [60] with a top mass $m_t = 171.4 \pm 2.1$ GeV are [62]:

<table>
<thead>
<tr>
<th>Channel</th>
<th>Cross Section</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-channel</td>
<td>$\sigma(pp \rightarrow tq)$</td>
<td>$150 \pm 6$ pb</td>
</tr>
<tr>
<td></td>
<td>$\sigma(pp \rightarrow \bar{t}q)$</td>
<td>$94 \pm 4$ pb</td>
</tr>
<tr>
<td>s-channel</td>
<td>$\sigma(pp \rightarrow t\bar{b})$</td>
<td>$7.80 \pm 0.70_{-0.60}$ pb</td>
</tr>
<tr>
<td></td>
<td>$\sigma(pp \rightarrow \bar{t}b)$</td>
<td>$4.35 \pm 0.26$ pb</td>
</tr>
<tr>
<td>Wt-channel</td>
<td>$\sigma(pp \rightarrow tW^-)$</td>
<td>$34.5 \pm 4.8$ pb</td>
</tr>
<tr>
<td></td>
<td>$\sigma(pp \rightarrow \bar{t}W^+)$</td>
<td>$34.5 \pm 4.8$ pb</td>
</tr>
</tbody>
</table>

1.3 Top quarks at the LHC

1.3.1 Parton distribution functions

The large centre-of-mass energy at the LHC will disclose proton-proton interactions with smaller momentum fractions $x$ and at larger energy scales $Q^2$ than before. In Figure 1.12
the ranges in $x$ and $Q^2$ covered by the LHC are compared to the ranges covered by the Tevatron, HERA, and fixed target experiments. These latter three have provided measurements of the gluon and quark densities inside the proton. The measurements constrain the PDF’s in the $x$ and $Q^2$ region accessible at these colliders. Also shown in Figure 1.12 are the rapidity ranges $y$ which can be reached by various masses $M$ produced at the LHC.

\textbf{Figure 1.12:} Ranges of the energy scale $Q^2$ and momentum fraction $x$ at the LHC compared to fixed target, HERA, and Tevatron experiments. The dotted lines indicate the rapidity $y$ for a massive system $M$ produced with a certain momentum fraction $x$. Figure (modified) taken from [64].

At the beginning of the LHC, a large range of $x$ and $Q^2$ values for the PDF’s will be unexplored territory. The well understood processes of $W^\pm$ and $Z$ boson production will act as reference processes in the calibration of the PDF’s. The CTEQ collaboration has

\footnote{HERA (Hadron Elektron Ring Anlage) was an $e^\mp p$ collider at DESY in Hamburg, Germany. Until June 2007 electron and positrons were accelerated up to 27.6 GeV and collided head-on with protons of 920 GeV.}

\footnote{Note that due to its forward acceptance ($1.9 < \eta < 4.9$) especially the spectrometer of the LHCb experiment is suitable for studying Drell-Yan processes at these $x$ and $Q^2$ ranges [65].}
suggested that $t\bar{t}$ production can also contribute here [66]. $W^\pm$ and $Z$ boson production depend mainly on the quark-antiquark initial state and hence on the quark densities. On the other hand, $t\bar{t}$ production predominantly takes place via gluon fusion and thus depends mainly on the gluon distribution. This leads to an anti-correlation between the $t\bar{t}$ cross section and the $W^\pm$ and $Z$ boson cross sections. This anti-correlation might help constrain the PDF’s further. At the same time it would reduce uncertainties in single top and Higgs production cross sections which in turn are correlated with the $t\bar{t}$ cross section.

The DGLAP equations are used to evolve PDF’s from a scale $Q^2_0$ to a higher scale $Q^2$. At the LHC, the equations may however not be adequate at small $x$ values. Large logarithms of the type $\log(1/x)$ then need to be resummed. Various equations have been constructed to handle this: BFKL (Balitsky-Fadin-Kuraev-Lipatov) [67, 68, 69], CCFM (Catani-Ciafaloni-Fiorani-Marchesini) [70, 71, 72] and LDC (Linked Dipole Chain) [73, 74]. As these methods predict a steeper rise of the proton’s gluon density towards lower $x$ values than DGLAP evolution, the typical small-$x$ behaviour could be detected at the LHC by observing enhancement of dijet production with large rapidity gaps [75]. Although momentum fractions for $t\bar{t}$ production are too large to affect it directly, the large $W + \text{jets}$ background could also be sensitive to small-$x$ effects [76, 77].

### 1.3.2 Top quark mass and the Higgs boson

A precise measurement of the top mass does not only improve the cross section determination it also helps to constrain the Higgs boson mass. The mass of the $W^\pm$ and $Z$ bosons depend on the top quark mass $m_t$ and the Higgs boson mass $m_H$ via radiative corrections $\Delta r$ [78]:

$$M_{W}^2 = \frac{\pi \alpha}{\sqrt{2} G_F} \cdot \frac{(1 + \Delta r/2)}{\sin^2 \theta_W} \quad \text{(with: } \sin^2 \theta_W \equiv 1 - M_{W}^2/M_{Z}^2)$$

The electromagnetic coupling constant $\alpha$, the Fermi constant $G_F$, and the weak mixing angle have been measured with great precision [33]. Important contributions to the radiative corrections are shown in Figure 1.13. The dependence of the radiative corrections on the top quark mass is quadratic $\sim m_t^2$, the dependence on the Higgs boson mass is logarithmic $\sim \ln (m_H^2)$. The electroweak precision data together with the measurements of $M_W$ and $m_t$ from LEP, SLD, and the Tevatron constrain the Higgs boson mass. The direct and indirect measurements of $M_W$ and $m_t$ and the latest fit to electroweak precision data are shown in Figure 1.14.

![Figure 1.13: Radiative corrections to $M_W$ and $M_Z$ from the top quark and Higgs boson.](image-url)
Chapter 1. Top quark physics

Figure 1.14: Left: direct (dotted ellipse) and indirect (solid ellipse) measurements of \( M_W \) and \( m_t \) at 68% C.L. together with isolines indicating possible Higgs boson masses (dark shaded area). Right: the ‘blueband’ plot showing the indirect determination of the Higgs boson mass from all electroweak precision data together with the excluded region (at 95% C.L.) of Higgs boson masses from direct searches by LEP (light shaded area). The preferred Higgs boson mass is \( m_H = 87^{+35}_{-26} \) GeV. Figures taken from [79].

Direct searches at LEP exclude a Standard Model Higgs boson mass below 114.4 GeV at 95% C.L. [80]. The mass range 162–166 GeV has been excluded recently at 95% C.L. by the Tevatron experiments [81]. The fit to electroweak precision data in the right plot suggests a Standard Model Higgs boson mass of \( m_H = 87^{+35}_{-26} \) GeV (68% C.L. corresponding to \( \Delta \chi^2 = 1 \)). The upper mass limit derived from this fit is 157 GeV at 95% C.L. (corresponding to \( \Delta \chi^2 = 2.7 \)). If the limit of \( m_H \gtrsim 114 \) GeV from LEP is included, the upper limit rises to 186 GeV. Including low \( Q^2 \) data from the NuTeV collaboration affects these results only slightly. Although this fit does not prove the existence of a Higgs boson, it shows that there is limited range of possible Higgs boson masses at which a Higgs boson could be discovered consistent with the Standard Model.

At the LHC it will be possible to search for the Higgs boson over a large range of possible masses \( m_H \). However, measuring the top quark mass to complement the electroweak precision data will still be useful to check the consistency with the Standard Model. But with an expected resolution of \( \Delta m_t \sim 1 \) GeV it will be challenging for ATLAS to improve in precision with respect to the Tevatron experiments [54].

1.3.3 Non-Standard Model top decays and \( t\bar{t} \) resonances

As mentioned in Section 1.2.2 top quarks decay predominantly via \( t \rightarrow W^+b \). However the value \( |V_{td}| = 0.99913^{+0.00044}_{-0.00043} \) [33] leaves some room for the CKM suppressed modes \( t \rightarrow W^+s \) and \( t \rightarrow W^+d \) which have CKM elements values of \( |V_{ts}| = 0.0407 \pm 0.0010 \) and
1.3. Top quarks at the LHC

$|V_{td}| = 0.00874^{+0.00026}_{-0.00037}$ respectively [33]. With the large number of top events expected at the LHC, there will be top quarks decaying via the latter two modes. Deviations from these branching ratios could indicate processes beyond the Standard Model. For example, the top quark might decay via a charged Higgs boson: $t \rightarrow H^+ b$, or, another scenario, the top quark decays via a flavour changing neutral current (FCNC) like $t \rightarrow Zq$ with $q = d, s$.

The polarisation of the $W$ boson in top decays can be used to test the $V - A$ structure of the electroweak interaction. Since only left handed particles interact weakly, the $W$ boson is polarised. This results in a characteristic distribution of the angle $\cos \theta^*$ between the $b$ quark and the lepton from the decay in the rest frame of the $W$ boson. Anomalous couplings due to interactions with eg. a $V + A$ structure could alter this distribution.

Although the top quark has an electric charge of $Q = \frac{2}{3} e$ according to the Standard Model, it might turn out that its charge is actually $Q = -\frac{4}{3} e$. The top quark would then decay via $t \rightarrow W^- b$. Measurements of DØ and CDF [82, 83] indicate however that an exotic top quark is not very likely (excluded with 92% C.L.).

The production of $t\bar{t}$ could be enhanced due to a heavy resonance like $gg \rightarrow \phi \rightarrow t\bar{t}$. The heavy resonance would interfere with normal $t\bar{t}$ production and lead to a visible distortion in the differential cross section $d\sigma/dM_{t\bar{t}}$ [84]. The spin of this heavy resonance can be deduced from the spin correlation of the $t\bar{t}$ pair.

1.3.4 Pair production with additional jets

Top pair production with additional jets are important to understand in great detail because these events may conceal yet unobserved processes with similar event topology. Associated Higgs production with top quarks is such a process, shown in Figure 1.15. This process is interesting because it offers the possibility to observe the Higgs boson and to simultaneously measure the top Yukawa coupling. The top Yukawa coupling $y_t$ indicates the strength of the coupling of the Higgs boson to the top quark and is given by:

$$y_t = \frac{m_t \sqrt{2}}{v} \approx 1$$

with $v$ the vacuum expectation value of 246 GeV. Since the top quark mass $m_t$ is approximately 175 GeV, the coupling is close to unity. Masses of the other fermions are substantially smaller than the top quark mass and therefore the Higgs boson coupling is strongest to the top quark.

Unfortunately, the predicted cross section for $t\bar{t}H$ at NLO is only 700 fb and the branching ratio $H \rightarrow b\bar{b}$ is 0.68 for a Higgs boson mass of 120 GeV [85, 86]. Thus the cross section is more than three orders of magnitudes lower than the NLO $t\bar{t}$ cross section. Hence, to be able to distinguish the $t\bar{t}H$ signal from the $t\bar{t}$ background one needs to understand the production of additional jets ($t\bar{t}jj$), and specifically additional jets containing b-quarks ($t\bar{t}bb$), extremely well. Uncertainties in the cross section predictions for these latter processes should be smaller than the cross section for $t\bar{t}H$.

As was discussed in Section 1.3.4, the $t\bar{t}$ cross section is known at NLO in analytical form [87] and approximations of the NNLO accuracy have been made [45, 46, 50]. In ad-
Chapter 1. Top quark physics

Figure 1.15: Two Feynman diagrams exemplifying the similarity of the $t\bar{b}b$ and $t\bar{t}H$ topologies.

dition, the $t\bar{t}$ + jet cross section is calculated at NLO level \cite{88} and two loop corrections are numerically determined for $t\bar{t}$ production in the quark–antiquark channel \cite{89}. With the substantial progress made \cite{90}, the determination of the top quark production cross section at NNLO seems to become within reach soon. Recently, NLO predictions for the $tib\bar{b}$ cross section \cite{91, 92} have shown that the scale uncertainties are much reduced. The large $K$-factor of 1.8 and its strong dependence on cuts used in the calculation indicate however that further NNLO corrections might be desired here too.