# Contents

1 Introduction 7  
1.1 Background 7  
1.2 Main results 10  
1.3 Brief history of the conjecture 11  
1.4 Approach 12  
1.5 Overview of the thesis 14  
1.6 Generalizations 15

2 Proof of the volume conjecture for Whitehead Chains 17  
2.1 Introduction 17  
2.2 Proof of the main theorem 19  
2.3 Proof of the lemmas 21

3 The volume conjecture for augmented knotted trivalent graphs 27  
3.1 Introduction 27  
3.2 Knotted Trivalent Graphs 32  
3.3 The colored Jones invariant of a KTG 36  
3.4 The geometry of the complement of an augmented KTG 43  
3.5 Conclusion 51

4 Asymptotics of classical spin networks 53  
4.1 Introduction 53  
4.2 Existence of asymptotics 56  
4.3 Asymptotics of 6j-symbols 59  
4.4 A generating function for spin network evaluations 62  
4.5 Plan of the paper 63  
4.6 Evaluation of spin networks 64  
4.7 G-functions and existence of asymptotic expansions 68  
4.8 Computation of asymptotic expansions using Borel transform 69  
4.9 Asymptotics of the 6j-symbol 71  
4.10 Computation of asymptotic expansions using the WZ method 79  
4.11 Chromatic evaluation of spin networks 81  
4.12 Challenges and future directions 84

5 Asymptotics of quantum spin networks at a fixed root of unity 87  
5.1 Introduction 87  
5.2 Evaluation of quantum spin networks 92  
5.3 Proof of Theorems 5.3, 5.4, 5.5 and 5.6 99